THE AZIMUTHAL BEAM-PLASMA INSTABILITIES IN STATIONARY PLASMA THRUSTERS

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Abstract

In recent theoretical investigations of Stationary Plasma Thrusters the analysis of excitement and time evolution of plasma oscillations appears as one of key problems in understanding the physical processes responsible for optimal performances. The presence of wide plasma oscillation spectrum achieving relatively high amplitude level is expected to main factor responsible as well for transport properties of SPT plasmas and for observed time variation of discharge current.

The analysis of experimental data [1] and the qualitative estimations of possible electric field high frequency fluctuations in SPT [2] tell us that the greatest level of high frequency fluctuation should correspond to the "spoke" mode or, in other words, current driven instability in the azimuth direction. This type of high frequency electric field fluctuations appears as the best candidate to be responsible for "anomalous" contribution to electron conductivity in SPT plasmas. The aim of this paper is to introduce high frequency electron fluctuations propagating in the azimuth direction and to analyse its effect on the macroscopic properties of SPT plasmas.

1. Introduction

Modelling plasmas in Stationary Plasma Thrusters we are faced with problem of the adequate approximation of plasma electric conductivity or, equivalently, the effective electron collision frequency. Classical approximation of electron collision frequency in transverse magnetic field, although including the effect of near wall conductivity, does not provide to current-voltage discharge characteristics close to the measured in laboratory tests, especially in the voltage range close to the ignition of discharge. Better approximation could be achieved when Bohm diffusion has been introduced, but in this case there appears necessity of introducing additional fitting parameter measuring the effective value of this diffusion coefficient.

Careful analysis of plasma oscillation spectra deduced by E.Y. Choueiri [1] on the basis of experimental data presented by V. Kim and A. Bishayev [3] has allowed the author to distinguish five frequency bands ranging from 1kHz up to 5MHz, where the plasma oscillations are excited. Following the theoretical analysis performed in paper [1] one can expect that most of modes in the classified in first four groups (i.e.: up to $\approx 0.5MHz$) should be contained in 2-D hybrid [7,8] (or fluid) models [see e.g.: of SPT plasmas, if sufficiently exact numerical procedure could be applied. In this considerations we focus our attention on the fifth (in Choueiri classification) frequency range: from some hundreds of kHz up to some MHz. Here we expect the necessity of analysing the kinetic background of possible fluid model, since e.g.: Bernstein modes could be derived in terms of Vlasov theory only [4 – 6].

The aim of this paper is to discuss the spectrum of magnetized plasma oscillations by dispersion relation analysis and check the possibility of determining the steady state oscillation level. Since the excitement of unstable modes propagating transverse to magnetic field needs, in general, the presence of destabilizing factors we take into account such of them like high velocity relative motion of electrons and ions and the presence of an electron beam in thermalized plasma. There remain, beside above, one important destabilizing factor i.e.: the electron temperature anisotropy [2, 6], which will be discussed in other paper. Having determined mode frequencies and increments we will try check if there exist simple mechanism of mode saturation. Here we present the application of quasi-linear theory of weak plasma turbulence [17-19] in describing the coupling of background plasma parameters with high frequency oscillations.

2. Dispersion relation analysis

Searching the high frequency SPT plasma fluctuations in frame of fluid model we limit our considerations to the analysis of dispersion relation for quasi-uniform plasmas in magnetic field. To be consistent with fluid description [13-16] of unperturbed plasma we assume, when deriving dispersion relation, the electron distribution function as a shifted maxwellian having mean velocity equal to electron drift velocity including electric and gradient drifts. Here we will neglect most of geometric effects resulting from the annular shape of discharge channel, although remaining the periodic boundary conditions along the "y" coordinate corresponding to azimuth direction in exact formulation. Hence, we analyse the linear disturbances of plasmas in magnetic field $\vec{B}=(0,0,B_0(x))$ and electric field $\vec{E}=(E_0(x),0,0)$ with unperturbed electron distribution:

