Development of 1D2V Magneto-static Two-Fluid Plasma Simulation of a Hall Effect Thruster

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Rupali Sahu* and C. A. Gonzalez†
Texas A&M University, College Station, Texas, 77840, USA

and

Kentaro Hara‡
Stanford University, Stanford, California, 94305, USA

A 1D-2V two fluid code is developed in an attempt to study the anomalous electron transport observed in Hall Effect Thrusters (HETs). Conservation of mass and momentum are solved for ions and electrons along with the Poisson equation to solve for the electric field. Global Lax Flux Vector Splitting is used to evaluate fluxes based on Monotonic Upwind Scheme for Conservation Laws (MUSCL) reconstruction of primitive flow variables at the cell interfaces. A third-order Strong-Stability Preserving Runge-Kutta (RK3-SSP) scheme is used for time-stepping. Moments of Maxwellian distribution are taken to derive the kinetic fluxes at the boundaries. This way, the desired fluxes at the boundaries are directly imposed instead of calculating the flow variables at the ghost cells. To verify the code, an initially quasi-neutral is simulated between two floating walls. Sheaths are developed near the walls as predicted by theory and the sheath width and sheath potential are found to be in good agreement with available literature. Hall thruster simulations are performed including the axial-azimuthal momenta, where gradients along azimuthal direction are neglected. The stabilization of breathing mode oscillations and a steady state discharge.

Nomenclature

n = particle number density
ξ = particle total energy
e = elementary charge
u_B = Bohm velocity
Γ = number density flux
Δt = time step
dx = grid size
ϕ = electrostatic potential
SE = sheath edge

*Research Assistant, Aerospace Engineering, rupalisahu@tamu.edu.
†Postdoc, Aerospace Engineering, caangonzalez@tamu.edu.
‡Assistant Professor, Aeronautics and Astronautics, kenhara@stanford.edu

I. Introduction

Hall effect Thrusters (HETs) are electric propulsion devices with axial electric and radial magnetic fields. Electrons are injected at the cathode, and are trapped in the azimuthal (E×B) direction. These trapped electrons ionize the xenon gas, and accelerate the resulting xenon ions to gain thrust. However, the confinement of electrons in the azimuthal direction is significantly less than that predicted by classical theories, i.e., the electron mobility varies as $1/B$ instead of $1/B^2$, where B is the magnetic field strength.[1] This directly and adversely affects the thruster performance. Various studies have been carried out to investigate this anomalous electron transport over decades by using fluid or hybrid models[2–4] and fully kinetic simulations.[5,6] Electron interactions with the walls of the thruster[7] and azimuthal instabilities (in particular, electron cyclotron drift instability)[8,9] are believed to be the major contributors to the enhanced cross-field transport. Recently observed azimuthally rotating spokes have led to a hypothesis that the resulting periodic azimuthal electric field also contributes to the electron transport.[10,11] However, these mechanisms are not properly understood, nor is their contribution to anomalous electron transport been quantified in an anode-to-cathode configuration. In simulations, electron collisions frequency is usually manipulated to get results in better agreement with experiments. The collision model used for this study is described in section III-B of this paper.

To simplify the calculations, it is common to use a drift-diffusion model for electrons which assumes negligible electron inertia and/or a quasi-neutral plasma. By solving Navier-Stokes equations for both the ion and electron fluids on a 1D-2V domain, the aforementioned assumptions are not imposed. Hence, it is expected that this model might give a better insight of electron dynamics in Hall thruster discharge plasma.

For code verification, an initially quasi-neutral plasma is modelled between two perfectly absorbing walls. As the model evolves, more electrons, due to their lower mass, diffuse out to the walls than the ions. This makes the wall negatively charged, and starts repelling electrons in its vicinity. This plasma sheath test case was selected due to the relatively easy implementation and the abundance of existing theoretical and numerical studies on plasma sheaths. The computational set-up and results are described in detail in section II-B and III-A of this paper, respectively. This is followed by details of Hall thruster modelling and discussion of results in section III-B. Final conclusions and future work are discussed in the final section.

