# AN ANALYTICAL STUDY OF THE THRUST IN MPD THRUSTERS

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## Nomenclature

*MPD* magnetoplasmadynamic

***Symbols***

- $A$ potential vector
- $\underline{\boldsymbol{B}}$ magnetic field
- $f$ volume force
- $F$ axial thrust
- $I$ current
- $\underline{j}$ current density
- $m$ mass
- $r$ radius
- $V$ voltage, volume where act electromagnetic forces
- $w$ velocity
- $z$ axial length
- $Z$ impedance
- $\mu$ magnetic permeability

***Subscripts***

- $a$ anode
- $b$ blowing
- $c$ cathode, compression
- $e$ exhaust
- $p$ pumping
- $r$ radial component
- $z$ axial component
- $\theta$ azimuthal component

## 1 INTRODUCTION

In the coaxial MPD thrusters the electromagnetic thrust (F), in the thruster axis direction, can be calculated by means of a well known theoretical expression [1], that some authors call "Maecker law",

$$F = \frac{\mu}{4\pi}\left[\ln\left(\frac{r_a}{r_c}\right) + C\right] i^2, \tag{1}$$

where $0.5 < C < 1$ is a constant depending on the cathode tip geometry and current distribution. It provides thrust values in good agreement with the experimental data. The formula derives from a two dimensional semi-empiric model and it is the result of the contribution of two electromagnetic force components. The first, called "blowing" component, is a force of mass while the second, called "pumping" component, is a pressure force; it is due to the pressure the electromagnetic forces induce inside the discharge [2].

This paper describes some considerations aimed to corroborate the validity of the previous formula.

### Experimental results

Experimental studies on the thrust of self-field coaxial MPD thrusters varying the engine scale [3], the thruster geometry [4] and the propellant properties [5], have shown that thrust is proportional to the power employed in the thruster. Then let us state first some items:

a) The conduction electrons in a steady discharge are influenced only by the electrostatic field between the cathode and the anode. As this field is conservative, the work that the discharge does to bring the electrons from the cathode to the anode is independent of the path of the electrons.

b) A sizable portion of the thruster power is deposited with the electrodes. The size of this portion depend on discharge current, voltage fall at the electrodes, electrodes material and electrodes surface area that is affected by the discharge [6]. The electrode voltage fall and the electrode material are independent of the discharge current; thus discharges having the same area on the surface of the electrodes and the same current intensity in any case produce the same electrode power deposition.

c) The power employed in Joule effect by the discharge, that is a function of the length of the discharge itself, can be already neglected [7]; the differences in power employed among discharges of different length are even more negligible.

d) The power employed in the dissociation and ionization of the propellant - given a certain propellant and a constant flux of the same - is a function only of the current intensity of the discharge.

In conclusion, if the discharge is steady, the thrust is a function only of the discharge current and of its width on the electrodes surface.

## 2 GENERAL HYPOTHESES

Generally speaking, the thrust of a rocket is the total force that the thruster exchanges with the propellant, initially at rest inside the thruster, allowing it to reach the final (exhaust) relative axial velocity ($w_e$), i.e.

$$F = \dot{m}\, w_e$$

In a pure MPD thruster the thrust is to be intended as that due to the contribution of all the electromagnetic forces the thruster exerts on the charged particles of the plasma jet; these can be written

$$F' = \int_V \underline{\boldsymbol{j}} \times \underline{\boldsymbol{B}}\; dV$$

in the regions where there is a current with a magnetic field and

$$F'' = q\; \underline{\boldsymbol{w}} \times \underline{\boldsymbol{B}}$$

in the regions where there is only a magnetic field. The latter, being perpendicular to the velocity, does no work; it only biases the direction of the velocity.

In order to develop the theory let us consider a coaxial thruster (Fig. 1) with a cylindrical discharge chamber and a short cylindrical cathode. Moreover cylindrical coordinates $r$, $\theta$, $z$) are assumed.