$$f_{0e}(\vec{v})=n_0(x)\left(\frac{m}{2\Pi\kappa T_e(x)}\right)^{3/2}\exp\left(\frac{m(v-V_d(x))^2}{2\kappa T_e(x)}\right) \quad , \tag{1}$$

where: $\vec{V}_d=\vec{V}_E+\vec{V}_{grad}+V_{ex}\vec{I}_x$; $\vec{V}_E=\vec{E}_0\times\vec{B}_0/B_0^2$; $\vec{V}_{grad}=\vec{\nabla}(n_0\kappa T_e/eB_0)\times(\vec{B}/B_0^2)$.
Similarly, the cold fluid approximation in case of ions correspond to ion distribution function of the form:

$$f_{0i}=n_0(x)\delta\left(\vec{v}-\vec{V}_i(x)\right)$$

Following the classical analysis [see 3 ÷ 5] we get the dispersion relation for disturbances propagating transverse to $\vec{B}_0$ (i.e. for: $\vec{k}=(k_x,k_y,0)$) in the form:

$$\frac{1}{\omega_{pe}^2}-\frac{\left(m/M\right)}{\left(\omega-\vec{k}\bullet\vec{V}_{Def}\right)^2}-\sum_{n=1}^{\infty}\frac{2I_n(\lambda)n^2}{\lambda\left(\omega^2-n^2\omega_{Be}^2\right)}=0 \tag{2}$$

where: ω_{Be}, ω_{pe} denote electron cyclotron and plasma frequencies respectively, I_n are Bessel functions of second kind and $\lambda=(k^2\kappa T_e)/m\omega_{Be}^2$. For computational convenience we performed the substitution $\omega\to\omega-\vec{k}\bullet V_d$, and introduce: $\vec{V}_{Def}=\vec{V}_i-\vec{V}_d$, which corresponds to the reference frame moving with the electron drift velocity. Here: $\vec{V}_{Def}=(V_i-V_{ez},-V_E-V_{grad},0)$. It has to be pointed out that in real (annular) geometry the electric and gradient drift velocities for $\vec{B}_0=(B_r(z)(r/R_1),0,0)$, $\vec{E}_0=(0,0,E_0(z))$ $V_{Def;y}$ would be proportional to r and therefore the scalar product $\vec{k}\bullet(\vec{V}_E+\vec{V}_{grad})$ in this geometry will not depend on r since $\vec{k}=(0,(2\Pi l)/(2\Pi r),k_z)$, where l - integer. Solutions ω of eq. (2) scaled by ω_{Be} depend on three dimensionless parameters:

$\mathrm{K}=\vec{k}\bullet\vec{V}_{Def}/\omega_{Be}$; $\mathrm{N}=\omega_{pe}/\omega_{Be}$; λ;

In case of SPT100 we can estimate [see 3]:

$$\omega_{pe}=(1\div 3)*10^{10}[s^{-1}];\quad \omega_{Be}=(0.2\div 3)*10^{9}[s^{-1}];\quad k_y=4*l[m^{-1}];$$
$$V_{Def}\approx 10^6[m/s];\quad v_{Te}=(0.5\div 1.5)*10^6[m/s];$$

Therefore discussing the solutions of eq. (2) we limit ourselves to the range: $-2.0 < \mathrm{Re}(\omega) < 2.0$ and $\vec{k} \bullet \vec{V}_{Def} < 2.0$. Electron to ion mass ratio has been taken as that for Xe i.e. $m/M = 4.19 * 10^{-6}$.

In the long wave limit $\mathrm{K} << 1.0$ the solutions are presented in fig. 1. We distinguish there two stable modes (“1” and ”4” with $Im(\omega) = 0$) and two modes (“2” and “3”) with imaginary parts of opposite signs, for $\mathrm{K} = \vec{k} \bullet \vec{V}_{Def} / \omega_{Be} \le 0.004$ in the N range: $\omega_{pe}/\omega_{Be} = 2.5 \div 6.0$.

