Magnetic Circuit Optimization for Hall Thrusters Design

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Abstract: Hall thruster performance is highly dependent on the topology of the applied magnetic field. This field is produced and shaped by a magnetic circuit, consisting of magnetic coils and ferromagnetic parts that confine the magnetic flux. Moreover, in magnetically-shielded Hall thrusters, in addition to ensuring efficient operation, the magnetic field is designed such that the erosion of the channel walls is minimized to significantly extend the thruster lifetime. Nevertheless, the magnetic circuit comprises a major part of the thruster mass. Furthermore, the coil power consumption is a factor influencing the total thruster efficiency and the operational temperature. Hence, it is important to provide an optimized circuit of minimized mass and power consumption, consistent with the requirements on thruster performance and lifetime. The aim of the present effort is to develop an optimization-based design tool for the magnetic circuit of shielded Hall thrusters. The work is conducted in two steps. As a first step, an analytical model of the circuit is developed through establishing an equivalent electrical circuit. This model is then coupled with an optimization algorithm to provide approximate set of solutions. The second step intends to obtain more accurate solutions, serving as an automated algorithm to design an optimum magnetic circuit able to generate the magnetic field of desired characteristics, magnitude and topology. This is performed in two phases using a multi-objective evolutionary algorithm of “Non-Dominating Sorting Genetic Algorithm (NSGA-II)”, developed by the authors. COMSOL Multiphysics is used as the field solver and iterates with the optimization code. The first phase aims at finding the optimum circuits in terms of mass and coils power, capable of providing the required magnetic field peak intensity on the channel centerline. In the second phase, the required shielding topology of magnetic field is achieved by optimizing some additional geometric features of the circuit.

Nomenclature

\[
\begin{align*}
B_z &= \text{axial component of magnetic field} \\
D &= \text{cathode housing diameter} \\
\tau_b &= \text{central base thickness} \\
L &= \text{channel length} \\
d &= \text{channel mean diameter} \\
W &= \text{channel width} \\
I &= \text{coil current} \\
X &= \text{decision vector of phase I}
\end{align*}
\]

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Electric plasma thrusters have demonstrated their potential in providing substantial mass savings by significantly reducing the amount of required propellant. Hall thrusters are among the most capable EP systems which enable a variety of mission scenarios, ranging from near-Earth communications and observations to deep-space exploration. This versatility of Hall thrusters is due to their capability of operating both in high-thrust and high-Isp modes.

The operation of a Hall thruster relies on a significant drop in electron axial mobility in the exit region obtained by means of a strong radial magnetic field. This field forces the electrons to drift azimuthally while trying to reach the anode. As a result of this increased resistance on the electron’s path to the anode, a strong axial electric field is generated at the channel exit. The hindrance that electrons, emitted from cathode, feel against their motion towards the anode extends their residence time sufficiently to collide and ionize the neutrals injected into the channel. At the same time, the ions,
almost unmagnetized due to their larger mass, are accelerated outward by the axial electric field, leading to generation of thrust.

The topology of the magnetic field has a significant influence on the performance of Hall thrusters. The magnetic field topology is usually characterized in terms of the location of the so-called “magnetic lens”, and the maximum magnetic field intensity along the channel centerline. In general, an important rule to design the magnetic topology is to ensure that the non-radial component of the field on the thruster channel centerline is very small in the lens region.

One major consequence of the topology of the magnetic field, specially the location of the lens, is related to the lifetime of Hall thrusters. In more detail, according to the concept of thermalized potential, as far as the electron temperature remains sufficiently low, the magnetic field lines are almost equipotential. This implies that under low electron temperature condition, having mostly-radial magnetic field near the exit ensures an almost axial electric field and axial acceleration of ions. However, the electron temperature in the acceleration region is shown to be high enough for a considerable deviation from equipotentiality to occur. In addition to the radial component of electric field due to this deviation, the formation of plasma sheath near the wall accelerates part of the ion population towards the channel walls. This effect causes significant erosion of the ceramic channel around the exit region. Eventually, as the poles become exposed to plasma, they will be also eroded. Erosion of the poles leads to departure of the magnetic topology from what was intended, and hence, the thruster is no more able to provide the expected performance. This marks the end of a Hall thruster operational life.

A viable solution to the issue of ceramic wall erosion, now known as “magnetic shielding”, enables significant extension of the thruster lifetime by at least one order of magnitude compared to thrusters with conventional magnetic topologies. Implementation of this method imposes further requirements on the topology of magnetic field. To this purpose, magnetic field line that is tangent to the wall near the exit, known as “grazing line”, is required to penetrate deep into the channel and approaches the surface of the anode. Being electron temperature relatively-low near the anode, this moderates the electron temperature on the grazing line to guarantee the equipotentiality along this line. This leads to the establishment of an electric field perpendicular to the wall in the exit region, where the line is almost tangent, retarding ions from reaching the wall. Besides, lower electron temperatures and large electric potential close to that of the anode cause lower potential drops perpendicular to the wall due to formation of a weaker sheath, thus, further reducing the ion flux to the wall.