The following general hypotheses are assumed:

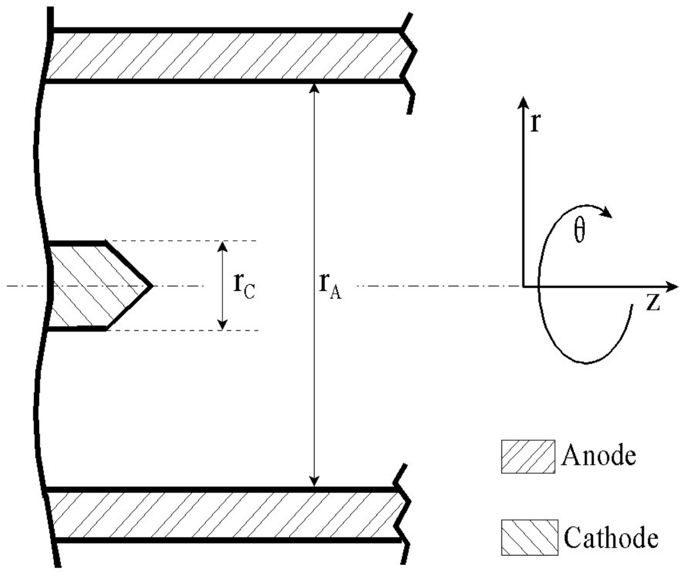


**Fig. 1 -** Cylindrical MPD Thruster with a short Cathode.

A) The space inside the thruster is linear, homogenous and isotropic, independent of its physical state.

B) The system composed by the thruster, the energy source and the transmission line connecting the two behaves as an Ohmic circuit, i.e. $V = Z\,I$.

C) The system works in a steady state, i.e. both the power absorbed by the thruster and the current intensity are constant.

D) The accelerated propellant is a fully single-ionized plasma.

E) The discharge inside the thruster and, as a consequence, the self-induced magnetic field should be axially symmetric, and moreover it should be $\dfrac{\partial}{\partial\theta} = 0$.

Let us introduce the potential vector $\underline{\boldsymbol{A}}$, defined by

$$\underline{\boldsymbol{B}} = \nabla \times \underline{\boldsymbol{A}}, \tag{7}$$

that in cylindrical coordinates becomes

$$\underline{\boldsymbol{B}} = \left( \frac{1}{r} \frac{\partial A_z}{\partial \theta} - \frac{\partial A_\theta}{\partial z} \right) \underline{\hat{\boldsymbol{r}}} + \left( \frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \right) \underline{\hat{\boldsymbol{\theta}}} + \frac{1}{r} \left( \frac{\partial (r A_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta} \right) \underline{\hat{\boldsymbol{z}}}. \tag{8}$$

On the basis of the A) hypothesis, the potential vector $\underline{\boldsymbol{A}}$ is parallel to the current density that for the E) hypothesis is

$$\underline{\boldsymbol{j}} = j_r \underline{\hat{\boldsymbol{r}}} + j_z \underline{\hat{\boldsymbol{z}}}; \tag{9}$$

so the potential vector $\underline{\boldsymbol{A}}$ can be written as

$$\underline{\boldsymbol{A}} = A_r \, \hat{\boldsymbol{r}} + A_z \, \hat{\boldsymbol{z}}. \tag{10}$$

On the basis of this equation and of the axial symmetry of the phenomena, equation (8) reduces to

$$\underline{\boldsymbol{B}} = \left( \frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \right) \underline{\hat{\boldsymbol{\theta}}} = B_\theta \, \underline{\hat{\boldsymbol{\theta}}}. \tag{11}$$

Hence the self-induced magnetic field has neither radial nor axial components. The azimuthal component $B_\theta$ of the magnetic field is a function of the geometry of the discharge and of the intensity of the current density.

## 3 THE ARC GEOMETRY

The discharge exerts a pressure on the electrodes but the only discharge portion entering the front face of the cathode contributes to the axial force. In the thrusters with short cathode it can be assumed that this portion represents nearly the total current. The whole current discharge will be assumed to be entering the front face of the cathode.

As previously stated, once the discharge area on the anode is fixed , any morphology can be assumed for the discharge in order to calculate the resulting thrust, as long as the "suitable" current intensity is used. In particular the form shown in Figure 2 is chosen.

