IEPC-2019-334 Low frequency ionization oscillation model with azimuthal dimension in a Hall thruster

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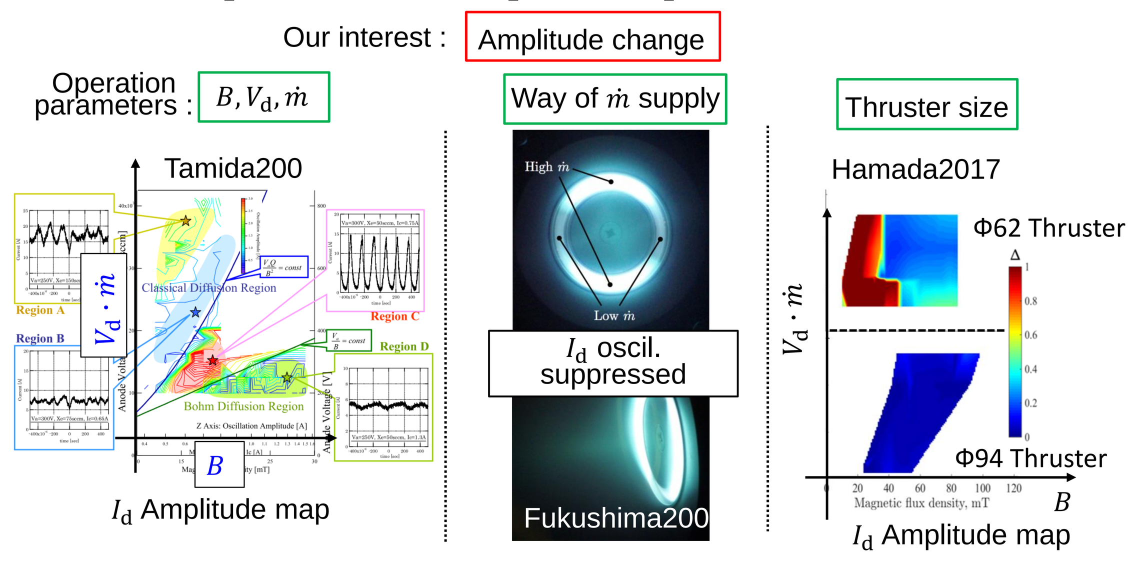
Research Overview

- Bulk discharge oscillations in a few tens kHz band.
  - Mainly coming from ionization oscillation, also called 'breathing oscillation'
  - Highly synchronized & oscillative and detrimental to thruster operation
- Specifically focused on
  - Taking azimuthal dimension into account for the model
  - Clarifying the relation of ionization frequency and amplitude
- Possible reasons of discharge current oscillation suppression

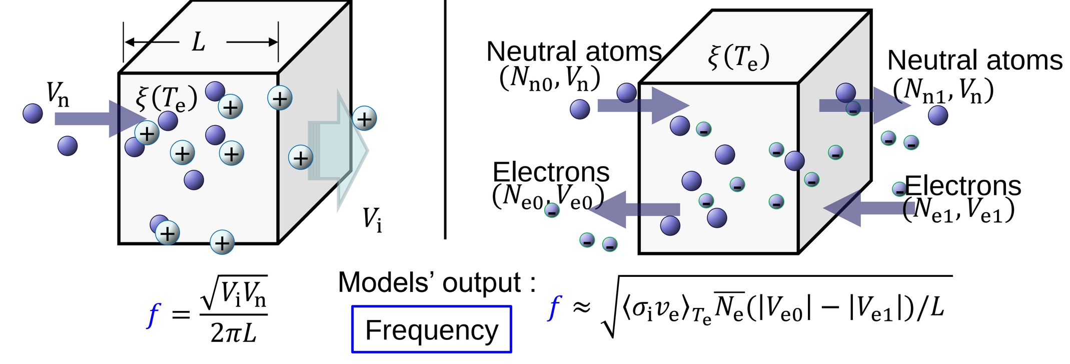
Total discharge current oscillation suppression
- Local oscillation suppression
- De-sync of locals
  - by difference φ
  - by difference f

