Electric Propulsion Mission Design with Minimal Solar Cells Radiation Degradation

IEPC-2019-270

Presented at the 36th International Electric Propulsion Conference
University of Vienna, Austria
September 15-20, 2019

Alexander Starchenko*
Research Institute of Applied Mechanics and Electrodynamics, Moscow, 125993, Russia

Abstract: We propose a new method for reduction of the radiation power degradation of a spacecraft solar array (SA) during a combined geostationary orbit insertion using a booster and an electric propulsion system. The essence of the method is to optimize the shape of the orbit transfer trajectory and the perigee argument of the intermediate orbit. The Pontryagin maximum principle is applied to the problem for optimization of the SA electrical power at the end of the spacecraft operational lifetime (EOL). The radiation SA power degradation is described with an additional phase variable and a corresponding differential equation of motion. The solution to the adjoint equation for SA relative power is obtained in closed form. Calculation of radiation SA power degradation was carried out using the models of charged particle fluxes of the Van Allen radiation belts, AE8 MAX and AP8 MAX. Depending on the intermediate orbit parameters we manged to increase the EOL SA power by 0.16–0.73% of the initial SA power at the beginning of the transfer. The additional costs of the characteristic velocity relatively to the minimum time trajectories do not exceed 400 m/s.

Nomenclature

SA	=	solar array
BOL	=	beginning of the operational lifetime
EOL	=	end of the operational lifetime
GEO	=	geostationary orbit
EPS	=	electric propulsion system
$\vec{f}$		discrepancies vector of the boundary value problem
$\vec{z}$		vector of the unknown parameters
$\vec{r}$		position of the spacecraft
$\vec{x}$		phase variables vector
$\vec{p}$		adjoint variables vector

*Mathematician, Spacecraft Trajectory Design Department, aleksandr.starchenko@phystech.edu.

I. Introduction

We consider the combined scheme of a spacecraft geostationary orbit (GEO) insertion using a chemical Fregat-like booster and an electric propulsion system (EPS). The spacecraft is considered to be equipped with triple-junction solar cells, which generate power for the electric propulsion system. After the intermediate orbit insertion with the booster the spacecraft continues the transfer to GEO with help of the electric propulsion only. During this many revolution operation the spacecraft passes the Van Allen radiation belts, which can result in substantial solar arrays output electric power degradation[1]. In order to compensate solar cells radiation power degradation and to guarantee the necessary power level at the end of the spacecraft operational lifetime (EOL), it is necessary to design the excess of solar arrays power at the begin of spacecraft lifetime (BOL). The increase in the initial solar array power will result in an increase of the solar array mass, the spacecraft complexity and cost. As an alternative variant to reduce the SA radiation degration one could increase the SA coverglass thickness, which will also result in SA total mass increase.

The another way to cope with this problem is to consider the problem of minimizing the EOL solar cell radiation degradation by spatial GEO insertion trajectory variation. This approach is described in several papers[2–6]. In this paper we propose to use necessary optimality conditions in a form of the Pontryagin maximum principle in the spatial trajectory variation approach.

II. Problem Statement

The following scheme is considered for launching a spacecraft into GEO. At the first stage, a Soyuz-type launch vehicle launches the spacecraft with a booster similar in characteristics to a Fregat booster into low intermediate orbit. Then, the booster delivers the spacecraft to intermediate orbit and is separated from the spacecraft. At the final stage, the spacecraft using the EPS is transferring along the many revolution trajectory to GEO.

We suppose that at the beginning of the final stage the spacecraft is located at the perigee of an intermediate orbit and has mass m_0. The perigee altitude of an intermediate orbit is $h_{\pi,0}$, the apogee altitude is $h_{\alpha,0}$ and the inclination of an intermediate orbit is i_0. The argument of perigee and longitude of the ascending node of the intermediate orbit are ω_0 and Ω_0, respectively. The target orbit of the spacecraft will be considered as GEO with altitude $h_{GEO} = 35793\,\text{km}$, zero eccentricity, and an inclination. The angular position of the spacecraft in the final orbit is not fixed. For given fixed values $h_{\pi,0}$, $h_{\alpha,0}$ i_0, ω_0, Ω_0 and m_0 we will find such transfer trajectories so that the degradation in the maximum SA electrical output power due to the influence of cosmic radiation during the transfer would be smallest. Since during the EOL the SA power only decreases and during the design of a spacecraft it is necessary to guarantee a sufficient level of energy input at the end of EOL, then it is reasonable to include SA degradation on GEO until the EOL. Then, the considered problem can be written as follows: $\Delta N = N_0 - N_{EOL} \to \min$, where N_0 is the initial SA power and N_{EOL} is the EOL SA power. For the fixed N_{EOL}, this problem is equivalent to the problem of maximizing the SA relative power at the EOL

$$\beta_{EOL} \equiv \frac{N_{EOL}}{N_0} \to \max\,.$$

Thus, further we will consider the problem of searching for trajectories of electric propulsion transfer with the maximum value of EOL SA relative power for each selected set of values $h_{\pi,0}$, $h_{\alpha,0}$ i_0, ω_0, Ω_0 and m_0.

For certainty we will suppose that the transfer from an elliptical intermediate orbit to the GEO is carried out with the help of two SPT-140 thrusters with the total thrust of 0.58 N. The thrusters' specific impulse is assumed to be 1780 s. It is assumed that after placing into GEO, the spacecraft spent at this orbit 15 years.

