# 2D (axial-azimuthal) Particle-In-Cell benchmark for ExB discharges

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## ExB discharges

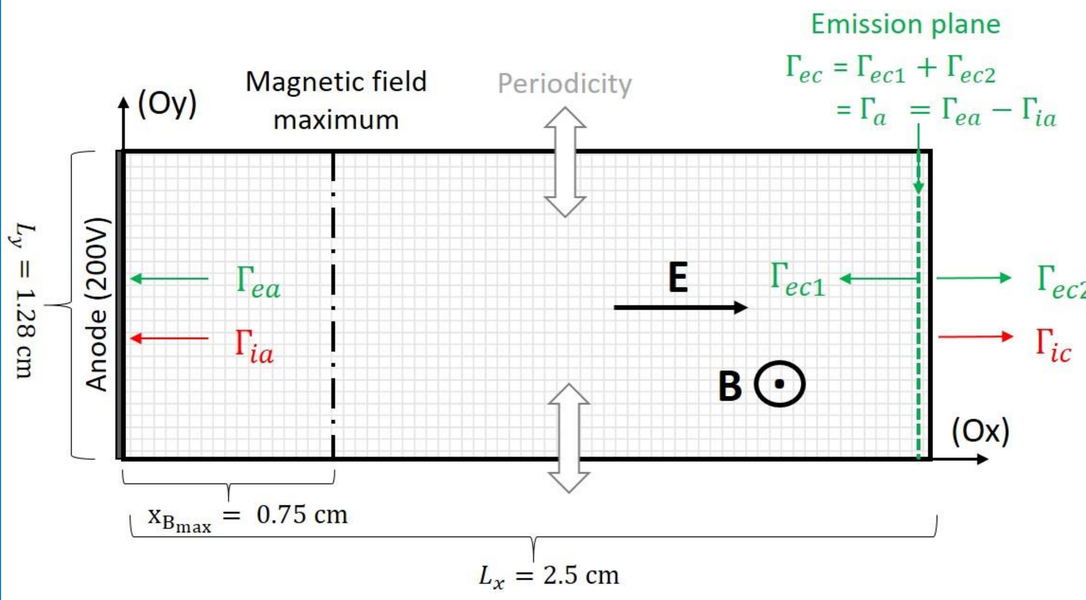
- **Low gas pressure** (typically between 0.1 and 10 mTorr)
- Plasma confinement by a static magnetic field ➔ **Electrons are magnetized – Ions are not magnetized**
- **Closed drift in azimuthal ExB direction ➔ Instabilities** in ExB direction due to large electron drift (Electron Cyclotron Drift Instability)
- In many systems, electron current across magnetic field is orders of magnitude higher than predicted by classical (collisional) theory ➔ **anomalous transport**
- Kinetic effects important ➔ **Need for Particle-In-Cell (PIC) simulations**
- Complex problems ➔ Increasing need for **Verification (benchmark) & Validation (experiments)**

**➔ Need new ways to validate our codes : LANDMARK project [2]**

## 7 independently developed codes

| Codes : | LPP | LAPLACE | CERFACS | RUB | Usask | TAMU | PPPL |
|---|---|---|---|---|---|---|---|
| **Algorithms** | | | | | | | |
| **Pusher solver** | Explicit | Explicit | Explicit | Implicit | Explicit | Explicit | Explicit |
| **Poisson solver** | HYPRE | Pardiso | Maphys | FFT / Thomas | FFT | HYPRE | HYPRE |
| **Floating-point precision** | Double | Single (pusher) – Double (Poisson) | Double | Single (pusher) – Double (Poisson) | Double | Double | Double |
| **Code acceleration** | | | | | | | |
| **Architecture** | CPU | CPU | CPU | GPU | CPU | CPU | CPU |
| **Parallelization** | MPI | MPI/OpenMP | MPI | CUDA | MPI | MPI | MPI/OpenMP |
| **Decomposition** | Domain | Particle | Domain | Both | Domain | Particle | Particle |
| **Language** | Fortran | Fortran | Fortran | C+Cuda C | Fortran | C++ | C |
| **Simulation times (days)** | | | | | | | |
| **Case 1 ($N_{ppc,ini}$ = 150)** | 8 (360 CPU) | 5 (108 CPU) | 7 (360 CPU) | 14 (1 GPU) | 21 (32 CPU) | 15 (300 CPU) | 2.5 (224 CPU) |
| **Case 2 ($N_{ppc,ini}$ = 75)** | 5 (360 CPU) | 3 (108 CPU) | 4 (360 CPU) | 9 (1 GPU) | 11 (32 CPU) | 11 (200 CPU) | 2.5 (112 CPU) |
| **Case 2 ($N_{ppc,ini}$ = 300)** | 14 (360 CPU) | 6 (180 CPU) | 13 (360 CPU) | 14 (2 GPU) | 20 (64 CPU) | 22 (400 CPU) | 2.5 (448 CPU) |

