EFFECT OF PLASMA-WALL RECOMBINATION AND TURBULENT RESISTIVITY ON THE CONDUCTIVITY IN HALL THRUSTERS

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Abstract

This paper deals with the operation of Hall plasma thrusters and the transversal to the magnetic field current in such systems. The problems of the transverse conductivity and related processes are treated in the framework of the well-known Morozov's model, the difference is in the addition of the mechanism of plasma recombination on the walls of the thruster and further volume ionization of the neutrals created at the walls. The distribution functions of the neutrals and secondary electrons generated by ionization are found, as well as the expression for the transverse current. The obtained result contains an additional correcting factor compared to the conventional Morozov's equation for this current and permits a better quantitative agreement of the theoretical expression and experimental results. We also estimate the importance of the generation of the transverse current due to turbulent resistance arising because of the strong Hall current in the thruster. Our estimates show that this phenomenon can play an important role.

I. Introduction

The near-wall conductivity was proposed by Morozov in 1968 [1] to explain the conductivity of collisionless plasma across the magnetic field in Hall thrusters. In the same work Morozov has noted that the conductivity could be explained by the presence of noise in plasma, however he dedicated the main attention to the near-wall conductivity (n.w.c.) arising from electron-wall collisions. The problem is more general, since it can refer to all the plasma systems with insulating walls transverse to the magnetic field and with applied electric field parallel to the wall. As a rule both the plasma density and the neutral density in these systems are low enough for the electron mean free path to be much larger than the inter-wall distance. Thus these plasmas can be considered as collisionless. Assuming electron-wall collisions, Morozov has developed a new approach for calculating the near-wall electron current [1]. This method consists in making use of integrals of motion for electrons in collisionless approximation. Thus having found the space-time evolution law of the electron velocity distribution function, the boundary conditions for this function were used, namely the electron velocity distribution function at the wall was chosen as maxwellian, with a certain temperature. Morozov [1] assumed that the temperature of the emitted electrons is different from that of the plasma electrons. Comparison with experiment brought about the need of choosing different shapes of the initial electron velocity distribution function [2], and of considering the secondary electron emission from the insulating wall [3]. We propose to consider two additional mechanisms in the framework of the original model. Despite of the experimental verification of the n.w.c. model [4], according to Morozov the problem of near-wall conductivity and related processes remains the main task to enhance the thruster efficiency and ensure longer lifetime and low level of noise [5].

The first additional mechanism we are going to consider in the present work is recombination-ionization in the near-wall plasma region and its influence on the electron current in Hall thrusters. These processes were used, in particular, to estimate the ion current in systems with a very strong magnetic field, so that both the ions and electrons were magnetized [6,7]. The second mechanism of generation of the transverse to the magnetic field current is the turbulent resistivity [8,9] that we will discuss in Sec. IV.

Let us first study the neutralization processes. We will use the approach developed by Morozov [1], adding the electron-ion wall recombination and volume ionization of neutrals emitted from the wall. An important question is the uncertainty concerning the energy of the neutral atoms leaving the wall.

Let us discuss in more detail the ion-wall interaction. Several processes are possible, depending on the energy and mass of the incident ions, as well as wall material and surface state. So we need to know the predominant processes for the interaction of Xe^+ ions and boron nitride walls for the energies of the incident particles in the range 30-500 eV. There are numerous papers devoted to the interactions of the noble gas ions with metal surfaces. The general conclusion is that the ions are completely neutralized, only about 0.01% of ions are reflected non-neutralized [10]. As for the electron ejection due to ion bombardment, it is also very low – around 0.005 electrons per ion [10]. Actually, we are dealing with insulating walls, so the previous data can only be used as a very rough estimate. However, even for the insulating surfaces the neutralization is close to 100% [11]. Sputtering can also play an important role in plasma-wall interactions starting from the energies of the order of ~100 eV. However, according to the experiment and numerical simulations [12], the sputtering for Xe^+ ions for these energies is negligible – it represents about 2 or 3% of the incident ion flux for the energy 500 eV. So, summarizing all the estimates and observations mentioned above, one can conclude that the main process in this energy range is the neutralization of the incident xenon ions and their subsequent reflection (probably after the adsorption by the wall).

