

Momentum Modeling of a Hall Thruster Plume

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Numerical simulation techniques were used for the first time to track the evolution of species-specific momentum throughout a Hall thruster plume. All significant particle interactions were included in the model, and specific emphasis was placed on the dependence of velocity distribution functions and total plume momentum on the density of background xenon gas. To obtain the results, the coordinate-dependent influence of plume parameters such as ion flux density, neutral density, electron temperature and plasma potential was included in the calculations. Results confirm that significant plume interactions occur, with initial ion beam momentum becoming substantially distributed between ion and neutral populations as the ion beam is attenuated, and increasing total momentum flux across a thruster-centric hemispherical surface at increasing downstream distance. Velocity distribution functions continue to change significantly with propagation distance beyond 1m and exhibit angular dependence. Momentum augmentation is due to collision-based reduction of average ion momentum as average neutral momentum increases, and the on-going acceleration of ion populations in the local electric field. The increase of plume momentum with rising background gas density is less than predicted by an earlier analytical study, and part of the discrepancy stems from the plasma potentials used as model inputs.

I. Introduction

High brightness, broad beam xenon ion sources operated in a ground-based environment generate a background of neutral xenon gas, due to finite pumping speed of the vacuum facility. Penetration of the neutrals into the ion beam plasma and interaction with its constituents may modify plasma potential, density, temperature, velocity and trajectory characteristics. Ultimately, thruster performance figures can be affected. This “facility effect” and others have produced numerous unexplained results over the years.¹

Propellant atoms are ionized efficiently in modern ion propulsion devices, the majority of which rely on xenon propellant. While the ion flux in the plume is much higher than neutral flux, neutral density normally dominates over plasma density. During ground operation, the density of background neutrals is established by thruster flow rate and pumping speed of the facility. In the present study we focus on the effects of neutral background gas density and its interaction with beam ions through charge exchange reactions. In addition to alteration of the plasma density profile a principal macroscopic effect of the interaction is modification of performance metrics of an operating propulsion device. Several thrust modification mechanisms exist that are associated with neutrals. Neutrals that

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diffuse into plume regions near a thruster exit plane may undergo ionization and acceleration. This process produces a more direct augmentation of thrust than charge exchange. Ion-neutral collisions impede neutrals from diffusing into the core of the ion beam and reduce the average energy of plume ions. A thrust enhancement will result if there is subsequent axial momentum gain for the just-formed slow xenon ion; the sum of beam and charge exchange ion pair momentum will exceed that of a beam ion that did not charge exchange.

Xenon neutrals from the background gas diffuse into the plume of a thrust-generating ion beam and undergo reactions of several types. The most important for thrust generation are charge exchange reactions with approximately zero momentum transfer, where a fast singly charged beam ion and slow neutral atom are efficiently converted into a slow ion and a fast neutral atom according to $\text{Xe}^+ (\text{fast}) + \text{Xe} (\text{slow}) \rightarrow \text{Xe} (\text{fast}) + \text{Xe}^+ (\text{slow})$. Ion-neutral and neutral-neutral collisions that produce scattering at various angles are also important, and their effects on plume divergence of Hall Effect Thrusters (HETs) have been studied before.²

Recent development and application of an analytical model³ to calculate neutral density in the operating xenon HET plume predicted neutral depletion would occur in the core of the ion beam. Analysis of ion-neutral charge exchange reactions concluded that thrust augmentation would occur due to collision-based reduction of average ion momentum with concurrent increases of average neutral momentum, and the on-going acceleration of ion populations in the local electric field to increase their momentum. Downshifting the average ion energy by flowing it into neutral energy augments plume momentum, because kinetic energy depends on the square of velocity whereas momentum is proportional to velocity, and a given energy change produces a greater velocity increment for low energy populations. The result is increased plume momentum that may be registered as thrust exerted by the HET.

Direct Simulation Monte Carlo (DSMC) techniques were used here for the first time to track the evolution of species-specific and total plume momentum throughout a Hall thruster plume. Detailed calculations were performed to determine velocity and density distribution functions for the population of ions and neutrals throughout the HET plume. A DSMC approach was used to model the collisions of the ions and neutrals, with Particle-in-Cell (PIC) modeling of ion transport in the plume electric field. All significant particle interactions were included in the model, and specific emphasis was placed on the dependence of velocity distribution functions and total plume momentum on the density of background xenon gas. To obtain the results, the coordinate-dependent influence of plume parameters such as ion flux density, neutral density, electron temperature and plasma potential was included in the calculations.

Ion velocity distribution functions have been measured and modeled previously for internal environments and near field plumes of lower power Hall thrusters, exploring time-dependent variation of the electric field and VDF.⁴ Neutral measurement and modeling studies have shown that the VDF evolves in the flow field, with ion-neutral and neutral-neutral interactions influencing the profile.^{4,5}

II. Hall Thruster Plume Model

The xenon plasma plume of a Hall thruster is composed of beam ions with velocities on the order of 20 km/s, low energy charge exchange ions, neutral atoms, and electrons. The total number density in the thruster is of the order of 10^{18} m^{-3} , placing the plasma in the rarefied flow regime. Computational analysis of Hall thruster plumes is regularly performed using a hybrid particle-fluid formulation.^{6,7} The direct simulation Monte Carlo (DSMC) method⁸ models the collisions of the heavy particles (ions and atoms). The Particle In Cell (PIC) method⁹ models the transport of the ions in electric fields. Overall, a hybrid approach is employed in which the electrons are modeled using a fluid description. In the present study, we employ a detailed treatment of the electron fluid by simultaneous solution of its three conservation equations.¹⁰

Plasma Dynamics

The most successful Hall thruster plume models are based on a hybrid approach in which heavy species are modeled using particles and the electrons modeled as a fluid. In the present study, a detailed fluid electron model is employed that uses all three electron conservation equations. The model has been described in detail elsewhere and applied to the near field of a 200 W Hall thruster operated in a test chamber.¹⁰

The electron continuity equation is¹¹

$$\frac{\partial}{\partial t}(n_e) + \nabla \cdot (n_e \bar{v}_e) = n_e n_a C_i \quad (1)$$

where $\bar{\mathbf{v}}_e$ is the electron velocity vector, n_a is the atomic number density, and C_i is the ionization rate coefficient. Assuming steady flow, this equation is transformed into a Poisson equation by introducing a stream function ψ using

$$n_e \bar{\mathbf{v}}_e = \nabla \psi \quad (2)$$

such that

$$\nabla^2 \psi = n_e n_a C_i \quad (3)$$

for which numerical solutions are obtained using the standard Alternating Direction Implicit (ADI) method. The spatial distribution of the ion particles gives the electron number density, n_e , under the assumption of charge neutrality. This allows the electron velocity vector to be determined through solution of (3). In the present study, the xenon ionization rate coefficient is set to zero due to the low temperatures involved.

In the absence of magnetic field effects, the electron momentum equation is¹⁰

$$\frac{\partial}{\partial t} (m_e n_e \bar{\mathbf{v}}_e) + m_e n_e (\bar{\mathbf{v}}_e \cdot \nabla) \bar{\mathbf{v}}_e = -en_e \bar{\mathbf{E}} - \nabla p_e + \bar{\mathbf{R}} \quad (4)$$

where m_e is the mass of an electron, $\bar{\mathbf{E}}$ is the electric field, p_e is the electron pressure, and $\bar{\mathbf{R}}$ is the friction term. It is further assumed that the electrons behave as a perfect gas ($p_e = n_e k T_e$), and that the friction term is given by:

$$\bar{\mathbf{R}} = \frac{en_e \bar{\mathbf{j}}}{\sigma} \quad (5)$$

where $\bar{\mathbf{j}}$ is the current density, and σ is the electrical conductivity.

Assuming a steady state, neglecting the inertial term on the left hand side of (4), and introducing the plasma potential $-\nabla \phi = \bar{\mathbf{E}}$, a generalized Ohm's law is obtained:

$$\bar{\mathbf{j}} = \sigma \left[-\nabla \phi + \frac{1}{en_e} \nabla (n_e k T_e) \right] \quad (6)$$

For given n_e , $\bar{\mathbf{v}}_e$, and T_e , the charge continuity condition:

$$\nabla \cdot \bar{\mathbf{j}} = 0 \quad (7)$$

is then solved to obtain the plasma potential. This equation is written as a Laplace equation with weak source terms and is again solved using an Alternating Direction Implicit (ADI) scheme.

The electron energy equation is given by¹¹

$$\frac{\partial}{\partial t} \left(\frac{3}{2} n_e k T_e \right) + \frac{3}{2} n_e (\bar{v}_e \cdot \nabla) k T_e + p_e \nabla \cdot \bar{v}_e = \nabla \cdot \kappa_e \nabla T_e + \bar{j} \cdot \bar{E} - 3 \frac{m_e}{m_i} \nu_e n_e k (T_e - T_H) - n_e n_a C_i \varepsilon_i \quad (8)$$

where m_i is the ion mass, ν_e is the total electron collision frequency, κ_e is the electron thermal conductivity, and T_H is the heavy particle temperature. Again assuming a steady state, and dividing by the thermal conductivity:

$$\nabla^2 T_e = -\nabla \ln(\kappa_e) \cdot \nabla T_e + \frac{1}{\kappa_e} \left(-\bar{j} \cdot \bar{E} + \frac{3}{2} n_e (\bar{v}_e \cdot \nabla) k T_e + p_e \nabla \cdot \bar{v}_e + 3 \frac{m_e}{m_i} \nu_e n_e k (T_e - T_H) + n_e n_a C_i \varepsilon_i \right) \quad (9)$$

where $\bar{j}$ is obtained¹⁰ after the plasma potential is calculated. Equation (9) is again a Laplace equation with weak source terms that is solved using the ADI approach.

