Progress in Hydrodynamic Modeling of Hall Thrusters

IEPC-2005-044

Presented at the 29th International Electric Propulsion Conference, Princeton University, October 31 – November 4, 2005

Michael Keidar*, and Iain D. Boyd†
University of Michigan, Ann Arbor MI

Abstract: In this paper we present a model that improves several aspects for Hall thruster simulations using a two-dimensional fluid formulation and describe several directions in which the current state-of-the-art can be advanced. The magnetic mirror effect is studied in the channel of a Hall thruster. It is shown that gradients in magnetic field affect the presheath structure and electric potential distribution. The length of the radial presheath region decreases in the presence of a magnetic field gradient. The two-dimensional potential shape can be affected by proper choice of the magnetic mirror ratio. In particular, it is possible to obtain a concave shape of the potential profile in the channel even in the case of a primarily radial magnetic field. This in turn can be used to effictively control the ion dynamics in the acceleration region. A new formulation of the original near-wall conductivity model that takes into account an axial electric field in the sheath is presented. It is shown that in the specific case of small ratio of the secondary electron temperature to the bulk electron temperature, the near wall conductivity can be significantly enhanced.

I. Introduction

A HALL thruster is a propulsion device in which ions are accelerated in a quasi-neutral plasma. An electric field in the Hall thruster discharge is sustained across the magnetic field. Because of this feature, Hall thrusters offer a much higher thrust density than other types of ion thrusters. Most of the potential drop in this configuration is concentrated in the region of large magnetic fields. This electric field is responsible for ion acceleration. Typically, the radial component of the magnetic field dominates in the acceleration region. Consideration of only the radial component of the magnetic field leads to a one-dimensional picture of the plasma flow in the Hall thruster channel. However it is an accepted notion that a concave shape of the magnetic field leads to many advantages in thruster operation.[1] A recent study further explored advantages of using a plasma lens configuration in which the magnetic field has a concave shape in the channel.[2] A plasma lens magnetic configuration leads to an additional effect of increased magnetic field magnitude near the channel walls in comparison with the channel centerline, the so-called magnetic mirror effect.[3] However, it is not understood how the magnetic mirror effect will affect the potential distribution and ultimately the ion dynamics in the Hall thruster. Very recent study of interior potentials structures in Hall thruster operation on xenon and krypton revealed some interesting unexpected features related to magnetic field effect.[4]

* Research Scientist, Department of Aerospace Engineering, keidar@umich.edu.
† Professor, Department of Aerospace Engineering.

Recently, many models were developed to describe various features of Hall thruster operation[5,6,7,8,9,10,11,1213,14]. Generally, models can be divided into three categories, namely fluid, particle and hybrid particle-fluid. Despite significant progress in modeling Hall thrusters, many key issues remain still unresolved that limits applicability and practical usage of the modeling in general. Among others, this list includes the electron transport mechanism across the magnetic field, wall interactions, and plume divergence mechanism. Many similar assumptions are employed in these different models, thus advancing some of the basic formulation in the framework of any existing model will help general advancement of all modeling. In this paper we improve several aspects for any model using a two-dimensional fluid formulation and describe several directions in which the current state-of-the-art can be advanced.

The magnetic field distribution is extremely important in Hall thrusters since the electric potential is governed by the magnetic field so that equipotential contours tend to line up with the magnetic field lines.[15] Indeed, with a correction of a logarithmic factor due to the possible density variation along the magnetic field, this is the case in most conventional Hall thruster channels.[16,17] In addition, it was recently shown that the electric potential may deviate from the thermalized potential due to radial gradients of magnetic field and electron temperature.[10,18] Other factors, such as segmented electrodes, can also affect the potential distribution inside the channel.[19] Understanding the coupling between the potential and magnetic field distributions has tremendous implication for our understanding of Hall thruster operation and may help to formulate any Hall thruster model. Current state-of-the-art Hall thruster models rely on certain assumptions about the potential distribution and therefore most predictions are limited to simplified cases.

In this paper, we attempt to develop a more general formulation for the potential distribution in a Hall thruster taking into account the magnetic mirror effect on the electron dynamics. Two related effects are considered, namely, the effect of the magnetic field gradient on the presheath structure in the 1D radial presheath formulation, and dependence of the 2D potential distribution in the Hall thruster channel on the magnetic field gradient. In addition, a new formulation of the original near-wall conductivity model that takes into account an axial electric field in the sheath will be presented. It will be shown that under certain conditions, i.e. small ratio of the secondary electron temperature to the bulk electron temperature, the near wall conductivity can be significantly enhanced.

II. Magnetic mirror effects

We start by formulating a model of the plasma flow between two dielectric walls for a Hall thruster taking into account magnetic field variation in the radial direction as shown schematically in Fig.1. We consider the magnetic mirror effect by taking into account only the radial component of the magnetic field.