II. Computational model

A. Governing Equations

Ions and electrons are modelled as fluids in thermal non-equilibrium, i.e., ions and electrons have different temperatures. For a 1D-2V model, all spatial variations along the y direction are neglected while the y momentum (e.g. v) is retained as a function of x. The magnetic field in the radial direction (z) is considered to be varying only along the x direction. Conservation of number density, momentum, and energy are solved along with Poisson equation to evaluate electric fields. Ideal gas law is assumed to close the system of equations. The governing equations are given as:

$$\frac{\partial}{\partial t} n_s + \frac{\partial}{\partial x}(n_s u_s) = \dot{n}, \tag{1}$$

$$\frac{\partial}{\partial t}(n_s u_s) + \frac{\partial}{\partial x}\left(n_s u_s^2 + \frac{P_s}{m_s}\right) = \frac{q_s}{m_s} n_s (E_x - v_s B_z) - n_s u_s \nu_{m,s}, \tag{2}$$

$$\frac{\partial}{\partial t}(n_s v_s) + \frac{\partial}{\partial x}(n_s u_s v_s) = \frac{q_s}{m_s} n_s (E_y + u_s B_z) - n_s v_s \nu_{m,s}, \tag{3}$$

$$\frac{\partial}{\partial t}(n_s \xi_s) + \frac{\partial}{\partial x}\left(u_s\left(n_s \xi_s + \frac{P_s}{m_s}\right)\right) = \frac{q_s}{m_s} n_s (u_s E_x + v_s E_y) + \dot{Q}_s, \tag{4}$$

$$\frac{\partial^2}{\partial x^2}\phi = -\frac{e(n_{ion} - n_{electron})}{\epsilon_0}, \tag{5}$$

$$P_s = n_s k_B T_s, \tag{6}$$

where s denotes the fluid species (ions or electrons), E_x and E_y are the electric fields along x and y axes

$$E_x = -\frac{\partial}{\partial x}\phi,$$

and ξ is the total energy

$$\xi = \frac{P}{mn(\gamma - 1)} + \frac{1}{2}(u^2 + v^2).$$

The primitive variables (n, u, v, P) are defined at cell centers. MUSCL[12] reconstruction is used to calculate the variables at cell interfaces to obtain a scheme that is stable and second order accurate in space domain. A Strong Stability Preserving Runge-Kutta (RK-SSP 3)[13] scheme is used to gain third order accuracy in time domain. Global Lax scheme[14] is used to evaluate fluxes at the cell interfaces due to its reported stability at "sonic points", i.e. when the fluid bulk velocity becomes zero or equal to its thermal velocity.

B. Boundary Conditions

To model the plasma between two walls, perfectly absorbing wall boundary conditions (BCs) need to be imposed at both ends. A perfectly absorbing wall is analogous to vacuum right outside the domain. Hence, number density and pressure in the ghost cells can be set to 0. Zeroth order Neumann boundary conditions are used for other fluid quantities . The ghost cell BCs are hence given as:

$$n_{s,ghost} = 0, \tag{7}$$

$$\frac{\partial u_{s,wall}}{\partial x} = 0, \tag{8}$$

$$\frac{\partial v_{s,wall}}{\partial x} = 0, \tag{9}$$

$$\frac{\partial T_{s,wall}}{\partial x} = 0. \tag{10}$$

In the case of Hall thrusters, the anode acts as a perfectly absorbing wall, hence BCs mentioned in equations (7)-(10) can be used. Electrons are injected at the cathode, while the ions exit supersonically. Hence, "outflow" BCs are used for the ions at the cathode. Outflow BCs are taken as zeroth order Neumann BCs. The injected electrons assume a Maxwellian distribution that has a bulk velocity of zero velocity and a temperature of 3 eV. Electron number density is specified such that quasi-neutrality is achieved in the last cell in the next time-step. This "quasi-neutral" BC is derived by taking the difference of continuity equations of ions and electrons at the last cell. This gives a condition on the electron number density flux Γ_e as given in equation (11).

$$\Gamma^k_{e,N+\frac{1}{2}} = (n^k_{e,N} - n^k_{i,N})\frac{\Delta x}{\Delta t} + \Gamma^k_{e,N-\frac{1}{2}} + \Gamma^k_{i,N+\frac{1}{2}} - \Gamma^k_{i,N-\frac{1}{2}}, \tag{11}$$

where superscript k denotes the current time step, subscript N is last cell center near the cathode and $N + \frac{1}{2}$ is the cathode interface.