Finally, the electron transport coefficients are evaluated using their basic definitions¹¹

$$\sigma = \frac{e^2 n_e}{m_e \nu_e} \quad (10)$$

$$\kappa_e = \frac{2.4}{1 + \frac{\nu_{ei}}{\sqrt{2} \nu_e}} \frac{k^2 n_e T_e}{m_e \nu_e} \quad (11)$$

where $\nu_e = \nu_{ei} + \nu_{en}$, ν_{ei} is the ion-electron collision frequency, ν_{en} is the neutral-electron collision frequency, and these frequencies are evaluated for the xenon system using cross sections obtained previously.¹¹ Note that, for each time step, the numerical scheme iterates several times through the solution of (7) and (9) due to the coupling between ϕ and T_e .

Collision Dynamics

The DSMC method uses particles to simulate collision effects in rarefied gas flows by collecting groups of particles into cells which have sizes of the order of a mean free path. Pairs of these particles are then selected at random and a collision probability is evaluated that is proportional to the product of the relative velocity and collision cross section for each pair. The probability is compared with a random number to determine if that collision occurs. If so, some form of collision dynamics is performed to alter the properties of the colliding particles.

There are two basic classes of collisions that are important in the Hall thruster plumes: (1) elastic (momentum exchange); and (2) charge exchange. Elastic collisions involve only exchange of momentum between the participating particles. For the systems of interest here, this may involve atom-atom or atom-ion collisions. For atom-atom collisions, the Variable Hard Sphere (VHS) collision model is employed.⁸ For xenon, the collision cross section is

$$\sigma_{EL}(Xe, Xe) = \frac{2.12 \times 10^{-18}}{g^{2\omega}}, \quad (12)$$

where the units are m^2 , g is the relative velocity, and $\omega=0.12$ is related to the viscosity temperature exponent for xenon.

Both atom-ion momentum and charge exchange processes are modeled using the same cross section that was developed by Boyd and Dressler⁷ based on a combination of theory and experiment:

$$\sigma_{CEX}(Xe, Xe^+) = (-13.6 \log_{10}(\varepsilon) + 87.3) \times 10^{-20}, \quad (13)$$

where $\varepsilon = \frac{1}{2} m^* g^2$ and is measured in eV, and m^* is the reduced mass. Reference 7 also recommends that charge

exchange cross sections for the interaction where a doubly charged ion captures two electrons from an atom are a factor of two lower than the values for the singly charged ions at corresponding energies. Charge-exchange collisions are simulated using the differential cross sections developed in Ref. 7. Momentum exchange collisions assume either the same differential scattering cross sections, or isotropic scattering.

Boundary Conditions

For the computations of Hall thruster plumes, boundary conditions must be specified at several locations: (1) at the thruster exit; (2) at the cathode exit; (3) along the outer edges of the computational domain; and (4) along all solid surfaces in the computational domain.

Several macroscopic properties of the plasma exiting the thruster are required for the computations. For the electrons, their current, temperature, and the plasma potential must be specified. For each of the heavy species (Xe, Xe^+ , Xe^{2+}) we require the number density, velocity, and temperature. In the real device, these properties vary radially across the exit plane.

The approach to determining these properties involves a mixture of analysis and estimation. The basic performance parameters of mass flow rate, thrust, and total ion current are assumed known. The neutrals are assumed to exit the thruster at the sonic speed corresponding to some assumed value for their temperature. Finally, divergence angles for the lower and upper edges of the exit channel must be assumed. Combining all this information then allows all species densities and the velocities to be determined. Determination of the properties of multiple charge states, for example Xe^{2+} is considered in the present study, requires knowledge of the current fraction of that state. The major plasma properties at the thruster exit are summarized in Table 1.

In the electron fluid model, the external cathode of the Hall thruster must be modeled. While the actual cathode provides essentially a point source of electrons that therefore involves a three dimensional flow, in the present study it is modeled within the axially symmetric framework of the code. This is not a bad assumption given the high mobility of the electrons that rapidly form a symmetric flow field. The boundary conditions required for the electrons at the exit of the cathode are the electron current, the plasma potential, and the electron temperature.

Both fluid and particle boundary conditions are required at the outer edges of the computational domain. The electron fluid conditions employ either Dirichlet (fixed value) or Neumann (fixed gradient) boundary conditions. For the simulation of a spacecraft plume in space, these conditions are difficult to determine. In the present case, the gradients of electron current normal to the domain edges are set to zero, and the plasma potential on all domain boundaries is set to 3 V. The electron temperature along the outer edges of the domain is usually set to 1.0 eV. The heavy particle boundary condition is to simply remove from the computation any particle crossing any domain edge.

The solid wall surfaces of the Hall thruster are also included in the computation. Along these walls, a fixed value (1 eV) of the electron temperature is usually employed. The front face of the thruster is sprayed with di-electric material and so a condition of zero current is employed to calculate the plasma potential. In terms of heavy particles, any ions colliding with the walls are neutralized. Both atoms and neutralized ions are scattered back into the flow field from the surface of the thruster wall assuming diffuse reflection at a temperature of 500 K.

The simulations include the finite background gas pressure of the experimental facility. This gas is initialized to a given pressure and density at a temperature of 300 K. The nominal facility pressure of 1.1×10^{-5} torr ($4 \times 10^{17} \text{ m}^{-3}$ is the corresponding density) is designated 1X. In this nomenclature 0X refers to space conditions and 2X refers to twice the nominal operational facility pressure. As the thruster plasma plume expands into the background gas, the collisional interactions are simulated. Total flow rate for the simulation was 15.9 mg/s and thruster input power was 4.5 kW (14.9 A at 300 V).

Some adjustments were made to boundary conditions to better match unpublished experimental data for a medium power Hall thruster device as facility background pressure was changed, increasing the “scatter” in plotted results. Electron impact ionization of neutral atoms in the plume was included in the simulation, but magnetic fields and transient behavior were not.

Table 1. Properties used in the simulation at the exit of the Hall thruster.

Inner Diameter (mm)	100
Outer Diameter (mm)	140
Plasma Density (m^{-3})	3.7×10^{17}
Neutral Density (m^{-3})	7.8×10^{18}
Ion Velocity (m/s)	18,000
Neutral Velocity (m/s)	280
Ion Temperature (eV)	2.0
Neutral Temperature (K)	750
Electron Temperature (eV)	6.0
Plasma Potential (V)	79

III. Results and Discussion

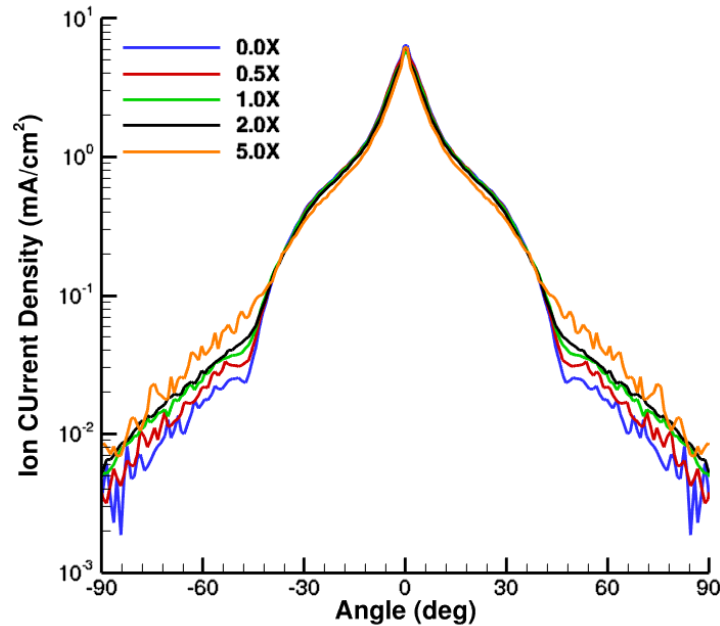


Figure 1. Angular dependence of plume ion flux, 1 m from thruster exit, at various background pressures (1X refers to facility pressure at the nominal 4.5 kW operating point).

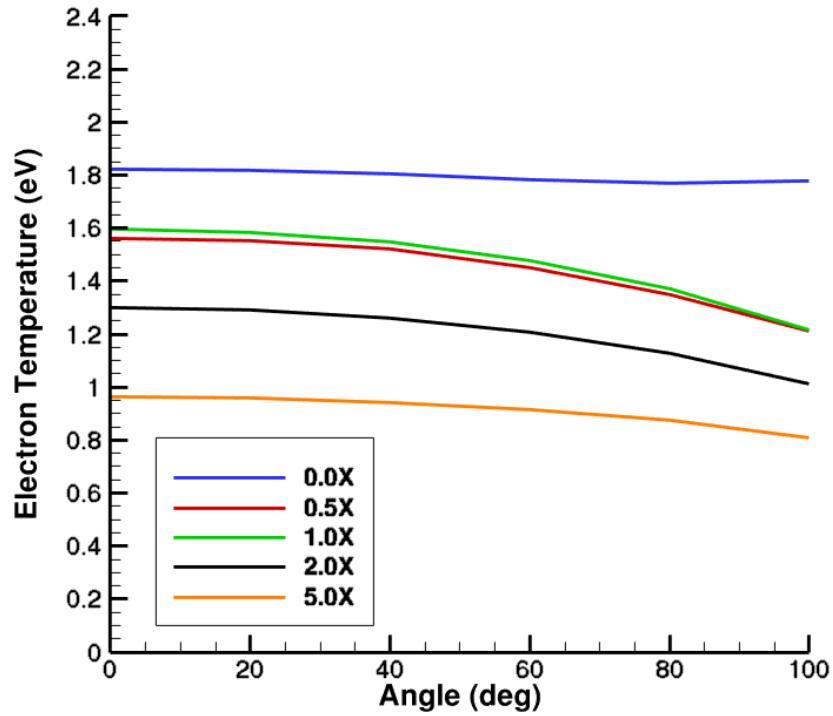


Figure 2. Angular dependence of plume electron temperature, 1 m from thruster exit, at various background pressures.