Motivations

- Interested parameter and operation parameters



- Some ionization models proposed

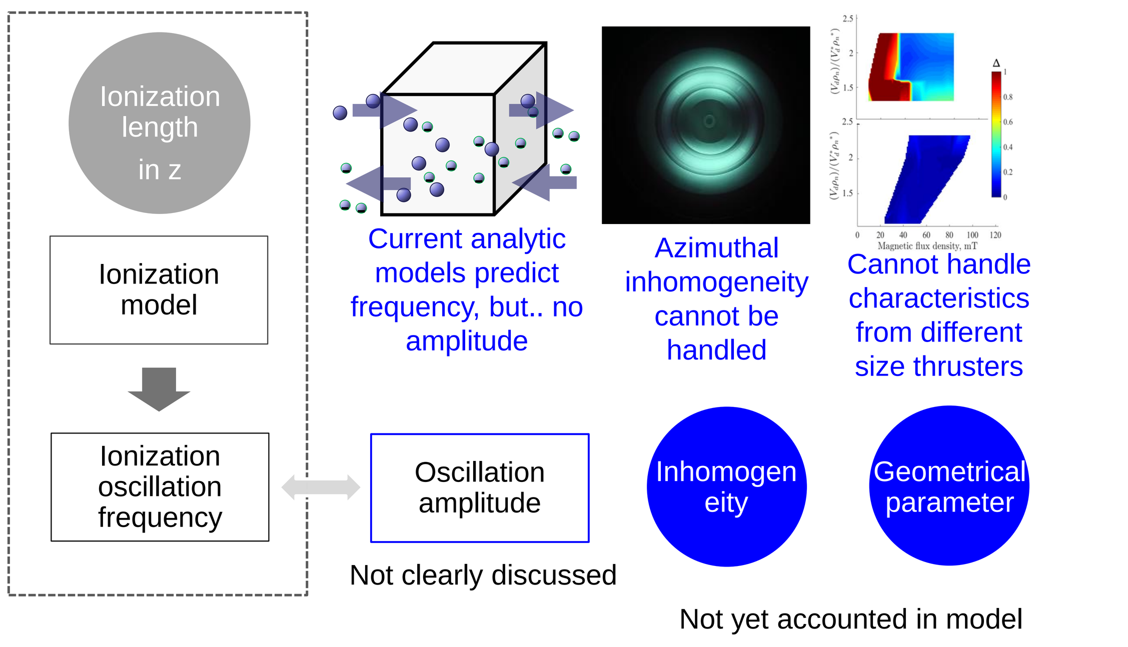


- Limitation of the proposed models and motivation

Our interest :	But, no relation with	Model's parameters are not given with operation parameters :	No consideration on
	Amplitude	$B, V_d, \dot{m}$ / Way of $\dot{m}$ supply	Thruster size

Current Limitations and Goal

- **Current limitation of ionization models**



- **Goal of this research**

Ionization model	INPUT
Operation parameters	B, V_d, $\dot{m}$
Way of $\dot{m}$ supply	Ξ
Geometrical parameters	L_z, R_w, L_θ

*Only with parameters which can be pre-determined as control variables

Ionization oscillation frequency ⇒ OUTPUT: Oscillation amplitude (Our interest)

