Numerical Simulation and Theoretical Analysis of Particle Acceleration in Traveling Magnetic Field Thruster

IEPC-2019-795

Presented at the 36th International Electric Propulsion Conference
University of Vienna, Austria
September 15-20, 2019

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The physics behind the thrust mechanism of a traveling magnetic field thruster has been studied extensively both theoretically and numerically. The main thrust mechanism was attributed to the formation of a Double Layer (DL). A novel method of incorporating the DL into the thruster's theoretical framework is proposed. A linear theory was thus established, by considering the DL as a wave disturbance in plasma, which then predicts a growth rate associated with the formation of a DL. The theoretical growth rate prediction was then validated numerically by 1D electrostatic particle-in-cell simulations and was found to be comparable to the theoretical growth rate. Since the DL was treated as a wave disturbance, a multi-resolution Dynamic Mode Decomposition (mrDMD) analysis was performed on the averaged particle velocity data. The mrDMD analysis reveals two growing wave modes with a growth rate which was also comparable to the theoretical growth rate and the numerical growth rate.

Nomenclature

B	= magnetic field
E	= electric field
γ	= growth rate
k	= wavenumber
m	= mass of an electron
M	= mass of an ion
μ	= adiabatic invariant
n	= particle density
ω	= wave frequency
ω_p	= plasma frequency
ϕ	= electric potential
q	= charge of an electron
T_e	= electron temperature
v_x	= particle velocity in the x–direction
v_{th}	= thermal velocity of the particle

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I. Introduction

ELECTRIC thrusters have been increasingly keenly researched in the past few decades for their potentially high efficiency. However, some type of electric thrusters, such as the ion thrusters, have acceleration grids for the charged particle acceleration, and these grids are subjected to grid erosion[1] due to the continuous interaction of the grid with the plasma. This challenge could be overcome by adopting electrodeless thruster concepts. In an electrodeless thruster the charged particles are accelerated by imparting kinetic energy to the plasma by methods such as inductive wave launching antennas[2], rotating electric fields[3], beating electrostatic fields[4] and so on. The momentum transfer by these methods will also result in Joule heating losses and various other losses such as wall losses and conduction losses. These losses, which are usually minimum initially, grow over time and can significantly influence the performance of the thruster. Research is being carried out to minimise the losses and improve the gain of kinetic energy by the plasma.

In this paper, we explore a type of electrodless thruster concept called as the traveling magnetic field thruster for effective thrust generation. The working principle is similar to that of a electrohydrodynamic pump where a conducting fluid is accelerated by the Lorentz force arising due to the interaction of the applied magnetic field and the self-generated electric fields within the fluid. The TMFT is a multistage thruster where the axially traveling magnetic pulse is utilised in the acceleration stage to accelerate the charged particles. Previous studies[5−8] have shown promising acceleration and thrust in propagating magnetic field configurations. These experiments show that the magnetic pulse or the magnetic piston is effective in accelerating the charged particles, and the background static magnetic field reduces the plasma-wall losses.

It can be seen from Ref. 9 that the acceleration of the charged particles is by means of a formation of a Double Layer (DL). The DL creates strong localized electric fields which are sufficient to accelerate both the ions and the electrons, and an ion beam was observed. The magnetic pulse, initially, pushes the electrons to the pulse front since the electrons are much mobile than the ions. The ions remain behind the pulse, leading to a charge separation event. The plasma then tries to bridge the separation by forming a DL. The phenomenon behind the plasma acceleration has been discussed briefly before but it is necessary to establish a theoretical model which accounts for the DL and a numerical simulation to validate the theory for a complete thruster framework.

This paper is separated into three major parts. The first section provides the overview of the thruster and uses a novel linear approach to incorporate the DL into the thruster framework. The linear theory provides a growth rate for the DL. The second section provides the numerical modeling setup of the simulation. The final section discusses about the growth rates obtained from theory, simulation and multi-resolution Dynamic Mode Decomposition (mrDMD). The influence of the DL growth rate on the thruster performance is also briefly mentioned.

II. Thruster concept

The TMFT consists of two stages. The first stage is the source stage where the propellant gas is ionized by means of a helicon discharge or a microwave discharge. The ionized gas is then injected into the second stage called as the acceleration chamber. The acceleration chamber has a cylindrical geometry and is enclosed by a system of coaxial coils producing progressively phase-shifted alternating current. Since the current in the coils are phase shifted progressively, the magnetic pulse appears to be travelling along the length of the thruster. The schematic of the TMFT is shown in Fig. 1.