Even though “unshielded” Hall thrusters are still suitable for Near-Earth missions, e.g. onboard large GEO telecommunications platforms, spacecrafts intended for interplanetary exploration or cargo missions require longer firing times and should be necessarily equipped with magnetically-shielded thrusters. As a result, on the one hand, the magnetic field topology of the thrusters intended for these missions is necessary to incorporate the requirements of magnetic-shielding scheme mentioned above. On the other hand, the suitable power range of the Hall thrusters for these missions is typically above 10 kW. In this respect, however, based on the existing scaling laws of Hall thrusters used to size the thruster’s channel\(^1\), the thruster mass increases significantly with discharge power. Consequently, compliance with the mission requirements concerning subsystems mass serves as an important motive to optimize the thruster design.

In this respect, design of the magnetic circuit is of great significance as it constitutes a major fraction of the entire thruster mass. In addition, the power given to the coils, although being a small percentage of the total thruster power consumption, noticeably influences total thruster efficiency. Also, as sources of heat dissipation, the coils, specially the inner one, affects the steady state operational temperature. This is because the inner coil is not in view of the ambient and therefore, heat cannot be directly radiated away. Hence, the circuit design should be optimized to ensure a proper operation of the thruster through its entire required life with minimum mass and power possible.

The magnetic circuit of Hall thrusters consists of the coils and ferromagnetic parts. The coils generate the magnetomotive force essential for the Hall thruster operation. The number of turns and currents into the coils can be adjusted to obtain the desired field magnitude and topology. Two typical arrangements of the coils exist to generate the required field: two coaxial inner and outer coils, or one inner and multiple outer column coils, positioned evenly-space along the azimuth on the exterior of thruster. In addition to the main coils, one or multiple smaller coils can be included for more refined modifications of the field topology. It is worth mentioning that for small Hall devices with discharge powers below 1 kW, permanent magnets are commonly used instead of magnetic coils.

The ferromagnetic parts provide a low-reluctance path through which the magnetic lines flow and as a result, helps in shaping the magnetic topology for the thruster operation. These are mainly comprised of an inner and outer pole together with a base. Magnetic screens, extensions to the thruster magnetic circuit that serve as better confining the magnetic lines, are also often included. The aim of this effort is to develop an optimization-based design tool for the magnetic

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circuit of magnetically-shielded Hall thrusters which has been conducted in two steps. “Step I” serves as an approximate but less computationally-demanding approach to estimate the mass-power Pareto front. The details of this approach are described in Section II. “Step II” includes two phases, the outcome of which is the final optimum design of the circuit based on the specified requirements. The first phase intends to find the optimum sizes of the main components of the circuit. In the second phase, the required topology of magnetic field is achieved by adjusting minor features of the circuit. In Section III, the implemented approach for Step II, including an overview of the optimization algorithm is presented. Finally, Section IV presents and discusses the results obtained from our analytical and numerical approaches in terms of the Pareto fronts for a generic 5 kW-class thruster with various requirements. This section also includes several trend analyses for different characteristics of the thruster magnetic circuit.

II. Step I: Approximate Approach to the Optimization of Circuit Geometry and Characteristics

A. Lumped Model of a Hall Thruster Magnetic Circuit

A lumped model was formulated as a versatile tool, applicable to Hall thrusters of any power class, to allow rapid assessment of the magnetic circuit characteristics and for the purpose of optimizing the relevant circuit design parameters, Figure 1.

![Magnetic Circuit Parametrization for Step I approach.](image)

As it is shown in Figure 2, the model relies on estimating the magnetic reluctances corresponding to each circuit component and air gap, incorporating them as resistances in an equivalent electrical circuit. In this equivalent circuit, the magnetic coils act as voltage generators.

![Equivalent electrical circuit of a Hall thruster magnetic circuit. Reluctances and magnetic fluxes are computed for:](image)

Figure 2. Equivalent electrical circuit of a Hall thruster magnetic circuit. Reluctances and magnetic fluxes are computed for: inner core (IC), inner pole (IP), inner screen (IS), inner base (IB), central base (B), outer base (OB), outer screen (OS), outer pole (OP), outer core (OC), inner core-to-inner screen air gap (IC-IS), inner pole-to-inner screen air gap (IP-IS), inner screen-to-outter screen air gap (IS-OS), outer pole-to-outter screen air gap (OP-OS), outer core-to-outter screen air gap (OC-OS), inner pole-to-outter pole air gap (IP-OP), inner pole-to-outter core air gap (IP-OC), inner core-to-outter base air gap (IC-OB).
The value of each reluctance shown in Figure 2 strongly depends on the magnetic circuit geometrical parameters defined above. Once the reluctances are computed, the unknown magnetic fluxes can be calculated by applying Kirchhoff’s laws, which are a direct consequence of Maxwell’s equations, Eq. (1) and Eq. (2).

