Acceleration by Oscillating EM Fields

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Abstract: The acceleration of charged particles by oscillating electric and magnetic fields perpendicular to each other is studied theoretically. It is shown that charged particles are accelerated perpendicular to both fields if there is a $\pi/2$ phase difference between the fields. Analytical expressions describe the particle dynamics. When there is such a phase difference, there is a secular term in the expression for the particle velocity that grows linearly in time. A parameter is identified that determines the direction and size of the acceleration.

Nomenclature

ω	=	frequency of oscillation
E_1	=	amplitude of the oscillating electric field
B_1	=	pressure coefficient
q	=	particle charge
m	=	particle mass
v	=	particle velocity
u	=	normalized particle velocity
ω_{c1}	=	cyclotron frequency related to the oscillating magnetic field
τ	=	normalized time

I. Introduction

There is a growing interest in developing electrode-less plasma thrusters to reduce erosion and extend life-time. In the Helicon Plasma Thruster (HPT) [1], a thrust is generated as the plasma heated by radio-frequency waves is ejected away, being accelerated through a magnetic nozzle. The excitation of a double layer at the exit of the HPT was also suggested as a way to impart momentum to the plasma [2]. Another method is imposing a travelling magnetic field [3]. Other methods are the introduction of a rotating magnetic field at the exit of a helicon source [4] and the direct wave-drive thruster [5]. In this paper, the acceleration of charged particles by oscillating electric and magnetic fields perpendicular to each other is studied theoretically. It is shown that charged particles are accelerated perpendicular to both fields if there is a $\pi/2$ phase difference between the fields. Analytical expressions describe the particle dynamics. When there is such a phase difference, there is a secular term in the expression for the particle velocity that grows linearly in time. A parameter is identified that determines the direction and size of the acceleration. In Section II, we present the solution of the equations of motion. In Section III we present numerical examples. Possible applications are discussed in Section IV.

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II. The model

We assume uniform-in-space time-varying electric field $\vec{E}$ and magnetic field $\vec{B}$ of the form:

$$\vec{E} = \hat{x} E_1 \cos(\omega t) \quad \vec{B} = \hat{y} B_1 \cos(\omega t - \phi). \qquad (1)$$

The equations of motion of particle of mass m and charge q are

$$m \frac{dv_x}{dt} = qE_1 \cos(\omega t) - qv_z B_1 \cos(\omega t - \phi), \qquad (2a)$$

and

$$m \frac{dv_z}{dt} = qv_x B_1 \cos(\omega t - \phi). \qquad (2b)$$

It is convenient to use dimensionless form:

$$\frac{du_x}{d\tau} = s\left[\cos(\tau) - u_z \cos(\tau - \phi)\right], \qquad (3a)$$

and

$$\frac{du_z}{d\tau} = su_x \cos(\tau - \phi). \qquad (3b)$$

Here,

$$\vec{u} \equiv \frac{\vec{v}}{E_1 / B_1} \quad \tau \equiv \omega t \quad s \equiv \frac{qB_1}{m\omega} = \frac{\omega_{c1}}{\omega} \quad \omega_{c1} \equiv \frac{qB_1}{m}. \qquad (4)$$

Two cases are now examined. One is when $\phi = 0$ and a second when $\phi = \pi / 2$.

A. **Phase difference zero.**

Let us assume that $\phi = 0$. Equations (3a)-(3b) can be integrated and u_x and u_z are periodic in time. When the velocity is zero at $\tau = 0$, the solutions are in particular simple,

$$u_z = 1 - \cos(s \sin \tau), \quad u_x = \sin(s \sin \tau). \qquad (5)$$

There is a net drift in the $E \times B$ direction, independent of mass or charge.

B. **Phase difference $\pi / 2$**

This is the interesting new case. Let us write the equations again, where we allow an initial phase φ. The equations are

$$\frac{du_x}{d\tau} = s\left[\cos(\tau + \varphi) - u_z \sin(\tau + \varphi)\right], \qquad (6a)$$

and

$$\frac{du_z}{d\tau} = su_x \sin(\tau + \varphi). \qquad (6b)$$

Note that φ, the initial phase, is different from ϕ, the phase difference between the electric and the magnetic fields.

We combine the equations to

$$\frac{df}{d\tau} = is\sin(\tau+\varphi)f + s\cos(\tau+\varphi), \qquad (7a)$$

where

$$f \equiv u_x + iu_z. \qquad (7b)$$

The solution of the homogeneous equation is

$$f_H = \exp\left[-is\cos(\tau+\varphi)\right]. \qquad (8)$$

The solution of the inhomogeneous equation for $f(\tau=0)=0$ is

$$f = s\exp\left[-is\cos(\tau+\varphi)\right]\int_0^\tau \exp\left[is\cos(\tau'+\varphi)\right]\cos(\tau'+\varphi)\,d\tau'. \qquad (9)$$

Expanding the exponents, we write the solution as

$$f = s\sum_{m=-\infty}^{\infty}\left(\frac{1}{i^m}J_m(s)\exp\left[im(\tau+\varphi)\right]\left\{\sum_{\substack{n=-\infty\\ n\neq 0}}^{\infty}\frac{J_n'(s)i^n}{n}\left\{\exp(in\varphi)-\exp\left[in(\tau+\varphi)\right]\right\} - iJ_0'(s)\tau\right\}\right). \qquad (10)$$