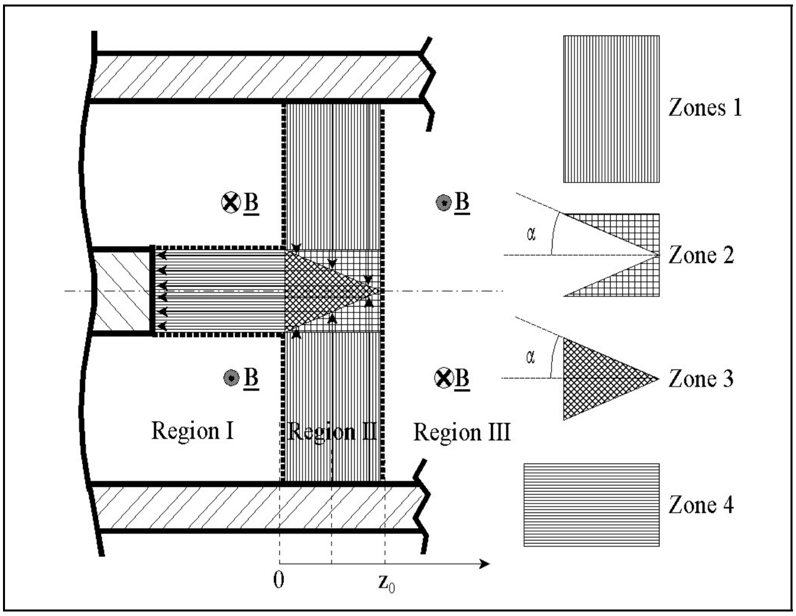


**Fig. 2 -** The space inside the thruster is divided in three regions by the discharge.

The space inside the thruster may be divided into three regions: the region *I* is that where the propellant flow has not yet crossed the discharge, the region *II* is the one occupied by the discharge itself, and region *III* is the one downstream of the discharge.

The region *II* can be in turn divided into four zones. In the different zones we have respectively:

in zone *1* $\quad 0 < z < z_0 \quad and \quad r_c < r < r_a \quad \Rightarrow \quad \underline{\boldsymbol{j}} = j_r \underline{\hat{\boldsymbol{r}}};$

in zone *2* $\quad \tilde{z} < z < z_0 \quad \left[ \tilde{z} = z_0 \left( 1 - \frac{z}{z_0} \right) \right] \quad and \quad 0 < r < r_c \quad \Rightarrow \quad \underline{\boldsymbol{j}} = j_r \underline{\hat{\boldsymbol{r}}};$

in zone *3* $\quad 0 < z < z_0 \quad and \quad 0 < r < \tilde{r} \quad \left[ \tilde{r} = r_c \left( 1 - \frac{r}{r_c} \right) \right] \quad \Rightarrow \quad \boldsymbol{j} = j_z \, \hat{\boldsymbol{z}};$

in zone *4* $\quad z_c < z < 0 \quad and \quad 0 < r < r_c \quad \Rightarrow \quad \boldsymbol{j} = j_z \, \hat{\boldsymbol{z}}.$

The boundary surface between zone 2 and zone 3 is assumed as conical.

## 4 CURRENT DENSITY

The arc current density must respect 3 conditions:

I In the whole region *II*, being a linear, homogenous and isotropic space, holds the equation of continuity of the current

$$\nabla \cdot \boldsymbol{j} = 0, \tag{16}$$

that is the vectorial field $\boldsymbol{j}$ is solenoidal and hence the current $I$ must be constant in every section of the flux;

II All the current must go through the internal surface of the anode, i.e.

$$i = \int_0^{z_0} j_r \, \hat{\boldsymbol{r}} \; 2 \, \pi \, r_A \, dz;$$

III All the current must go through the front surface of the cathode, i.e.

$$i = \int_0^{r_C} j_z \, \hat{\boldsymbol{z}} \; \pi \, r \, dr.$$

Two different current distributions will be analyzed:

A The radial current density in zone *1* and *2* is constant along the axis, i.e.

$$\boldsymbol{j} = j_r(r) \; \hat{\boldsymbol{r}} = - \frac{i}{2 \, \pi \, r \, z_0} \; \hat{\boldsymbol{r}}. \tag{19}$$

B The axial current density in zone *3* and *4* is constant along the radius, i.e.

$$\boldsymbol{j} = j_z \; \hat{\boldsymbol{z}} = - \frac{i}{\pi \, r_c^2} \; \hat{\boldsymbol{z}}. \tag{20}$$

### *4.1 CASE A*

To respect condition *I* along the boundary surface between zone *2* and zone *3,* the normal components of the current density on the two sides of the surface must be equal each other, i.e.

$$for \quad r = \tilde{r} \quad \Rightarrow \quad j_z = \frac{j_r}{\tan \alpha} = - \frac{i}{2 \, \pi \, r \, z_0 \tan \alpha}. \tag{21}$$

Condition *III* requires

$$i = \int_0^{r_c} \int_0^{2\pi} \frac{i}{2 \, \pi \, r \, z_0 \tan \alpha} \; (r \, dr \, d\theta) = \frac{i \, r_c}{z_0 \tan \alpha}; \tag{22}$$

since for a conical surface the following always holds

$$\frac{r_c}{z_0 \tan \alpha} = 1, \qquad \textbf{(23)}$$

the latter condition is respected too.