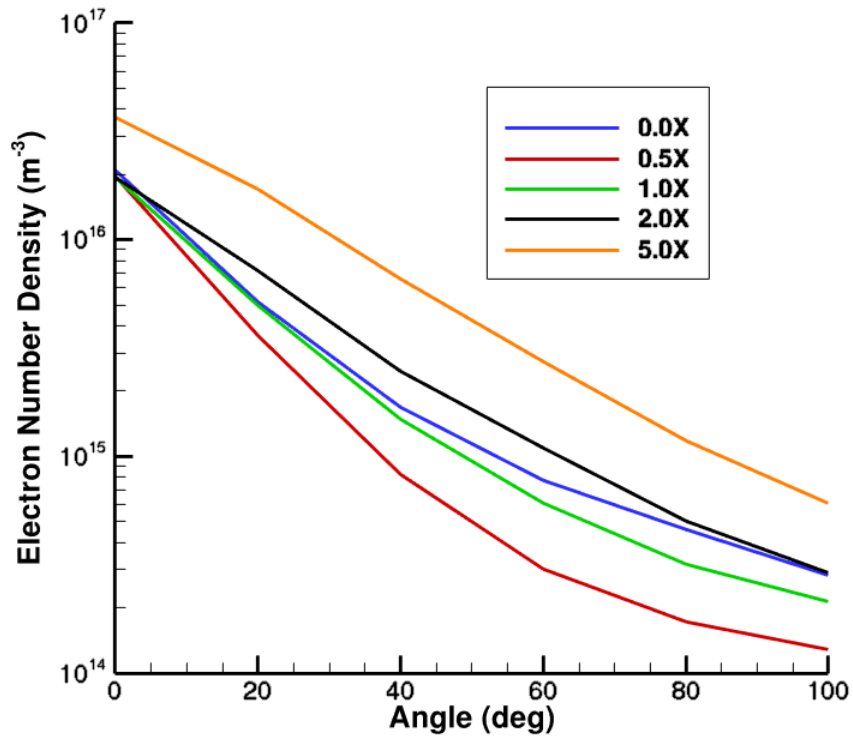


Figure 3. Angular dependence of electron density, 1 m from thruster exit, at various background pressures.

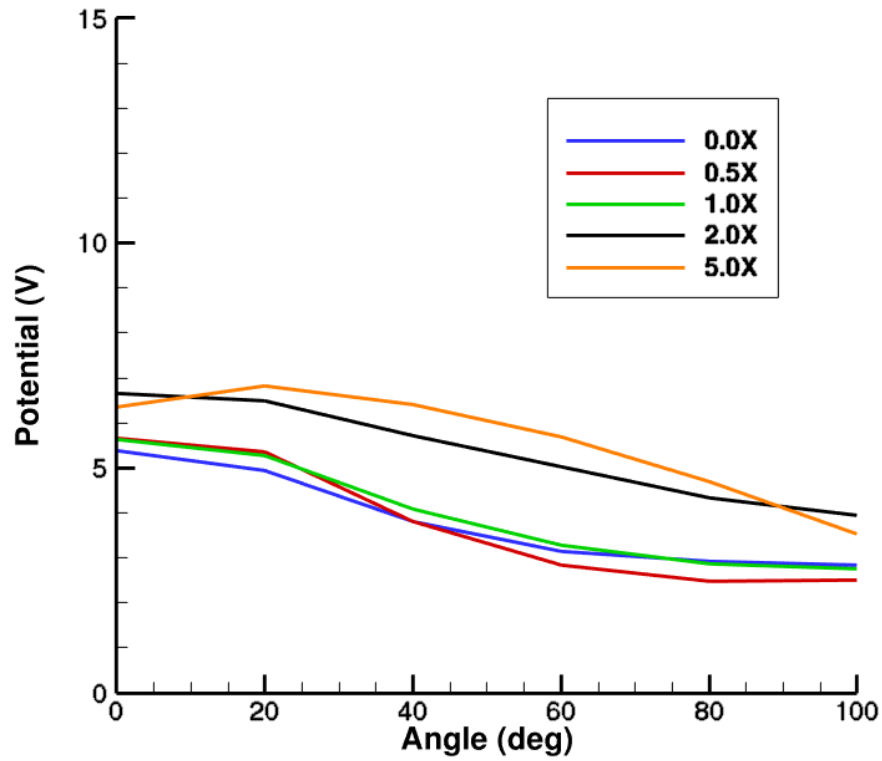


Figure 4. Angular dependence of plasma potential, 1 m from thruster exit, at various background pressures.

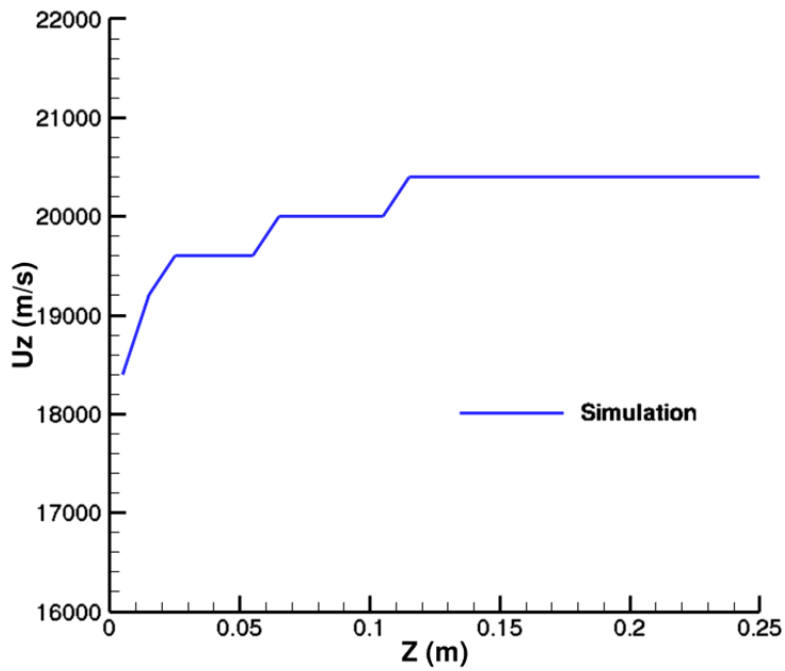


Figure 5. Calculated peak axial velocity for Xe^+ beam ions from exit to 25-cm downstream, for 1X case.

Angular dependence of the plume ion flux at 1-m distance is plotted in Figure 1 as a function of xenon background pressure. The core of the ion beam is uninfluenced by background pressure, but pressure dependence is substantial outside of ± 40 degrees. In the plume wings there is nearly $\times 3$ greater ion current density at 5X pressure compared to 1X, and the 1X to 5X flux progression may be nonlinear (i.e., $\Delta J/\Delta P$, where J is current density and P is pressure, increases with decreasing pressure). These simulation results compare well with available experimental data.¹⁰

The angular dependences of electron temperature and density at 1-m distance, also for selected background pressures from 1X to 5X, are plotted in Figures 2 and 3, respectively. The electron temperature shows little angular dependence below 40 degrees, but drops with increasing density of background xenon. There is roughly a factor of 2 ratio between electron temperatures at 0X and 5X, with T_e about 1.8 eV across the entire 0X angular range. The electron number density is significantly elevated with xenon background density. Here there is as much as $10\times$ more electron density at 5X pressure, for angles equal to or greater than 40 degrees. In the small-angle core of the plume, the electron density depends relatively little on background xenon pressure. Ion current density is well known in the ion beam core, therefore electron density is also quite well known there if the electron temperature is known.

The simulated angular plasma potential curves exhibit both angular and pressure dependence, as shown in Figure 4. The angular dependence is similar at all pressures, with angular profiles trending upward with pressure. Scatter in the data is increased by minor changes of boundary conditions made to improve comparisons with some of the experimental results. Simulated potential results are in contrast to experimental data trends regarding pressure dependence, as the measurements produced inverse variation of potential with background pressure. Behavior of the plasma potential presents the principal discrepancy between simulation and experimental results.

Ion velocity at the exit plane and just downstream can be related directly to the local plasma potential and its delta with respect to where the ion was born. Comparison of the simulation result, shown in Figure 5, with Xe^+ laser induced fluorescence data indicates good agreement up to 10 cm from the exit. Xe^+ axial velocity of 18 km/s at the exit corresponds to about 221 eV of kinetic energy, so the plasma potential there would be 79V if the ions were born at 300V (not realistic due to combined effects of anode fall and the negative floating potential of the cathode; anode to cathode ΔV is fixed at 300V) and 49V for ions born at 270V, a more reasonable value. Plasma potential and

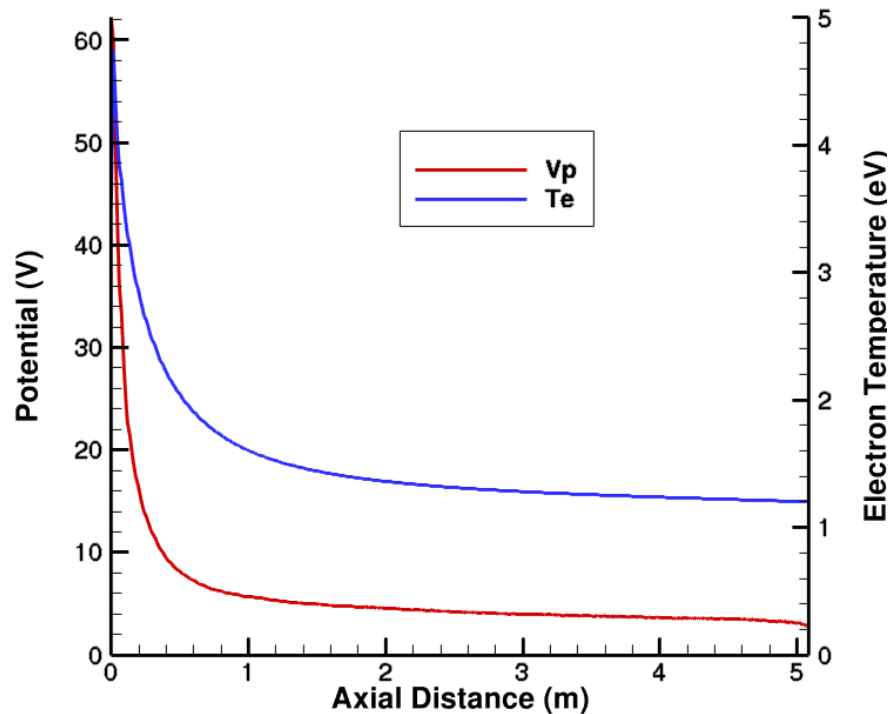


Figure 6. Plasma potential and electron temperature axial profiles.