\[
\int \mathbf{v} \cdot \mathbf{B} \, dV = 0 \quad \Rightarrow \quad \Phi = \text{const} \quad \text{in every branch}
\]

\[
\sum \Phi_i = 0 \quad \text{in each node}
\]

\[
\int \mathbf{v} \times \mathbf{B} \cdot dS = \int \mu \, j \, dS \quad \Rightarrow \quad \sum R_i \Phi_i = N_j \, I \quad \text{along the } j\text{-th closed loop}
\]

\[
R_i = \frac{\int dl}{\mu \cdot A} \quad \text{is the } i\text{-th branch reluctance}
\]

The ferromagnetic and gap reluctances are calculated according to Eq. (3):

\[
R = \int_0^s \frac{dl}{\mu_0 \mu_r \cdot A(l)}
\]

where \(dl\) is the path along which the magnetic lines flow, and \(A(l)\) is the flux area as a function of the path coordinate \((l)\). Even though the calculation of the reluctances for the ferromagnetic components is straightforward, finding a reasonable expression for the path length and flux area of the air gaps is a challenge. Nevertheless, considering the typical magnetic field topologies of Hall thrusters and some general properties of magnetostatic fields, the following considerations and assumptions were made in order to approximate the gap reluctances:

- Magnetic lines are perfectly axisymmetric.
- They always exit perpendicularly from the high-magnetic-permeability ferromagnetic components,
- Magnetic flux tubes fill all 3D space, and,
- For the sake of simplicity, the boundary of every magnetic flux tube is described by a combination of straight lines and circular arcs.

The resulting geometrical shapes used to calculate the gap reluctances are shown in Figure 3, in which the blue lines represent the path followed for the integration over \(dl\), and the green lines represent the axisymmetric flux area \(A(l)\). Eq. (3) is integrated numerically to obtain the approximate values of the magnetic reluctances in the ferromagnetic components and air gaps.

Figure 3. Schematic of magnetic flux tube geometries used to estimate the gap reluctances in Hall thruster magnetic circuit.

The reluctances of all gaps and ferromagnetic components are calculated by means of Eq. (3) and according to Figure 3, except for the following:

\[
\int \mathbf{v} \cdot \mathbf{B} \, dV = 0 \quad \Rightarrow \quad \Phi = \text{const} \quad \text{in every branch}
\]

\[
\sum \Phi_i = 0 \quad \text{in each node}
\]
As Figure 4 shows, the inner pole-to-inner screen (R_{IP-IS}) and outer pole-to-outer screen (R_{OP-OS}) gap reluctances are each modeled as two reluctances in parallel according to Eq. (4):

\[ R_{IP-IS} = \left( \frac{1}{R_{IP-IS,1}} + \frac{1}{R_{IP-IS,2}} \right)^{-1} \quad (4) \]

- Referring to Figure 5, due to the effect of the charge flux produced by the electromagnets, the inner core-to-inner screen (R_{IC-IS}) and outer core-to-outer screen (R_{OC-OS}) gap reluctances are obtained to be twice the ones calculated using Eq. (3), see Eq. (5).

\[
R_{IC-IS} = \frac{1}{\mu_0 \cdot \mu_r \cdot \left[ \cosh \left( \frac{2\pi}{\mu_r \cdot K \cdot \left( \frac{1}{A_{IC}} + \frac{1}{A_{IS}} \right) \cdot L \right) - 1 \right]} \sim \frac{k}{\mu_0 \cdot \pi \cdot L} = 2 \cdot \int_0^{w_i} \frac{dr}{\mu_0 \cdot A} \quad (5)
\]

where \( k = \ln \left( \frac{B}{2} + \tau_i + w_i \right) - \ln \left( \frac{B}{2} + \tau_i \right) \)

Performing the calculations as described above, Eq. (6) shows the resulting linear system of equations to be solved in order to determine the unknowns of the equivalent electrical circuit depicted in Figure 2. The inputs to the presented to the lumped model are the magnetic circuit design parameters defined in Figure 1, from which each element of the reluctance matrix and of the magnetomotive force vector is computed. The model output are the 16 unknown magnetic fluxes, which are then used to calculate the average value of magnetic field induction (B) in each circuit component and air gap.
B. Magnetic Circuit Optimization: Objective and Constraints

The lumped model described above is used to evaluate the nonlinear constraints imposed to the magnetic circuit throughout the optimization process. These constraints must be imposed in order to prevent magnetic saturation in the circuit, to ensure a symmetric magnetic lens at the discharge channel exit, and to provide the required value of magnetic field peak along the channel centerline. Mathematically, these constraints translate into the following relations, Eq. (7) to Eq. (9)

\[
\max \left\{ \frac{\Phi_{IC}}{A_{IC,min}} ; \frac{\Phi_{OC}}{A_{OC,min}} ; \frac{\Phi_B}{A_{B,min}} ; \frac{\Phi_{OB}}{A_{OB,min}} ; \frac{\Phi_{IS}}{A_{IS,min}} ; \frac{\Phi_{OS}}{A_{OS,min}} ; \frac{\Phi_{IP}}{A_{IP,min}} ; \frac{\Phi_{OP}}{A_{OP,min}} \right\} < B_{SAT},
\]

\[
\Phi_{IP-IS} = \Phi_{OP-OS},
\]

\[
\frac{\Phi_{IP-OP}}{A_{IP-OP}} \geq B_{PEAK}.
\]

However, it must be noted that the lumped model is only able to provide average magnetic field values over each flux tube area. This implies that magnetic field intensity near the edges of the ferromagnetic components is not predicted, and that the saturation level of the magnetic circuit cannot be estimated accurately. The same applies to the magnetic field along the channel centerline; the imposed constraint only refers to the “mean” value of magnetic field induction intensity along the channel centerline, meaning that a considerably higher peak is obtained when the magnetic field is calculated more accurately (for example, using the open-source FEMM software).