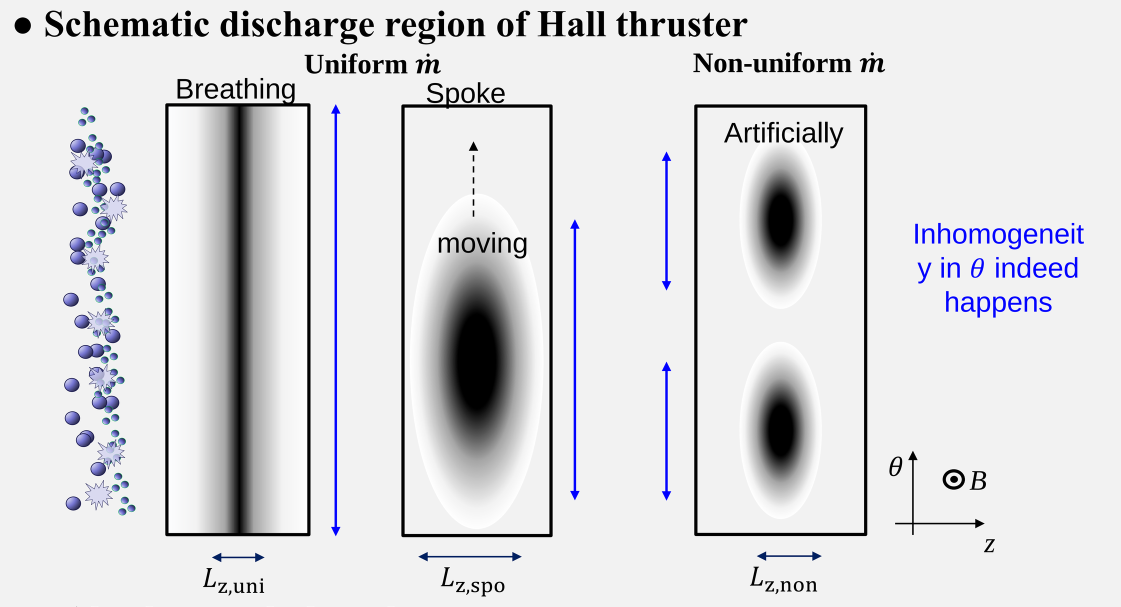
Objectives

1. Development of ionization model with decisive operation parameters taking azimuthal dimension into account
2. Connecting a physical constraint coupling ionization frequency and amplitude
   - ➤ Build a confined-model which can predict ionization oscillation amplitude tendency from common operation parameters

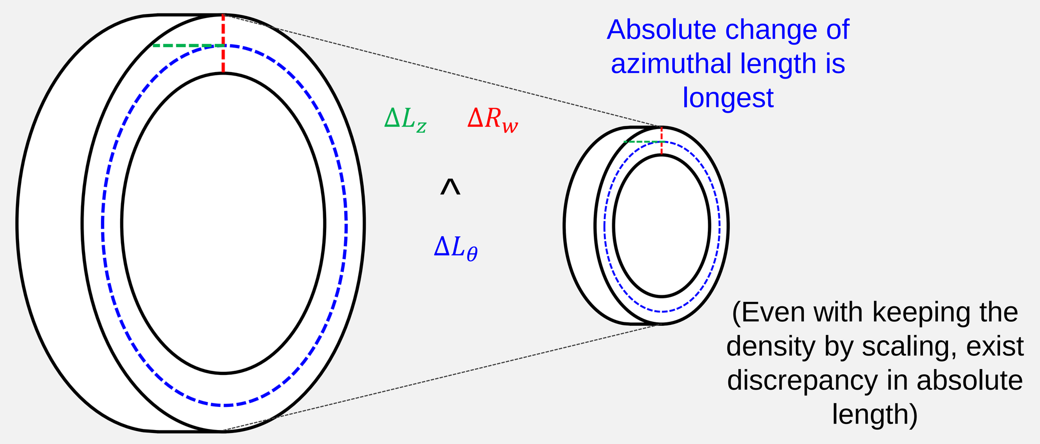
Azimuthal Dimension in Hall Thrusters and Modeling of Species on Ionization

1. Hall thruster with azimuthal n_n inhomogeneity

- **Schematic discharge region of Hall thruster**



- **Absolute scale-length**

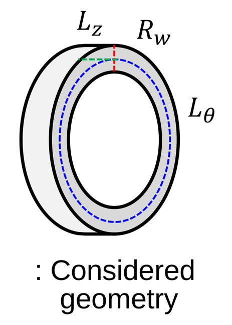


2. Inhomogeneity and geometrical parameters

- **Continuity equation of neutrals and electrons**

$$\int_V \frac{\partial n_n}{\partial t}dV + \int_S n_n v_n dS = -\int_V n_n n_e \langle\sigma_{di} v_e\rangle_{T_e} dV$$

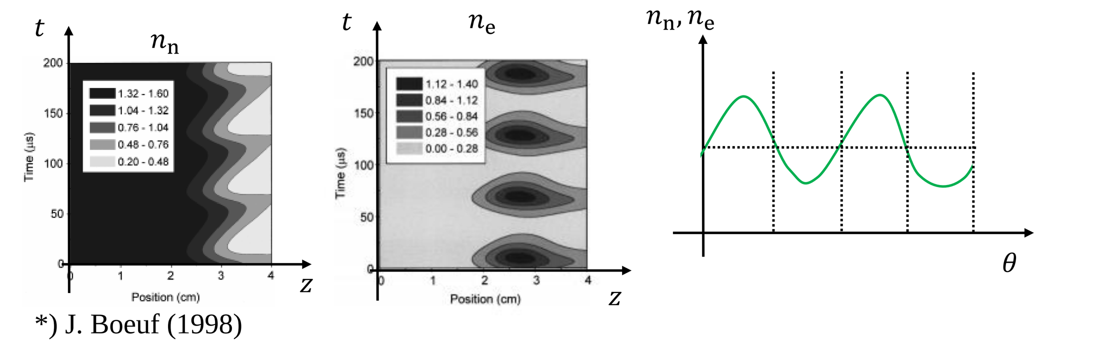
$$\int_V \frac{\partial n_e}{\partial t}dV + \int_S n_e v_e dS = \int_V n_n n_e \langle\sigma_{di} v_e\rangle_{T_e} dV$$