III. Results and Discussion

A. Two wall plasma problem

An initially stationary, quasi-neutral plasma of number density 1×10^{16} m^{-3}, electron temperature of 10 eV and Hydrogen ion temperature of 1 eV is modeled between two floating walls. Domain is divided into 1000 cells. The distance between the walls is 256 Debye lengths (based on initial data, approximately 0.06 m). Wall potential is taken as 0 V. Particles diffusing out to the walls are re-injected back into the domain in the form of ionization source. The ionization source term in the mass and energy conservation equations are taken proportional to the local electron density. The steady state results are plotted in figure A.

The sheath edge is defined as the point where ion bulk velocity is equal to the local ion acoustic speed, u_B. Electron to ion number density ratio at this sheath edge is 0.96. The sheath width is 5.28 Debye lengths, which is in agreement with theoretical deductions that sheath width is of the order of a few Debye lengths. The sheath potential, calculated as the potential drop between the sheath edge and the wall, is found out to be 13.46 V. At the sheath edge, the electron temperature is 6.896 eV. For a floating wall, sheath theory predicts a sheath potential of $\phi_{SE} = T_{eV} \log(\frac{m_{ion}}{2\pi m_{electron}})$ under an isothermal assumption, which turns out to be 16.98 V for this case. The difference in the numerical and theoretical values of sheath potential can be attributed to the isothermal assumption in theories, which is clearly violated in the simulation (see Fig. A(c)).

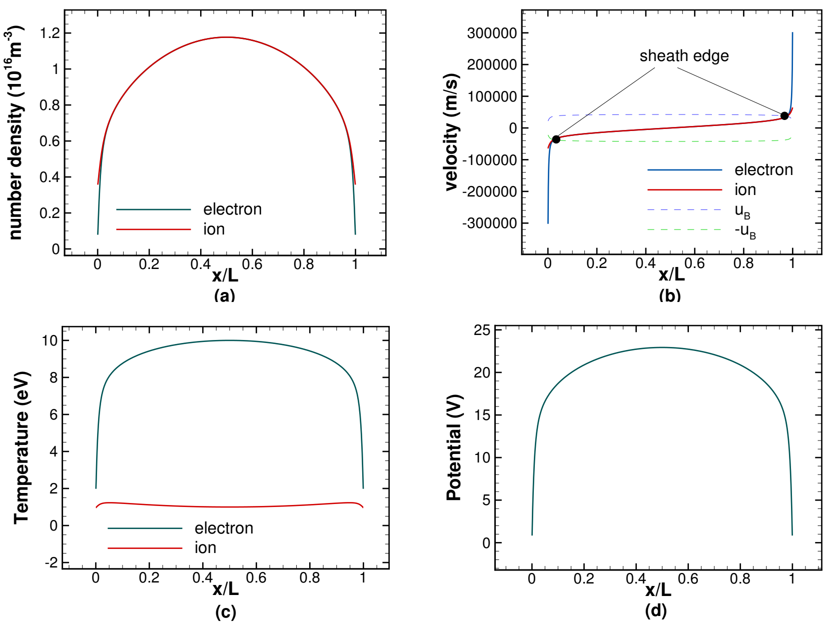

Figure 1. Plasma between two walls. Steady-state results plotted for distribution of (a) number density (b) bulk velocity (c) temperature of both fluids and (d) electrostatic potential.