A. Equations of Motion

In order to avoid singularities near zero eccentricity and inclination, the equations in equinoctial elements[7–10] will be used to describe the spacecraft motion in orbit. Equinoctial elements h, e_x, e_y, i_x, i_y and F can be expressed through focal parameter p of the osculating orbit, eccentricity e, argument of the pericenter ω, inclination i, longitude of the ascending node Ω and true anomaly ν as follows: $h = \sqrt{p/\mu}$, $e_x = e\cos(\Omega+\omega)$, $e_y = e\sin(\Omega+\omega)$, $i_x = \mathrm{tg}(i/2)\cos\Omega$, $i_y = \mathrm{tg}(i/2)\sin\Omega$, $F = \nu+\Omega+\omega$. Then equations of spacecraft motion can be written in the following form:

$$\frac{dh}{dt} = a_Y \frac{h^2}{\xi} \tag{1}$$

$$\frac{de_x}{dt} = \frac{h}{\xi}\left\{a_X \xi \sin F + a_Y \left[(\xi+1)\cos F + e_x\right] - a_Z e_y \eta\right\} \tag{2}$$

$$\frac{de_y}{dt} = \frac{h}{\xi}\left\{-a_X \xi \cos F + a_Y \left[(\xi+1)\sin F + e_y\right] + a_Z e_x \eta\right\} \tag{3}$$

$$\frac{di_x}{dt} = a_Z \frac{h}{\xi}\frac{\tilde{\varphi}}{2}\cos F \tag{4}$$

$$\frac{di_y}{dt} = a_Z \frac{h}{\xi}\frac{\tilde{\varphi}}{2}\sin F \tag{5}$$

$$\frac{dF}{dt} = \frac{\xi^2}{h^3} + a_Z \frac{h}{\xi}\eta, \tag{6}$$

where a_X, a_Y and a_Z are radial, transversal and normal components of disturbing acceleration acting on the spacecraft in orbit and $\mu = 3.98600436$ m^3/s^2 is Earth's gravitational parameter. Moreover, Eqs. (1)–(6) use the following notations: $\xi = 1 + e_x \cos F + e_y \sin F$, $\eta = i_x \sin F - i_y \cos F$, and $\varphi = 1 + i_x^2 + i_y^2$. To describe the orientation of the EPS thrust vector we will use pitch and yaw angles. The pitch angle is the angle between the projection of the thrust vector on the plane of the spacecraft osculating orbit and transversal direction. The yaw angle is the angle between the thrust vector and plane of the spacecraft osculating orbit. Using definitions of yaw and pitch angles and notion of a switching function $\delta(t)$ of EPS (for $\delta = 0$ the propulsion system is switched off, for $\delta = 1$ it is switched on), we get the components of disturbing acceleration from the EPS thrust and the second zonal harmonic of the expansion of Earth's:

$$a_X = \delta \frac{P}{m}\sin\vartheta\cos\psi - K g_r, \tag{7}$$

$$a_Y = \delta \frac{P}{m}\cos\vartheta\cos\psi - K g_\tau, \tag{8}$$

$$a_Z = \delta \frac{P}{m}\sin\psi - K g_n, \tag{9}$$

where

$$K = \frac{6 J_2 R_E^2}{\tilde{\varphi}^2}\frac{\xi^4}{h^8}, \quad g_r = \frac{\tilde{\varphi}^2}{4} - 3\eta^2,$$

$$g_\tau = \left(i_x^2 - i_y^2\right)\sin 2F - 2 i_x i_y \cos 2F, \quad g_n = \eta\left(2 - \tilde{\varphi}\right),$$

as well as, $J_2 = 1.08263 \times 10^{-3}$ is the coefficient of the second zonal harmonic and $R_E = 6378.14$ km is Earth's equatorial radius. To complete the equations of motion we introduce equation for a current spacecraft mass m

$$\frac{dm}{dt} = -\delta\frac{P}{w}, \tag{10}$$

where $w = I_{sp} g_0$ is the exhaust velocity of EPS and $g_0 = 9.80665$ m/s^2 is the standard value of gravity acceleration on Earth's surface.

B. Objective Function

There are several approaches for solar cell radiation degradation estimation. In this paper we use the equivalent flux approach[1]. The idea of this method is to quantify damage from arbitrary space radiation spectrum of protons and electrons in terms of one type particles flux with fixed energy and incidence direction, so this flux can be easily recreated in a laboratory. The equivalent flux approach consists of converting an omnidirectional flux of cosmic radiation protons and electrons through a set of coefficients into a monoenergetic normally incident flux of electrons with the energy of 1 MeV, which will create the same amount of single-type solar cell damage per unit time as the initial fluxes. Thus, the level of damage caused by cosmic radiation to solar cells can be measured in an equivalent total amount (fluence) of 1 MeV electrons normally

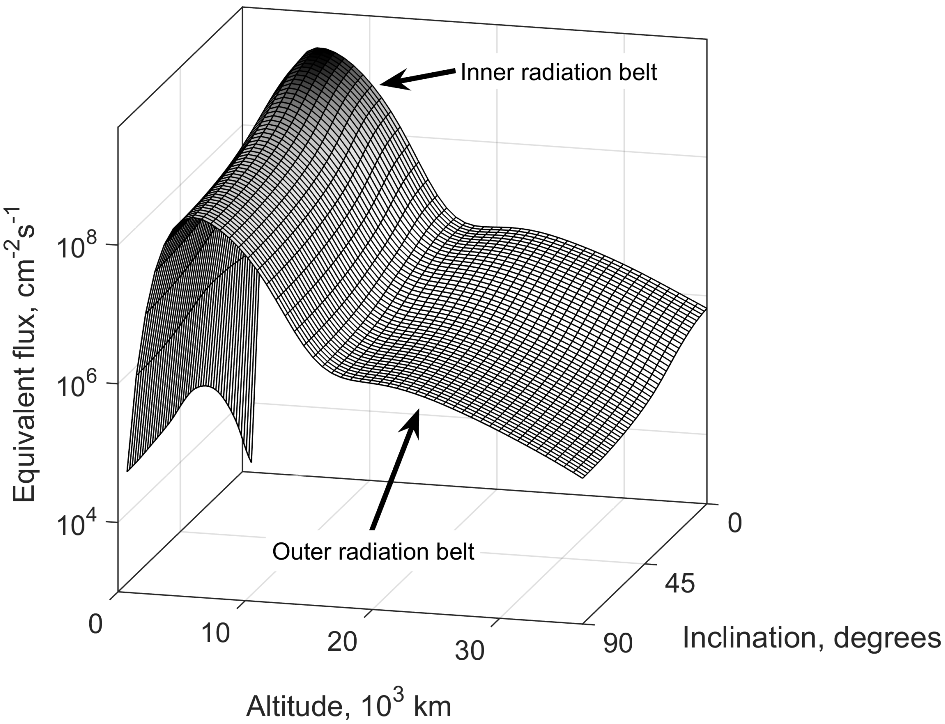

Figure 1.