### *4.2 CASE B*

Applying the same reasoning as in the previous case the proposed distribution respects all the conditions if

$$j_r = - \frac{i}{\pi\, r_c} \frac{1}{r} \left( 1 - \frac{z}{z_0} \right) \tan \alpha \,. \qquad \textbf{(24)}$$

## 5 SELF-INDUCED MAGNETIC FIELD

The self-induced magnetic field can be obtained in every region of the space as a function of the current density, applying the Maxwell equation

$$\oint_C \underline{\boldsymbol{B}} \cdot d\underline{\boldsymbol{l}} = \mu \int_S \underline{\boldsymbol{j}} \cdot \underline{\hat{\boldsymbol{n}}}\; dS. \qquad \textbf{(25)}$$

As an example the calculus of the expression of the magnetic field inside the zone *1*, due to the radial current density, will be described. This magnetic field becomes zero at section $z'$ of the duct, where $z'$ depends on the distribution of the current density; in symbols

$$[\underline{\boldsymbol{B}}(r)]_{z'} = 0. \qquad \textbf{(26)}$$

The Maxwell equation can be applied to the curve $C$ composed by two half-circumferences with radius $r_c < r < r_a$ and by two segments parallel to the axis of the duct with extremes $z'$ and $z' < z < z_0$:

$$\int_0^{\pi} B_\theta (r, z')\, r\, d\theta + \int_{z'}^{z} B_z\, dz + \int_{\pi}^{0} B_\theta (r, z)\, r\, d\theta + \int_{z}^{z'} B_z\, dz = \mu \int_0^{\pi} \int_{z'}^{z} j_r\, r\, d\theta\, dz, \qquad \textbf{(27)}$$

that for (11) and (26) becomes

$$\int_{\pi}^{0} B_\theta (r, z)\, r\, d\theta = \mu \int_0^{\pi} \int_{z'}^{z} j_r\, r\, d\theta\, dz. \qquad \textbf{(28)}$$

For the axial symmetry, both the magnetic field and the current density are not a function of $\theta$, as well as the radius; it can be then written

$$B_\theta (r, z)\, r \int_{\pi}^{0} d\theta = \mu \int_{z'}^{z} j_r\, r\, dz \int_0^{\pi} d\theta. \qquad \textbf{(29)}$$

Since the radius is also independent of $z$, the expression of the magnetic field due to the radial current density is

$$B_\theta (r, z) = - \mu \int_{z'}^{z} j_r\, dz \qquad \textit{for} \quad 0 < z < z_0 \quad \textit{and} \quad r_c < r < r_a. \qquad \textbf{(30)}$$

A similar expression can be deduced for every zone of the space. For reasons of conciseness, only the results will be reported and only where needed.

## 6 THRUST IN REGION I

In Region *I* the magnetic field is equal to

$$B_\theta(r,z) = \frac{\mu}{r}\int_0^{r_c} j_z\, r\, dr + \mu \int_0^{z'} j_r\ dz \qquad for \quad z_c<z<0 \quad and \quad r_c<r<r_a . \tag{31}$$

where the first term is due to the axial current flowing in the zone *4* of the region II and the second term to the radial current flowing in the zone *1* of the region *II*.

Since in region *I* there is no current, the propellant is not ionized; then no force is acting on the propellant.

## 7 THRUST IN REGION II

The current density and the magnetic field interact generating a volume force on the propellant

$$\boldsymbol{f} = \boldsymbol{j}\times\underline{\boldsymbol{B}} = j_r\, B_\theta\, \hat{\boldsymbol{z}} - j_z\, B_\theta\, \hat{\boldsymbol{r}}, \tag{32}$$

### *7.1 BLOWING*

The thrust blowing component is equal to first term of (32) integrated over the volume occupied by zone *1* and zone *2*. Zone 1 occupies a volume equal to

$$V_1 = \int_0^{2\pi}\int_0^{z_0}\int_{r_c}^{r_a} r\, dr\, dz\, d\theta , \tag{33}$$

and zone *2* a volume equal to

$$V_2 = \int_0^{2\pi}\int_0^{z_0}\int_{\tilde{r}}^{r_c} r\, dr\, dz\, d\theta . \tag{34}$$

The thrust due to the Blowing component will be calculated using both the proposed current density distributions.