Another important question is the temperature, or, more precisely, energy distribution of the atoms created during neutralization of the incident ions. We did not find the required data for Xe atoms. However, we can have an idea of a possible energy distribution from some speculations and, possibly, from numerous papers concerning the interaction of hydrogen atoms with the walls [13,14]. Since the Xe atoms are heavier than B or N (mass ratio is about 0.1), the direct reflection of the incident ions is relatively small, in other words we have some interaction of the incident ions and the wall material, i.e. thermalization, the energy of the hitting projectiles is mainly transferred to the atoms of the wall. As for the interaction of hydrogen with the C walls (the same mass ratio ~0.1), the energy of the reflected atoms is about 0.1 of their incident energy [13,14]. In other words, the energy of the reflected neutrals is essentially lower than their initial incident energy, the excess is deposited to the wall.

Let us assume that plasma moves along the magnetic field lines to the wall and represents a collisionless fluid having a mean velocity equal approximately to the ion acoustic velocity. As a matter of fact we are dealing with the expansion of plasma into vacuum, since the plasma disappears at the wall due to instant recombination. This process takes place on the scale comparable to Debye length, which is much less than the ionization length, λ_i:

$$\lambda_i = \frac{1}{n_e \langle \sigma V_e \rangle} V_n \qquad (1)$$

where n_e is the plasma density, $\langle \sigma V_e \rangle$ is the rate coefficient for ionization, V_n is the thermal velocity of the neutrals leaving the wall. Let us estimate the ionization length for the typical parameters of Hall thrusters using Xenon as working gas: $n_e \approx 10^{11} cm^{-3}$, $\langle \sigma V_e \rangle \approx 10^{-7} cm^3 s^{-1}$, for electron temperature in the range from 10 to 50 eV.

(a) If the neutral temperature is low (close to the wall temperature, ≈ 0.1 eV) the ionization length λ_i is a few mm. For the same plasma parameters the Coulomb collision length is larger by a factor 10^3. Thus the ionization length is comparable with the channel width, electron-ion collisions being negligible. At last, let us estimate the electron-neutral collision length. Assuming an upper limit for the neutral density of $10^{14} cm^{-3}$, we obtain for the electron-neutral collision length several tens of cm, and only for neutral density $10^{15} cm^{-3}$, we would obtain this length comparable to the channel width. These estimates show that the shortest length is the ionization length, and it should be considered in further discussion. Of course the Debye length, which is of the order of $2 \cdot 10^{-2} cm$ is much smaller than the ionization length. Similar estimates of the ionization length were made by Bugrova *et al* [15], but they compare this length to the longitudinal size of ionization and acceleration layers, which are shorter than the channel length. They require that the ionization length should be lower than the longitudinal size of the ionization and acceleration layers. Note that the channel length is 2 to 3 times larger than the channel width. It means that the necessary condition for the thruster operation, according to Bugrova *et al* [15], corresponds to the requirement that the ionization length is less or comparable to the channel width.

An open question is the near-wall neutral density. In steady state the plasma flux to the wall and the return flux of neutrals from the wall should be equal. This allows us to obtain the upper estimate for the near-wall neutral density n_{n0}:

$$n_{n0}V_n = n_e c_s,\ n_{n0} = \sqrt{\frac{T_e}{T_W}} n_e \qquad (2)$$

Here T_W is the wall temperature, T_e is the electron temperature, c_s is the ion acoustic velocity, M is the ion mass. This estimate gives the well-known result that the neutral density is approximately one order of magnitude larger than the plasma density.