incident on solar cells. Knowing the value of the equivalent fluence of 1 MeV electrons, it is possible from experimental data to estimate the value of the degradation of various electrical characteristics of solar cells, including the maximum electrical power. To calculate the equivalent fluence from the spectra of the Van Allen belts electrons and protons incident on solar cells, the EQFRUX program[1] was used. When calculating the decrease in SA power, it was considered that the photosensitive elements of solar cells are behind a layer of coverglass, which partially weakens the flux of energetic the Van Allen belts particles incident on the solar cells. The thickness of the covergalss influences the conversion coefficients for calculating the equivalent fluence. It was assumed when calculating that solar cells are located behind a layer of silica coverglass with a thickness of $150\,\mu$m and a density of $2.2\,\mathrm{g/cm^3}$ (as in the report[1]). It was assumed that the back surface of solar cells is covered with structural elements of SA with the same weakening properties as in silica coverglass with a thickness of $500\,\mu$m. According to report[1], to consider the solar cell damage caused by the particle flux falling from both the front and back SA surfaces, it is necessary to calculate separately equivalent fluences for particles incident on the front and back surfaces of SA, and then add it together.

The integral equivalent fluence of 1-MeV electrons accumulated by the unit of solar cell area from the beginning to the end of the transfer can be calculated as follows:

$$\Phi(T) = \int_0^T U\left(r(s), i(s)\right) ds, \tag{11}$$

where T is the electric propulsion transfer duration, $U\left(r(s), i(s)\right)$ is the equivalent flux of 1 MeV electrons depending on current time t and position $\vec{r}(t)$ of the spacecraft in near-Earth space. The function U is calculated numerically using the IRENE[13] software package as described above. In order to obtain the necessary smoothness of the equivalent fluence function U we calculated equivalent flux on 1200 circular orbits and fitted the obtained data with high order two dimensional spline. The data for the fit was obtained as the mean equivalent flux on a circular orbit for several days. The resulting spline $U(r, i)$ is shown in Fig. 1. According to the report[1], for the relative EOL SA power we have the following semi-empirical formula

$$\beta_{EOL} \equiv \frac{N_{EOL}}{N_0} = 1 - C\ln\left(1 + \frac{\Phi(T) + \Phi_{GEO}}{\Phi_X}\right), \tag{12}$$

where $C = 0.1037$ and $\Phi_X = 2.432 \times 10^{14}\mathrm{cm}^{-2}$ are the approximation constants for experimental data on the solar cell degradation from the documentation of the solar cell manufacturer, $\Phi_{GEO} = 3.5 \times 10^{14}\mathrm{cm}^{-2}$ is the equivalent fluence of 1-MeV electrons accumulated by solar cell after GEO insertion and until EOL. The Φ_{GEO} is calculated using the IRENE software package for the period from January 1, 2021 to January 1, 2036. Combining Eq.(11) and Eq.(12), we have an additional equation for the spacecraft motion associated with the SA output power

$$\frac{d\beta}{dt} = -CV\exp\left(-\frac{1-\beta}{C}\right), \quad \beta(0) = 1 - C\ln\left(1 + \Phi_{GEO}/\Phi_X\right), \tag{13}$$

where $V(r,i) = U(r,i)/\Phi_X$. Thus, as a objective function in this paper, we consider the value of relative power of SA at the EOL $\beta(T)$.

III. Optimal control

The problem of maximizing the EOL SA relative power could be formalized as the minimum time orbit transfer problem[7] with the additional phase variable $\beta(t)$ satisfying Eq. (13) and an additional constraint at the end of the transfer:

$$\beta(T) = \beta_T. \tag{14}$$

We consider a series of the minimum time problems with the constraint (14), where β_T is constant in every problem, but gradually increasing from problem to problem. In practice there is always a maximum value of β_T for which there is still a solution to the boundary value problem. Thus, gradually increasing β_T and solving the resulting boundary value problems it is possible to find the transfer trajectory with the highest possible value of the EOL SA relative power. Similarly to the paper[7], we use the Pontryagin maximum principle to obtain an optimal in terms of transfer time control for the extended system of Eqs. (1)–(13). For the Hamiltonian of this system we have

$$H = -1 - \delta\frac{P}{w}p_m + \delta\frac{P}{m}\frac{h}{\xi}\left(A_\tau\cos\vartheta\cos\psi + A_r\sin\vartheta\cos\psi + A_n\sin\psi\right) + H_F + H_{J2} + H_\beta \tag{15}$$

where

$$A_\tau = hp_h + \left[(\xi+1)\cos F + e_x\right]p_{ex} + \left[(\xi+1)\sin F + e_y\right]p_{ey},$$
$$A_r = \xi\left(p_{ex}\sin F - p_{ey}\cos F\right),$$
$$A_n = \eta\left(e_x\,p_{ey} - e_y\,p_{ex}\right) + \frac{\tilde{\varphi}}{2}\left(p_{ix}\cos F + p_{iy}\,\sin F\right) + \eta p_F$$
$$H_F = p_F\frac{\xi^2}{h^3}, \quad H_\beta = -p_\beta\,CV\exp\left(-\frac{1-\beta}{C}\right), \quad H_{J2} = -K\frac{h}{\xi}\left(\vec{g},\vec{A}\right),$$
$$\vec{g} = \left(g_\tau, g_r, g_n\right)^T, \quad \vec{A} = \left(A_\tau, A_r, A_n\right)^T.$$