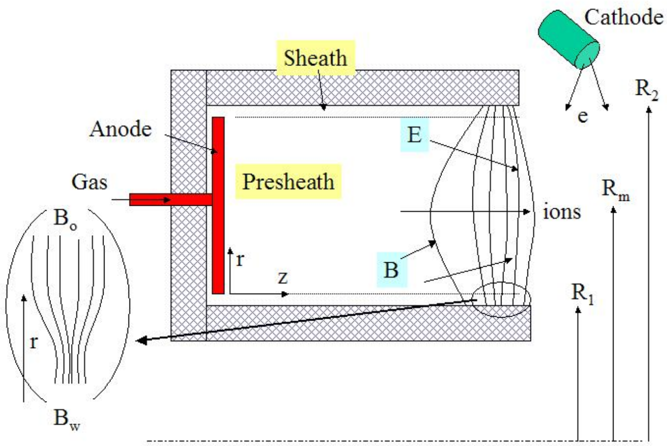

Figure 1. Schematic of the Hall thruster. In the enlarged area the simplified magnetic field geometry is shown.

Increase of the magnetic field magnitude from the channel centerline towards the wall leads to an additional force that acts on the electrons. Let us consider a weakly divergent magnetic field, i.e radial magnetic field variation along *r* independent of *z* as shown schematically in Fig. 1. In this case, the adiabatic invariant is conserved along the magnetic field that leads to the so-called magnetic mirror effect. Under these conditions it is simple to show that the additional force acting on the electrons depends on the electron energy.[20] In this case the electron momentum equation along the magnetic field in the absence of current along the magnetic field can be written as follows:

$$0 = en\nabla\varphi - nkT_e \frac{1}{B}\nabla B - \nabla(nkT_e) \tag{1}$$

where n is the electron density, φ is the potential, B is the magnetic field, and T_e is the electron temperature. In the case of small density gradient, the potential drop is such that the potential is more positive in the high magnetic field region[21]

$$\varphi_{\max} - \varphi_{\min} = \frac{kT_e}{e}\ln\frac{B_{\max}}{B_{\min}} \tag{2}$$

Based on this effect a so-called End-Hall ion source was developed.[20] Let us start from analysis of the quasi-neutral plasma (presheath) problem in the Hall thruster. It was shown previously that the presheath scale length becomes comparable to the channel width under typical conditions of the Hall thruster plasma flow.[8] Thus, a model for the quasi-neutral plasma region is extended up to the sheath edge in order to provide boundary conditions at the plasma-sheath interface as shown in Fig. 1. In this model, we will consider a 1D presheath structure in the radial direction between the plasma bulk and the dielectric wall.

The presheath model is based on the assumption that the quasi-neutral region length (i.e. the channel width, Fig. 1) is much larger than the Debye radius and therefore we will assume $Z_i n_i = n_e = n$, where Z_i is the ion mean charge, n_i is the ion density and n_e is the electron density. For simplicity, only single charge ions are considered in this paper (Z_i = 1). A condition that the magnetic field has only a radial component, $B_r = B$, is imposed. In the hydrodynamic model it is additionally assumed: (i) the system reaches a steady state, (ii) the electron component is not inertial, i.e. $(\boldsymbol{V_e}\nabla)\bullet\boldsymbol{V_e}=0$, (iii) ionization is neglected. Under these assumptions, the following system of equations describes the ion component of the quasi-neutral plasma:

$$nm_i(V_i\nabla)V_i = enE - \nabla P_i - nm_i V_i \nu_c \quad (3)$$

$$\nabla(nV_i) = 0 \quad (4)$$

where P_i is the ion pressure and ν_c is the total ion collision frequency. By combining Eqs. 1,3,4 one can obtain the equation for the ion velocity in the presheath:

$$(V - \frac{C_s^2}{V})\frac{dV}{dr} = -\nu_c V - \frac{kT_e}{m_i}\frac{1}{B}\frac{dB}{dr} \quad (5)$$

where C_s is the Bohm velocity, $C_s = \sqrt{\frac{kT_e}{m_i}}$. Along the magnetic field, the equation for the potential distribution reads:

$$0 = \frac{d\varphi}{dr} - \frac{kT_e}{eB}\frac{dB}{dr} - \frac{kT_e}{en}\frac{dn}{dr} \quad (6)$$

We use the following normalized variables: $u=V/C_s$, $\phi=e\varphi/kT_e$, $\alpha=\nu_c\rho_e/C_s$, $\rho_e=m_eC_s/(eB)$, $\chi=r/\rho_e$. After normalization, the system of equations will have the following form:

$$\frac{1-u^2}{u}\frac{du}{d\chi} = \alpha u + \frac{d\ln B}{d\chi} \quad (7)$$

$$\frac{d\phi}{d\chi} = \frac{d\ln B}{d\chi} + \frac{d\ln n}{d\chi} \quad (8)$$

In the solution of the system of equations (7-8), the plasma cannot overcome the ion sound speed as follows from Eq. 7. Therefore, the system of equations (7-8) determines the plasma parameter distribution in the presheath up until the presheath-sheath interface.

The potential distribution in the presheath is shown in Fig. 2 with magnetic field gradient as a parameter. One can see that the potential drop in the presheath decreases with increased magnetic field gradient. For instance when $\frac{1}{B}\frac{dB}{d\chi}$ increases from 0 to 0.5 the potential drop decreases by a factor of 2. In addition, the length of the presheath decreases with increase of the magnetic field gradient as shown in Fig. 2.