It has been found that the main thrust generation mechanism of the thruster is the formation of a DL due to charge separation caused by the magnetic pulse. The DL accelerates the charged particles and forms an ion beam thus generating thrust.

A. Linear Theory

Let us consider the DL to be a wave disturbance propagating in plasma. The equilibrium quantities are denoted by subscript 0, the perturbed quantities by subscript 1, electrons by the subscript e, and ions by the subscript i. We assume that the perturbed quantities vary as $\exp[i(kx - \omega t)]$. The magnetic moment of the electrons is considered to be an adiabatic invariant, and the electrons are considered to be isothermal. We consider a magnetic pulse traveling through a plasma column and the dynamic response of electrons given by the linearized equation of motion as:

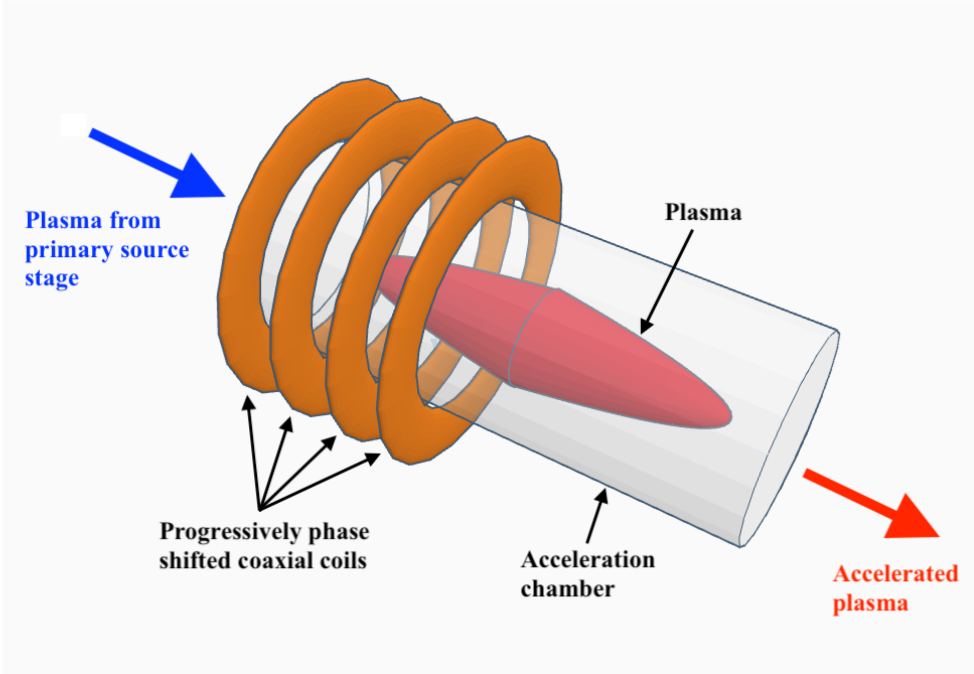

Figure 1: Schematic of the TMFT.

$$m\frac{\partial v_{1,e}}{\partial t} = -qE_1 - \mu\frac{\partial B}{\partial x} - \frac{\partial n_0 KT_e}{\partial x}. \tag{1}$$

Taking Fourier transform of (1) and using $E_1 = -\frac{\partial \phi_1}{\partial x}$,

$$-im\omega v_{1,e} = ikq\phi_1 - ik\mu B - ikn_0 KT_e,$$

$$v_{1,e} = \frac{k}{m\omega}[-q\phi_1 + \mu B + n_0 KT_e]. \tag{2}$$

From the equation of motion of ions,

$$M\frac{\partial v_{1,i}}{\partial t} = qE_1, \tag{3}$$

$$v_{1,i} = \frac{kq\phi_1}{M\omega}. \tag{4}$$

The continuity equation for electrons and ions is:

$$\frac{\partial n_{1,e,i}}{\partial t} + n_0\frac{\partial v_{1,e,i}}{\partial x} = 0,$$