#### 7.1.1 Case A

Since the radial current density is constant along the axis *z*, the intensity of the self-induced magnetic field due to said density inside zone *1* and zone *2* is linear and symmetric with respect to the middle of the discharge, in symbols

$$B_\theta(r, z<z') = -B_\theta(r, z>z') \quad and \quad z' = \frac{z_0}{2}. \tag{35}$$

As a consequence, the integral along the axis *z* from *0* to $z_0$ of this component of the magnetic field is always zero, and hence, since $J_r$ is constant along the *z* axis, also the blowing due to this component of the magnetic field is zero.

In zone *1* and *2* the magnetic field induced by the axial current density of zone *3* is present.

##### *7.1.1.1 Zone 1*

The magnetic field due to the $J_z$ of zone *3* is

$$B_\theta = \frac{\mu}{r}\int_0^{\tilde r} J_z\, r\, dr = \frac{\mu}{r}\int_0^{\tilde r} -\frac{I}{2\pi r z_0 \tan\alpha}\, r\, dr\,; \tag{36}$$

solving the integral we obtain

$$B_\theta = -\frac{\mu I}{2\pi r z_0 \tan\alpha}\int_0^{\tilde r} dr = -\frac{\mu I \tilde r}{2\pi r z_0 \tan\alpha}. \tag{37}$$

Integrating the first term of the (32) on $V_1$, the thrust due to the blowing is equal to

$$F_{b1} = \int_0^{2\pi}\int_0^{z_0}\int_{r_c}^{r_a} J_r\, B_\theta\, r\, dr\, dz\, d\theta = \int_0^{2\pi}\int_0^{z_0}\int_{r_c}^{r_a}\left(-\frac{I}{2\pi r z_0}\right)\left(-\frac{\mu I \tilde r}{2\pi r z_0 \tan\alpha}\right) r\, dr\, dz\, d\theta\,; \tag{38}$$

taking out from the integral the part constant and solving the integral for $\theta$, we obtain:

$$F_{b1} = 2\pi\left(\frac{\mu I^2}{4\pi^2 z_0^2 \tan\alpha}\right)\int_0^{z_0}\tilde r\int_{r_c}^{r_a}\frac{1}{r^2}\, r\, dr\, dz. \tag{39}$$

Simplifying and solving the integral for $r$, the blowing component is then

$$F_{b1} = \frac{\mu I^2}{2\pi z_0^2 \tan\alpha}\int_0^{z_0}\tilde r\, \ln(r)\Big|_{r_c}^{r_a} dz = \frac{\mu I^2}{2\pi z_0^2 \tan\alpha}\int_0^{z_0}\tilde r\, \ln\left(\frac{r_a}{r_c}\right) dz\,; \tag{40}$$

that substituting the value of $\tilde r$, becomes

$$F_{b1} = \frac{\mu I^2}{2\pi z_0^2 \tan\alpha}\int_0^{z_0} r_c\left(1-\frac{z}{z_0}\right)\ln\left(\frac{r_a}{r_c}\right) dz = \frac{\mu I^2}{2\pi z_0^2 \tan\alpha}\, r_c \ln\left(\frac{r_a}{r_c}\right)\int_0^{z_0}\left(1-\frac{z}{z_0}\right) dz \tag{42}$$

Solving the integral for $z$

$$F_{b1} = \frac{\mu I^2}{2\pi z_0^2 \tan\alpha}\, r_c \ln\left(\frac{r_a}{r_c}\right)\left(z-\frac{z^2}{2 z_0}\right)\Bigg|_0^{z_0} = \frac{\mu I^2}{2\pi z_0^2 \tan\alpha}\, r_c \ln\left(\frac{r_a}{r_c}\right)\left(z_0-\frac{z_0}{2}\right), \tag{43}$$