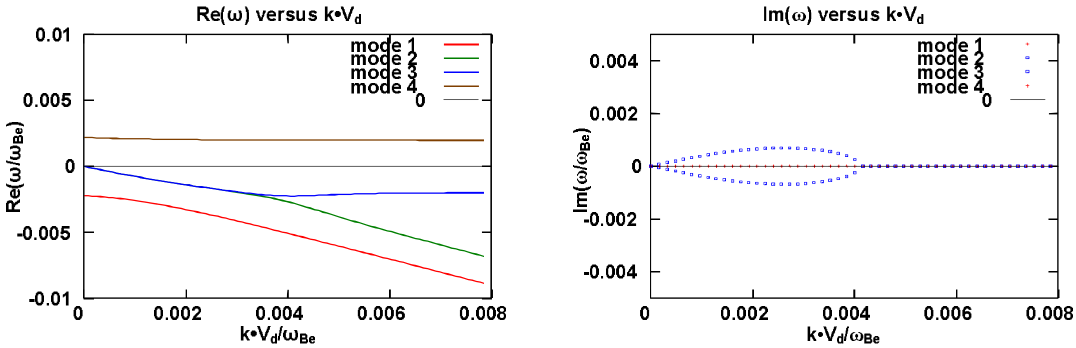

Fig.1a Dispersion relation for: $\omega_{pe}/\omega_{Be} = 2.5$

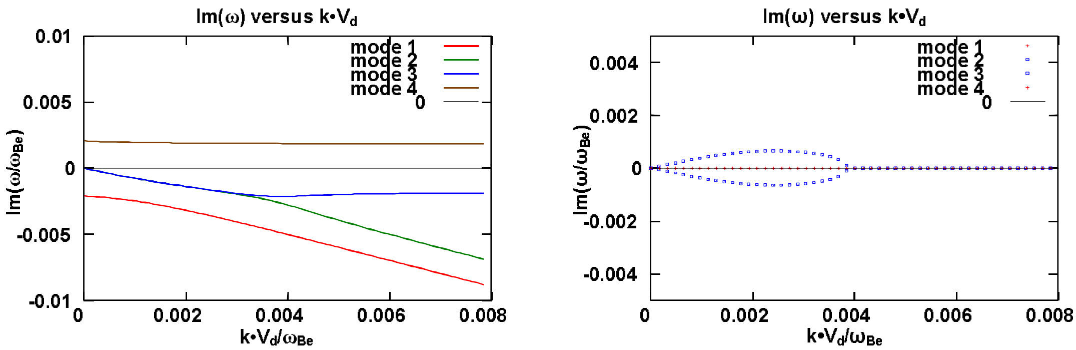

Fig.1b Dispersion relation solutions for: $\omega_{pe}/\omega_{Be} = 6.0$

In the range $0.005 < \vec{k} \bullet \vec{V}_{Def} / \omega_{Be} \le 1.0$ all four modes remain stable. Having in mind that in above analysis the collisional damping has been neglected, one can expect only the unstable mode “3” to be excited. Its maximal increment $Im(\omega) \approx 0.0006\omega_{Be} \approx 2*10^{6}s^{-1}$ appears as comparable with typical electron collision frequency. The respective mode frequency $Re(\omega)$ is proportional to $(-\vec{k} \bullet \vec{V}_{Def})$. Hence, in the unstable $\vec{k} \bullet \vec{V}_{Def}$ range and for $60Gauss < B < 180Gauss$ only $k_y = 4,8,12\,[m^{-1}]$ are allowed in case of SPT100 (not more than three periods along azimuth). The mode frequencies are then in the range $(0.5 \quad 2.3)*10^{-3}\omega_{Be}$; i.e.: $(0.5 \div 7.0)MHz$. Therefore this mode could be identified with that, which presence in SPT has been deduced from analysis of plasma measurements in thrusters channel by E.Y. Choueiri [1]. Esipchuk et al. [9,10] performed and analysed the probe measurements in SPT channel. They proved experimentally the presence of modes of frequencies $0.7 \div 10MHz$ and propagating in azimuth direction. It has to be noted also that in order to derive unstable modes we had to introduce in the equation (2) the contribution of small deviation of $\vec{k}$ vector from transverse to $\vec{B}$ plane i.e: $k_z = k\cos\theta;\quad where:\ \cos\theta < (m/M)^{1/2}$. In real geometry this deviation could result as the effect of centrifugal force. In case presented above we put $\cos\theta = 10^{-4}$.