Furthermore, it is also worth mentioning that other purely-geometrical constraints are also applied to increase the robustness of the lumped model, to guarantee a minimum housing diameter for a central cathode, and to allow for a magnetic-shielding field topology.

The optimization is performed by coupling the lumped model to a global optimization algorithm based on Sequential Quadratic Programming (SQP) methods. A scalarizing function is used to represent the optimization objective as indicated in Eq. (10)

\[
OBJ = \frac{P}{P_{ref}} + \frac{M}{M_{ref}}
\]

where the resulting circuit mass \(M\) is calculated by means of simple geometrical formulae, and the power consumption of the electromagnets \(P\) is computed from their corresponding input currents and cable lengths. The resistance of the cables is calculated by considering an inner coil temperature of 500°C and an outer coil temperature of 350°C.

The circuit mass vs. power consumption Pareto front is then computed by sweeping the reference values of the circuit mass and coils power consumption \((M_{ref} \text{ and } P_{ref})\) used to normalize the scalarizing function. As such, the followed optimization approach falls within the category of constrained single-objective optimization algorithms.
III. Step II: Exact Approach to the Optimization Problem

A. Overview of the Methodology and the NSGA Optimization Algorithm Description

Many real-world design processes involve more than one objective. In a multi-objective optimization problem, the objectives typically conflict with each other, meaning that reduction of one leads to the enhancement of the other in a way that all objectives cannot be optimized simultaneously. There are two general approaches for multi-objective optimization. One is to combine all objectives into one objective function and treat the problem as a single-objective optimization. Considering the typical conflicting nature of the optimization objectives, the optimized solution is highly affected by the definition of the single function which itself depends on the relative significance of the individual objectives in any specific problem. However, there are cases in which the user needs to have a broader range of optimized choices in order to perform a trade-off between the targets and decide according to their preferences. This possibility is provided in the second method which treats objectives separately and obtains a set of solutions referred to as the Pareto optimal set. This approach works on the basis of the concept of domination in a n-dimensional space. In a problem in which all objectives are to be minimized, a solution A is said to be dominated by solution B if and only if the values of all objective functions corresponding to B are less than those for A, meaning that B is absolutely a better solution than A with respect to all objectives. However, if there exists at least one objective which has a lesser value for A than B, without further information, there is no way to prefer one solution over the other. Optimization algorithms proceed, through multiple iterations, in a way to find the non-dominated solutions representing the Pareto optimal set. Corresponding to the Pareto optimal set in the decision variables space is the Pareto front which is presented in the objective domains. The shape of the Pareto front is such that the improvement of one objective always leads to degradation of at least one other objective.

In case of Hall thruster magnetic circuit, mass and coil power consumption could be considered as conflicting objectives. This is because in order for the coils to provide a required magnetic field peak, we need to either add to the number of turns which leads to larger mass and power or increase the carried current which increases the coils power. Although, the first case increases both mass and power, it is favorable in terms of power relative to the second case due to the squared proportionality of power with current. Hence, considering the conflicting behavior of coils mass and power, and also the fact that they comprise significant mass of the circuit, mass and power should be minimized as two separate objectives.

In addition, the relative significance of minimization of either circuit mass or coils power is determined by the specific missions for which the thruster is designed. Therefore, it is preferred to provide a diverse range of optimal configurations for the thrusters so as to allow the best compromise between mass and power according to the mission.

To this purpose, the multi-objective evolutionary algorithm of the “non-dominating sorting genetic algorithm (NSGA-II)”[^3,4] is used.

Genetic algorithms in general works based on randomly searching the feasible decision variables space through the principal genetic operators of crossover, mutation and selection, evolving towards the optimum solutions through iterative process until the convergence criterion is met. The random-based search allows GA to be able to find the global optima without getting stuck in local optima.

NSGA varies from simple single-objective GAs in its selection operator. Being of multiple objectives, the sorting of the populations’ members, and therefore selecting the superiors among them, is more complicated than that in an ordinary GA. Besides, the selection must be performed in a way that maintains the diversity of solutions leading to an almost uniform distribution of population over the predicted Pareto front. In the following the optimization algorithm is briefly discussed.