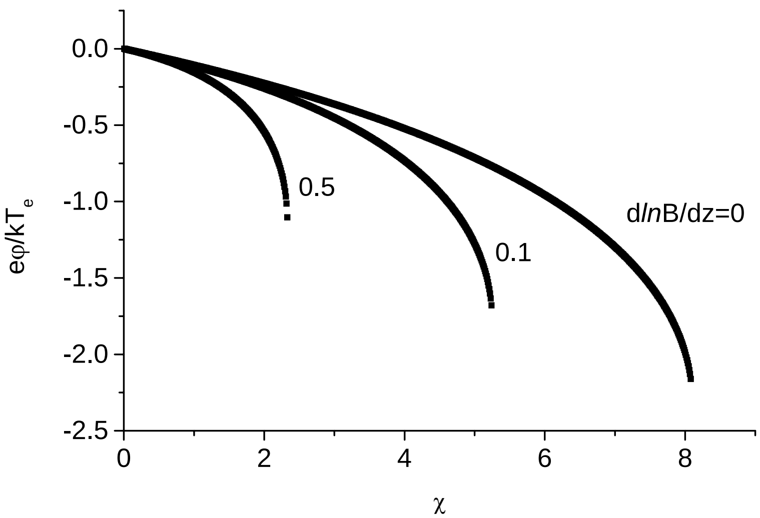

Figure 2. Potential distribution in the presheath.

It is important to note that the potential distribution along the magnetic field can be determined from Eq. 6. This is the generalized form of the so-called "thermalized" potential introduced by Morozov.[14,15] It is important to understand how the magnetic field gradient affects the potential distribution along the magnetic field, since it directly affects the ion dynamics in the Hall thruster channel.

In order to study this effect, we use a two-dimensional model of the plasma flow in a Hall thruster described elsewhere[8] and having the following features. A hydrodynamic model is employed in a 2-D domain assuming that the system reaches a steady state. In order to simplify the problem without missing the major physical effects, we consider one-dimensional flow of the neutrals. The momentum and mass conservation equations for ions and neutrals and energy balance are considered. Electron conductivity is specified to be governed by Bohm type anomalous conductivity across the magnetic field.[8] We modified this model by taking into account the magnetic field gradient along the magnetic field. In this case the 2D potential distribution is calculated according to Eq. 6.

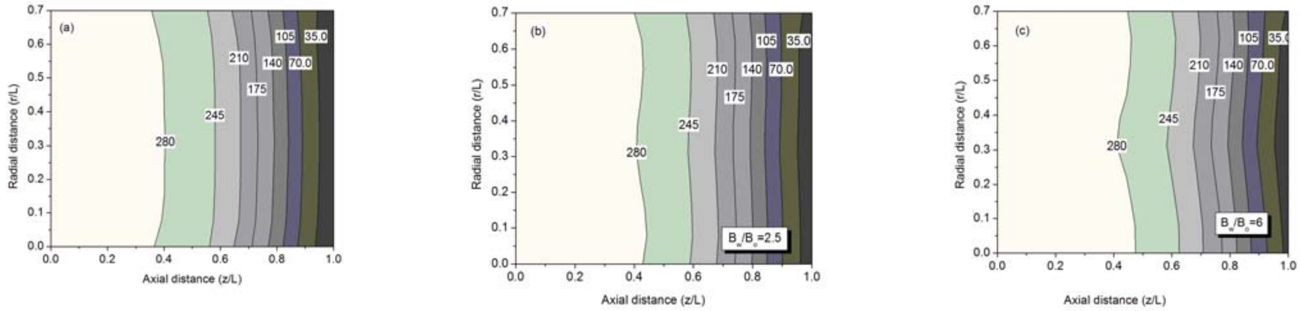

Figure 3. 2D potential distribution in the Hall thruster channel. *L* is the length of the Hall thruster channel. a) B_w/B_o=1; b) B_w/B_o=2.5; c) B_w/B_o=6;

Calculated 2D potential distribution in the Hall thruster channel is shown in Fig.3 with magnetic mirror ratio (ratio of the magnetic field near the wall to the magnetic field in the center of channel, B_w/B_o) as a parameter. One can see that the magnetic field gradient significantly affects the potential distribution. A higher magnetic field gradient leads to more positive potential in the high magnetic field region. In turn, this changes the shape of the equipotential lines. Usually, the magnetic mirror ratio is about 2-3 (Refs. 2,3). However taking into account the concave shape of the magnetic field (i.e. extending our consideration beyond the simplified case with only radial magnetic field component) the magnetic mirror ratio can be up to 6 along a magnetic field line.[2,3] It should be noted that a high magnetic mirror ratio leads to a concave potential profile even in the case of a perfectly radial magnetic field.

In addition, a model of the plasma flow in a Hall thruster channel was developed that takes into account the two-dimensional current conservation effect and relies on some experimental input parameters, such as magnetic field and electron temperature distribution.[10] The model was an attempt to explain the experimentally found non-uniform potential distribution across the thruster channel. This effect was explained by the change of the electron mobility across a magnetic field due to the magnetic field gradient and due to the electron current along the magnetic field driven by the electron temperature gradient.[10]

In the next section, the electron transport mechanism in Hall thrusters is studied. A new formulation for near-wall conductivity is proposed that takes into account details of the near wall sheath.