- **Neutrals and electrons behavior**

$$n_n = n_{n0}\exp[-z/\lambda_{\mathrm{mfp}}]\,h(\theta) + \tilde{n}_n h(\theta)\exp[-i\omega t]$$

$$n_e = n_{e0} h(\theta) + \tilde{n}_e h(\theta)\exp[-i\omega t]$$



Time varying oscillation + Axial decay | Time varying Standing wave | $h(\theta)=1-\Xi\cos(2\theta)$, Ξ = 0.0 to 1.0 : representing azimuthal distribution, and Ξ is non-uniformity strength

Results and Discussion

1. Dispersion relation

$$\begin{bmatrix} -i\omega + n_{e0}\chi\xi & n_{n0}\chi\xi G(a) \\ -n_{e0}\chi\xi & -i\omega \end{bmatrix}\begin{bmatrix}\tilde{n}_n \\ \tilde{n}_e\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

$$\omega^2 + i\chi\xi n_{e0}\omega - \chi^2\xi^2 n_{n0} n_{e0} G(a) = 0 \qquad \omega = \omega_r + i\gamma$$

- **Frequency (Re[ω]/2π)**

$$f = \frac{1}{4\pi}\cdot\chi(\Xi)\cdot\nu_{\mathrm{ion}}(T_e)\cdot\sqrt{4\cdot\kappa\cdot G(a) - 1}$$

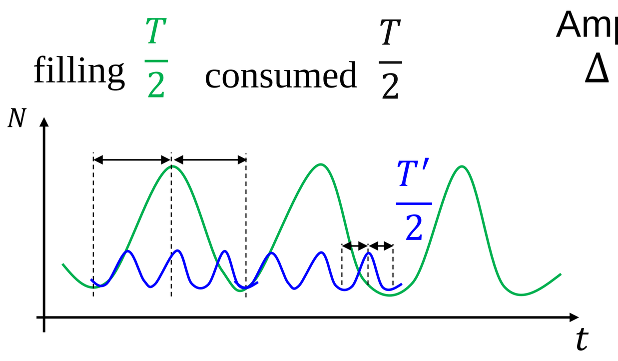
Ionization model with pre-determined parameters

■ : Geometrical parameters ■ : Input parameters

$\chi(\Xi) \equiv \frac{\Xi^2+2}{2}$	: Effect due to azimuthal non-uniformity Ξ	
$\nu_{\mathrm{ion}}(T_e) \equiv n_{e0}\langle\sigma_{di} v_e\rangle_{T_e}$	: Ionization collision frequency	With common relation $T_e \sim 0.1V_d$: $\nu_{\mathrm{ion}} = \nu_{\mathrm{ion}}(T_e) \to \nu_{\mathrm{ion}}(V_d)$
$\kappa \equiv \frac{n_{n0}}{n_{e0}}$	: Equilibrium density ratio between neutrals and electrons	From input $\dot{m} = n_{n0} m_n v_n 2\pi R_w L_\theta$: $n_{n0} = n_{n0}(\dot{m})$, $\kappa \to \kappa(\dot{m})$
$G(a) \equiv (1-e^{-a})/a$, where $a \equiv L_z/L_{\mathrm{ion},z}$	: Effect of the ratio of representative ionization box axial length to axial geometric length	Assuming $L_{\mathrm{ion},z} = 2r_L(B, T_e)$, *Length scale that energy transferred by electrons: $G = G(a) \to G(B, V_d)$

2. Relation between frequency and amplitude

- **Connection between frequency and amplitude**



Amplitude

$$\Delta = \alpha V_{\mathrm{ionz.\,box}} = \alpha L_\theta R_w L_{z,\mathrm{ion}}$$