In case of $K = \vec{k} \bullet \vec{V}_{Def} / \omega_{Be} > 1.0$, which for SPT plasmas should correspond to $k_y > 10^3 [m^{-1}]$ and thus to the wave length of the order of an electron Larmor radius, we also recover the unstable modes. The solution of equation (2) are presented in fig. 2.

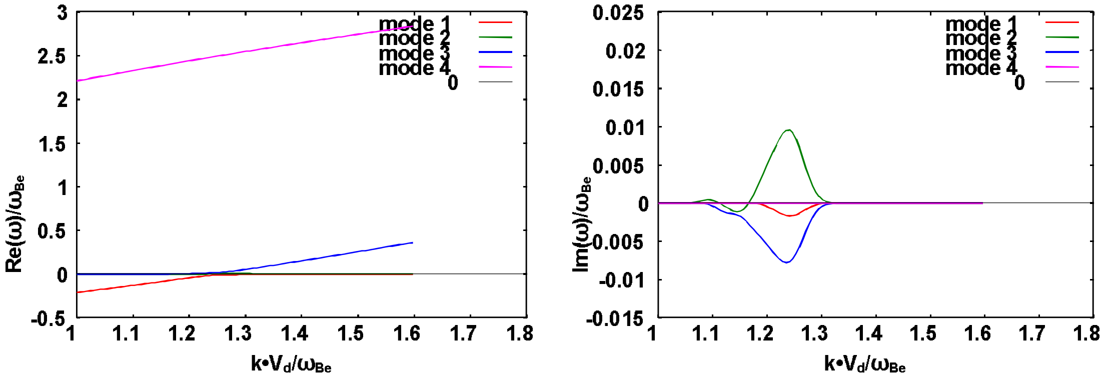

Fig.2a Real and imaginary parts of dispersion relation for $\omega_{pe} / \omega_{Be} = 6.0$

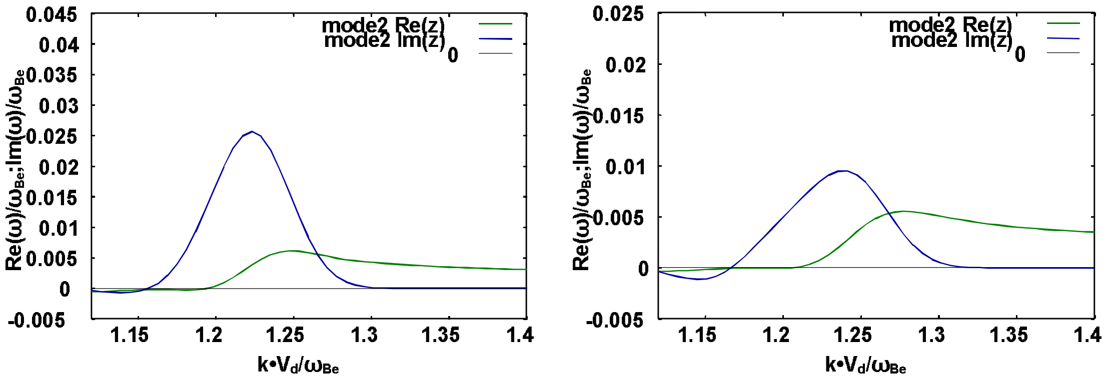

Fig.2b Real and imaginary parts of dispersion relation for mode „2" ($\omega_{pe} / \omega_{Be} = 3.0$)