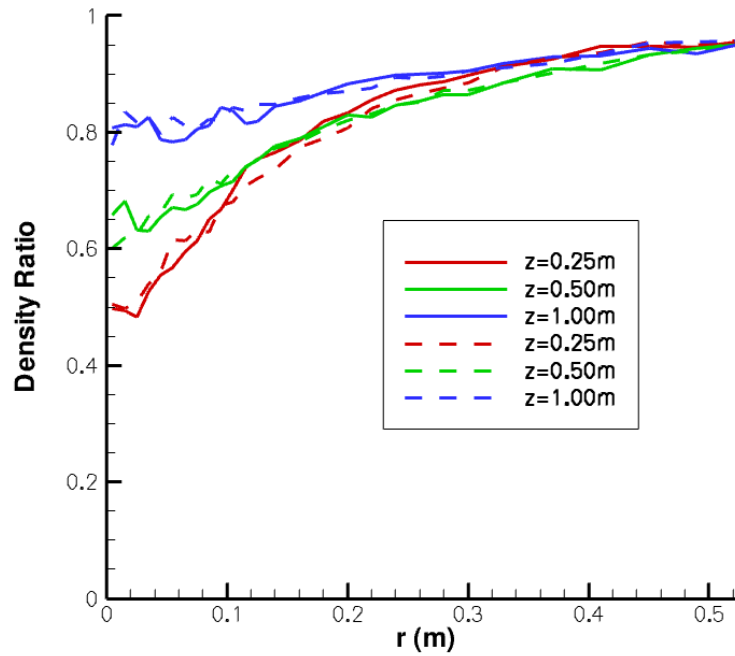


Figure 7. Radial profiles of background gas density ratio as a function of axial distance. Dashed lines = background gas is only source of neutrals; solid lines = thruster source flow is present.

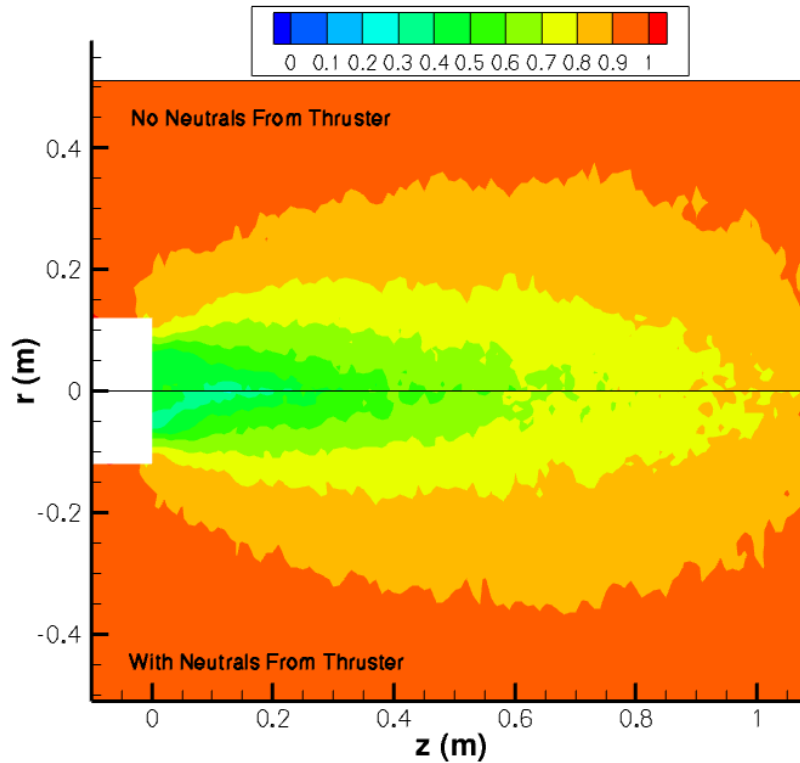


Figure 8. Contours of background gas density ratio with and without the inclusion of neutrals flowing out of the thruster.

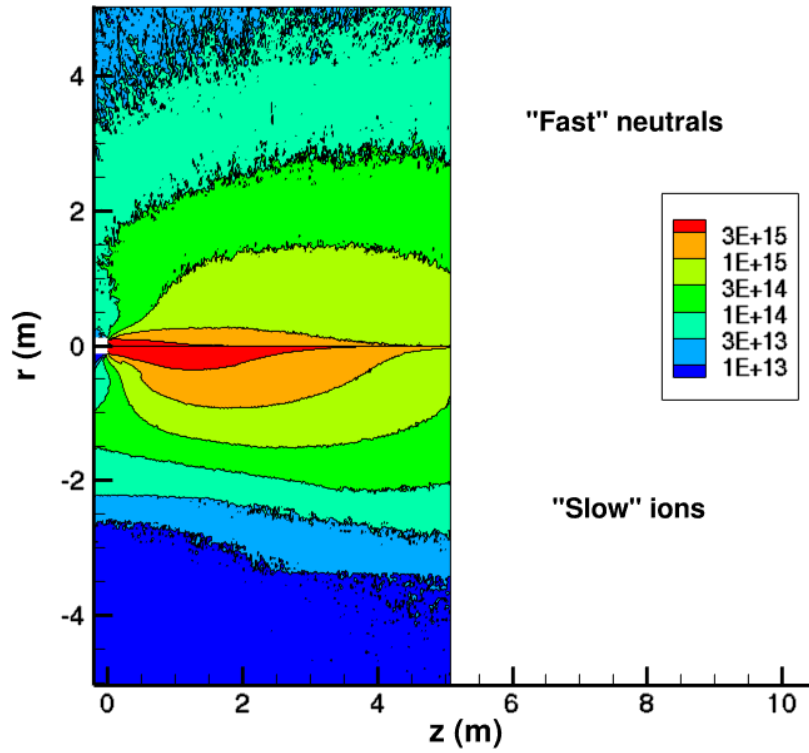


Figure 9. Number density contours for fast neutral and slow ion charge exchange products.

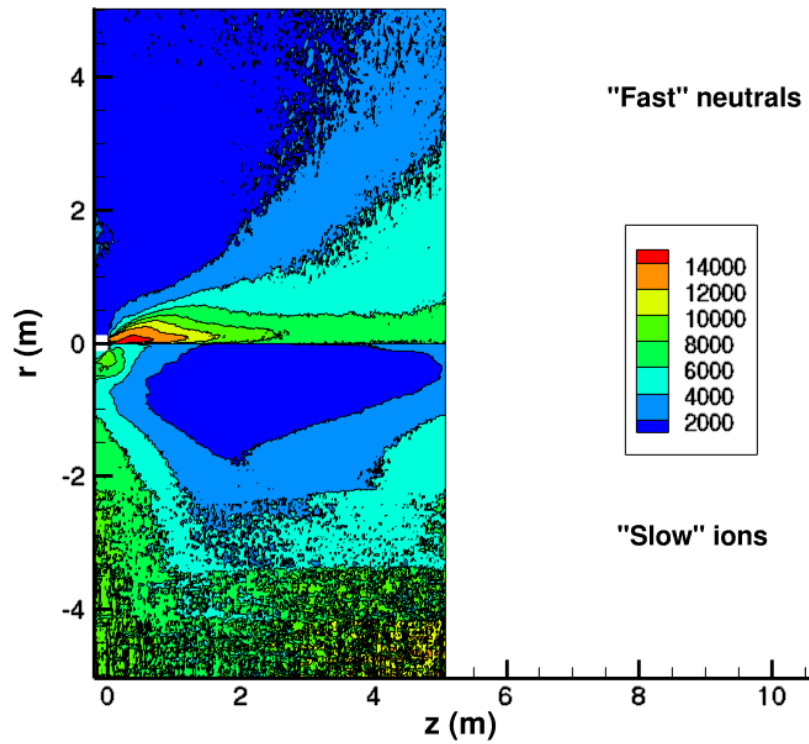


Figure 10. Velocity contours for fast neutral and slow ion charge exchange products.

electron temperature axial profiles are given in Figure 6. Plasma potential drops quickly in the near field region, down to about 12V at 30 cm and 6V at 100 cm, decreasing slowly from there to the boundary of the computational region. Electron temperature also drops quickly at first and stabilizes at about the same distance, eventually becoming rather insensitive to increasing distance and angular coordinate, but dropping with increased background pressure as was shown in Figure 2. In contrast, the calculated plasma potential trended up with background pressure and was moderately sensitive to angular coordinate, as seen in Figure 4. The features of electron temperature behavior are similar to experimental observations,¹² with the exception that at lower (1X) pressure experimental data indicate much greater angular dependence than at (2X or 5X) higher pressure, which is not matched by the simulations. The experimental plasma potential showed a definite decreasing trend with increasing background pressure,¹² opposite to the numerical results but similar to other experimental studies.¹³

As confirmation of the analytical model for radial profiles of neutral xenon in the plume due to diffusion of background gas,³ simulated radial profiles obtained from the present calculations show similar behavior. The ratio of neutral density along radial cuts through the plume, relative to chamber wall density, is plotted in Figure 7. There is no significant difference at any axial distance, with and without inclusion of thruster source flow. As expected, a minimum exists on the plume axis that becomes progressively more shallow with increasing axial distance. At 25-cm the centerline density is about 50% of the wall density.

Relative background density contours are displayed in Figure 8, where it is clear that source neutral flow has little if any effect on neutral density throughout the plume. Neutral density is about 30% of wall density at the minimum locations, which are within 15 cm of the exit plane.