III. Near-wall conductivity

The idea of near wall conductivity (NWC) stems from the fact that typically, in the Hall thruster channel, the mean free path for electron neutral collisions is about 1 m, while the distance between walls is about 1 cm. Therefore electron collisions with the wall happen much more often than collisions with neutral particles (the same conclusion is true for electron–ion collisions, electron-electron collisions since ionization degree is about 0.01 or less). Without the presence of the axial electric field,electron reflection in the sheath is mirror-type and therefore cannot contribute to the conductivity. This makes the axial electric field one of the most important factors in determining the electron transport.

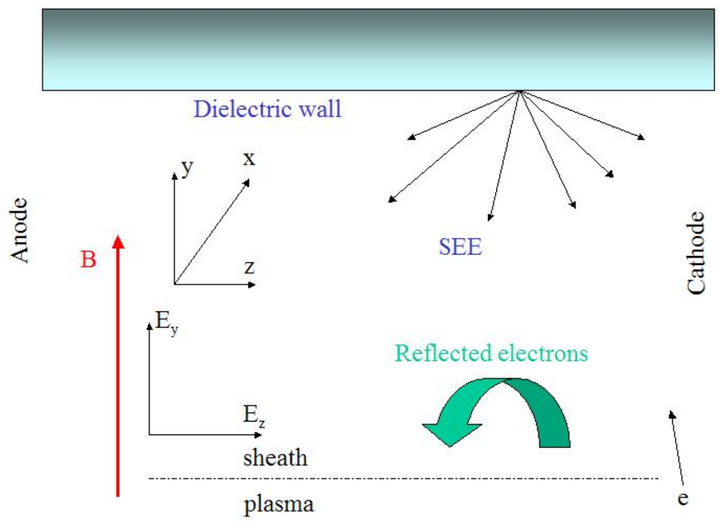

Figure 4. Schematic of electron interactions in the sheath

Possible electron trajectories in the sheath near the dielectric are shown schematically in Fig.4 dependent on the initial velocity at the sheath edge. Two electron populations exist dependent on the electron energy distribution function and sheath potential drop. Reflected electrons are the low energy population of the energy distribution having energy smaller than the potential drop in the sheath. On the other hand, energetic electrons transit through the sheath and collide with the wall thus leading to secondary electron emission (SEE) as shown schematically in Fig.4. For typical Hall thruster parameters (electron temperature 20-30 eV, wall material is boron nitride) the SEE coefficient is about 1. Therefore, the sheath reaches the space charge saturated regime associated with a non-monotonic potential profile. In this case, the sheath voltage drop U_w is relatively small and is about T_e. [22] In this case, the fraction of electron current colliding with the wall is large. One should therefore consider the SEE effect on the transport across the magnetic field. SEE electrons have an angular distribution that depends on the energy of the primary electrons and angle of incidence. In the presence of the axial electric field in the sheath, the SEE angular distribution should change so that electrons would have some preferable injection in the direction of the electric field. This effect should also contribute to electron transport across the magnetic field. The frequency of electron collisions with walls can be estimated as:

$$\nu_{ew} = n_a < \sigma V_e > \sim < V_e > / h \tag{9}$$

where $<V_e>$ is the average electron velocity and h is the distance between walls. Previously, Baranov *et al*.[23] proposed to take into account that only a fraction of electrons will collide with walls due to reflection in the sheath, i.e

$$\nu_{ew} = \frac{\langle V_e \rangle}{h} \exp\left(-\frac{\Delta\varphi_y}{T_e} \right) \tag{10}$$

However, taking into account that electron reflection from the sheath boundary can also contribute to the near-wall conductivity (as will be shown below), one can conclude that the collision frequency should be close to $<V_e>/h$. In the next section, we describe the model of the near-wall conductivity taking into account details of electron interactions in the sheath.

IV. Analysis of the near-wall conductivity in a Hall thruster channel

If the mean free path in the Hall thruster channel is much larger than the channel width, as happens typically in Hall thrusters, the electron collisions with the channel wall start to play a significant role. Morozov developed a model that calculates the near-wall current.[24] The method consists in solving equations for integrals of motion for electrons in a collisionless approach. The electron distribution function at the wall was chosen to be Maxwellian. The model was further developed over the last decades.[25,26] Recently, a new formulation of the NWC problem based on Morozov's approach was proposed[27] taking into account ion neutralization near the dielectric wall. It was shown that the resulting NWC current contains a correction factor and permits better quantitative agreement with experiment. Indeed, recent two-dimensional simulations of the ion dynamics in the Hall thruster demonstrated a significant effect of the ion neutralization near the walls.[28]

Phenomenologically, the NWC is the result of electron collisions with walls and consequent cycloidal motion along the magnetic field. Spatially oscillating currents is the essence of the NWC. Electrons reflecting from the wall are not monoenergetic and therefore the resulting current oscillation will rapidly decay with distance from the wall with most electron current concentration in the near wall region giving the appropriate name for this effect, near wall conductivity.