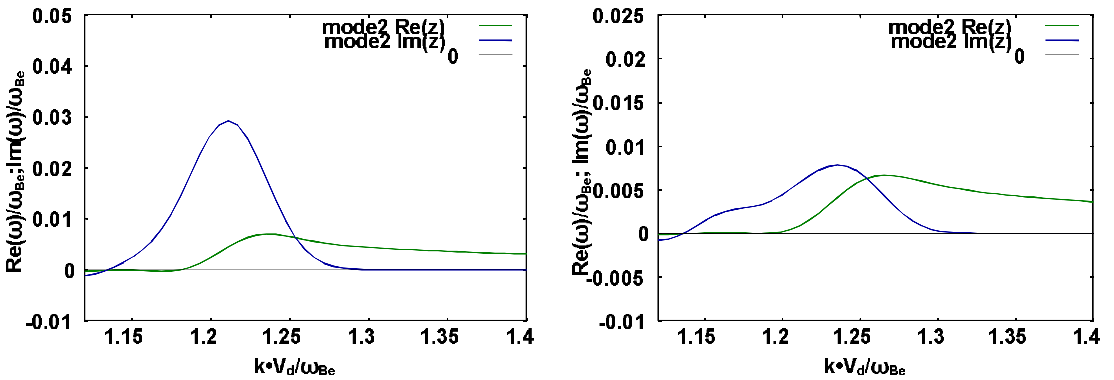

Fig.2c Real and imaginary parts of dispersion relation for mode „2" ($\omega_{pe} / \omega_{Be} = 6.0$)

Mode "2" having frequency $\approx 0.005\omega_{Be}$ and maximal increment $\approx 0.03\omega_{Be}$ appears as strongly unstable. Hence one can expect fluctuations in the band $(0.004 \div 0.005)\omega_{Be}$ and the wavelength of the order electron Larmor radius to be excited.

Several authors (see e.g. [2,11,12,13]) showed, that in SPT plasmas electron distribution function could not be thought as a thermalized maxwellian distribution. One can distinguish the bulk plasma described by electron maxwellian distribution and the electron beam composed with those electrons which accelerated in the cathode-channel exhaust potential drop did not loose their energy in the inelastic collisions. In order to examine also this effect we add on the left hand side of eq. (2) the contribution of electron beam assuming its temperature negligible, its density $n_1 = \alpha\, n_0$ and its velocity equal to $\vec{V}_d$. In the figure 3 we present the solution of dispersion relation including the electron beam contribution:

$$\frac{1}{\omega_{pe}^2} - \frac{\left(m/M\right)}{\left(\omega - \vec{k}\bullet\vec{V}_{Def}\right)^2} - \frac{\alpha}{\left(\left(\omega - \vec{k}\bullet\vec{V}_{Def}\right)^2 - \omega_{Be}^2\right)} - \sum_{n=1}^{\infty} \frac{2I_n(\lambda)n^2}{\lambda\left(\omega^2 - n^2\omega_{Be}^2\right)} = 0 \qquad (3)$$