The superscript T in the last two equations means the transposition operation. Thus, vectors $\vec{g}$ and $\vec{A}$ are column vectors. This notation will be used in a sequel. The control that maximize the Hamiltonian in Eq. (15) have the following form

$$\delta \equiv 1, \quad \cos\vartheta = \frac{A_\tau}{\sqrt{A_\tau^2 + A_r^2}}, \quad \sin\vartheta = \frac{A_r}{\sqrt{A_\tau^2 + A_r^2}}, \tag{16}$$

$$\cos\psi = \frac{\sqrt{A_\tau^2 + A_r^2}}{A}, \quad \sin\psi = \frac{A_n}{A}, \quad A = \sqrt{A_\tau^2 + A_r^2 + A_n^2}. \tag{17}$$

Since $\delta(t) = 1$, $\forall t \in [0;T]$, we can omit the equations for m and p_m from the system and use closed form solution for the mass dependence on time $m = m_0 - (P/w)t$. If we introduce vector notations $\vec{x} =$

$(h, e_x, e_y, i_x, i_y)^T$ and $\vec{p} = (p_h, p_{ex}, p_{ey}, p_{ix}, p_{iy})^T$, then for the full system of equations of optimal motion we get

$$\frac{d\vec{x}}{dt} = P\frac{k}{A}\left(A_\tau\frac{\partial A_\tau}{\partial \vec{p}} + A_r\frac{\partial A_r}{\partial \vec{p}} + A_n\frac{\partial A_n}{\partial \vec{p}}\right) + \frac{\partial H_{J2}}{\partial \vec{p}}, \tag{18}$$

$$\frac{dF}{dt} = \frac{\xi^2}{h^3} + Pk\eta\,\frac{A_n}{A}, \tag{19}$$

$$\frac{d\beta}{dt} = -CV\exp\left(-\frac{1-\beta}{C}\right), \tag{20}$$

$$\frac{d\vec{p}}{dt} = -P\left[\frac{\partial k}{\partial \vec{x}}A + \frac{k}{A}\left(A_\tau\frac{\partial A_\tau}{\partial \vec{x}} + A_r\frac{\partial A_r}{\partial \vec{x}} + A_n\frac{\partial A_n}{\partial \vec{x}}\right)\right] - \frac{\partial H_F}{\partial \vec{x}} - \frac{\partial H_\beta}{\partial \vec{x}} - \frac{\partial H_{J2}}{\partial \vec{x}}, \tag{21}$$

$$\frac{dp_F}{dt} = -P\left[\frac{\partial k}{\partial F}A + \frac{k}{A}\left(A_\tau\frac{\partial A_\tau}{\partial F} + A_r\frac{\partial A_r}{\partial F} + A_n\frac{\partial A_n}{\partial F}\right)\right] - \frac{\partial H_F}{\partial F} - \frac{\partial H_\beta}{\partial F} - \frac{\partial H_{J2}}{\partial F}, \tag{22}$$

$$\frac{dp_\beta}{dt} = -p_\beta\frac{\partial H_\beta}{\partial \beta} = p_\beta V\exp\left(-\frac{1-\beta}{C}\right). \tag{23}$$

To improve numerical stability of a boundary value problem which will be formulated in the next section we use the motion averaging method[7]. According to this method the true longitude F is a fast variable and the other variables are slow. The equation for the fast variable could be omitted. The right parts of the equations for the slow variables are time-averaged on a one revolution according the following formula

$$\frac{d\vec{y}(t)}{dt} = \frac{1}{T_c}\int_t^{t+T_c}\vec{f_e}\left(\vec{y}, F, s\right)ds = \frac{n}{2\pi}\int_0^{2\pi}\vec{f_e}\left(\vec{y}, F, s\right)\frac{ds}{dF}dF, \tag{24}$$

where $\vec{y} = \left(\vec{x}^T, \beta, \vec{p}^T\right)^T$, t is current time, T_c is the osculating period of revolution of the satellite, $\vec{f_e}\left(\vec{y}, F, t\right)$ is the right parts of the equations of optimal motion, $n = \mu^{-1}h^{-3}\left(1 - e_x^2 - e_y^2\right)^{3/2}$ is the mean motion and $ds/dF \equiv dt/dF = h^3/\xi^2$. Since the Hamiltonian of an averaged system of equations does not depend on F, from Eq. (22) we have $p_F = const$. Moreover, since the true longitude is not fixed at the end of the orbit transfer, from the transversality conditions we have $p_F(T) = 0$. Thus, for the averaged system of equations $p_F \equiv 0$ and Eq. (22) could be omitted from the system, as well as, term H_F from the Hamiltonian (15).