Using (2) and continuity equation for electrons:

$$-i\omega n_{1,e} = -ikn_0 v_{1,e},$$

$$n_{1,e} = \frac{k^2 n_0}{m\omega^2}[-q\phi_1 + \mu B + n_0 KT_e]. \tag{5}$$

Using (4) and continuity equation for ions:

$$n_{1,i} = \frac{kn_0}{\omega}\left(\frac{kq\phi_1}{M\omega}\right). \tag{6}$$

From Poisson equation,

$$\epsilon_0(\frac{\partial E_1}{\partial x}) = -q(n_e - n_i),$$

$$-(ik)^2\epsilon_0\phi_1 = -\frac{q^2k^2n_0}{m\omega^2}\Bigg[-\bigg(1+\frac{m}{M}\phi_1\bigg)+\frac{\mu B}{q}+\frac{n_0KT_e}{q}\Bigg].$$

We obtain the dispersion relation as,

$$1 = \frac{\omega_p^2}{\omega^2}\Bigg[1-\frac{\mu B}{q\phi_1}-\frac{n_0KT_e}{q\phi_1}\Bigg]. \tag{7}$$

Using sheath approximation[10], $\phi_1 \approx \frac{KT_e}{2q}$, the dispersion relation becomes:

$$\omega = \omega_p\sqrt{1-2\bigg(\frac{\mu B+n_0KT_e}{KT_e}\bigg)}. \tag{8}$$

When $2(\frac{\mu B+n_0KT_e}{KT_e}) > 1$, there exists a complex solution to the dispersion relation and the growth rate is represented as,

$$\mathrm{Im}\bigg(\frac{\omega}{\omega_p}\bigg) = \sqrt{2\bigg(\frac{\mu B+n_0KT_e}{KT_e}\bigg)}. \tag{9}$$

Usually, $n_0KT_e << \mu B$ and hence,

$$\mathrm{Im}\bigg(\frac{\omega}{\omega_p}\bigg) = \sqrt{\frac{2\mu B}{KT_e}}. \tag{10}$$

By definition, $\mu = \frac{mv_x^2}{2B}$ and we can reduce (9) to:

$$\mathrm{Im}\bigg(\frac{\omega}{\omega_p}\bigg) = \frac{v_x}{v_{th}}. \tag{11}$$

where $v_{th} = \sqrt{KT_e/m}$, the thermal energy of the electrons. Equation (10) says that the instability should grow at a rate which is equal to the ratio of kinetic energy (v_x) to the thermal energy of the electrons (v_{th}) around the DL region. The electric field in the plasma should then theoretically grow at the rate predicted by Eq. (10).

B. Numerical Modelling

DLs are electrostatic structures, and hence their properties can be studied by simple electrostatic numerical simulations. Therefore, an 1D electrostatic particle-in-cell (PIC) simulation was carried out. The electron equation of motion was given by Eq. (1), but the pressure term was neglected due to its minimal influence on the growth rate as can be seen from the linear theory. The ion equation of motion was given by Eq. (3). Equations (1) and (3) were integrated in time by a leaping algorithm. The velocity was estimated at half time steps and the position of the particles were computed at integer time steps. The electric field and electric potential are related and were calculated from the charge density by using Poisson equation.

The electric field is obtained by solving Poisson equation at the grid node points by using a central difference scheme. The contribution of the forces and fields were interpolated to the grid node points by a fourth-order weighting scheme. The field boundary conditions were Dirichlet type boundary conditions. When an ion crosses the boundary a new electron-ion pair is introduced inside the domain as the particle boundary condition.

The simulation domain is 0.2 m in length and is sub-divided into 2,000 grid points. Grid convergence studies were done for higher number of grid points but the changes were insignificant, and hence 2,000 grid points was chosen as the optimal grid resolution. Initially, the electron and the ion pairs were loaded uniformly across the length of the simulation domain. The ion to electron mass ratio is set to be 1836. The initial ion and electron temperatures follow Maxwellian distributions. The electron temperature was set at

0.5 eV, and the ion temperature at 0.1 eV. There are 120,000 super-particles with 40,000 particles aggregated into a single super-particle thus giving an effective plasma density of 4.8×10^9 m^{-3}. Thus a sufficient number of super-particles was arranged in each cell.

The magnetic pulse used has a peak magnitude of 0.04 T and a pulse width of 0.006 m. The pulse travels at a constant velocity of 30 km/s along the length of the domain. The magnetic pulse enters the domain at 5 μs and the entire simulation is carried out for 15 μs.