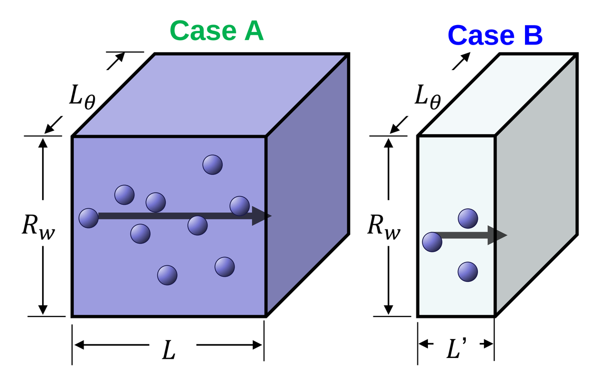
* $L_{z,\mathrm{ion}}$ is the length neutrals can propagate with the velocity, $v_{n,z}$, during the filling sequence, $T/2$. *Wei2012

$$L_{z,\mathrm{ion}} = v_{n,z}\frac{T}{2} = v_{n,z}\frac{1}{2}\frac{1}{f}$$

constant

$$\Delta = \mathrm{func}(L) = \mathrm{func}(L(f)) = \mathrm{func}(f)$$

Δ and f are coupled by $v_{n,z}$



$$\Delta = \alpha L_\theta R_w v_{n,z}\frac{1}{2}\frac{1}{f}$$

Non-varying

$$\therefore \Delta = \frac{\alpha'}{f}$$

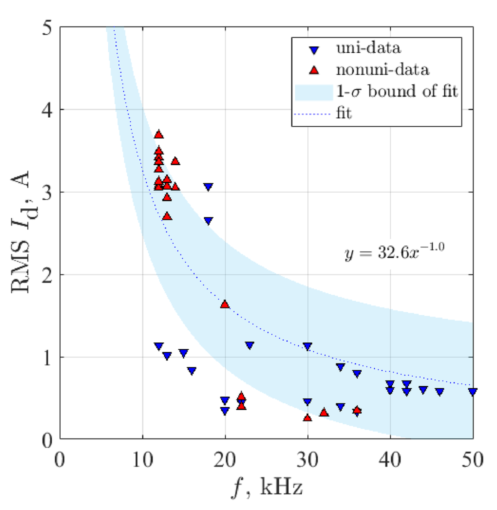
*Simplified analysis

When frequency is obtained from the model, amplitude can be connected

3. Exp. observation and comparison to analysis

- **Fitting of azimuthal distribution of V_p and n_e**

Analysis is for peak |P1| > 0.1 cases from FFT, which is considered there exists the breathing mode



Fitting function: $\Delta = a * f^{-1}$: From simple analysis

- Negative correlation is confirmed
- Some discrepancy between two are expected to come from
  1. Variation of axial location of ionization region along azimuth
  2. Variation of axial length of ionization box along azimuth

  , which would result in time phase-off to be collected at anode as I_d

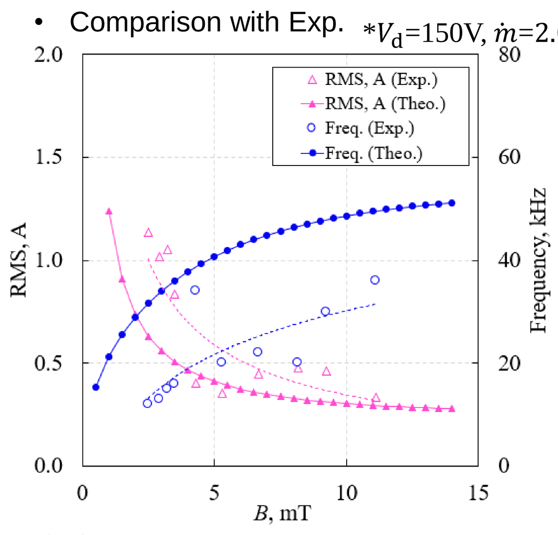
*Uniform $\dot{m}$ data : V_d=150V and 200V, $\dot{m}$=2.09mg/s
*Non-uniform $\dot{m}$ data : V_d=150V, $\dot{m}$=2.09mg/s, Ξ=0.0-1.0

$$f = \frac{1}{4\pi}\cdot\chi(\Xi)\cdot\nu_{\mathrm{ion}}(V_d)\cdot\sqrt{4\cdot\kappa(\dot{m})\cdot G(B) - 1}$$