It is easy to prove that the adjoint Eq. (23) has a closed form solution. Indeed, one can notice that Eqs. (20) and (23) have common multiplier $V\exp\left(-(1-\beta)/C\right)$. Using this fact we obtain the solution with the separation of variables method:

$$p_\beta(t) = p_\beta(0)\exp\left(-\frac{\beta(t) - \beta(0)}{C}\right). \tag{25}$$

Substituting Eq. (25) into Eqs. (21) and (15), we have the system of averaged equations of optimal spacecraft motion

$$\frac{d\vec{x}}{dt} = P\frac{k}{A}\left(A_\tau\frac{\partial A_\tau}{\partial \vec{p}} + A_r\frac{\partial A_r}{\partial \vec{p}} + A_n\frac{\partial A_n}{\partial \vec{p}}\right) + \frac{\partial H_{J2}}{\partial \vec{p}}, \tag{26}$$

$$\frac{d\beta}{dt} = -CV\exp\left(-\frac{1-\beta}{C}\right), \tag{27}$$

$$\frac{d\vec{p}}{dt} = -P\left[\frac{\partial k}{\partial \vec{x}}A + \frac{k}{A}\left(A_\tau\frac{\partial A_\tau}{\partial \vec{x}} + A_r\frac{\partial A_r}{\partial \vec{x}} + A_n\frac{\partial A_n}{\partial \vec{x}}\right)\right] - p_\beta(0)C_0\frac{\partial V}{\partial \vec{x}} - \frac{\partial H_{J2}}{\partial \vec{x}}, \tag{28}$$

and the Hamiltonian

$$H_{\text{opt}} = -1 + PkA - K\left(\vec{g}, \vec{A}\right) - p_\beta(0)C_0V, \tag{29}$$

where $C_0 = C\exp(-(1-\beta(0))/C) = const$.

IV. Boundary Value Problem

To satisfy the conditions of the maximum principle, it is necessary to solve the boundary value problem for the system of averaged equations of optimal motion (26)–(28) with the following boundary conditions at the begining and the end of the transfer

$$\vec{x}(0) = \vec{x}_0, \quad \beta(0) = 1 - C\ln\left(1 + \Phi_{GEO}/\Phi_X\right), \tag{30}$$

$$\vec{x}(T) = \vec{x}_T, \quad \beta(T) = \beta_T, \quad \bar{H}_{opt}(T) = 0 \tag{31}$$

where $\vec{x}_0$ and $\vec{x}_T$ can be expressed through the intermediate orbit parameters $h_{\pi,0}$, $h_{\alpha,0}$, i_0, ω_0, Ω_0, m_0 and GEO altitude $h_{GEO} = 35793\,\mathrm{km}$. The notation $\bar{H}_{opt}(T)$ refers to the expression for Hamiltonian (29) averaged according to the scheme (24) at the end of the electric propulsion phase of the transfer. Equations (31) contain the values of the equinoctial elements, the relative SA power, and the Hamiltonian at the instant $t = T$. To calculate them, it is necessary to numerically integrate the system of equations of optimal motion (26)–(28) over interval $t \in [0; T]$ with initial conditions (30) and using the averaging scheme (24). In order to perform this integration it is necessary to know the transfer duration T and the initial values of the adjoint variables $\vec{p}(0)$ and $p_\beta(0)$. Let $\vec{z} = (\vec{p}(0), p_\beta(0), T)^T$ and $\vec{f}(\vec{z}, \tau) = (\vec{x}(T) - \vec{x}_T, \beta(T) - \beta_T, \bar{H}_{opt}(T))^T$. Then, the boundary value problem of the maximum principle could be reduced to the problem of solving a system of nonlinear equations:

$$\vec{f}(\vec{z}\,) = 0 \tag{32}$$

To solve the nonlinear system (32) in this paper, we use the method of numerical continuation of the solution[7,11,12]. According to this method the problem is reduced to solving the Cauchy problem for the system of ordinary differential equations

$$\frac{d\vec{z}}{d\tau} = -\left[\frac{\partial \vec{f}(\vec{z}, \tau)}{\partial \vec{z}}\right]^{-1}\left(\vec{b} + \frac{\partial \vec{f}(\vec{z}, \tau)}{\partial \tau}\right), \quad \tau \in [0; 1], \quad \vec{z}(0) = \vec{z}_0, \tag{33}$$

where τ is the continuation parameter, $\vec{f}(\vec{z}_0\,) = \vec{b}$ and function $\vec{f}$ in general case depends also on τ. The Cauchy problem (33) is solving numerically using the Dormand-Prince method of 7(8) order. Therefore the solution of (32) is just $\vec{z}^* = \vec{z}(1)$.

It is known that there are reliable methods[7] for solution the minimum time orbital transfer problems. Thus, it is seems to be reasonable to use this solution as the initial guess $\vec{z}_0$. Moreover, for $p_\beta(0) = 0$ the averaged equations of optimal spacecraft motion (26)–(28) and the Hamiltonian (29) coincide with the averaged equations of optimal motion and the Hamiltonian for the averaged minimum time problem. Thus, as initial approximation we will use $\vec{z}_0 = (\vec{p}^*(0), 0, T^*)^T$, where $\vec{p}^*(0)$ and T^* are obtained from the solution to the minimum time GEO transfer problem. Furthermore, in order to perform a continuous transition from the minimum time problem to the problem with fixed EOL SA relative power, we set

$$\beta_T = (1 - \tau)\beta_{\min T} + \tau\beta_{\max}, \tag{34}$$

where $\beta_{\min T}$ is the relative EOL SA power at the minimum time trajectory, and $\beta_{\max}$ is a constant, such that $\beta_{\min T} < \beta_{\max} < \beta(0) < 1$. Then, for the the derivative of $\vec{f}$ with respect to the continuation parameter we have

$$\frac{\partial \vec{f}(\vec{z}, \tau)}{\partial \tau} = (0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \beta_{\min T} - \beta_{\max} \quad 0)^T. \tag{35}$$

V. Numerical results

Consider several intermediate orbits whose parameters are listed in Table 1. This table defines the energy parameters of intermediate orbits, but does not fix for each orbit the longitude of ascending node Ω_0 and the argument of perigee ω_0. In the minimum time orbit transfer problem (in the central field of Earth's attraction) for the transfer between non-coplanar elliptical and circular orbits, the optimal value of the perigee argument of the intermediate orbit is $\omega_0 = 0$, but value Ω_0 does not affect the transfer time. For the averaged equivalent flux, due to symmetry relative to the axis of Earth's rotation, value will not affect the relative power at the end of EOL. Therefore, we will consider further the minimum time problem accounting