B. Hall thruster discharge plasma

An axial simulation of SPT-100 Hall thruster[4] is attempted using the 1D-2V model, where it is assumed that flow variables do not change along the θ, or the azimuthal direction. Simulation domain is 0.05m, with anode at $x = 0$ m and cathode at $x = 0.05$ m. The thruster exit is located at $x = 0.025$ m. The applied potential difference is 300V. Neutral xenon gas is injected at the anode at a mass flow rate of 5 mg/s. Electron temperature is kept constant in time. Profiles for electron temperature and magnetic field is taken from steady state results in Ref. 4. Ion and neutrals have a constant temperature of 0.1 eV. The electron collision model is also taken from the same reference, where total electron momentum transfer collision frequency ν_m is given by equation 12.[15]

$$\nu_m = \nu_{en} + \nu_{wall} + \nu_{ano}, \tag{12}$$

where ν_{en} is the electron-neutral collision frequency taken as

$$\nu_{en} = 2.5 \times 10^{-13} n_{neutral},$$

ν_{wall} is wall collision frequency, and ν_{ano} is the anomalous electron collision frequency. The profile for ν_{ano} is taken from Ref. 15. Ions are assumed to be collisionless and non-magnetized. It is also assumed that ions hitting the anode neutralize with the electrons and are pumped back in the domain along with neutral flux. The simulation is run for 500μs. Since a steady state is not achieved during the course of simulation, averaged flow parameters are plotted in figure 2. The discharge current trace is shown in figure 3.

It can be seen that quasi-neutral BCs are effective in providing smooth profiles for electron number density, axial velocity and the electric field at the cathode. More commonly used "net current conservation" BC usually develops a sheath like structure near the cathode.[1] This is undesirable since cathode is designed to act as an injection plane for electrons, not a wall close to which sheaths usually form. However, a sheath forms near the anode, as expected since anode is, in fact, modelled as a wall. The sheath potential is 8.25 eV, which is less than the sheath potential that corresponds to a dielectric wall: 15.82 eV assuming 3 eV electrons and xenon ions at the anode. This allows the electron current to flow through the anode. Comparing the results with Ref. 4, it can be seen how resolving the sheath leads to a higher electric field and electron axial velocity at the anode. It is also seen that Global Lax scheme used for calculating fluxes successfully