(b) If the neutral temperature is high (comparable to the energy of ionization) the ionization length according to Eq.(1) is increased by approximately a factor of 10 and exceeds the channel width. Actually, as soon as the ionization length exceeds the channel width the plasma density will go down because the ionization process is hindered. The fast neutrals after multiple collisions with the walls acquire the wall temperature, the ionization length will go down and the case of low neutral atom temperature will be attained.

The purpose of this paper is to calculate the electron transverse current generated by the volume ionization of neutrals produced due to plasma-wall recombination process and to make an estimate of the influence of turbulent resistivity on this current.

II. Neutral atoms distribution function.

Let us introduce a coordinate system traditionally used in plasma thrusters, namely axis x is directed along the magnetic field, *i.e.* across the channel, with its origin at the external wall, the axis z is along the applied electric field, *i.e.* along the channel (see Fig. 1). Since the scale of the processes under consideration is much smaller than the radius of the thruster, we have chosen the Cartesian coordinate system. We will consider the problem of ionization of the neutral flux from the wall by the volume plasma. The plasma electrons have an energy of 2 - 3 times the ionization potential so they can ionize the neutrals. The secondary electrons formed by ionization have an energy of several eV [16].

The formation of the secondary electrons and ions is related to the density of neutral atoms at the point where ionization occurs. Since there are no collisions the neutrals move from the wall along a straight line, until the moment of their ionization. The equation describing the evolution of the neutral number is as follows:

$$\frac{dN_n}{dt} = -N_n n_e \left\langle \sigma V_e \right\rangle = -\nu_i N_n \qquad (3)$$

where ν_i is the ionization frequency depending on the electron distribution function and the ionization cross section, N_n is the number of neutrals. Since the velocity of the neutrals remains constant until the ionization moment we obtain for the steady state case in one-dimensional approximation the following:

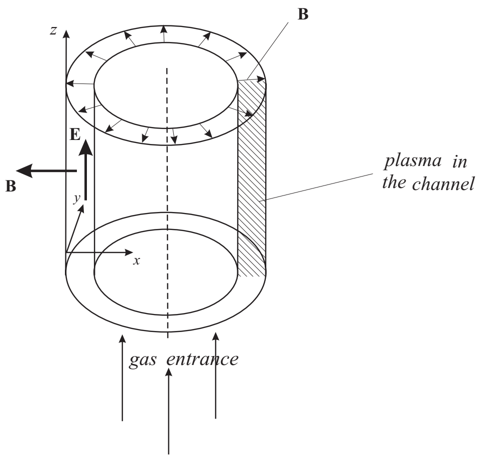

Fig.1. Schematic diagram of the plasma thruster, coordinate system.

$$V_{0n}\frac{dN_n}{dx} = -N_n n_e \left\langle \sigma V_e \right\rangle \qquad (4)$$

Hence the ionization length is given by the equation:

$$\lambda_i = \frac{V_{0n}}{n_e \left\langle \sigma V_e \right\rangle} \sim \frac{V_{0n}}{V_{Te}}\frac{1}{n_e\sigma} \ll \frac{1}{n_e\sigma} \qquad (5)$$

since the velocity of the neutral particles is much less that the electron thermal velocity V_{Te}. Our estimates in Sec. I show that the ionization occurs at rather short distances (less than the channel width). On the other hand the neutral and plasma densities are sufficiently small in order to neglect the influence of collisions along the neutral particle trajectories.

Let us consider now the steady state neutral distribution function corresponding to the stationary neutral flux from the wall. Then in the kinetic equation for the distribution function

$$\frac{\partial f_n}{\partial t} + \mathbf{V}_n \frac{\partial f_n}{\partial \mathbf{r}} = -\nu_i f_n \qquad (6)$$

one can omit the time derivative and find the solution, which becomes maxwellian at the wall :

$$\mathbf{V}_n \frac{\partial f_n}{\partial \mathbf{r}} = -\nu_i f_n \qquad (7)$$