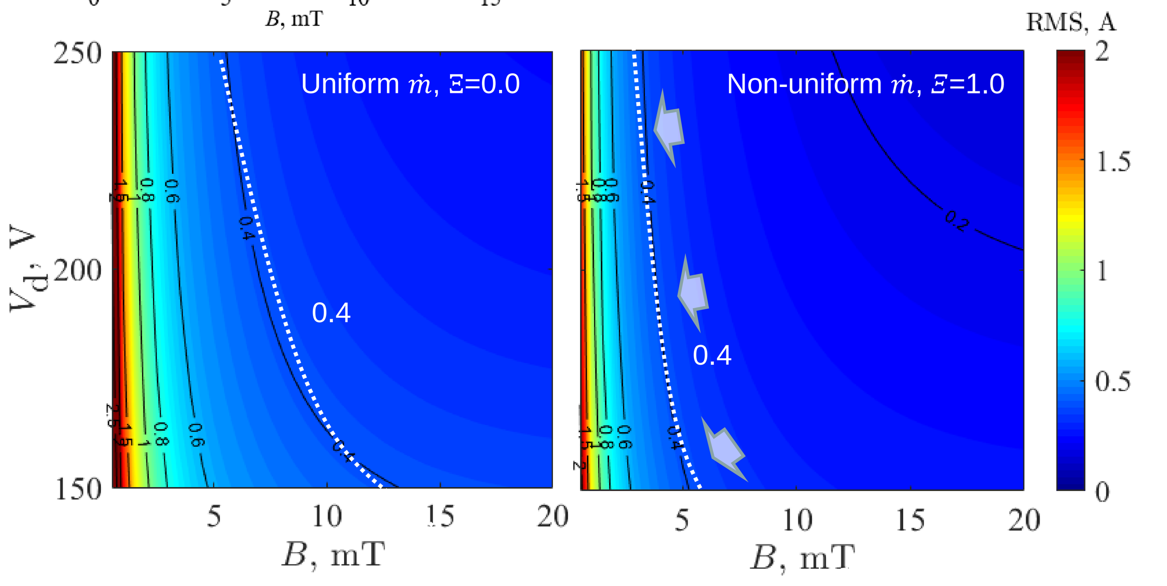
$$\mathrm{RMS} = \mathrm{RMS}(f) = 32.6\cdot f^{-1}$$

Ξ, V_d, $\dot{m}$, B (Operation parameters) ⇒ Ionization model ⇒ Ionization oscillation frequency ⇒ Oscillation amplitude

- **Simulated oscillation map with parameters $(B, V_d, \dot{m})$ + Ξ**
  - Comparison with Exp. *V_d=150V, $\dot{m}$=2.09mg/s; Uniform



- ❖ When it comes to the breathing mode,
  - Frequency is overestimated, but quantitatively agrees with experimental trend
  - Dependence on operation parameters also show acceptable agreement
- ❖ Using I_d instead of I_b could have led discrepancy
- ❖ Not including transition to azimuthal mode



Conclusion

- **Built an ionization model requiring pre-known parameters**
  - Parameters which can be pre-determined, including common operation parameters, geometric parameters, and azimuthal inhomogeneity
- **Observed a physical constraint coupling ionization frequency and amplitude**
  - Frequency and amplitude are in couple through the ionization box size and neutral velocity for neutral refilling cycle
- **Built a combined-model which can predict low-frequency oscillation tendency from common operation parameters**
  - Simple 0-D revised model qualitatively well predicts breathing ionization oscillation characteristics

Selected References

[1] J. M. Fife, M. Martinez-sanchez, and J. Szabo, 33rd AIAA/ASME/SAE/ASEE Jt. Propuls. Conf. Exhib. (1997). https://doi.org/10.2514/6.1997-3052
[2] N. Yamamoto, K. Komurasaki, and Y. Arakawa, J. Propuls. Power 21, 870 (2005). https://doi.org/10.2514/1.12759
[3] J. P. Boeuf and L. Garrigues, J. Appl. Phys. 84, 3541 (1998). https://doi.org/10.1063/1.368529
[4] W. Liqiu, H. Ke, W. Chunsheng, L. Hong, Z. ChaoHai, and Y. Daren, J. Vac. Sci. Technol. A Vacuum, Surfaces, Film. 30, 061304 (2012). https://doi.org/10.1116/1.4758788
[5] L. Wei, C. Wang, H. Ke, and Y. Daren, Phys. Plasmas 19, 012107 (2012). https://doi.org/10.1116/1.4758788

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* Acknowledgments : This work was supported by JSPS KAKENHI Grant Number 18J14592.