An initial population consisting of \(N_p\) members (chromosomes), each being a vector in a n-dimensional space of decision variables (genes), is randomly generated. The crossover and mutation operators are applied respectively on \(N_c\) and \(N_m\) numbers of members, randomly selected from the existing population. In crossover, pairs of members called “parents” are combined to create new solutions known as “offspring” which is expected to inherit the favorable “genes” from parents improving the fitness of solutions. The mutation operator is responsible to introduce new members in the population pool slightly different from the existing ones. This way, the diversity of population is ensured which allows searching the entire feasible space and escape from local optima. In each iteration, all population members consisting of those generated either randomly during initialization (in the first iteration) or through crossover and mutation must be evaluated and ranked. To assign ranks to the population members, the “dominance rule” is applied to population members according to the values of their corresponding objective functions. This leads to dividing the population into non-
dominated fronts in the objective functions’ domain with specific ranks \( (F_i, i \in \{1, 2, ..., k\}) \). The ranking is carried out in a way that all solutions belonging to a single front with the same rank are dominated by equal number of other solutions without being dominated by each other. This means that fronts of lower ranks contain fitter solutions. Besides, the first front \( (F_1) \) corresponds to those solutions which are not dominated by any other, representing the most promising solutions in each iteration, i.e., being closest to the optimal ones.

A second order ranking is performed based on calculating the relative “crowding distance” of solutions and accordingly sorting the members of a front of an equal rank \( (F_{ij}, j \in \{1, 2, ..., l\}) \). Based on crowding distance criterion which aims at a uniform spread of solutions, on each front, the solutions located in a less crowded regions on a certain front are preferred over the ones placed in more concentrated areas.

The next step is selecting the best solutions for the next iteration through replacing the initial population by the N best rank members. The selection begins from the lowest rank front \( (F_i) \) and the least crowded-distant members and continues until N members are selected. This way, NSGA-II ensures that the “elitist” solutions are maintained in the new population while keeping the diversity of the population.

Proceeding through iterations, the first-ranked front \( (F_1) \) approaches the eventual Pareto front. Finally, after a predefined number of iterations or when a stopping criterion is met, the obtained \( F_1 \) front in the last iteration is presented as an approximation of actual Pareto front of the problem.

The code consists of two phases with the ultimate goal of developing an automated design tool of magnetic circuit optimized in terms of mass and power capable of providing the required magnetic field magnitude and topology. Both phases follow multi-objective optimization-based scheme for which NAGA-II is implemented in MATLAB. As the magnetic field solver COMSOL Multiphysics is connected to MATLAB. A 2D axisymmetric parametric model is generated in COMSOL with the general circuit configuration according to the user’s requirements which receives the specific dimensions from the optimization code, evaluates the fields and return the required values back to the code.

Based on these values, the code evaluates the problem constraints which are defined in terms of implicit objective function(s) in addition to the explicit objectives. According to the values of all objective functions whether explicit or implicit the candidate circuits will be sorted and selected for the following iteration. The description of each phase is presented in the following.

A.1. **Phase I: Overall Circuit Geometry and Characteristics Optimization**

The objective of this phase is to obtain general characteristics of optimized circuits of minimum mass and coil power consumption capable of generating a required magnetic field peak magnitude along the channel centerline \( (B_{peak}) \). Apart from the peak, the field intensity inside the circuit must remain below a certain limit to prevent the saturation of the ferromagnetic material which causes substantial deviations of field lines from the designed paths. Also, the field is required to be almost symmetrical with respect to channel centerline in the channel and near-plume, particularly in the lens region. As a matter of fact, there are many different circuit geometries of low mass and power which are able to generate the required magnetic field in terms of magnitude; whereas, their overall field topologies might be far the typical field of Hall thruster. Imposing a constraint that somehow quantifies the suitability of generated field is essential to avoid domination of such improper configurations over the those capable of producing acceptable fields but with higher mass or/and power. The symmetry of lens could be a criterion for the solutions fitness.

Therefore, the mass and coil power are defined as the objectives of optimizations distinctively. The rest of requirements on magnetic field namely, field peak value, symmetry and saturation are the problem constraints, which are accounted for as implicit optimization objectives incorporated through the following penalty functions:

\[
f_1(X) = C_1 \left[ \frac{B_p}{B_{peak}} - 1 \right]^2 + \left( \max \left( 0, \frac{B_{max}}{B_{sat}} - 1 \right) \right)^2 \tag{11}
\]

\[
f_2(X) = \frac{1}{B_p^2} \left[ C_2 \sum_{i=1}^{m} (Bz_{R_i} - Bz_{L_i})^2 + (Br_{R_i} - Br_{L_i})^2 \right] + \int_{c.L} B_z^2 \, dz \tag{12}
\]

\( f_1 \) includes the constraints on field peak value and saturation of the circuit. \( f_2 \) is responsible for ensuring the magnetic field symmetry with respect to the centerline. The symmetry is measured through calculating the squared difference of
magnetic field components on m pairs of points located symmetrically on both sides of the channel centerline, mainly concentrated in the lens region, together with the integral of the squared of the axial magnetic field component along the centerline. $Bz_R, Br_R, Bz_L, Br_L$ denote the axial and radial components of magnetic field on the points on right and left sides of the channel centerline, respectively. The constants $C_1$ and $C_2$ need to be adjusted so to make sure each term in the objective functions is weighted appropriately.

The design constraints such as those regarding the channel size are implemented in the code as directly controlling the optimization parameters or restricting the decision variable space.