Table 1. Intermediate orbits parameters

No.	SC mass m_0, kg	Apogee $h_{\pi 0}$, km	Perigee $h_{\alpha 0}$, km	Inclination i_0, degrees
1	2100	1300	82300	32.0
2	2200	800	76300	38.0
3	2250	800	77300	42.0
4	2300	800	71300	46.0
5	2400	800	55300	47.5

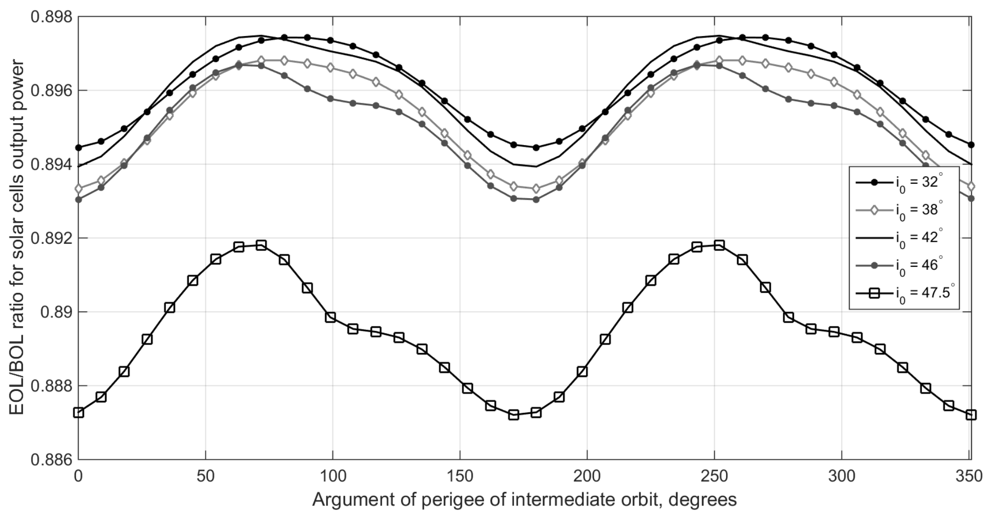

Figure 2.

for the second zonal harmonic of the expansion of the Earth's gravitational potential for each intermediate orbit from Table 1 for and different values ω_0. Figure 2 shows the dependence of the averaged relative power of SA at the end of EOL on the argument of perigee for all intermediate orbits from Table 1. A periodic character of the dependence of the averaged relative power on the argument of the perigee of the intermediate orbit is seen. The period of this dependence is 180 deg.

For each intermediate orbit from Table 1, the value of the perigee argument of the intermediate orbit with the minimum relative EOL SA power on minimum time trajectory was chosen and the boundary value problem was solved reduced earlier to Cauchy problem (33). The constant $\beta_{\max}$ in all obtained boundary value problems was assumed to be equal 0.906, which is less than $\beta(0) = 0.90679615$. At a certain value of the continuation parameter $\tau = \tau_f < 1$, a significant increase in the residuals at the right end was observed. In this case, the corrector after each step could not vanish the residuals on the right end. At this point, integration over the continuation parameter was interrupted. It should be noted, however, such interruption of integration of system (33) does not guarantee that the increase in residuals at the right end occurred due to the lack of solution to the boundary value problem for But, nevertheless, the obtained trajectories allow us to estimate the maximum possible SA power at the EOL

The values of $\vec{z}(\tau)$ obtained during solution (33), were used to integrate the non-averaged equations of optimal motion (19), (26)–(28) and calculate the non-averaged relative EOL power of SA using the IRENE software package. Figure 3 presents the results of the calculation of the averaged and non-averaged relative EOL SA power on the obtained non-averaged trajectories. All calculations presented in Fig. 3 correspond to the choice of ω_0 with the minimum relative EOL SA power on the minimum time trajectory. The averaged relative power is presented on all graphs of Fig. 3 by light lines, non-averaged by dark lines. The caption of each graph in Fig. 3 indicates the inclination and the selected value of the perigee argument of the intermediate orbit. On the abscissas of the figures, the value of the additional characteristic velocity for EPS

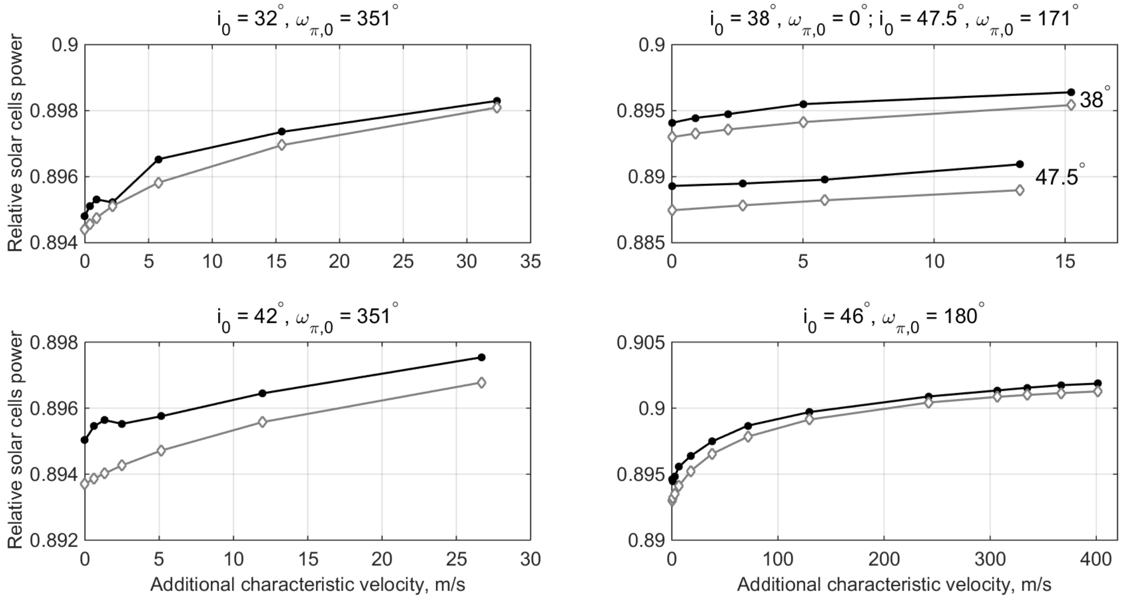

Figure 3.