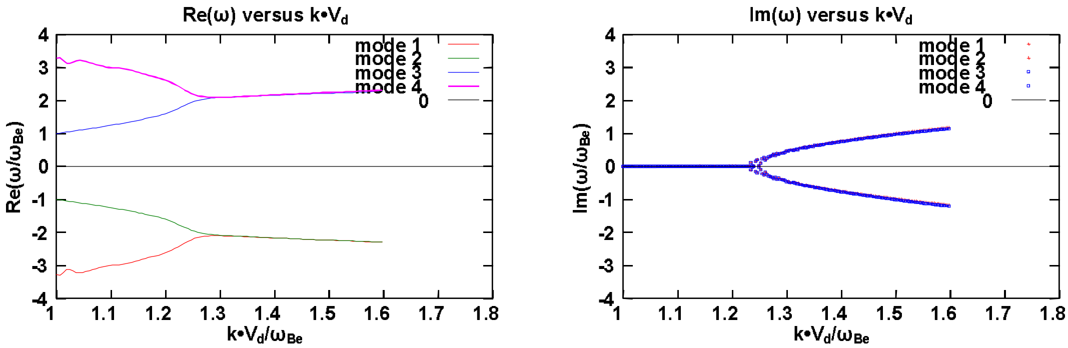

Fig.3a Dispersion relation solutions for: $\omega_{pe}/\omega_{Be} = 6.0;\ \alpha = 10^{-4}$

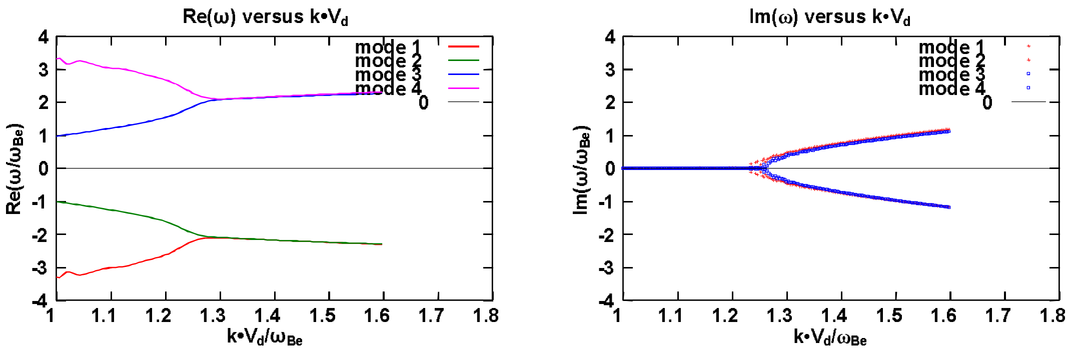

Fig.3b Dispersion relation solutions for: $\omega_{pe}/\omega_{Be} = 6.0;\ \alpha = 10^{-2}$

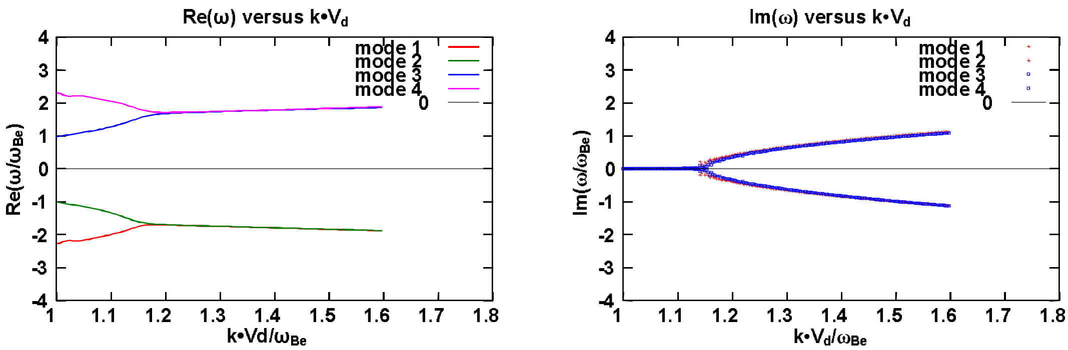

Fig.3c Dispersion relation solutions for: $\omega_{pe}/\omega_{Be}=3.0, \alpha=10^{2}$

The presence electron beam will then result in appearance of unstable modes "1" and "4" of frequencies close to $\pm 2\omega_{Be}$ and increment growing with $\vec{k}\bullet\vec{V}_{Def}$ up to $\mathrm{Im}(\omega) \approx \omega_{Be}$ for $\vec{k}\bullet\vec{V}_{Def} = 1.6\omega_{Be}$. The specific feature of these modes is the presence of instability threshold, which position varies from $\vec{k}\bullet\vec{V}_{Def} \approx (1.1 \div 1.3)\omega_{Be}$ depending on ω_{pe}/ω_{Be} ratio.