The solution of this generally speaking 3D equation can be found in two ways. The first one is based on the hypothesis that the ionization length is small compared to the characteristic dimensions of the wall**.** Hence the distribution function depends only on the coordinate x perpendicular to the wall. The Eq. (7) becomes one-dimensional:

$$V_{nx} \frac{\partial f_n}{\partial x} = -\nu_i f_n \qquad (8)$$

and its solution is:

$$f_n \sim \exp(-\nu_i \frac{x}{V_{nx}}) \left(\frac{M}{2\pi T_n} \right)^{3/2} n_{n0} \exp(-\frac{M}{2T_n} \left(V_{nx}^2 + V_{ny}^2 + V_{nz}^2 \right)) \qquad (9)$$

Here T_N is the temperature of the neutrals, which is close to the temperature of the wall. The second approach consists in considering the motion of the neutrals along the characteristics in arbitrary directions. The neutral particle does not loose either momentum or energy, and the probability of its existence on the given trajectory decreases exponentially with time. Therefore one can consider an arbitrary point P located at the distance x from the surface and find the distribution function at this point (see Fig. 2). The particles arrive to this point along straight lines, *e.g.* from points A and B of the wall. The distribution function along these lines decreases exponentially, therefore for a given absolute value of the neutral velocity $|\mathbf{V}_n|$ we obtain:

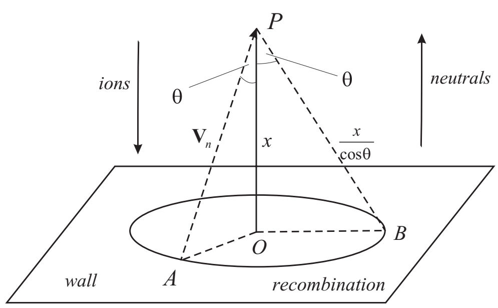

Fig.2. Schematic representation of calculation of neutral density in point P.

$$f_n(P) \sim \exp(-\nu_i \frac{x}{\cos\theta} \frac{1}{|\mathbf{V}_n|}) \left(\frac{M}{2\pi T_n} \right)^{3/2} n_{n0} \exp(-\frac{M}{2T_n} |\mathbf{V}_n|^2 \left(V_{nx}^2 + V_{ny}^2 + V_{nz}^2 \right))$$

Indeed the path of the particle is $\frac{x}{\cos\theta}$ and its velocity is $|\mathbf{V}_n|$. On the other hand since $|\mathbf{V}_n| \cos\theta = |V_{nx}|$, we come to the same result as that of Eq. (9).

Now we know the distribution function of the neutrals. In the following section, we will find the distribution function of the electrons produced by ionization.

III. The distribution function of electrons

Vlasov equation for electrons has the standard form:

$$\frac{\partial f_e}{\partial t} + \mathbf{V}_e \frac{\partial f_e}{\partial \mathbf{r}} - \frac{e}{m} \left(\mathbf{E} + \frac{1}{c} \mathbf{V}_e \times \mathbf{H} \right) \frac{\partial f_e}{\partial \mathbf{V}_e} = 0 \qquad (10)$$

It is known that the distribution function remains constant on the characteristics, which are defined by the equation of motion of electrons. Since we have constant applied electric and magnetic fields, the solution of this equation is the motion with constant velocity $c E/H$ along the y axis and cyclotron rotation around magnetic force line. As for the electron motion along the magnetic field, it is also occurring with constant velocity along the x axis. Hence

$$V_{ez} = V_{\perp} \sin(\omega_{He} t + \alpha)$$
$$V_{ey} = V_{\perp} \cos(\omega_{He} t + \alpha) + c\frac{E}{H} \qquad (11)$$
$$V_{ex} = V_{\|}$$