The general scheme of the magnetic circuit, assumed for the optimization in this work, consists of coaxial inner and outer coils, screens, inner and outer poles, and a base. Figure 6 indicates the geometry of the circuit to be optimized in the first phase, including the dimensions that, together with the coils current, constitute the decision vector, $X = [x_1, \ldots, x_{15}]$. The distance between screens must be constrained such as to accommodate channel of required dimensions. Besides, some spaces around the coils are left for coil’s supports and possible gaps for thermomechanical considerations.

As a multi-objective optimization problem, there is no single optimized solution. Apart from the objective functions representing the constraints which must be met, the other ones accounting for mass and power are not optimum in the same solution. Selection of one configuration among all acceptable optimized solutions depends on the relative importance of the objectives in each specific case, which is determined by high-level requirements derived from a particular mission. The selected configuration serves as the input to the next phase.

A.2. Phase II: Field Topology Optimization and Circuit Refinement

In the second phase, the focus is on modifying the topology of the magnetic field to provide shielding. In fact, the aim is to refine the design of the circuit to obtain a field with the desired topology, while ensuring that the constraints of the previous phase remain satisfied. To this purpose, minor features are added to the circuit, including auxiliary coil(s) and finer geometrical elements.

Besides, minor refinements in some parts of the circuit, fixed in previous phase, is allowed such as recession of poles from the channel centerline, small variation in screens height and main primary coils’ current. The sizes of the refinements constitute the decision vector $Y$.

To superimpose the desired field topology on the model, a set of points are defined in regions of interest on which the field lines are forced to be aligned with those in the target topology. The target can be the field topology of a previously-designed magnetically-shielded thruster of even different power level.

To be able to use the magnetic field topology of a thruster with a different power level, and therefore different dimensions as a reference, positions of the control points are needed to be specified parametrically as functions of the channel dimensions.
dimensions and relative distance from channel wall or centerline. Moreover, as the general scheme of the circuit is fixed in the last phase, the requirements regarding the magnitude of magnetic field are almost set. Hence, the objective function must be defined in a way to only impose the reference field topology not the magnitudes, especially when a thruster of different power level is chosen as reference.

The objective function quantifies the deviation of the achieved magnetic field topology from that of the target at the control points described by the following expression. As the channel aspect ratio in terms of channel length ($L$) over channel width ($W$) is different from the reference thruster, the scaling factors $\frac{W}{W_{\text{ref}}}$ and $\frac{L}{L_{\text{ref}}}$ are incorporated into the objective function, $f_1(Y)$ as

$$f_1(Y) = \sum_{i=1}^{n} \left( \frac{W}{W_{\text{ref}}} \frac{L}{L_{\text{ref}}} \left( \frac{B_{ri}}{B_{zi}} - \frac{B_{r\text{ref}i}}{B_{z\text{ref}i}} \right) \right)^2.$$  (13)

As mentioned before, the distribution of the control points is decided according to relative importance of field topology in various regions of thruster.

As the magnetic shielding concept is based on being the grazing line of low temperature and high potential. To make sure this is the case, this line shall penetrate deep enough into the channel, otherwise there will be deviation from the conditions necessary for shielding. As the magnetic field intensity decreases almost monotonically from the exit to near anode, the magnitude of the field along the centerline on the grazing line ($B_{grz}$) could be an indicator of how close the grazing line gets to the anode, and therefore introduces a criterion quantifying the level of shielding capability of the magnetic field. The requirement of achieving a certain level of shielding poses a constraint on the maximum value that $B_{grz}$ can assume. A separate objective function can be defined to enforce such constraint such as $f_4$ in which $B_0$ is the field magnitude on the grazing line on the channel centerline which needs to be less than a percentage of the field peak value.

Finally, we need to make sure that while modifying the topology, the $B_{\text{peak}}$ remains above the minimum limit required for the thruster operation and also maintain the magnetic field below the saturation limit in the circuit. It is worth mentioning that $B_{\text{peak}}$ here might not be the same as in previous phase. As in the process of reaching a shielded magnetic field topology the field peak may drop by a fraction ($a$) depending on the shielding strategy. In phase I the required $B_{\text{peak}}$ needs to be taken larger than that considered in phase II which corresponds to what is actually needed for the nominal operation of the thruster.

The constraints are incorporated as additional objective functions as below

$$f_3(Y) = C_3 \left( \max \left( 0, 1 - \frac{B_p}{B_{\text{peak}}} \right) \right)^2 + \left( \max \left( 0, \frac{B_{\text{max}}}{B_{\text{sat}}} - 1 \right) \right)^2,$$  (14)

$$f_4(Y) = C_4 \left( \max \left( 0, \frac{B_0}{a \times B_{\text{peak}}} - 1 \right) \right)^2.$$  (15)

At the end, the final design is the one that satisfies all constraints with minimum deviation of magnetic field topology with respect to the reference.