However, we want to point out an additional important effect that was not considered. Secondary electrons interact with electric fields (both axial and radial) in the sheath and thus their energy distribution function (EDF) is modified. In this section, the NWC problem taking into account this effect is formulated. It should be pointed out that EDF modification due to the radial electric field was very recently considered using a similar formalism. We adopt the mathematical description proposed originally by Morozov[23]. The present model is based on the following assumptions:

a) plasma properties are spatially uniform
b) The EDF of the secondary electrons is Maxwellian with temperature T_w which is different from the bulk electron temperature T_e.
c) Electrons are accelerated in the sheath by potential drops $\Delta\varphi_z$ and $\Delta\varphi_y$ in both axial and radial directions.

Let us describe the NWC mechanism taking into account SEE. The main idea is that the distribution function of the emitted electrons is shifted by the electric field in the sheath. Since both axial and radial electric fields are present, the distribution function is shifted in both directions and the EDF is centered along the direction of the electric field. The electron dynamics can be fully described by the collisionless kinetic equation for distribution function $f(t,\boldsymbol{r},\boldsymbol{V})$:

$$\frac{\partial f}{\partial t} + V\frac{\partial f}{\partial r} - \frac{e}{m}(E + V \times B)\frac{\partial f}{\partial v} = 0 \tag{11}$$

where E is the electric field, B is the magnetic field. The following distribution function of the emitted electrons from the wall is assumed [23]:

$$f(v) = n_0 \left(\frac{m}{2\pi k T_w}\right)^{3/2} \exp\left(-\frac{mV^2}{2kT_w}\right) \tag{12}$$

where n_o is the electron density, T_w is the temperature of emitted electrons, which is unknown and remains a free parameter of the problem. Further consideration is based on the fact that the distribution function is constant along the characteristics, which are the equation of motion for electrons.[23] Assuming constant electric and magnetic field, the solution of the equation of motion is the electron drift with constant velocity *E/B* along the *x*-axis and cyclotron rotation. Let us use the following variables

$$V_E = \frac{E_z}{B};\ d_w = \frac{m}{2kT_w};$$

We take into account that typically, in Hall thrusters, the electron Larmor radius is much larger than the Debye length. Therefore, we neglect effect of the magnetic field in the sheath. The equations of motion for the electron (characteristics) have the following form:

$$\frac{dV_x}{dt} = -\omega V_z \tag{13}$$

$$\frac{dV_y}{dt} = 0 \tag{14}$$

$$\frac{dV_z}{dt} = -\frac{eE_z}{m} + \omega V_x \tag{15}$$

where ω is the electron cyclotron frequency. Using the above assumptions and conditions one can integrate the equation for characteristics:

$$V_{ez} = (V_{ex}{}^{o} - V_E)\sin(\omega t) + V_{ez}^{o}\cos(\omega t) \qquad (16)$$

$$V_{ex} = V_E + (V_{ex}{}^{o} - V_E)\cos(\omega t) - V_{ez}^{o}\sin(\omega t) \qquad (17)$$

$$V_{ey} = \sqrt{V_{ey}^{0} + \frac{2e\Delta\varphi_y}{m}} \qquad (18)$$

where $V_{ex}{}^{0}$, $V_{ez}{}^{0}$, $V_{et}{}^{0}$ are the velocity at the wall. It was taken into account that electrons are accelerated across the sheath having potential drop of $\Delta\varphi_y$. The current density (z component) can be calculated as follows:

$$j_{ez} = \int_{-\infty}^{\infty}\int_{\alpha_y}^{\infty}\int_{\alpha_z}^{\infty} f(v)V_z dV_x dV_z dV_y \qquad (19)$$

where $\alpha_y = \sqrt{\frac{2e\Delta\varphi_y}{m}}$ and $\alpha_z = \sqrt{\frac{2e\Delta\varphi_z}{m}}$. Now we substitute velocity components according to equation of characteristics (Eq.16-18). In this case, one can arrive at the following expression for the electron current density:

$$j_{ew} = n_0 \frac{E}{B}\left(\frac{m}{2\pi kT_w}\right)^{1/2} \exp\left(\frac{e\Delta\varphi_z}{kT_w}\right)\exp\left(\frac{e\Delta\varphi_y}{kT_w}\right) \int_{\sqrt{\frac{2e\Delta\varphi_y}{m}}}^{\infty} \exp\left(-\frac{mV_y^2}{2\pi kT_w}\right)\sin\left(\omega\frac{y}{V_y}\right)dV_y \quad (20)$$

Let us introduce the following new variables:

$\alpha = \frac{V_y}{\sqrt{\frac{2kT_w}{m}}}$ and $s = \frac{\omega y}{\alpha\sqrt{\frac{2kT_w}{m}}}$. In this case the a new function can be introduced which is the integral in Eq. 20

$$Q(s) = \int_{\sqrt{\frac{2e\Delta\varphi_y}{m}}}^{\infty} \exp(-\alpha^2)\sin\left(\frac{s}{\alpha}\right)d\alpha \qquad (21)$$

where *s* is the non-dimensional distance from the wall (y direction) and the function *Q(s)* determines the current distribution as a function of that distance. In essence, this function is similar to one introduced originally by Morozov[23] with one exception, i.e. the potential drop in the sheath is taken into account. In that sense, the present approach is similar to recent work of Barral et al.[29]. The difference is that, in addition, we take into account electron acceleration in the sheath along the axial electric field component. The current density due to NWC can be expressed as follows:

$$j_{ew} = n_0 \frac{E}{B}\left(\frac{m}{2\pi kT_w}\right)^{1/2} \exp\left(\frac{e\Delta\varphi_z}{kT_w}\right)\exp\left(\frac{e\Delta\varphi_y}{kT_w}\right)\times Q(s) \qquad (22)$$

One can see that the function Q(s) provides the dependence of the current density on the distance from the wall. The calculated dependence of Q(s) is shown in Fig. 5.

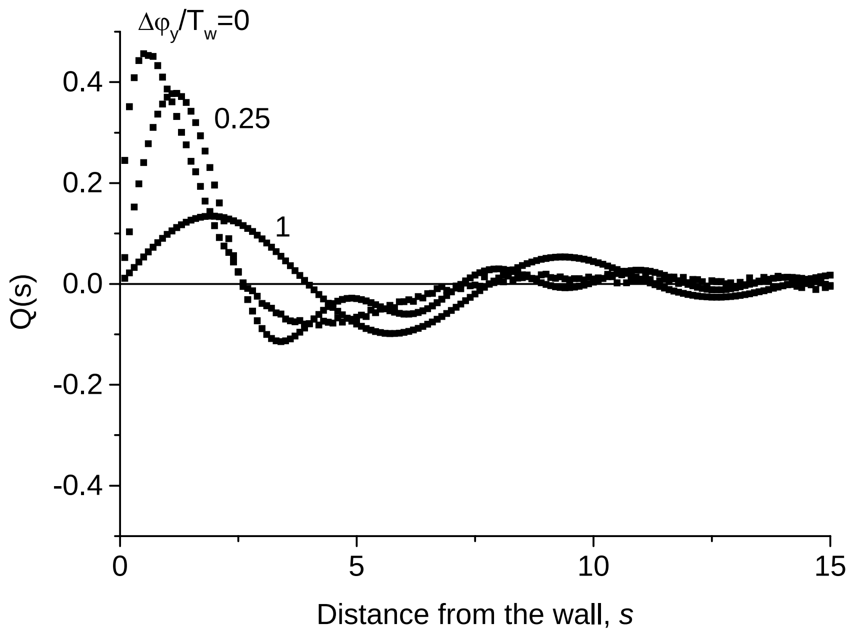

Fig. 5. Calculated function Q(s) as a distance from the wall with sheath potential drop as a parameter

One can see that current is concentrated near the wall in the simplest case of $\Delta\varphi_y$=0. This case corresponds to Morozov's original solution [23]. One can see that, generally, the sheath voltage leads to decreased current concentration near the wall and to more uniform current distribution across the channel between the two walls. This was a reason that led some authors to conclude that NWC may be a misnomer [28].

However it was indicated by some authors that in this formulation, the NWC current underpredicts the measured values [26]. Below we will consider additional an effect associated with electron interactions in the sheath that lead to enhancement of the NWC current density. The main idea is that the electric field along the wall can affect the near-wall current by producing an additional velocity shift in the axial direction. Typically in the Hall thruster channel the axial electric field is E_z=2-3x10^4 V/m, which is smaller than the typical radial electric field in the sheath. However it will be shown that in some cases the axial electric field can be an important factor contributing to NWC.

The current density increases by a factor of $\exp\left(\frac{\Delta\varphi_z}{T_w}\right)$ as can be seen from Eq.20 in which we have to know the potential drop $\Delta\varphi_z$ in the axial direction. In order to estimate the effective potential drop in the axial direction, let us consider in some detail the electron motion in the sheath. It was stated above that typically, in a Hall thruster acceleration channel, the saturated space charge sheath occurs.[30] In this case, the potential distribution in the sheath has a minimum. In the location of the minimum, the corresponding (y) component of the electric field is zero. On the other hand, there is an axial electric field component which arises from the axial potential distribution in the plasma bulk. The presence of the axial component was discussed previously.[31] It was suggested that the electric field along the dielectric is close to the electric field in the plasma bulk due to high dielectric strength of the wall material. The potential distribution in the axial direction can be obtained from the electron momentum equation as

$$\varphi_z = zE_z = E_z^2 \frac{y}{E_y} \tag{23}$$

The total potential drop that electrons are experiencing while being in the sheath can be estimated as follows:

$$\Delta\varphi_z = E_z^2 \int_{y_{\min}}^{y_{\max}} \frac{dy}{E_y(y)} \approx E_z^2 \frac{L_D}{E_y} \tag{24}$$

In the last expression, it was assumed that the sheath thickness is about Debye length. However, since the space charge limited sheath is considered, the electric field is not uniform in the *y* direction. The largest influence of the axial electric field is near the potential well, since typically the electric field in the *y* direction is much larger than in *z*, i.e. E_y/E_z>>1, except near the potential well where $E_y/E_z \leq 1$. The spatial extension of the potential well is about Debye length L_D (Ref.32). Thus, the potential drop can be estimated with satisfactory accuracy as

$$\Delta\varphi_z \approx E_z L_D \tag{25}$$

Taking Eq. 25 into account, one can estimate the NWC current enhancement ($\exp\left(\frac{E_z L_D}{T_w}\right)$), which depends on the bulk to wall electron temperature ratio as shown in Fig. 6. In typical conditions in the Hall thruster channel, we can find that this factor may be about 10.