Therefore simple analysis of transverse mode in plasma under the action of crossed electric and magnetic fields provides to appearance of unstable azimuth modes if the destabilizing factors like relative motion of electron and ions and/or the plasma – beam interaction take place. The azimuth current caused by electron drift could provide the excitation of long-wave unstable modes in the frequency band

$(0.5 \div 2.3) * 10^{-3}\, \omega_{Be}$ or the short-wave mode ($k \approx 1/r_{Le}$) in the frequency band $(0.003 \div 0.004)\omega_{Be}$. The presence of electron beam of velocity greater that the bulk plasma thermal velocity will result in appearance of unstable modes with $k < 1/r_{Le}$.

In this analysis the effect of mode propagation along magnetic field and therefore the appearance of Landau damping was completely neglected. This is equivalent to assuming that all disturbances propagating toward the dielectric walls are damped and do not affect the sheath. In more detailed analysis the question of sheath stability would appear.

3. Non-linear correction to mode evolution and interaction with background plasma.

In order to find the effect of excitation of unstable modes on the background plasma parameters we have to examine derivation 1-D three fluid model equations [e.g.13-15] introducing short time scale fluctuation propagating in azimuth direction following the quasi-linear theory of weakly turbulent plasmas [16-18]. Electron fluid equations are now derived from the kinetic equation of the form:

$$\frac{\partial F_0}{\partial t} + v_x \frac{\partial F_0}{\partial x} - \frac{e}{m}\left(\vec{E}_0 - \vec{B}_0 \times \vec{v}\right) \bullet \frac{\partial F_0}{\partial \vec{v}} = C_{coll} + \frac{e}{m}\left\langle \tilde{E}_{1y} \frac{\partial \tilde{f}_1}{\partial v_y} \right\rangle \equiv C_{coll} + C_{fluct} \tag{4}$$

where: $F_0 = F_0\left(x, \left(\vec{v} - \vec{V}_d\right)^2/2, t\right)$ is the electron distribution function averaged over y and z, and $\vec{v} = \left(v_x, v_y\right)$. C_{coll} denotes classical collision term, and the last term represents effect of electron scattering on the charge density fluctuations. f_1 is the solution of linearized kinetic equation:

$$f_1 = -\frac{e}{m}\Phi_k \frac{\partial F_0}{\partial \varepsilon}\left\{ \sum_{n,m=-\infty}^{\infty} \left[\delta_{n,m} - \frac{\omega(k) - kV_d}{\omega(k) - kV_d - n\omega_{Be}} \right] J_n\left(\frac{kv}{\omega_{Be}}\right) J_m\left(\frac{kv}{\omega_{Be}}\right) \exp\left(-i(n-m)(\phi - \Pi/2)\right) \right\}$$

where: $v_y = v\sin(\phi)$. Fluctuating electric field $E_{1y} = -ik_y \Phi_k$ is here determined by mode evolution equation:

$$\frac{\partial |\Phi_k|^2}{\partial t} = 2\,\mathrm{Im}\left(\omega(k), \{F_0\}\right)|\Phi_k|^2 \tag{5}$$

where it was used a fact, that after averaging performed in eq. (4), only $|\Phi_k|^2$ term will appear. Nonlinear correction to mode evolution will enter via functional dependence of $\mathrm{Im}\left(\omega(k), \{F_0\}\right)$ on F_0. Now we assume that the additional term in eq. (4) does not change the functional form of electron distribution function, but only affects its parameters ($\vec{V}_d, T_e$). Passing to respective electron fluid equations we get the corrections to the source terms:

for electron momentum azimuth component:

$$\int d^2v\, v_y C_{fluct} = -\sum_k \varepsilon_0 k |k\Phi_k|^2 \frac{\omega_{pi}^2 \gamma_k}{m\omega_k^3} \equiv -\nu_{fluct} V_{dy} \tag{6}$$

for electron energy

$$\int d^2v \frac{m\left(v_y - V_d\right)^2}{2} C_{fluct} = \sum_k \varepsilon_0 |k\Phi_k|^2 \frac{\omega_{pi}^2 \gamma_k}{\omega_k^2} \equiv \nu_{fluct} \frac{mV_d^2}{2} \tag{7}$$

where: $\mathrm{Re}(\omega\ ) \to \omega$; $\mathrm{Im}(\omega\ ) \to \gamma$.

In the first case of discussed above dispersion relation i.e.: in the “long-wave limit”, the sum over k is reduced small number of terms. Suppose that the collision frequency is such, that only one mode say k_0

is excited. Then assuming, that quasi-linear relaxation is much faster than macroscopic fluid characteristic time scale $\tau \quad >> t_{quasi\,linear}$, we are faced with an auxiliary problem of finding asymptotic solution of:

$$\frac{\partial V_d}{\partial t} = -\sum_k \varepsilon_0 k |k\Phi_k|^2 \frac{\omega_{pi}^2 \gamma_k}{m\omega_k^3} \approx -\frac{|\Phi_{k0}|^2}{AV_d^3}\gamma_{k0}(V_d) \tag{8}$$

$$\frac{\partial |\Phi_{k0}|^2}{\partial t} = 2\gamma_{k0}(V_d)|\Phi_{k0}|^2 \tag{9}$$

which can be solved in terms of $\Psi(V_d) \equiv \gamma_{k0\max}\int_{V_d(0)}^{V_d} \frac{Ax^3}{\gamma_{k0}(x)}dx$, since Ψ is the solution of:

$$\frac{\partial \Psi}{\gamma_{k0\max}\partial t} = C - \Psi; \quad where: \quad C = |\Phi_{k0}(0)|^2 + \Psi(0)$$

from which we can deduce that time scale of quasi-linear relaxation is $\gamma_{k0\max}^{-1}$ and the saturation level of electric potential fluctuation is $|\Phi_k|^2_{satur} = C - \Psi(V_{d1}), \quad where\ V_{d1} : \gamma_{k0}(V_{d1}) = 0$.

In case of fluctuations in the wave range of the order of electron Larmor radius similar estimations need more complicated solution method. This will be the aim of our future paper.

4.Conclusions

Dispersion relation for plasma in crossed static electric and magnetic fields has been analysed in the ω range up to second electron cyclotron resonance and for the wave numbers up to those of the order of inverse electron Larmor radius. Focusing attention on the transverse to both fields disturbance propagation direction and taking the electron-ion relative motion as the most important destabilizing factor we found two bands of unstable modes:

- one in the range of relatively long waves comparable with channel dimensions, which have the frequencies (in case of SPT100) in the range from hundreds of kHz up to some MHz, and maximal increment of the order of $10^5 s^{-1}$.
- and the second in the wave number range comparable with inverse electron Larmor radius. they are characterised by frequency of the order of some MHz and maximal increment approaching the order of magnitude of $10^7 s^{-1}$.

Using the quasi-linear theory of weak plasma turbulence we are able to estimate the relaxation of electric field fluctuation to the saturation level and its characteristic time at least in the “long wave regime”. The similar analysis in case of “short wave length” fluctuations needs more developed mathematical methods of solutions.

In above analysis the most important approximation is the reduction of real geometry of thrusters channel to rectangular cross section with additionally imposed periodic boundary conditions. More developed approach should contain better description of individual electron trajectories in crossed radial and axial fields. Also the question, if the sheath at the isolating walls could be treated as quasi-static, remain the opened one. In other case sheath instability could strongly affect the mode characteristics. The above mentioned problems will be discussed in our future papers.

Acknowledgements

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