and, being $z_0 \tan\alpha = r_c$,we finally obtain

$$F_{b1} = \frac{\mu i^2}{4\pi}\ln\left(\frac{r_a}{r_c}\right). \tag{45}$$

### *7.1.1.2 Zone 2*

The magnetic field due to the $j_z$ of zone *3* present in zone *4* is equal to

$$B_\theta = -\frac{\mu I \tilde r}{2\pi r z_0 \tan\alpha}. \tag{46}$$

Integrating on the volume $V_2$, the blowing component is equal to

$$F_{b2} = \int_0^{2\pi}\int_0^{z_0}\int_{\tilde r}^{r_c} J_r\, B_\theta\; r\,dr\,dz\,d\theta = \int_0^{2\pi}\int_0^{z_0}\int_{\tilde r}^{r_c}\left(-\frac{I}{2\pi r z_0}\right)\left(-\frac{\mu I \tilde r}{2\pi r z_0 \tan\alpha}\right) r\,dr\,dz\,d\theta. \quad \textbf{(47)}$$

Proceeding like for zone *1* we obtain for zone *2*

$$F_{b2} = \frac{\mu\, i^2}{8\pi}. \quad \textbf{(48)}$$

#### *7.1.1.3 Total blowing in case A*

The total component of blowing, sum of zone *1* and zone *2* contributes, is then

$$\boxed{F_{bA} = \frac{\mu\, i^2}{4\pi}\left[\ln\left(\frac{r_a}{r_c}\right) + \frac{1}{2}\right].} \quad \textbf{(49)}$$

### 7.1.2 Case B

A procedure analogous to case *A* leads, for the component of blowing in the case *B*, to the expression

$$\boxed{F_{bB} = \frac{\mu\, i^2}{4\pi}\left[\ln\left(\frac{r_a}{r_c}\right) + \frac{1}{4}\right].} \quad \textbf{(50)}$$

## *7.2 PUMPING*

The volume force (32) compresses the space occupied by the discharge. That is the ions of the plasma are subjected to a compression force equal to:

$$\underline{\boldsymbol{f_c}} = j_r\, B_\theta(j_r)\, \hat{\underline{\boldsymbol{z}}} - j_z\, B_\theta\, \hat{\underline{\boldsymbol{r}}}. \quad \textbf{(51)}$$

The electric repulsion force between the ions equilibrate the compression force; in other words inside the flux exists a pressure gradient that counteracts the compression force [2]. This pressure gradient is proportional to both the current density and the magnetic field intensity: if either of them varies along the axis of the discharge, the pressure gradient varies also; and hence a pressure gradient is established along the axis of the discharge.

Since the net flux of ions through the discharge is statistically zero[1], the forces acting inside the discharge must be in equilibrium [2]. As a consequence the pressure on the cathode must be equal to the pressure on the boundary section between zone 2 and zone 3. The latter is equal to the difference between the pressures due to zone 3 minus the pressure due to zone 2. In symbols:

$$p_c(r) = p_1(r) - p_2(r). \quad \textbf{(52)}$$

The force acting on the cathode is then equal to

[1] If the net flux of ions through the discharge would not be statistically zero, the resistance of the discharge would change during the functioning of the thruster, and hence the discharge current would not be constant, i.e. the thruster would not be steady.

$$F_p = \int_0^{2\pi} \int_0^{r_c} \left[p_1(r) - p_2(r)\right] r \, dr \, d\theta, \tag{53}$$

### 7.2.1 Case A

If the $j_r$ is constant along the $z$ axis, the pressure $p_{1A}$ is equal to

$$p_{1A}(r) = \frac{3}{8} \frac{\mu \, i^2}{\pi^2 z_0^2} \ln\left(\frac{r_c}{r}\right), \tag{55}$$

and the pressure $p_{2A}$ is equal to

$$p_{2A}(r) = \frac{\mu \, i^2}{8 \pi^2 r_c^2} \left[\frac{r_c}{r} - 1\right], \tag{57}$$

Substituting in (52), we obtain:

$$F_{pA} = \int_0^{2\pi} \int_0^{r_c} \left[\frac{3}{8} \frac{\mu \, i}{\pi^2 z_0^2} \ln\left(\frac{r_c}{r}\right) - \frac{\mu \, i^2}{8 \pi^2 r_c^2} \left(\frac{r_c}{r} - 1\right)\right] r \, dr \, d\theta, \tag{58}$$

that leads to

$$\boxed{F_{pA} = \frac{\mu \, i^2}{16 \pi}.} \tag{59}$$

### 7.2.2 Case B

In this case

$$p_{1A}(r) = \frac{3}{8} \frac{\mu \, i^2}{\pi^2 r_c^2} \left(1 - \frac{r^2}{r_c^2}\right), \tag{60}$$

and

$$p_{2B}(r) = \frac{\mu \, i^2}{8 \pi^2 r_c^2} \left(1 - \frac{r^2}{r_c^2}\right). \tag{61}$$