Plume neutrals, as well as ions, can be differentiated by their velocity characteristics, since “fast” neutrals and “slow” ions are efficiently produced in pairs by charge exchange, and background neutrals are themselves moving very slowly in comparison. Fast neutral and slow ion density contours are displayed in Figure 9, where it is apparent that the slow ions have significantly higher density – presumably due to their relatively low velocity. Both species have highest density near the exit plane of the thruster, decreasing in both radial and axial directions. Roughly speaking, one to two orders of magnitude drop in density occurs in axial and radial directions, respectively, over a range of several meters. The velocity contours for these species are plotted in Figure 10, showing that the

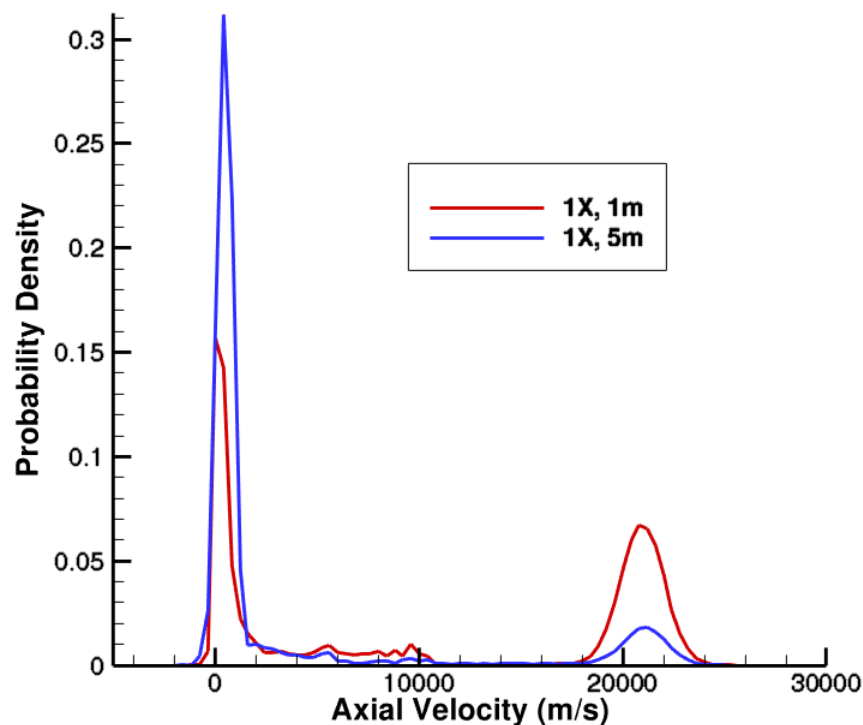


Figure 11. Axial velocity distribution function for ions at 1 and 5 m distances, with 1X facility pressure, on linear scale.

fast neutrals start out with much higher velocity than slow ions, near the exit plane. In the mid-field region of 1-2 meters from the thruster, it appears that the slow ions go through a minimum average velocity region. This may result from high production rates of both slow and elevated velocity ions near the exit plane, combined with significant conversion of these into the slow category as they migrate downstream. High average velocity and density is produced near the exit plane, low velocity and medium density in the mid-field region, and low density with modestly higher velocity at greater distances. These far-field properties occur because few ions are born at large distance and ions coming from upstream have experienced a long acceleration. Mid-field properties occur because most ions there were birthed at low potential relative to the near-field and accelerate moderately in both axial and radial directions due to modest electric fields. In the near-field region production is high and electric fields are relatively large. Fast neutrals formed near the exit plane typically will not retain their high velocity indefinitely as they travel away, due to collisions with background neutrals. With a mean free path that is on the order of several meters, multiple collisions are possible within the computational domain.

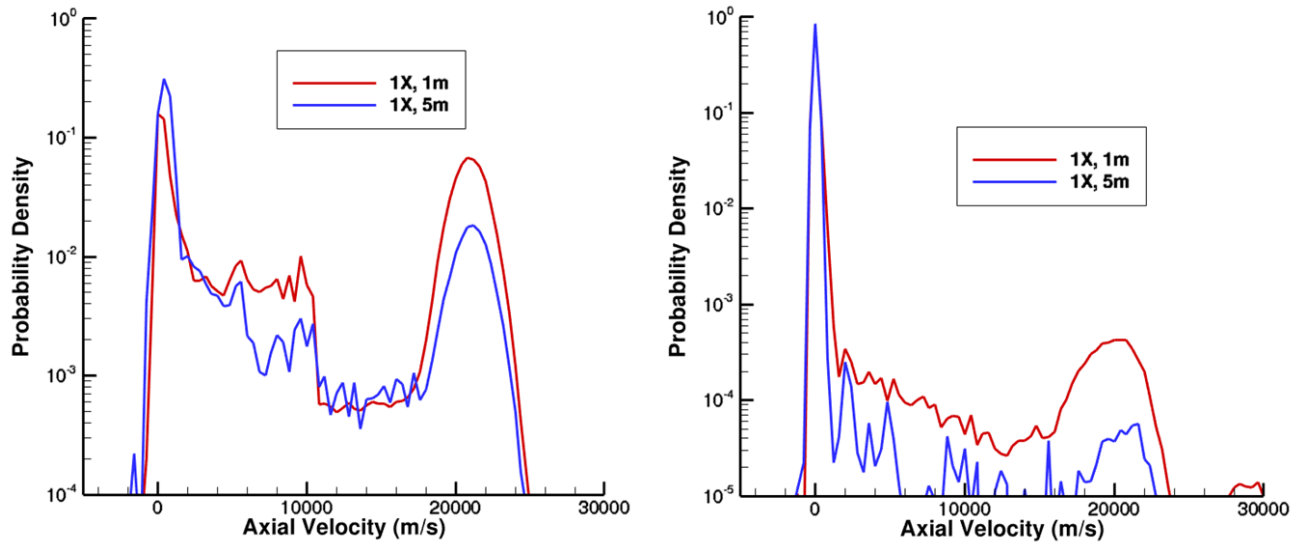


Figure 12. Axial VDF for ions (left) and neutrals (right), 1X pressure at 1 and 5 m.

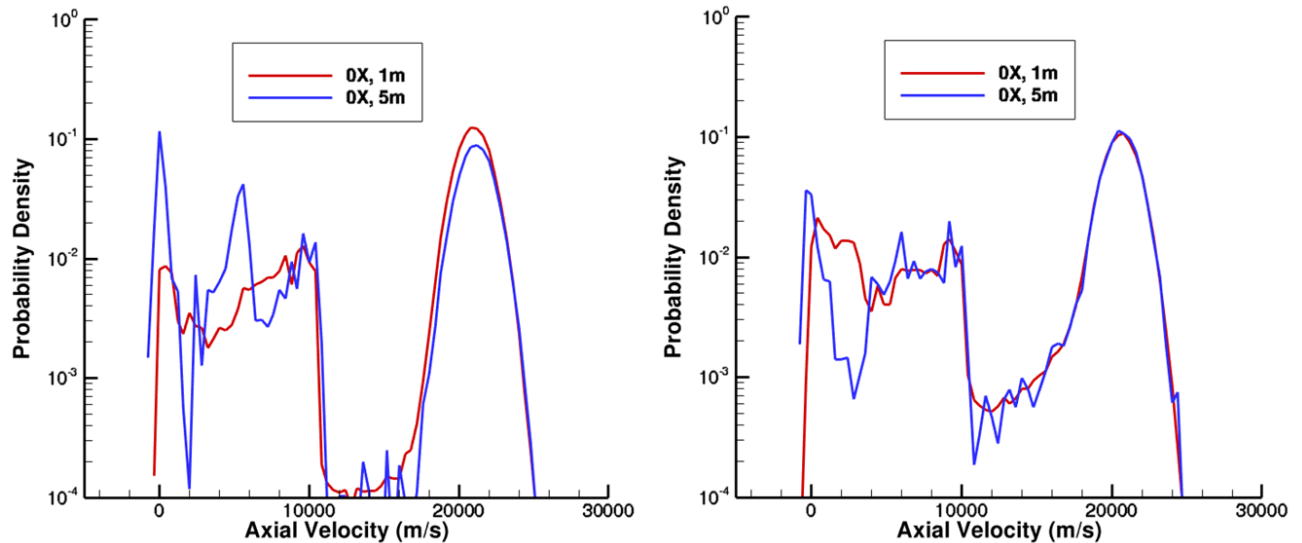


Figure 13. Axial VDF for ions on the thrust axis (left) and 20 degrees off-axis (right), 0X pressure at 1 and 5 m.

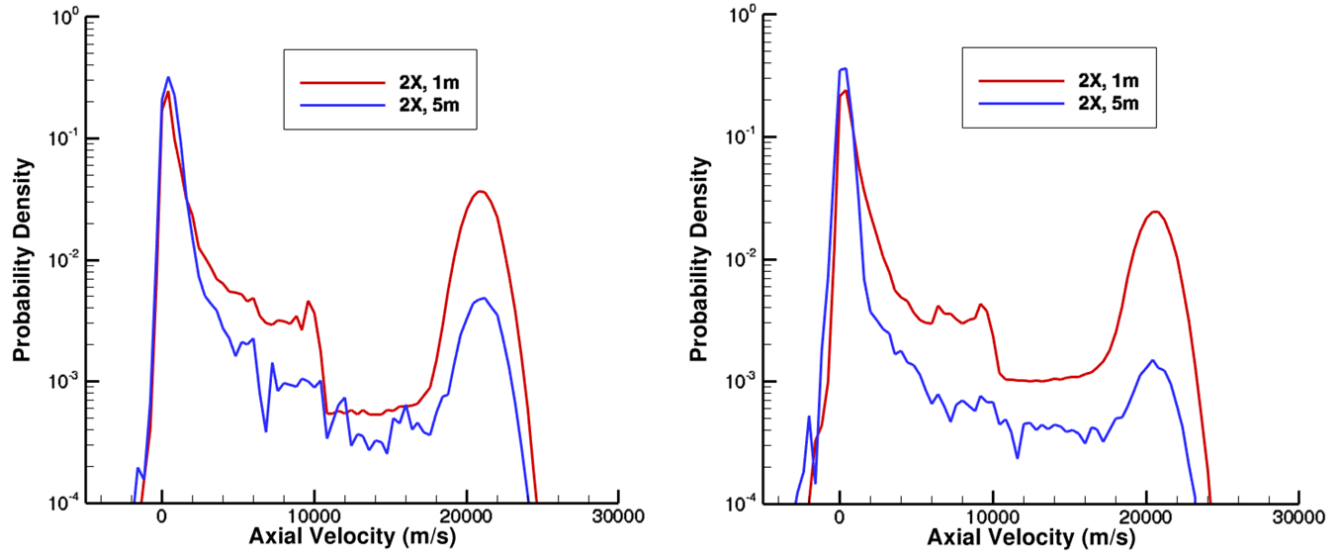


Figure 14. Axial VDF for ions on the thrust axis (left) and 20 degrees off-axis (right), 2X pressure at 1 and 5 m.