## Test case description



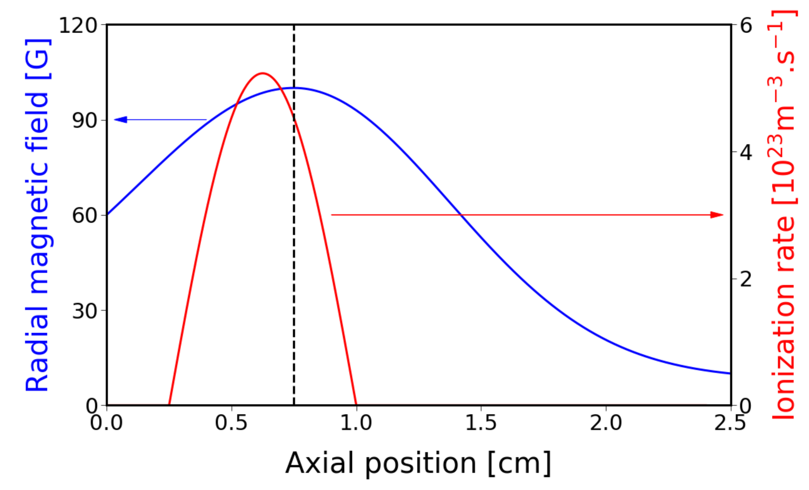

*Simulation domain. x is the axial direction, y the (periodic) azimuthal direction.*

Typical numerical parameters :

- Mesh : 500 x 256
- $\Delta x = \Delta y = 5.10^{-5}$ m
- $\Delta t = 5.10^{-12}$ s
- $T_{final} = 20\ \mu s$
- $T_{e,ini} = 10\ eV$
- $T_{i,ini} = 0{,}5\ eV$
- $U_d = 200V$

Simplifying assumptions :

- No collision taken into account
- Ionization from an imposed cosine profile
- Fixed radial magnetic field
- Cathode emission according to current equality




*Axial profiles of the imposed radial magnetic field and ionization rate. Dashed line corresponds to the position of maximum magnetic field.*

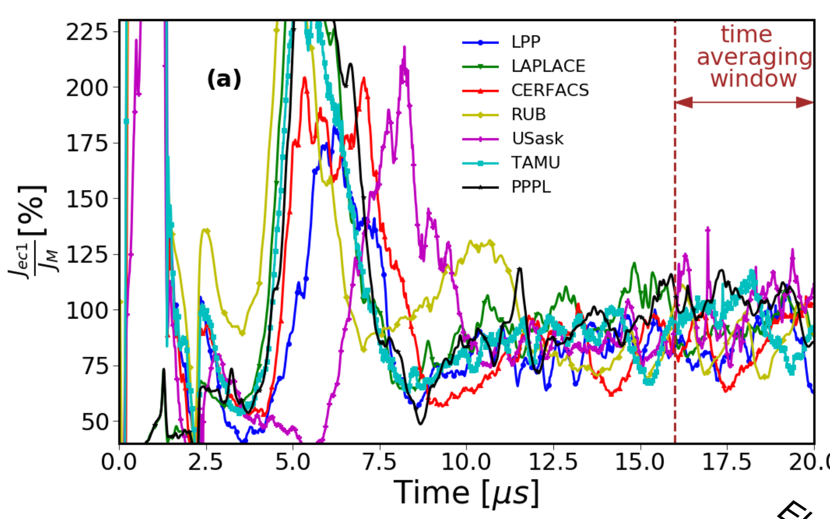
| Case | $N_{ppc,ini}$ at initialization | $N_{ppc,fin}$ at steady state |
|---|---|---|
| 1 (nominal) | 150 | 550 |
| 2 | 75 | 275 |
| 3 | 300 | 1100 |

*3 benchmark cases*

3 simulation cases to check numerical convergence according to the number of macroparticles per cell

## Averaged profiles

- We used the ratio between the electron current entering the discharge $J_{ec1}$ and the total ion current $J_M$ to quantify electron anomalous transport.
- A steady state is reached after 10 μs.