III. Results

A. Numerical Simulation

Growth of Electric Field

As the pulse travels through the plasma, a DL is generated. The DL gives rise to strong localized electric fields which, in turn, leads to the formation of an ion beam similar to the ion beam observed in Ref. 9. Based on the theory, the electric field associated with the DL has to grow exponentially at a rate given by Eq. (10). In order to obtain the growth of electric field, a spatial location was selected (x = 0.023 m), and the magnitude of electric field was graphed with respect to time. By doing so, we obtain the plot shown in Fig. 2 which provides a growth rate of γ_E=1.007. This plot is only for the linear regime, and after 54.5 μs the growth of the DL becomes non-linear and hence deviates from the linear theory predictions. By the end of the non-linear phase, the DL starts dissipating energy to the surrounding plasma and vanishes. This process of formation, growth and dissipation occurs repeatedly at different spatial locations as the magnetic pulse interacts with the plasma at the corresponding locations.

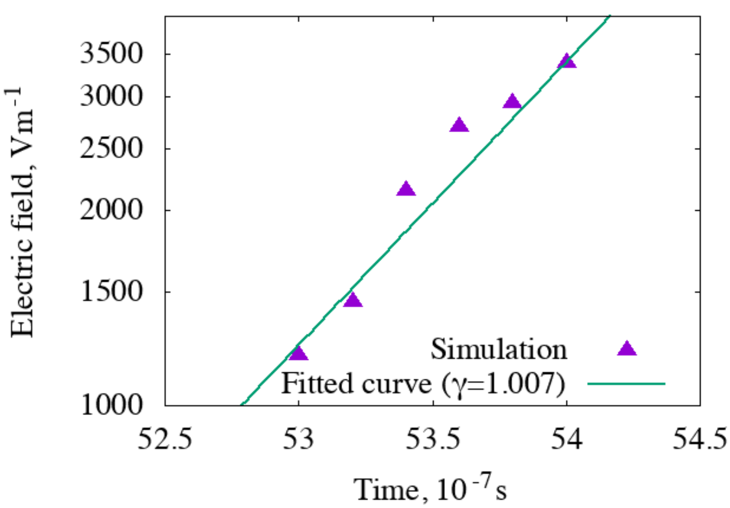

Figure 2: The growth rate of the electric field intensity.

Dynamic Mode Decomposition

Since the linear theory approximates the DL as a wave disturbance in the plasma, there should be a corresponding wavelength and frequency associated with it. Hence, in order to find the wave parameters and also to verify the presence of actually growing wave modes, a multi-resolution (mr)DMD analysis is carried out. mrDMD is a robust, purely data based algorithm which decomposes data into a set of dynamic modes that are derived from snapshots or measurements of a given system in time. The mrDMD has been shown[11] to handle transient phenomenon and moving (translating/rotating) structures in the data. mrDMD is useful in the current analysis since the phenomenon of interest is both time transient and translating in space. Hence, a data set was made comprising of the average particle velocity taken at constant time intervals. The particle velocity was averaged to the corresponding local grid points by the same fourth-order weighting used for the interpolation of the forces and fields. The mrDMD analysis was then done on the averaged particle velocities taken in the duration between the ion beam formation phase and the ion beam evolution phase (from 5 μs till 6.3 μs).

Figure 3 shows the eigenvalues or the growth rates of few of the different waves present in the plasma. As can be observed, there are two waves with eigenvalues $\lambda_{1,2}$=1.0014$\pm$0.0223j. The real part of the eigenvalues is the growth rate of the waves, and the imaginary part suggests that these two waves are travelling in opposite directions. The presence of two waves is also predicted by the theory and can be easily derived from the equation of motion of electrons:

$$m\frac{\partial v_x}{\partial t} = -qE - \mu\frac{\partial B}{\partial x}.$$

Taking the Fourier transformation and using $E = -\frac{\partial \phi}{\partial x}$ by expanding μ,

$$-i\omega m v_x = ikq\phi - ik\frac{mv_x^2}{2},$$

$$v_x^2 - \left(\frac{2\omega}{k}\right)v_x - \left(\frac{2q\phi}{m}\right) = 0. \quad (12)$$

Equation (12) shows a quadratic relationship for the velocity and indicates the existence of electrons with opposite velocities thus suggesting that the observed two eigenvalues in mrDMD are justifiable. The time evolution of the waves corresponding to the eigenvalues are shown in Fig. 4.