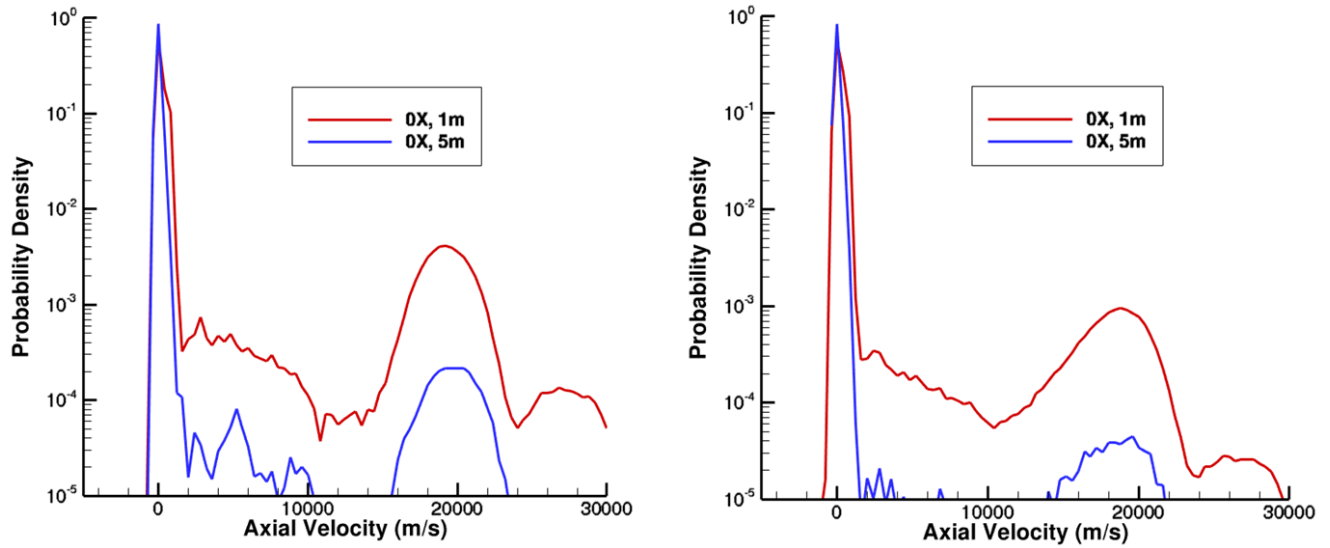


Figure 15. Axial VDF for neutrals on the thrust axis (left) and 20 degrees off-axis (right), 0X pressure at 1 and 5 m.

Outcomes of the various plume ion interactions span a wide spectrum. Because of the broad initial velocity distribution of beam ions, emitted over a broad angular distribution with varying plasma potential, electric field and neutral density throughout the plume, the tracking of individual nuclei on their journey and classifying the various particles with their respective velocity and density distributions is complex. Here we will focus on the velocity distribution functions for whole classes of particles: ions and neutrals. The total number of ions and neutrals in the plume varies little, since plume ionization is modest and we neglect the relatively slow process of recombination. It is instructive to study how the velocity distribution function varies within the plume, and it is a necessary quantity for calculating the total momentum crossing an arbitrary surface. By simulating the conversion of fast ions to slower ions and their subsequent acceleration by local fields throughout the plume, and summing the momentum, thrust augmentation associated with plume processes can be estimated.

An axial VDF for ions at 1 m and 5 m distances for the 1X pressure case is plotted on a linear scale in Figure 11. Ions are emitted from the thruster with velocity approaching 20 km/s, with a broad peak in that portion of the velocity spectrum due to the distribution of birth potentials. Ions with lower velocity are primarily formed via ion-neutral interactions that often involve charge exchange. These lower energy ions are apparent in the VDF, with a large peak at very low velocity relative to beam ion velocity. They are formed by elastic charge exchange reactions. Significant probability density also exists for medium energy ions formed through scattering processes. These have a very broad distribution with respect to velocity and much lower probability per velocity increment compared to the ion groups at the very low and very high ends of the spectrum. Comparison between 1 m and 5 m distributions shows that beam ions are efficiently converted to low energy ions as they propagate away from the thruster. The low energy population has ~ 0.5 km/s higher axial velocity at 5 m than at 1 m, and a slightly broadened velocity distribution, indicating that ions produced upstream have accelerated along their path to the 5 m point. The low energy population at 1 m has a tail on the high velocity side, which seems to be missing at 5 m. Although barely noticeable, the high energy peak at 5 m has its maximum shifted to slightly higher velocity than the corresponding 1 m peak, for the same reason as the low energy peak. Assuming the same energy gain applies to these cases, the velocity increment for the high energy peak would be much less, as observed.

Ion and neutral axial VDFs are plotted at 1 and 5 m for the 1X pressure case in Figure 12. The ion VDF is identical to the plot given in Figure 11, except done on a log scale. It is immediately apparent in the right panel that the neutral low energy peak is greatly dominant over the high energy peak at both 1 and 5 m. This occurs because of the abundance of thermal neutrals from the background, with other velocities produced only through a collisional interaction. At 5 m the probability density of the high energy peak is much less than at 1 m, probably because most fast neutrals formed upstream have high angle trajectories and are quickly lost, while the on-axis production rate of fast neutrals is much lower at 5 m than at 1 m. Because of plume expansion and beam ion losses, the current density at the 5 m location is far less.

The ion on-axis VDF for the 1 m and 5 m 0X case is plotted in Figure 13 (left), with 5m having a larger low energy peak but otherwise little difference between 1 m and 5 m. The corresponding 20 degrees off-axis result, shown in Figure 13 (right), exhibits subtle differences. Appearance of the trough between 10,000 and 17,000 m/s is one example.

Comparing the on-axis 0X plot in Figure 13 (left) with the corresponding 1X plot in Figure 12 (left), we see there is attenuation of the high energy peak in the latter case and a substantially higher probability density for the 1X low energy peak vs 0X.

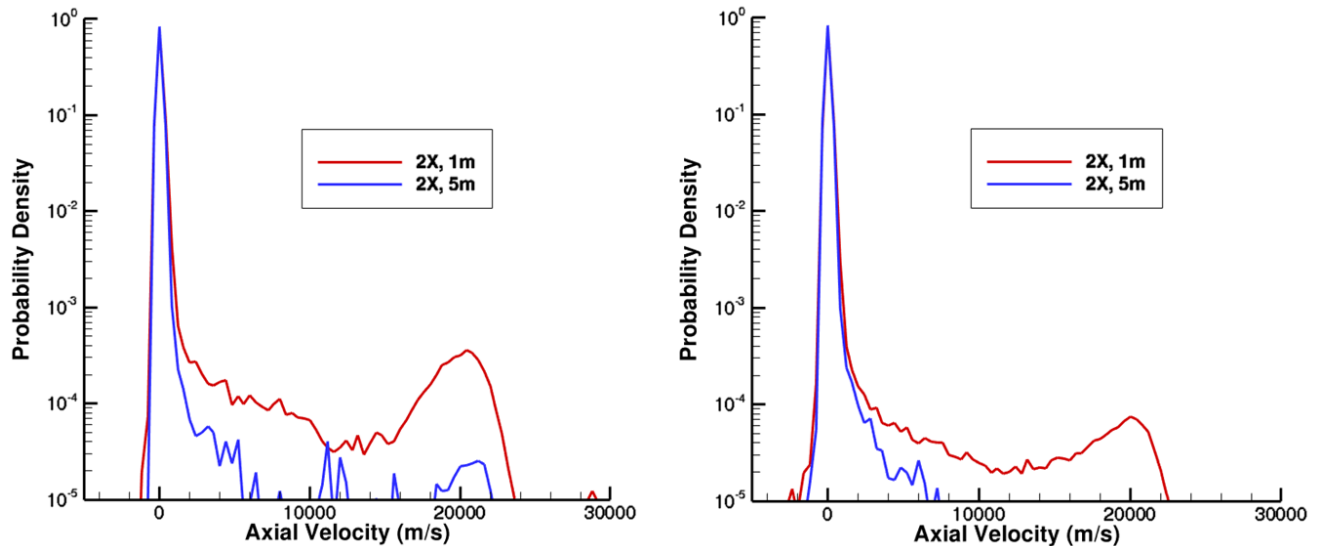


Figure 16. Axial VDF for neutrals on the thrust axis (left) and 20 degrees off-axis (right), 2X pressure at 1 and 5 m.