Let us establish the relation between the three constants in Eqs. (11) and the distribution function at the wall. At the moment $t = 0$ (corresponding to the ionization moment) the electron has the velocities V_{ex}^0, V_{ey}^0, V_{ez}^0. On the other hand we obtain from Eq. (11) that these velocities are equal to $V_{\|}$, $V_{\perp}\cos\alpha + c\frac{E}{H}$, $V_{\perp}\sin\alpha$ respectively. In other words,

$$V_{ex} = V_{\|} = V_{ex}^0$$
$$V_{ey} = V_{\perp}\cos(\omega_{He} t + \alpha) + c\frac{E}{H} = (V_{ey}^0 - c\frac{E}{H})\cos(\omega_{He} t) - V_{ez}^0 \sin(\omega_{He} t) + c\frac{E}{H} \qquad (12)$$
$$V_{ez} = V_{\perp}\sin(\omega_{He} t + \alpha) = (V_{ey}^0 - c\frac{E}{H})\sin(\omega_{He} t) + V_{ez}^0\cos(\omega_{He} t)$$

In the system of equations (12) the velocity components depending on time are expressed in terms of the initial velocity of the electron produced by ionization. We assume for simplicity that the initial electron distribution function is maxwellian, with a certain temperature $T_e^* \sim T_e \gg T_W \sim T_n$. As for the rate of electron generation, it is proportional to the rate of decrease of the neutral density. Let us calculate more precisely the rate of the electron generation. In order to determine the necessary factors, let us make use of the fact that these factors are defined by the product of the probabilities that a neutral will arrive to the point x_0 and the probability of ionization of this neutral in an interval dx_0. The first probability is $\exp(-\nu_i \frac{x_0}{V_{nx}})$ and the second one is $\frac{dx_0}{\lambda_i} = \frac{dx_0}{V_{nx}}\nu_i$. It should be pointed out that the factor $\langle \sigma V_e \rangle$ entering in ionization rate is averaged over all the plasma electrons responsible for the ionization of the neutrals. So it does not depend on velocities of newly born electrons. Thus, using the distribution function for the neutrals (9), the required electron distribution function has the following form in the vicinity of the point x_0:

$$df_e = n_{n0}\exp(-\nu_i \frac{x_0}{V_{nx}})\left(\frac{M}{2\pi T_n}\right)^{3/2}\exp(-\frac{MV_n^2}{2T_n})\nu_i \frac{dx_0}{V_{nx}} d^3\mathbf{V}_n \times$$
$$\times\left(\frac{m}{2\pi T_e^*}\right)^{3/2}\exp(-\frac{m\left(V_e^0\right)^2}{2T_e^*})d^3\mathbf{V}_e^0 \qquad (13)$$

To obtain the contribution to the electron current dJ_z from the layer of thickness dx_0, located near the point x_0, we have to multiply df_e by $-e$ and V_{ez} given by (12).The integration over z and y components of neutrals and newly born electrons is trivial and gives us

$$dJ_z = en_{n0}\nu_i dx_0 2\left(\frac{M}{2\pi T_n}\right)^{1/2}\int_0^\infty \frac{dV_{nx}}{V_{nx}}\exp(-\frac{MV_{nx}^2}{2T_n} - \nu_i\frac{x_0}{V_{nx}})c\frac{E}{H}\times$$

$$\times\left(\frac{m}{2\pi T_e^*}\right)^{1/2}\int_{-\infty}^{\infty} dV_{ex}^0 \sin\left(\omega_{He} t\right)\exp(-\frac{m\left(V_{ex}^0\right)^2}{2T_e^*}) \tag{14}$$