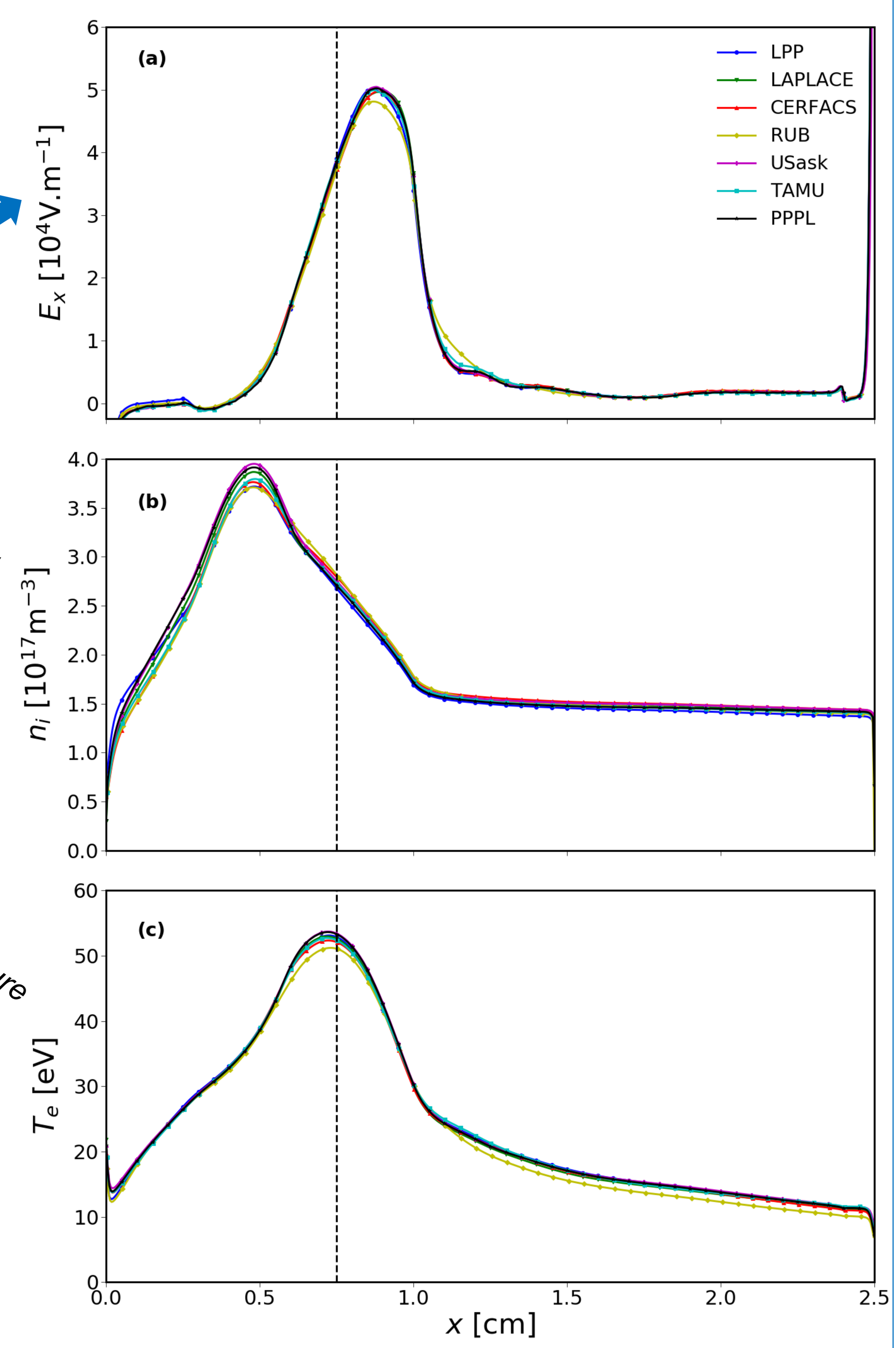



*Time evolution of $\frac{J_{ec1}}{J_M}$ for case 1.*

Axial electric field

Ion density

Electron temperature




*Azimuthally and time averaged profiles*

- We averaged azimuthally and between 16 and 20 μs to compare 3 parameters : axial electric field, ion density and electron temperature.
- Similar results have been obtained for the two other cases with $N_{ppc,ini}$ = 75 and $N_{ppc,ini}$ = 300 (see [1]).