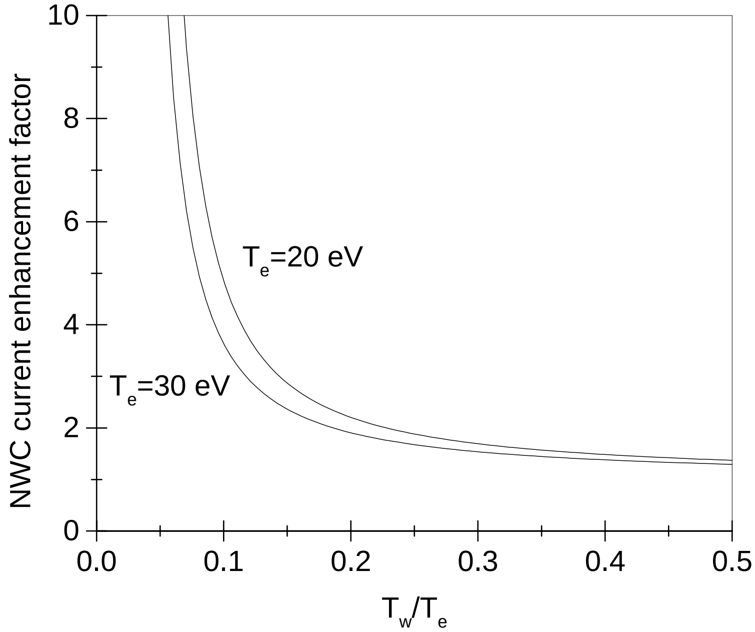

Figure 6. Enhancement of the NWC by axial electric field effect. The typical conditions are considered: E_z=3x10^4 V/m; n_o=10^{17} m^{-3}.

If we take into account possible enhancement factor due to axial electric field in the sheath, the predicted NWC current will be close to that experimentally measured.[26] Thus it can be concluded that, in general, NWC can explain the high electron mobility in a Hall thruster. However, it is quite interesting to point out that higher NWC current is expected in the case of small T_w as follows from Fig. 6. On the other hand, according to this model prediction, this is the case in which NWC current is not decaying from the wall as shown in Fig. 6, thus putting a question mark on the near wall nature of the current. Bearing this in mind, we can conclude that the full picture of electron transport in the Hall thruster is far from completion and further investigation is needed. One approach would be to use more detailed analysis of the plasma instabilities with application to a specific thruster configuration.

IV. Conclusions

In this paper, we considered an effect of the magnetic mirror on several aspects of the plasma flow in a Hall thruster channel. The magnetic field gradient affects several important flow characteristics, such as the radial presheath potential drop, the presheath length, and the two-dimensional potential shape in the channel. In particular, it was shown that a concave shape of the potential can be obtained even in the case of a primarily radial magnetic field.

This effect is important for ion dynamics since ions follow the electric field in the absence of collisions as happens typically in the acceleration region. Consideration of the magnetic field gradient in the model formulation represents an important departure from the current state –of-the-art modeling level for Hall thrusters.

A new formulation of the original near-wall conductivity model that takes into account an axial electric field in the sheath was presented. It was shown that under certain conditions, i.e. small ratio of the secondary electron temperature to the bulk electron temperature, the near wall conductivity can be significantly enhanced.

Acknowledgement

Authors thank Prof. Alec Gallimore, Dr. Richard Hofer, Jessy Linnell, Dr. Yevgeny Raitses and Prof. Isak I. Beilis for very fruitful discussion. This work was supported by AFOSR Grant No. FA9550-05-1-0042.