Since the electron is moving along the magnetic field with a constant velocity V_{ex}^0, the time in the system of Eqs. (12) can be replaced by $t=\left|\frac{x-x_0}{V_{ex}^0}\right|$. Let us note that $\left|x-x_0\right|$ is of the order of the Larmor electron radius, in other words much smaller than the ionization length, since the time t is measured in ω_{He}^{-1} for electron integral. Thus we have

$$dJ_z = en_{n0}c\frac{E}{H}\nu_i dx_0 2\left(\frac{M}{2\pi T_n}\right)^{1/2}\int_0^{\infty}\frac{dV_{nx}}{V_{nx}}\exp(-\frac{MV_{nx}^2}{2T_n}-\nu_i\frac{x_0}{V_{nx}})\times$$
$$\times\left(\frac{m}{2\pi T_e^*}\right)^{1/2}\int_{-\infty}^{\infty} dV_{ex}^0 \sin\left(\omega_{He}\left|\frac{x-x_0}{V_{ex}^0}\right|\right)\exp(-\frac{m\left(V_{ex}^0\right)^2}{2T_e^*}) \tag{15}$$

To obtain the total current it is necessary to integrate dJ_z over x and multiply by $2\pi R$, R is the mean channel radius.

$$I_z = 2\pi Ren_{n0}c\frac{E}{H}\nu_i 2\left(\frac{M}{2\pi T_n}\right)^{1/2}\int_0^L dx_0\int_0^{\infty}\frac{dV_{nx}}{V_{nx}}\exp(-\frac{MV_{nx}^2}{2T_n}-\nu_i\frac{x_0}{V_{nx}})\times$$
$$\times\left(\frac{m}{2\pi T_e^*}\right)^{1/2}\int_{-\infty}^{\infty} dV_{ex}^0 \exp(-\frac{m\left(V_{ex}^0\right)^2}{2T_e^*})\int_0^L dx \sin\left(\omega_{He}\left|\frac{x-x_0}{V_{ex}^0}\right|\right) \tag{16}$$

Using the same approximations as Morozov [1], i.e., $\rho_L \ll x_0 \ll \lambda_i \ll L$, in other words, the fact that the current flows only in the near-wall area, we obtain

$$I_z \approx 4\sqrt{\pi}Rn_{n0}ec\frac{E}{H}\left(\frac{2T_e^*}{m}\right)^{\frac{1}{2}}\frac{1}{\omega_{He}} \tag{17}$$

IV. Turbulent resistivity current.

Another mechanism that could generate some current transverse to the magnetic field is any sort of process that creates an effective collision frequency ν_{eff}. Indeed, the equation of motion for an electron in this case is

$$0=-e(\mathbf{E}+\frac{1}{c}[\mathbf{v},\mathbf{H}])-\nu_{eff}m\mathbf{v}.$$

The solution of this equation is

$$\mathbf{v}=\frac{c}{H}\frac{\left[\mathbf{E},\mathbf{h}\right]}{1+\frac{\nu_{eff}^2}{\omega_{He}^2}}-\frac{\nu_{eff}/\omega_{He}}{1+\frac{\nu_{eff}^2}{\omega_{He}^2}}\frac{c}{H}\mathbf{E}, \tag{18}$$

$$\mathbf{h}=\frac{\mathbf{H}}{H}.$$

So, depending on the effective collision frequency ν_{eff} we will have a certain small component of the Hall current in the direction parallel to the electric field:

$$\mathrm{v}_{ez} \approx c\frac{E}{H}\frac{\nu_{eff}}{\omega_{He}},\ \nu_{eff} \ll \omega_{He}.$$

However the frequency of collisions of electrons with various particles is very small, in other words we are dealing with collisionless plasma. So ν_{eff} is just some effective frequency that appears, for example, because of the turbulent resistance in Hall plasmas. Let us estimate the contribution of this resistivity to the creation of the current transverse to magnetic field.