The pumping component of the thrust in the case $B$ is

$$\boxed{F_{pB} = \frac{\mu \, i^2}{8 \pi}.} \tag{62}$$

## *7.3 TOTAL THRUST*

The total thrust due to Region *II* is equal to the sum of the terms of blowing and pumping. In both cases A and B the total thrust results equal to

$$F = \frac{\mu i^2}{4\pi}\left[\ln\left(\frac{r_a}{r_c}\right) + \frac{3}{4}\right]. \tag{63}$$

## 8 REGION III

Downstream of the discharge (region *III*) there is only the self-induced magnetic field due to the radial component of the discharge, which decreases with $z$:

$$\underline{\boldsymbol{B}} = B_\theta (j_r, z)\ \underline{\hat{\boldsymbol{\theta}}} \tag{64}$$

Because the plasma coming from the region *II* is moving away from the thruster, the magnetic field that it is crossing becomes weaker: since the total speed remains always the same (the Lorentz force does no work on the plasma, and hence the components of the speed change, but not the speed itself), such speed should be, sooner or later, equal to the local Alfvèn speed. The plasma is then no longer influenced by the magnetic field [8].

Actual arc geometries should lead to a velocity with radial and axial components. The Lorentz force acting on the plasma flowing with a velocity $\underline{\boldsymbol{w}}\ (w_r,\ 0,\ w_z)$ is

$$\underline{\boldsymbol{F}} = q\ \underline{\boldsymbol{w}} \times \underline{\boldsymbol{B}} = q\ [w_r B_\theta\ \underline{\hat{\boldsymbol{z}}} - w_z B_\theta\ \underline{\hat{\boldsymbol{r}}}]. \tag{65}$$

Taking into account that the curvature radius is much larger than the region *III* dimension, this force acts on the plasma so that (Fig. 3):

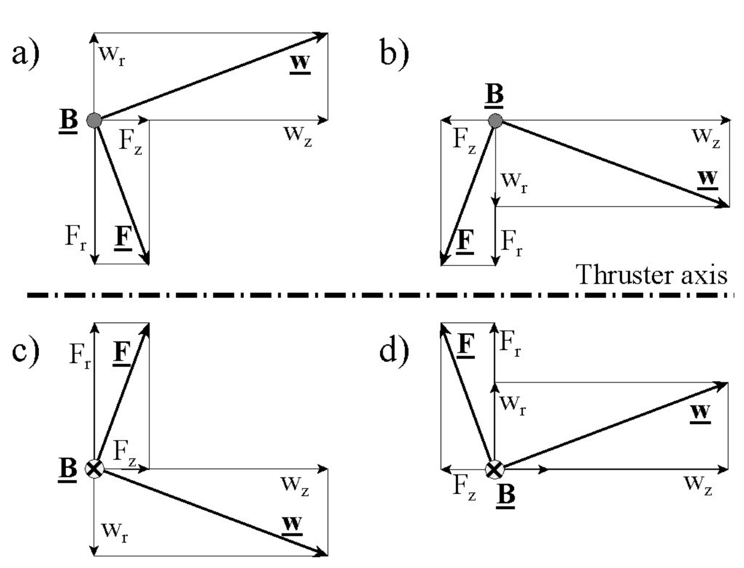


**Fig. 3 -** Lorentz force acting on the exhaust jet in Region *III*.

- if the radial component of the velocity is positive [toward the anode, cases a) and c) in figure] the axial speed increases while the radial speed reduces; the latter may become negative;
- if the radial component of the velocity is negative [toward the axis, case b) and d) in figure] the axial speed decreases while the radial speed increases; when the ions cross the thruster axis, they fall within the previous case.

As a consequence the plasma axial average speed decreases. The decrease in axial speed is smaller the higher the average positive radial component, and vice versa.

The previous theoretical thrust (62) represents the thrust in the case of purely axial exhaust velocity; actually it represents the upper limit value of the thrust.

## THANKS

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