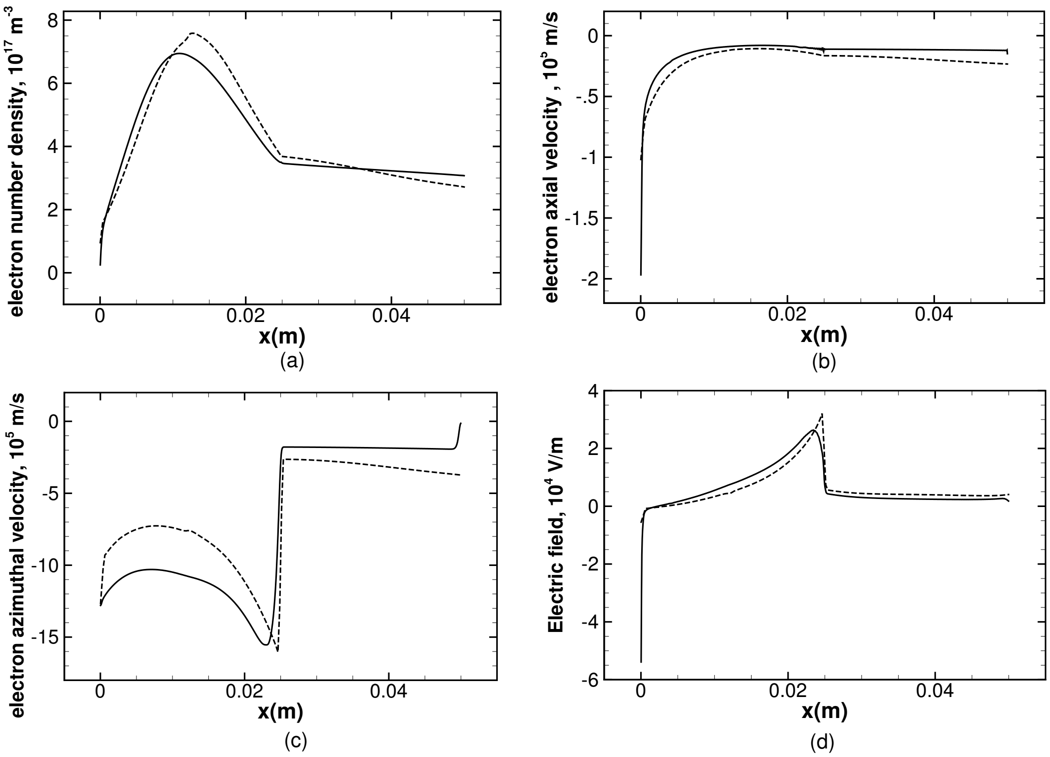

Figure 2. Time averaged profiles for (a) electron number density (b) electron axial velocity (c) electron azimuthal velocity and (d) electric field in the domain. Solid lines represent results from the two fluid code, dashed lines are results from Ref. 4 which uses a quasi-neutral, drift-diffusion model for electrons.

eliminates "sonic point oscillations" usually associated with Steger-Warming scheme.[4,16] In Ref.. 4, these oscillations are removed by developing an "electron pressure coupled method" which can be used only for quasi-neutral plasma discharges.

Breathing mode oscillations are observed due to the initial difference between the quasineutral drift-diffusion result and the two-fluid model. A modified predator-prey model in Ref. 17 derives an expression for the decay rate of these oscillations, for a constant electron temperature, as

$$\gamma = -\frac{1}{2}\frac{n_{neutral,injected}}{n_{neutral,injected} - n_{neutral,exit}} n_i \zeta_{ionization}, \tag{13}$$

where $\zeta_{ionization}$ is the ionization rate coefficient, which is a function of electron temperature. Simulations show a decay rate of ~ 7800 s^{-1}, while the analytical value of the decay rate for this set-up turns out to be 5780 s^{-1}. The oscillation frequency is 22.8 kHz, which is very close to the value predicted by the Predator-Prey model in Ref. 18 for a potential difference of 300 V and maximum magnetic field of 18 mT, which is 20 kHz. The oscillation frequency is predicted to be higher for a lower value of maximum magnetic field. The magnetic field for this study has a maximum value of 15 mT, hence an oscillation frequency greater than 20 kHz is expected. The time-averaged discharge current after $T = 500\mu s$ is 6.23 A, while the steady state current in the referred paper[4] is 7.3 A. The sensitivity of the two fluid model to the wall collision frequency model used, and the effect of dynamic electron temperature are a subject of further investigation.

IV. Conclusion

Details of magneto-static two fluid code developed to study plasma dynamics in complex devices such as Hall effect thrusters are provided. The code resolves a 1D2V domain, and can accurately capture magnetic and collisional effects. Simulation of plasma between two absorbing walls gives results in good agreement with theory. Numerical results show that isothermal assumption used in sheath problems is inaccurate.

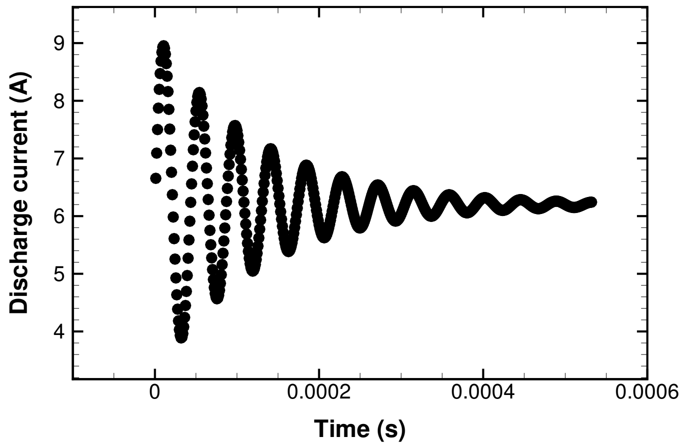

Figure 3. Discharge current in the Hall thruster showing breathing mode oscillations

Hall thruster simulations give profiles for flow variables in compliance with similar studies. Breathing mode oscillations are observed with an oscillation frequency and decay rate in compliance with existing Predator-Prey models. Efforts are being made to incorporate dynamically varying ion and electron temperatures in Hall thruster simulations, which is expected to help gain a better insight into the electron dynamics of the discharge.

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