Let us study in some more detail the mechanism of this instability. The flow of electrons rotating in crossed magnetic and electric field with the velocity $u = cE/H$ can generate an ion acoustic instability if the velocity of the electrons is larger than the ion acoustic velocity [8,9].

$$c_s < u < \mathrm{v}_{Te}$$

This instability, in turn, generates a group of hot ions, which causes the reduction of the growth rate and the instability stops. It limits the velocity of the electron flow to a value inferior to cE/H and in this way we have an effective turbulent resistance, and hence the collision frequency ν_{eff}. The general equation of the balance for the instability is [8,9]

$$0 = \omega_{pi}W\left[\frac{u}{v_{Te}} - \frac{\omega}{kv_{Te}} - a\frac{\omega}{kv_{Te}} - d\left(\frac{T_i}{T_e}\right)\frac{W}{n_0T_e}\right], \qquad (19)$$

$$d = \frac{k}{\Delta k} \cong 1,\ \nu_{eff} = \omega_{pe}\frac{W}{n_0T_e},$$

$$a = \alpha - 1,\ \alpha = 5.36\cdot\left(\frac{M}{m}\right)^{1/4}$$

Here W is the energy density of noise in plasma, $W \ll nT_e$; the first term corresponds to the excitation of the waves by the electron flow with the velocity u, the second one – to Landau damping, the third one – to the reduction of the instability due to hot ions and the last term represents non-linear Landau damping.

From the equation (19) we can see that two instability modes are possible: quasilinear mode if the last term is negligeable (and $u \approx \alpha c_s$) and non-linear mode when $u \gg \alpha c_s$ and the nonlinear term plays an important role.

We can estimate the effective collision frequency ν_{eff} from quasilinear mode approximation. The electron rotation velocity in this case is limited by (cf. eq. (18))

$$u = \alpha c_s = c\frac{E}{H}\frac{1}{1+\frac{\nu_{eff}^2}{\omega_{He}^2}}$$

and

$$\nu_{eff} = \omega_{He}\sqrt{\frac{cE}{Hu} - 1}.$$

To calculate the velocity of the electrons u we need to use the complete balance equation (19). Unlike hydrogen the last nonlinear term is important in xenon. So bearing in mind that $T_i / T_e \approx c_s / u$ and for the instability $\omega \sim kc_s$, we rewrite the eq. (19) as follows:

$$0 = \frac{u}{\mathrm{v}_{Te}} - \alpha \frac{c_s}{\mathrm{v}_{Te}} - \frac{c_s}{u} \frac{\omega_{He}}{\omega_{pe}} \sqrt{\frac{\frac{cE}{H}}{u} - 1}. \qquad (20)$$

For the small values of the ratio $\omega_{He} / \omega_{pe}$ we can put in the last term $u \approx \alpha c_s$ and the average electron velocity along the magnetic field is

$$\mathrm{v}_E = \alpha c_s \cdot \left(c \frac{E}{H} \frac{1}{\alpha c_s} - 1 \right)^{1/2} \qquad (21)$$

So the transverse current contribution due to the turbulent resistance is

$$I_t \approx en2\pi RL \cdot \alpha c_s \left(c \frac{E}{H} \frac{1}{\alpha c_s} - 1 \right)^{1/2} \qquad (22)$$

Let us now compare the two currents – classic Morozov expression and the turbulent current:

$$\frac{I_t}{I_0} \approx \frac{\sqrt{\pi} L \cdot \alpha c_s \left(c \frac{E}{H} \frac{1}{\alpha c_s} - 1 \right)^{1/2}}{c \frac{E}{H} \frac{\mathrm{v}_{Te}}{\omega_{He}}} \qquad (23)$$

We can estimate the maximum value of the square root:

$$\left(\frac{c \frac{E}{H}}{\alpha c_s} - 1 \right)^{1/2}_{\max} = \frac{\nu_{eff\,\max}}{\omega_{He}} \le \frac{\gamma_{inst}}{\omega_{He}} \approx \frac{u}{v_{Te}} \cdot \frac{\omega_{pi}}{\omega_{He}} \approx 5.36 \cdot \left(\frac{m}{M} \right)^{3/4} \frac{\omega_{pe}}{\omega_{He}}$$