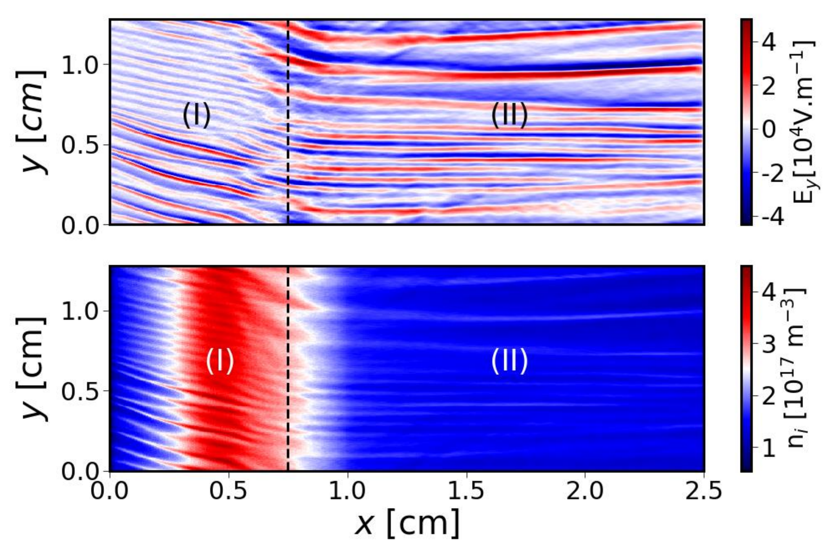
**➔ The 7 codes displays good agreement on averaged profiles.**

## Azimuthal instabilities

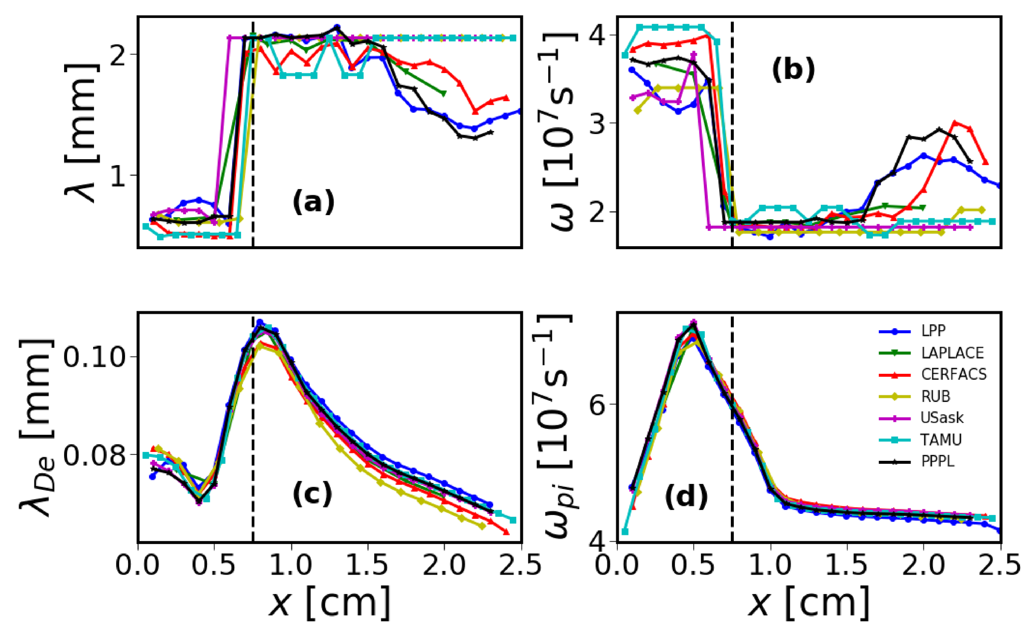
- High-frequency and short-wavelength instabilities are propagating in the azimuthal direction.
- Important influence on the electron axial anomalous transport.