transfer compared to the minimum time trajectory for the selected parameters of the intermediate orbit is plotted. All figures show a monotonous increase in relative power with increasing costs of characteristic velocity. The maximum increase in the EOL power of SA varies from 0.16% to 0.73% of the SA power in the initial orbit depending the parameters of the intermediate orbit. The additional characteristic velocity do not exceed 400 m/s. The increase in the SA power at the end of electric propulsion transfer is 0.33–1.6% of the initial SA power.

Figure 4 shows the dependences of the required thickness of the protective glass on the duration of the flight at various relative fixed powers of SA at the end of EOL. The relative power values fixed at the end of the EOL are indicated on isolines. The graph is obtained by varying the thickness of the front safety glass of SA on the trajectories obtained for the case of transfer from the intermediate orbit with inclination $i_0 = 46°$ and argument of perigee $\omega_0 = 180°$.

Figure 5 shows how the flight trajectory changed from the intermediate orbit with inclination $i_0 = 46°$ and argument of perigee $\omega_0 = 180°$ as the continuation parameter increased and, accordingly, the relative SA power increased by the end of EOL. The trajectories for each value of the continuation parameter are represented in the radius-inclination axes. The two dashed lines in each graph show the dependence of the altitude of the perigee and apogee on the inclination on the minimum time trajectory. Two continuous lines correspond to the dependence of the altitude of the perigee and apogee on the inclination on the trajectory obtained for a given value of the continuation parameter. The grayscale on each graph shows the value of the equivalent 1 MeV electron flux incident on SA. The graphs show a tendency for the perigee sections of the trajectory to be made higher than the boundary of the internal the Van Allen belts, as well as to an increase in the maximum altitude of the apogee. It can be also noted in Fig. 5 that as the final relative power of SA increases, the orbital inclination changes less and less in the area of intersection of the internal the Van Allen belts by the spacecraft trajectory and more and more outside this area. This fact partially confirms the correctness of the calculations, since it is clear that with decreasing inclination the equivalent flux inside the radiation belts increases, and therefore it is more efficient in terms of radiation damage to cross the Van Allen belts (in other words, to increase altitude) in the area of large inclinations.

VI. Conclusion

Using the approach of spatial variations of the transfer trajectory and the Pontryagin maximum principle we managed to increase the EOL SA power by 0.16–0.73% of the initial SA at the intermediate orbit. depending on the parameters of intermediate orbits. The additional costs of the characteristic velocity

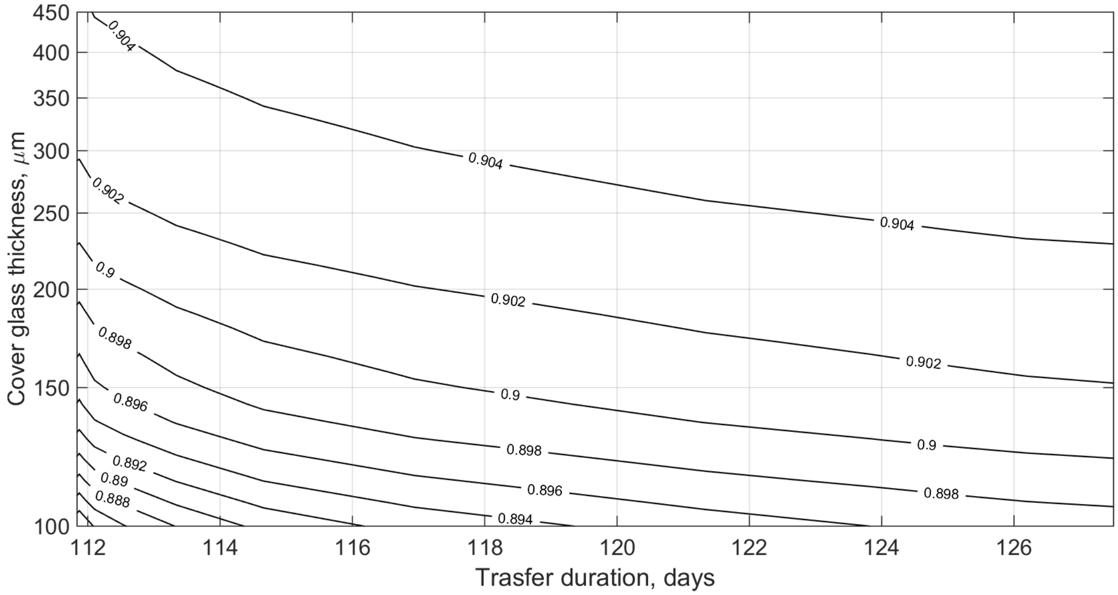

Figure 4.

increased relative to the minimum time trajectories by no more than 400 m/s. The increase in the SA power at the end of electric propulsion transfer is 0.33–1.6% of the initial SA power. A relatively small increase in the final power of SA is due to the higher radiation resistance of triple-junction solar cells compared to silicon ones. The another fact that could explain the obtained small effect is that most of the transfer time the spacecraft spents in a region with small equivalent flux, since only the internal the Van Allen belt has significant effect on the degradation of triple-junction solar cells. The paper considers the formulation of the problem, where the radiation degradation of the EOL SA power is described with an additional ordinary differencial equation, added to the spacecraft equations of motion. The maximum principle is applied to the obtained problem. The adjoint equation to the EOL SA relative power is solved in a closed form.