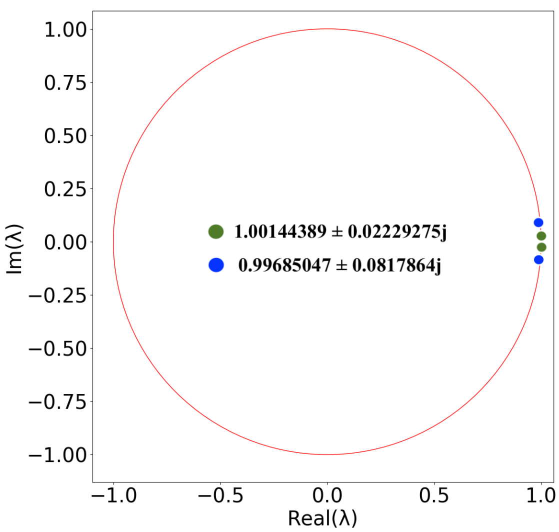

Figure 3: Eigenvalues showing two growing (green) and two decaying (blue) modes

B. Theory

The theoretical result obtained in Eq. (11), which says that the growth rate is the ratio of the axial velocity to the thermal velocity of the electrons, can now be used to obtain the growth rate for the simulated case. The local (around the DL) v_x and v_{th} values, however, has to be determined from the simulation since these two values are usually determined empirically. Hence we again go back to the spatial location where the electric field magnitude was monitored (at $x = 0.023$ m) and to the time when the electric field magnitude is at its peak (at 54 μs). Then the averaged values of v_x and v_{th} in that corresponding spatial location is obtained. The growth rate thus obtained is γ_T= 1.004.

The growth rates $(\gamma_E, \text{Re}(\lambda_{1,2}), \gamma_T)$ obtained from all the 3 methods above are comparable with each other and the theory is in good agreement with the simulation. One of the possible reasons for the relatively smaller value of Re$(\lambda_{1,2})$, as compared to γ_E and γ_T, might be due to the data set used for mrDMD which had the averaged velocities of the electrons and not the actual velocities of electrons themselves. There can also be differences between theory and simulations due to minor numerical round-off errors.

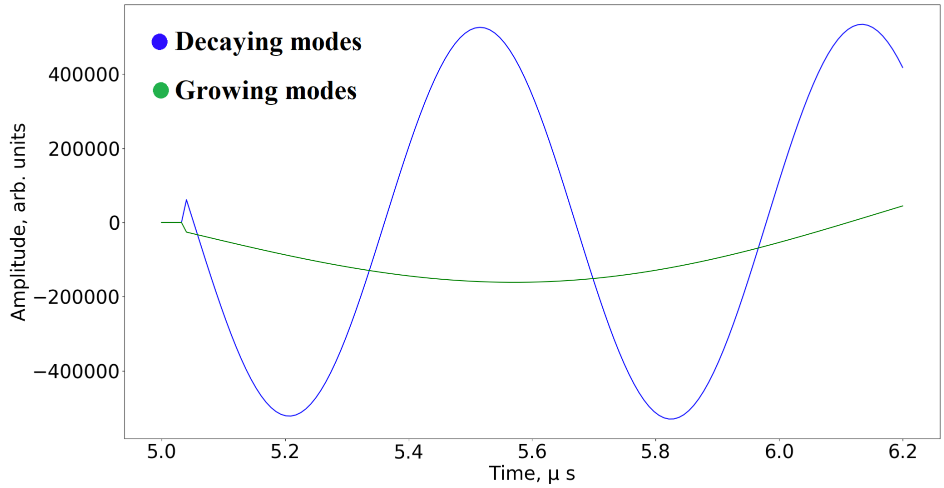

Figure 4: Time evolution of the growing and decaying modes.

IV. Conclusion

The PIC simulations and the mrDMD analysis confirm the presence of growing waves and thus indicate that a DL, formed by the interaction of the plasma and a traveling magnetic pulse, is a plasma instability and not just a conventional plasma sheath. There is a specific growth rate associated with the formation of a DL. The application of this insight extends well beyond the thruster context and into space plasmas. DLs are found almost everywhere in space plasmas from solar flares to auroras which are a result of interaction of the charged particles with a non-stationary magnetic field. But within the context of this thruster, work for an extensive theoretical model is underway which will assist in determining the wave number and the oscillation frequency of the wave disturbance theoretically. The current thruster configuration is limited to only one ion beam formation, but the extended theoretical model might provide valuable insights on the possible improvements that can be made on the thruster leading to plasma structures similar to multiple ion beam formations and hence increasing the thrust generation.

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