And after that the eq. (23) can be rewritten as

$$\frac{I_{t\max}}{I_0} \approx \frac{\sqrt{\pi} L \cdot \alpha c_s 5.36 \cdot \left(\frac{m}{M} \right)^{3/4} \frac{\omega_{pe}}{\omega_{He}}}{c \frac{E}{H} \frac{\mathrm{v}_{Te}}{\omega_{He}}} \approx \sqrt{\pi} 5.36 \cdot \left(\frac{m}{M} \right)^{3/4} \frac{\omega_{pe}}{\mathrm{v}_{Te}} L \qquad (24)$$

For most of the Hall thruster systems $n \sim 10^{11} cm^{-3}$, $T_e \sim 20eV$, $B \sim 200Gs$, $L \sim 1cm$ and the ratio (24) is about 0.1. This estimate is obtained in the conditions of quasilinear approximation and weak turbulence. For higher values of electric field the obtained ratio can be essentially increased. So, we can see that the currents are comparable, so this mechanism can be also added to explain the difference between the original Morozov's result and experiment.

V. Discussion.

In Sec. I we have made the estimates of maximum neutral density n_{n0} in the vicinity of the wall (see Eq.(2)). Substituting this expression to Eq. (17) we arrive to the following expression of the transverse current:

$$I_z = 2\pi R \frac{n_e ec}{\sqrt{\pi}} \frac{E}{H} \left(\frac{2T_e^*}{m} \right)^{\frac{1}{2}} \frac{1}{\omega_{He}} 2 \left(\frac{T_e^*}{T_N} \right)^{\frac{1}{2}} \qquad (25)$$

This expression has an additional factor $2\left(\frac{T_e^*}{T_N}\right)^{1/2}$ compared to the result in [1]. The coefficient "2" has appeared because the electrons after the ionization of neutrals move in all the directions, while in the original Morozov's model they are emitted from the wall only into a half-space. Let us estimate this factor. Since T_e^* is the temperature of the electrons produced by ionization and is of the order of plasma electron temperature T_e, and the temperature of the neutrals T_N is of the order of wall temperature T_W, as we discussed in Sec. I, this factor is of the order of 10.

Let us compare the obtained results with experiment. If the n.w.c. model is valid the currents in different size thrusters should be proportional to the mean radius of the channel for the same plasma conditions. Unfortunately systematic measurements are unavailable and comparison of different thrusters at the same plasma parameters cannot be effected. Therefore we can limit ourselves to comparing the thoroughly studied thrusters, such as SPT-50 [17] and SPT-100 [18]. For SPT-50 we have the discharge current of 1A, $D = 50mm$, plasma density $n_e \approx 10^{11} cm^{-3}$, the electron temperature 10 to 20 eV, the average electric field 100 V/cm, the magnetic field 300 Gauss [17]. According to the n.w.c. model [4] the calculated current is a few tenths of Ampere, *i.e.* one order of magnitude less than measured. Similar estimates for SPT-100 show that the calculation also underestimates the current by a factor of ten. Since the results obtained in this paper give a required corrective factor of ten to the standard n.w.c. results, one can consider that the main features of the Hall thruster were correctly described by the original method. Taking into account the plasma recombination at the wall followed by volume ionization allows attaining a better quantitative agreement.

A second conclusion following from this work is the importance of using of a heavy gas as the working fluid. Since the ionization length is inversely proportional to the square root of the ion mass, for light gases the ionization length can exceed the channel width ($11-15mm$). The ionization potential of these gases is also higher than in heavy gases, thus prohibiting the operation of thrusters with existing design in these gases.

Another important mechanism of the current generation is the turbulent resistance arising because of the instability generated by the Hall electron flow in the channel. We have estimated the importance of this phenomenon and made some calculations in the simple case of a weak turbulence. The results show that this mechanism should be taken into account while considering the phenomena taking place in the channel of a Hall thruster. It is a volume process of the current generation, unlike the other wall related mechanisms mentioned above.

Acknowledgement.

This work was carried out within Groupement de Recherche CNRS/CNES/SNECMA/ONERA N° 2232. We gratefully acknowledge the enlightening discussions with E. Behrisch and E. Parilis.

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