Figure 14 and its comparison with Figures 12 and 13 shows clearly that there is a regular ion VDF trend with increasing pressure, marked by a growing low velocity peak and shrinking high velocity peak. The same trend holds for 20 degree off-axis distributions also. Similarly, for neutrals, increasing background pressure corresponds to a

growing low energy peak and increasing attenuation of the high energy peak, accentuating amplitude differences there between 1 m and 5 m locations, as shown by Figures 15 and 16. For the 0X case, the low energy and high energy neutral peak amplitudes are closest. As pressure is increased, the low energy neutral peak becomes more dominant. The amplitude ratio between these peaks further increases at 20 degrees off-axis, relative to the axial points of reference. The explanation may be relatively low fast neutral production off axis, since ion flux is much lower there.

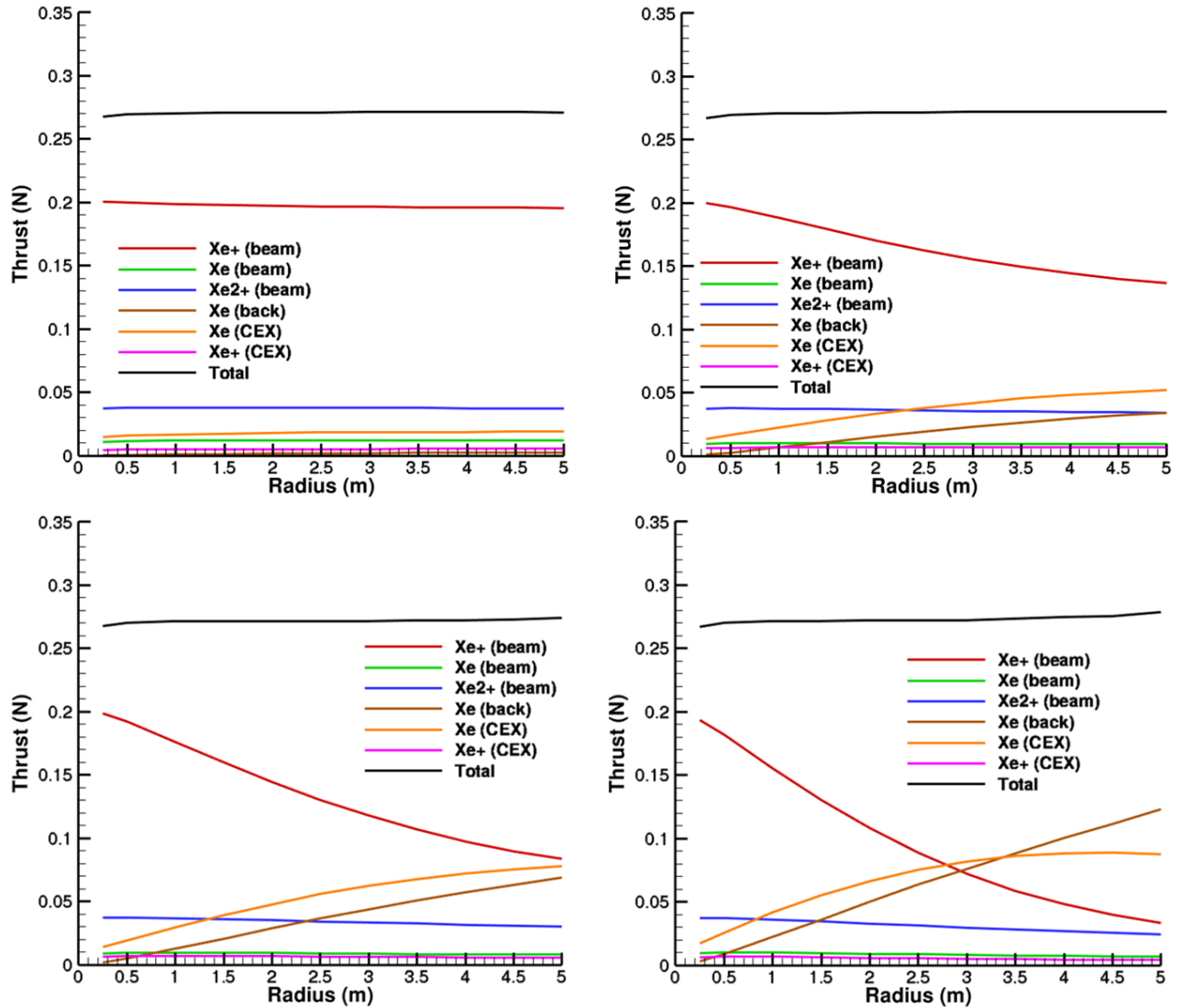


Figure 17. Calculated thrust contributions as a function of distance from thruster, for 0X (top left), 0.5X (top right), 1X (bottom left) and 2X (bottom right) background pressures. Xe (CEX) = fast atoms; Xe+ (CEX) = slow ions; Xe (beam) = neutrals emitted by thruster; Xe+ (beam) = singly charged ions emitted by thruster; Xe (back) = background atoms; Xe2+ (beam) = doubly charged ions emitted by thruster, and unlike Xe+ (beam) the profile includes both beam and CEX particles.

Species-specific thrust contributions for four different background pressures are plotted in Figure 17. In addition to the total thrust, as determined from the summed momentum flux crossing a thruster-centric spherical surface with 5-m radius, individual contributions are included in the plot. In the Figure, Xe (back) denotes thrust contribution from background atoms that underwent beam-ion scattering without charge exchange, Xe (CEX) denotes atoms that

were beam ions before undergoing a CEX event and becoming neutralized, and Xe^+ (CEX) denotes ions produced by CEX collisions involving neutrals from the source and background ensemble. If these ions undergo another CEX collision they will then be counted as beam species – about one-tenth do this. The summed contributions of ions and neutrals, as a function of background pressure, are given in Figure 18. It is apparent that ion momentum is being transferred into neutral momentum, with efficiency that depends nonlinearly on background xenon density. The 1X case corresponds to roughly 50% momentum transfer, with neutrals the dominant momentum carrier for the 2X case. Average ion velocity is decreasing as the relative population of slow CEX ions increases and the average neutral velocity rises. The mean axial ion velocity on the plume axis, plotted in Figure 19, illustrates the trend.

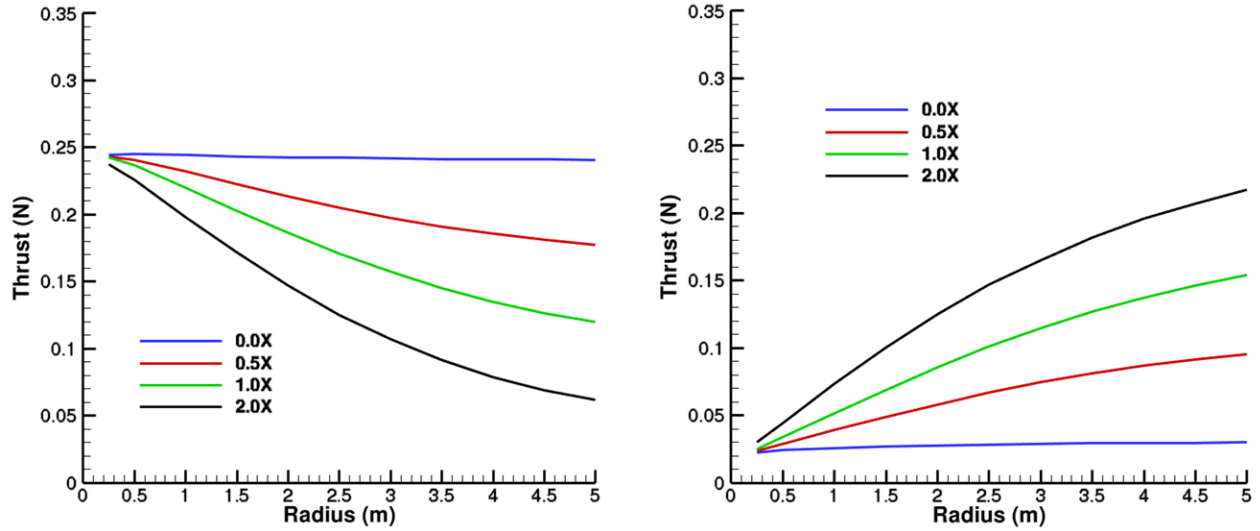


Figure 18. Calculated thrust contributions for ion (left) and neutral (right) ensembles as a function of integrating sphere radius and background pressure.

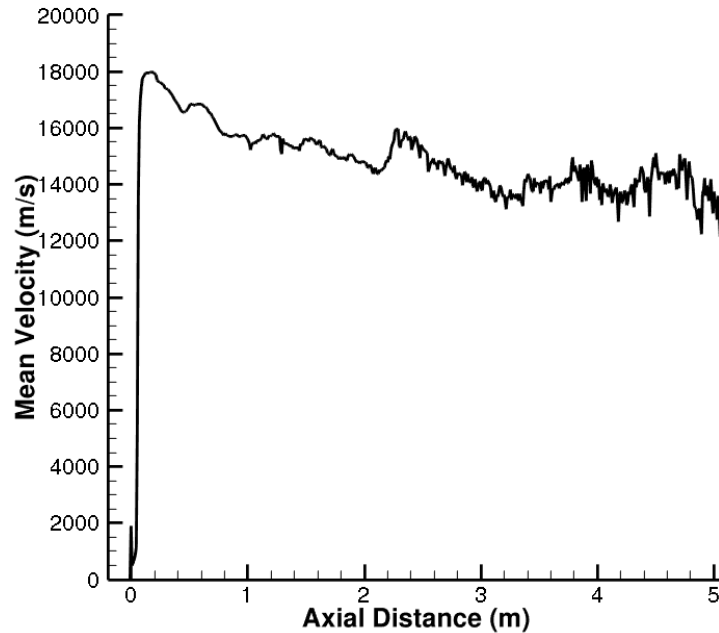


Figure 19. Mean axial ion velocity on plume centerline, as a function of axial distance.