The application of the described method is demonstrated through optimization of magnetic circuit of a generic magnetically-shielded 5kW class Hall thruster in four different cases, corresponding to different requirements on the channel size.
IV. Results and Discussions

To present the capabilities of the described approaches, the optimization results for a 5 kW-class Hall thruster at two operating discharge voltages of 300V and 450V, and with different requirements on channel dimensions, are presented. The following table summarizes the design requirements corresponding to the four cases studied. The d/W represents the ratio of the channel mean diameter over the channel width. In the first two cases, no limitation is imposed on this ratio, whereas in next two ones, the ratio is set to fixed values. The channel area is scaled according to well-established Hall thruster scaling laws and is kept as a fixed value (within 5% error) in each case.

Table 1: Operating conditions and requirements for the optimization case studies on a 5 kW-class Hall thruster

<table>
<thead>
<tr>
<th></th>
<th>Case A</th>
<th>Case B</th>
<th>Case C</th>
<th>Case D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power (W)</td>
<td>5000</td>
<td>5000</td>
<td>5000</td>
<td>5000</td>
</tr>
<tr>
<td>Voltage (V)</td>
<td>300</td>
<td>450</td>
<td>450</td>
<td>450</td>
</tr>
<tr>
<td>d/W ratio</td>
<td>free</td>
<td>free</td>
<td>5.5</td>
<td>6.5</td>
</tr>
</tbody>
</table>

Assuming that the output of the second optimization phase should be a circuit corresponding to the magnetic-shielding scheme, capable of providing a peak field of $B_{\text{peak}}$ along the channel centerline, which is fixed to a specific value from the range of 22mT to 28mT, in both the Step I and Step II, phase I, the required $B_{\text{peak}}$ for all cases above is taken to be 1.2$B_{\text{peak}}$. Furthermore, as mentioned in Section II.B, the coils power consumption is calculated from the resistance of the coils cable in their corresponding temperatures. In addition, the inner and outer coils currents are assumed to be the same. Moreover, the coils number of rows and columns are an integer multiple of coil cable diameter. In the following sections, the Pareto fronts and the trends of several optimization parameters, obtained from the approximate and exact approaches, are reported.

Final refinements of the magnetic circuit to achieve the desired field topology is carried out in the phase II on one of the optimized configurations of phase I. The result of the second phase in terms of the field topology before and after the optimization are shown.

A. Overall Circuit Geometry Optimization Results

The results of the Step I and the first phase of the Step II optimization are presented in Figure 7 and Figure 8, respectively, in terms of the coils power versus circuit mass Pareto fronts. The points on each graph correspond to all optimized circuit geometries that respect the constraints and requirements. Comparing the Pareto fronts in Figure 7, corresponding to the approximate approach, with those of the exact approach (Figure 8), it is clear that the results are quite consistent. This implies the representativeness and acceptable accuracy of the approximate magnetic circuit model, as introduced in Section II.A.
Figure 7: Coils power consumption vs. magnetic circuit mass Pareto fronts from the Step I, approximate approach; (a) Case A, (b) Case B, (c) Case C, and (d) Case D.

Figure 8: Coils power consumption vs. magnetic circuit mass Pareto fronts from the Step II, Phase I; (a) Case A, (b) Case B, (c) Case C, and (d) Case D.
In addition, as it was expected from the scaling laws of Hall thrusters, higher discharge voltages, or equivalently lower channel areas, lead to lower circuit mass and power. Besides, results presented in Figure 7 and Figure 8 coherently show that for a certain channel area, prescribing a ratio between the channel mean diameter and channel width (d/W) affects the solutions such that for a smaller ratio, the Pareto front extends to lower masses, whereas for a larger ratio, the front tends to extend towards lower powers. These conclusions are consistent with the trends of variations of the channel mean diameter and the channel width with mass and power for cases with free d/W ratio, Figure 9 and Figure 10. According to the results, having a fixed channel area (~dW), the general trend is such that widening the channel (reducing d) favors mass and is against power. However, increasing d (reducing W) leads to less coils power consumption but heavier circuits. In fact, to accommodate wider channels, the poles need to be farther, which means that to generate a specific field peak near the channel exit region, the coils require more power. Also, thrusters of larger channel diameters tend to occupy a greater envelop in radial extent, thus, leading to heavier circuits.

Figure 11 shows the trends of the coils current vs. the number of turns for the four optimization cases. As it can be seen from this figure and as it was intuitively expected, increasing the number of turns results in the reduction of the coils current for a fixed value of magnetic field peak. Similar trends to those shown here are also obtained from Step I optimization approach, which are presented in the appendix. The appendix section also includes more trends from Step II, Phase I related to the dependencies of the circuit mass and coils power to the current and number of turns of the electromagnets. It is noteworthy that obtaining the trends between the optimized magnetic circuit parameters, mass and the electromagnets power consumption, enabled by implementing the optimization tool, serves as one of the major outcomes of the present work. In fact, these data provide scaling insights into the problem of properly designing a magnetic circuit, which are of great importance in the preliminary development phases of a Hall thruster.