References

[1] V. Kim, J. Prop. Power 14 736 (1998).
[2] R. Hofer, P. Peterson, A.D. Gallimore, R.S. Jankovsky, 27th International Electric Propulsion Conference, Pasadena, CA, October 15-19, 2001 (Electric Rocket Propulsion Society, Cleveland, OH 2003), IEPC Paper 2001-036.
[3] R. Hofer and A.D. Gallimore Proceedings of 38th Joint Propulsion Conference and Exhibit, July 2002, Indianapolis, IN (American Institute of Aeronautics and Astronautics, Reston, VA 2002), AIAA Paper-2002-4111.
[4] Linnell, J. A. and Gallimore A.D., "Internal Plasma Structure Measurements of a Hall Thruster Using Xenon and Krypton Propellant," 29th International Electric Propulsion Conference, Princeton, NJ, October 31 - November 4,2005.
[5] A. Fruchtman, N. J. Fisch, 34th AIAA Joint Prop. Conference, Cleveland OH, 1998, (American Institute of Aeronautics and Astronautics, Washington DC, 1998), AIAA-1998-3500.
[6] J.M. Fife, Hybrid-PIC modeling and electrostatic probe survey of Hall thrusters, Ph.D Thesis, (MIT, 1998).
[7] K. Makowsky, Z. Peradzynski, N. Gascon, and M. Dudeck, 35th AIAA Joint Prop. Conference, Los Angeles, CA, 1999, (American Institute of Aeronautics and Astronautics, Washington DC, 1999), AIAA-99-2295.
[8] G. J. M. Hagelaar, J. Bereilles, L. Garrigues and J.-P. Boeuf, J. Appl. Phys. 93, 67 (2003).
[9] M. Keidar, I.D. Boyd, and I.I. Beilis, Phys. Plasmas, 8, 2001, p. 5315
[10] E. Ahedo, P. Martinez, and M. Martines-Sanches, Phys. Plasmas, 8, 2001, p. 3058.
[11] M. Keidar, A.D. Gallimore, Y. Raitses and I.D. Boyd, *Applied Physics Letters*,85, No. 13, 2004, p. 2481.
[12] S. Barral, K. Makovsky, Z. Peradzynski, N. Gascon, and M. Dudeck, Phys. Plasmas, 10, 4137, 2003.
[13]L. Garrigues, G.J.M. Hagelaar, J. Bareilles, C. Boniface and J.P. Boeuf, Phys. Plasmas, 10, 4886, 2003.
[14] M. Keidar, I.D. Boyd and I.I. Beilis, *Physics of Plasmas,* 11, 2004, p. 1715.
[15] A.I. Morozov, Sov. Phys. Dokl., 10 775 (1966)
[16] A.I. Morozov and V.V. Savelyev, in *Review of Plasma Physics*, Ed. By B.B. Kadomtsev and V.D. Shafranov (Consultant Bureau, New York, 2000), Vol. 21, p. 203.
[17] V.V. Zhurin, H.R. Kaufman and R.S. Robinson, Plasma Sources Sci. Technol., 8 1 (1999)
[18] Y. Raitses, M. Keidar, D. Staack and N.J. Fisch, J. Appl. Phys., 92 4906 (2002)
[19] Y. Raitses, D. Staack, L. Dorf and N.J. Fisch, Proceeding of 39th Joint Propulsion Conference, July 20-23, 2003, Huntsville, AL (American Institute of Aeronautics and Astronautics, Reston, VA 2002), AIAA paper 2003-5153.
[20] H. Muta, N. Itagaki, Y. Kawai, Vacuum, 66 209 (2002)
[21] H. Kaufman, R.S. Robinson and R.I. Seddon, J. Vac. Sci. Technol., A5(4) 2081 (1987)

[22] G.D. Hobbs and J.A. Wesson, Heat flow through a Langmuir sheath in the presence of electron emission, Plasma Phys., 9, pp. 85- 87,1967.
[23] V.I. Baranov, Yu.S. Nazarenko, V.A. Petrosov, A.I. Vasin, and Yu.M. Yashnov, 25th Inter. Conf. Electric Propulsion, Cleveland, OH, (The Electric Rocket Propulsion Society, Worthington, OH, 1997), IEPC-97-060
[24] A.I. Morozov, Sov. Phys. Dokl., 10, 775 (1966)
[25] A.I. Morozov and A. P. Shubin, Electron kinetics in the near-wall conductivity regime, Fizika Plazma Vol. 10, 1984, 1262-1269.
[26] A.I. Morozov and V.V. Savel'ev, Theory of the near-wall conductivity, Plasma Phys. Reports, Vol. 27, No. 7, 2001, pp. 570-575.
[27] A.A. Ivanov, A.A. Ivanov Jr and M. Bacal, "Effect of plasma-wall recombination on the conductivity in Hall thrusters," *Plasma Phys. Control. Fusion*, vol. 44, pp. 1463-1470, 2002.
[28] Levchenko, I.; Keidar, M, Visualization of ion flux neutralization effect on electrical field and atom density distribution in Hall thruster channel., IEEE Trans. Plasma Science, Vol. , No 2, Part 1, Apr 2005 Pp. 526 - 527
[29] S. Barral, K. Makowski, Z. Peradzynski and M. Dudeck, Is near-wall conductivity a misnomer, AIAA 2004
[30] Y. Raitses, D. Staack, A. Smirnov, and N. J. Fisch, Space charge saturated sheath regime and electron temperature saturation in Hall thrusters, Phys. Plasmas Vol. 12, 2005, 073507.
[31] V.V. Egorov,V. Kim, A.A. Semenov, I.I. Shkarban, Near –wall process and its influence on operation of accelerators with closed electron drift, in book Ion injectors and plasma accelerators, Moscow, Energoatomizdat, 1990 (in Russian)
[32] L. A. Schwager, Effects of secondary and thermoionic electron emission on the collector and source sheaths of a finite ion temperature plasma using kinetic theory and numerical simulation, Phys. Fluids B5 (2) 1993, pp. 631-645.