Here, J_m are Bessel functions. This expression includes a secular, non-oscillating, term:

$$u_{z,\mathrm{secular}} = sJ_0(s)J_1(s)\tau. \qquad (11)$$

In dimensional units, the velocity is

$$v_{z,\mathrm{secular}} = J_0(s)J_1(s)\frac{qE}{m}t. \qquad (12)$$

III. Numerical examples

We present a numerical calculation of the velocities for $s=0.5$ when the phase difference between the electric and the magnetic field was $\pi/2$ (section IIB). Figure 1 shows the two components of the velocity as a function of the normalized time. The velocities were obtained by a numerical integration of the equations of motion. Also plotted is the secular velocity, Eq. (11), and it is seen that the velocity is dominated by this term (except for oscillations around it). Figure 2 shows the normalized trajectories:

$$x_n = \int_0^\tau u_x d\tau', \quad z_n = \int_0^\tau u_z d\tau' \qquad (13)$$

Which are found through a numerical integration. Also shown is the coordinate in the z direction through the analytic expression:

$$z_{anal} = \mathbf{0.5} s J_0(s) J_1(s) \tau^2. \qquad \textbf{(11)}$$

It is clear from Figure 2 that z_{anal} and z_n are almost identical. Because of the oscillatory nature of the motion in the x direction, the particle practically only moves in the z direction.

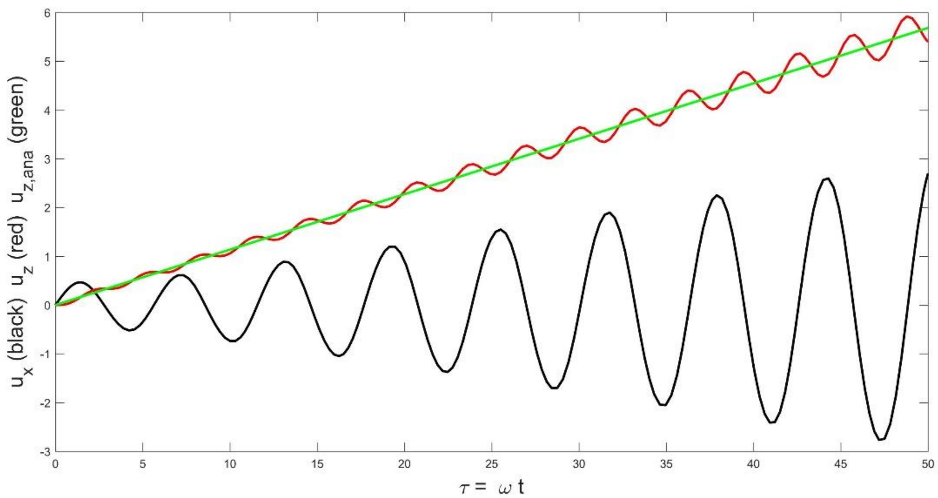

Figure 1. The normalized velocities as a function of normalized time. ($s = 0.5$)

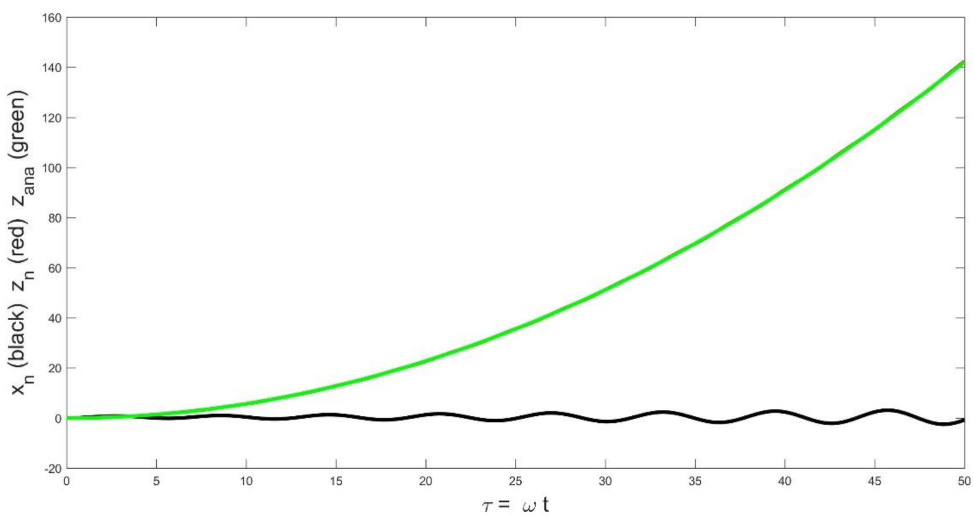

Figure 2. The normalized coordinates as a function of normalized time. ($s = 0.5$)

IV. Conclusion

We have shown that charged particles subject to crossed electric and magnetic fields with a phase difference between them are accelerated. This mechanism can be the basis for a thruster. The parameters should be adjusted so that ions are accelerated. The electron dynamics should be examined for this case to verify that they do not inhibit the plasma flow. In addition, ways to create such fields inside the plasma should be devised. The acceleration has interesting dependence on the charge and on the mass.

Acknowledgments

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