Figure 9: Trends of the channel width vs. circuit mass and coils power from Step II, Phase I; (a) width vs. mass for Case A, (b) width vs. mass for Case B, (c) width vs. power for Case A, (d) width vs. power for Case B.
Figure 10: Trends of the channel mean diameter vs. circuit mass and coils power from Step II, Phase I; (a) mean diameter vs. mass for Case A, (b) mean diameter vs. mass for Case B, (c) mean diameter vs. power for Case A, (d) mean diameter vs. power for Case B.
Figure 11: Trends of the current vs. number of turns for the inner and outer electromagnets from Step II, Phase I; (a) Case A, (b) Case B, (c) Case C, (d) Case D.

B. Phase II Optimization Results

As an input to this phase, one of the optimized configurations corresponding to “case A” with an intermediate mass (~ 8 kg) and power (~ 130 W) is selected. The magnetic flux intensity inside the circuit, the field topology, and its magnitude along the centerline for this circuit geometry are shown in Figure 12.

Figure 12: Selected configuration from phase I for the purpose of field topology optimization (Step II, Phase II); (a) Magnetic field topology, (b) Magnetic field flux intensity inside the circuit, (c) profile of field intensity along the channel centerline.
The field topology of SITAEL’s HT5k LL Hall thruster is taken as the reference. The shielded topology must provide the required $B_{\text{peak}}$. The output of this phase is a set of solutions which results in minimum values for the defined objective functions. However, as mentioned in Section III.A.2, one of these functions is the actual objective of the optimization and the other two are the problem constraints. Hence, the optimized solution is the one which satisfies the constraints with minimum value of the objective function, which is the deviation of field topology from that of the target. Figure 13 presents the field of the optimized circuit. The discrepancy error of the achieved field topology with respect to the target, defined through the following function, is less than 5%.

$$ f(Y) = \frac{1}{N} \sum_{i=1}^{n} \left( \frac{W - \frac{W_{\text{ref}}}{L_{\text{ref}}}}{\frac{B_{r,\text{ref}}}{B_{z,\text{ref}}}} - 1 \right)^2. $$

(16)

Comparing Figure 13 and Figure 12, it can be observed that the modifications introduced in the circuit through the second optimization phase have successfully resulted in changes in the magnetic field characteristics that are aligned with those of the shielding topology. The two main features are the existence of the grazing field line, tangent to the channel chamfer near the exit (Figure 13(a)), and the downstream shift in the magnetic field peak intensity towards the plume (Figure 13(b)).

V. Conclusions

The principal idea behind the work presented in this paper was to develop an optimization-based design-aiding tool in order to largely automate the process of the design of Hall thrusters’ magnetic circuit. In this respect, we followed two approaches, which were referred to as Step I and Step II. The efforts within the Step I were mainly focused on developing an approximate analytical model of the magnetic circuit. The aim of this model was to be able to obtain rapid, reasonable estimates of the circuit characteristics. This is achieved through the discretization of the problem domain to calculate magnetic reluctances and fluxes, and the establishment of an equivalent electrical circuit. Coupling this model to a global-search optimization algorithm, the overall consistency of outcomes with the exact solutions of Step II demonstrated the representativeness of the analytical model. This model is currently being further improved to increase the accuracy of its predictions.

Step II principal focus was on developing an advanced genetic algorithm optimization code, which was coupled to a FEM-based magnetic field solver. The application of an exact model is in line with the purpose of automation of the circuit design. In this respect, Step II system allowed automatic refinement of the circuit in order to achieve the magnetic-shielding field topology. In any case, this approach is intrinsically more computationally-costly, and thus time-
consuming. Thus, to lower the computational cost, various definitions of objective functions are being reviewed to obtain the best set of functions that minimize the required number of iterations needed for the convergence.

Following the promising results of the developed tool, it was successfully applied in the preliminary design phase of the SITAEL’s 5 kW and 20 kW Hall thrusters.

Finally, it should be also mentioned that the optimization-based model provides valuable insights into the scaling of the magnetic circuit and can be used as a tool for trade-off between various characteristics of the circuit.

Appendix

a. More Trends from Step II optimization approach, Phase I

Figure 14: Trends of the coils current vs. the magnetic circuit mass; (a) Case A, (b) Case B, (c) Case C; (d) Case D
Figure 15: Trends of the coils current vs. coils power consumption; (a) Case A, (b) Case B, (c) Case C; (d) Case D

Figure 16: Trends of the Channel mean diameter vs. channel width; (a) Case A; (b) Case B
Figure 17: Trends of the coils number of turns vs. magnetic circuit mass for the inner and outer electromagnets; (a) Case A, (b) Case B, (c) Case C, (d) Case D.
Figure 18: Trends of the coils number of turns vs. power consumption for the inner and outer electromagnets; (a) Case A, (b) Case B, (c) Case C, (d) Case D.

b. Trends from Step I optimization approach
Figure 19: Trends of the channel width vs. circuit mass and coils power; (a) width vs. mass for Case A, (b) width vs. mass for Case B, (c) width vs. power for Case A, (d) width vs. power for Case B.

Figure 20: Trends of the current vs. number of turns for the inner and outer electromagnets from Step I; (a) Case A, (b) Case B, (c) Case C, (d) Case D.

References