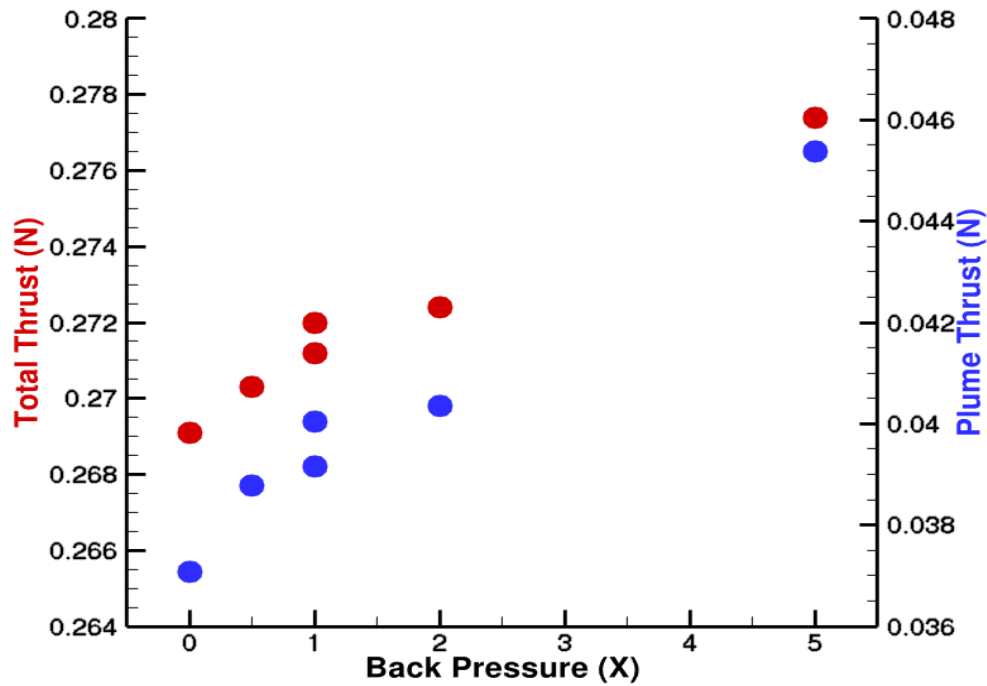


Figure 20. Integrated momentum flux crossing the spherical surface at 5-m distance, and the plume-only contribution.

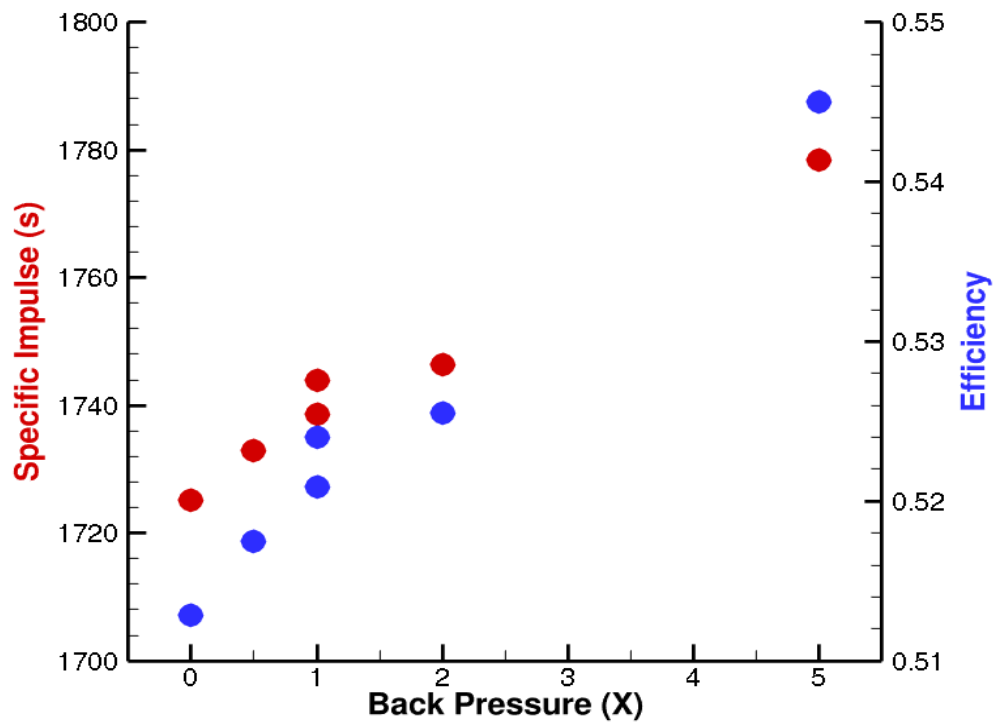


Figure 21. Specific impulse and efficiency as a function of the background pressure factor.

The only acceleration process in the plume simulation is due to electrostatic forces acting on the ions. By summing the associated velocity changes for every ion particle in the simulation, and averaging over time, the total momentum imparted to the plasma can be evaluated directly. Using this approach, the integrated momentum flux crossing the surface of a sphere of 5-m radius, centered on the thruster, is plotted in Figure 20 as a function of the xenon back pressure factor. The result is less nonlinear with background pressure than thrust predictions made previously by the analytical model. The magnitude of thrust augmentation over the 0X to 5X range is 3% for the numerical model, which is less than one-third of the variation found by the analytical model. However, there is a crucial difference with respect to plume potential inputs and the way total momentum was calculated. Plasma potential inputs to the analytical model were higher than potentials calculated and utilized by the numerical model in the 0.35 – 2.0 m region where the majority of plume interactions occur that go on to produce momentum gain. In the 0.0 – 0.25 m region simulation potentials used to determine momentum results were higher than corresponding analytical model potentials, but the summed momentum contributions of this relatively small region are minor in comparison to the total. In addition, the numerical method only calculated momentum change up to simulation potentials along the 5-m arc, rather than to a zero potential surface. A crude estimate of the correction to numerical results if experimental potentials were used instead, suggests the two methods are about a factor of two apart in their predictions of 0X to 5X thrust augmentation. Zero referencing the potential increases the terminal plume momentum at all pressures, and using the experimentally-derived potential with its increasing trend as pressure decreases, acts to steepen the initial momentum increase and flatten the curve at elevated pressure.

The analytical model made several approximations that were not necessary for the more sophisticated treatment used here. These include a single-collision assumption, uniform current density across radial cuts, neglect of Xe^{2+} and its reactions, and neglect of collisions where there is non-negligible momentum transfer between nuclei. Given the various approximations involved, the agreement between models is quite good. The numerical model, using accurate potentials, should provide better results than the analytical model.

The specific impulse and efficiency, obtained from the standard definitions $I_{sp} = T/\dot{m}g$ and $\eta_T = T^2/2\dot{m}P_{in}$, respectively, where $\dot{m}$ is mass flow rate, T is thrust and P_{in} is input power, are plotted in Figure 21. Both increase steadily with xenon background pressure, as has been observed in a number of past studies with various thrusters.^{1,14}

As mentioned, slow ion production from electron impact ionization in the plume was included in the simulations. Its momentum contribution was found to be substantially smaller than the sum of CEX and scattering components at 1X pressure, but not negligible. It would be of interest to determine relative contributions at each of the pressure conditions.

We have made no attempt to include background gas ingestion into the thruster itself, as the goal was to evaluate the effects of background gas diffusion into the plume only. As mentioned earlier, neither the magnetic field nor transient effects of the thruster (e.g. breathing mode oscillations) have been treated by the simulations. If either or both of these factors can significantly change plume potential, it could explain the discrepancy between experimental and numerical potentials. It has been observed before that crossed fields give rise to increasing potential as pressure is lowered, contrary to expected classical mobility predictions.¹³

IV. Conclusions

Numerical simulation techniques were used for the first time to track the evolution of species-specific and total plume momentum throughout a Hall thruster plume. Detailed calculations were performed to determine velocity and density distribution functions for the population of ions and neutrals within the plume volume. A DSMC approach was used to model the collisions of the ions and neutrals, with PIC modeling of ion transport in the plume electric field. All significant particle interactions were included in the model, and specific emphasis was placed on the dependence of velocity distribution functions and total plume momentum on the density of background xenon gas. To obtain the results, the coordinate-dependent influence of plume parameters such as ion flux density, neutral density, electron temperature and plasma potential was included in the calculations. Results confirm that significant plume interactions occur, with initial ion beam momentum becoming substantially distributed between ion and neutral populations as the ion beam is attenuated, and increasing total momentum flux across a thruster-centric hemispherical surface at increasing downstream distance. Velocity distribution functions continue to change significantly with propagation distance beyond 1m and exhibit angular dependence. Momentum augmentation is primarily due to collision-based reduction of average ion momentum as average neutral momentum increases, and the on-going acceleration of ion populations in the local electric field. The increase of plume momentum with rising background gas density is less than predicted in an earlier analytical study, with part of the difference attributable to relatively low plume plasma potentials found by the DSMC model. The difference in plume potentials in the two

studies – one based on heavy extrapolation of mid-field experimental data and the other on DSMC simulations - is substantial, and the difference is unexplained at this time. If the average plume potential varies inversely with background pressure as suggested by experimental data, the time varying nature of the thruster plasma and/or effects of the plume magnetic field may have to be included in the numerical simulation to realize similar behavior.

Plume momentum contributions from ions as a function of their birth location tail off with distance from the thruster, due to ion beam attenuation and decreasing local plasma potential at birth. As a result, 5-m facility dimensions do not appear to be required to realize similar results. The plasma potential distribution is critical, obviously, in the plume momentum calculation. Equating plume momentum augmentation with increased thrust level, the specific impulse and efficiency increases with background pressure. The improved performance results from plume ingestion of additional propellant and conversion to slow ions that gain more momentum in the local electric field than fast ions. Average ion momentum decreases as average neutral momentum increases. Ground-test performance tends to increase with thruster power, due to this mechanism, other factors remaining constant.

Further work needs to be done in this area, with much remaining to be learned. For the present study, the inclusion of magnetic field effects and additional efforts to ensure the internal consistency of simulation results could be next steps. Experimental measurements of near-field potentials, electron temperature and ion/neutral VDFs and neutral density, including transient behavior, will be valuable.

Acknowledgments

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