- Identification of the dominant mode at each axial position and comparison of its characteristics between the different codes.
- Similar results have been obtained for the two other cases with $N_{ppc,ini}$ = 75 and $N_{ppc,ini}$ = 300 (see [1])

**➔ The 7 codes displays satisfactory agreement on azimuthal instabilities.**




*2D axial-azimuthal maps of the azimuthal electric field (top) and ion density (bottom) obtained with code LPP at t = 20 μs.*




*Axial evolution of dominant mode characteristics for azimuthal electric field (azimuthal instabilities). (a) Wavelength. (b) Frequency. (c) Debye length. (d) Ion plasma frequency. Dashed line corresponds to the position of maximum magnetic field.*

## Numerical convergence

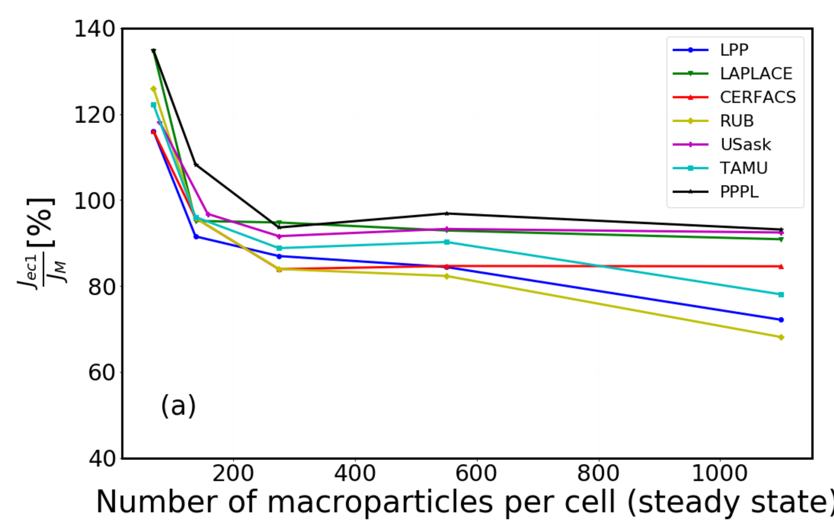
In a PIC simulation, we consider finite-sized particles and hence, numerical collisions can lead to fluctuations induced by thermal noise and have an impact on the study of the azimuthal instabilities and the related anomalous electron transport.

- Okuda and Birdsall [2] defined a frequency for these numerical collisions in 2D simulations:

$$\nu_{num} \sim \frac{\pi \omega_{pe}}{16\, N_D}$$

  with $N_D$ the number of macroparticles in a Debye sphere.
- In our nominal case, we have $\nu_{num} \sim 5.6 \times 10^{-5} \omega_{pe}$, and hence numerical collisions can be neglected according to the criteria of Turner [3] : $\frac{\nu_{num}}{\omega_{pe}} \leq 10^{-4}$.
- We assessed numerical convergence by varying the number of macroparticles per cell.

**➔ The chosen benchmark case has reached convergence.**




Mean value (between 12 and 20 μs) of $\frac{J_{ec1}}{J_M}$ depending on $N_{ppc,fin}$. The 3 benchmark cases correspond to $N_{ppc,fin}$ = 550 (Case 1), $N_{ppc,fin}$ = 275 (Case 2) and $N_{ppc,fin}$ = 1100 (Case 3)

## Conclusion & Future work

We managed here to:

- ✓ Define a simplified test case representative of an ExB discharge.
- ✓ Obtain a good agreement between 7 independently developed PIC codes on averaged profiles and azimuthal instabilities.
- ✓ Challenge numerical parameters and assess numerical convergence.

More work could be done in including more complex effects (collisions) or looking at the influence of specific parameters.

**➔ If you want to perform this benchmark, please read [1] (input parameters and data in supplementary materials) and don't hesitate to send us an email!**

## References

[1] T. Charoy, J. Boeuf, A. Bourdon, J. Carlsson, P. Chabert, B. Cuenot, D. Eremin, L. Garrigues, K. Hara, I. Kaganovich, T. Powis, A. Smolyakov, D. Sydorenko, A. Tavant, O. Vermorel and W. Villafana, submitted to Plasma Sources Sci. Technol. (2019).

[2] https://www.landmark-plasma.com

[3] Okuda H and Birdsall C 1970 Phys. Fluids 13 2123.

[4] Turner M 2006 Phys. Plasmas 13 033506.