Acknowledgments

The author would like to express his gratitude to professor V. G. Petukhov for his valuable remarks made during the discussion of this paper. This research was supported by the Russian Science Foundation grant (Agreement No. 16-19-10429).

References

[1]Tada, H. Y., Carter, J. R., Anspaugh, B. E., Downing, R. G. Solar "Cell Radiation Handbook," Third Edition, JPL Publication 82-69, 1982.

[2]Redding, D. "Avoiding the Van Allen Belt in Low-Thrust Transfer To Geosynchronous Orbit," 21st Aerospace Sciences Meeting, AIAA Paper 1983-0195. Reno, Nevada: 1983. January 10–13.

[3]Dutta, A., Choueiri, E. "Minimizing Radiation Fluence during Time-Constrained Electric Orbit Raising," 23Rd International Symposium On Space Flight Dynamics, Pasadena, CA: 2012.

[4]Jehn, R. "Radiation Optimum Solar-Electric-Propulsion Transfer From GTO to GEO," 24Rd International Symposium On Space Flight Dynamics, Laurel, MD: 2014.

[5]Dutta, A., Kasdin, N. J., Choueiri, E. "Minimizing Proton Displacement Damage Dose During Electric Orbit Raising of Satellites," Journal of Guidance, Control, and Dynamics, 2016, Vol. 39, No. 4, pp. 960–967.

[6]Starchenko, A. E. "Trajectory Optimization of a Low-Thrust Geostationary Orbit Insertion for Total Ionizing Dose Decrease," Cosmic Research, 2019, Vol. 57, no. 4, pp. 289–300.

[7]Petukhov, V. G. "Optimization of Multi-Orbit Transfers Between Noncoplanar Elliptic Orbits," Cosmic Research, 2004, Vol. 42, No. 3, pp. 250–268.

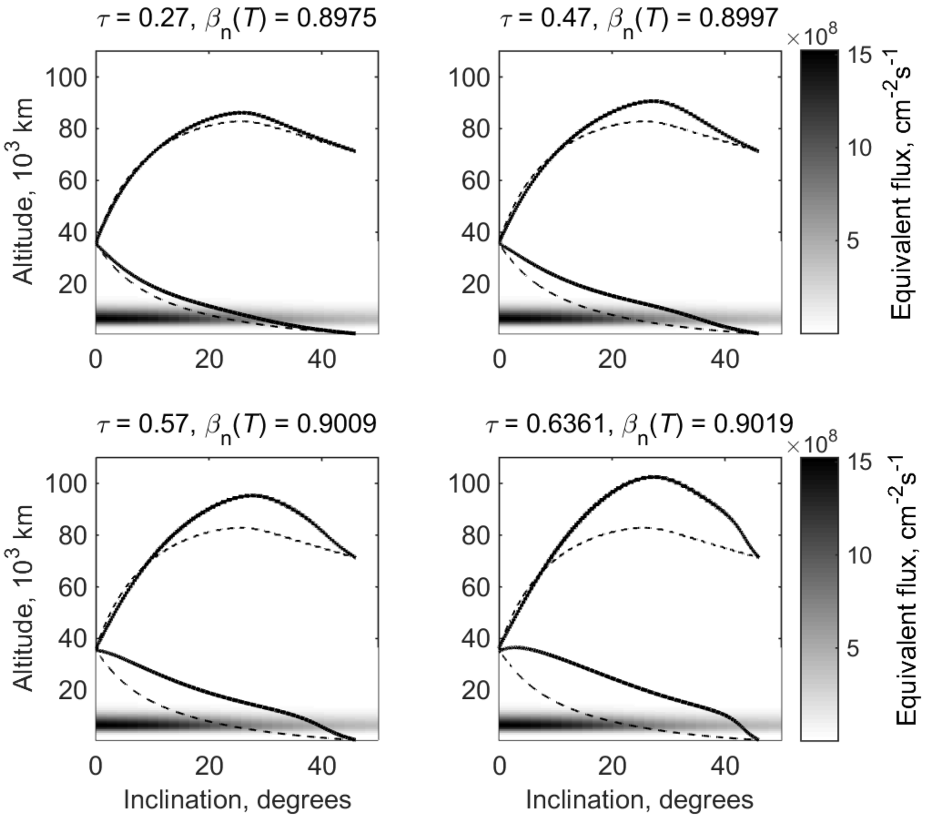

Figure 5.

[8]Brouke, R. A., Cefola, P. J. "On the Equinoctial Orbital Elements," Celestial Mechanics, 1972, No. 5, pp. 303–310.

[9]Battin, R. H. "An Introduction to the Mathematics and Methods of Astrodynamics. Revised edition," Reston, Virginia: AIAA, 1999.

[10]Kéchichian, J. A. "Applied Nonsingular Astrodynamics: Optimal Low-Thrust Orbit Transfer," Ed. by W. Shyy, V. Yang, Cambridge Aerospace Series No. 45, Cambridge: Cambridge University Press, 2018.

[11]Petukhov, V. G. "Method of continuation for optimization of interplanetary low-thrust trajectories," Cosmic Research, 2012, Vol. 50, No. 3, pp. 249–261.

[12]Shalashilin, V.A., Kuznetsov, E.B., "Metod prodolzheniya resheniya po parametru i nailuchshaya parametrizatsiya (The Method of Parametric Continuation and Optimal Parametrization)," Moscow: Editorial URSS, 1999.

[13]IRENE: International Radiation Environment Near Earth,
https://www.vdl.afrl.af.mil/programs/ae9ap9/downloads.php.