

# Numerical procedure permitting calculation of how a vacuum chamber influences Electric Propulsion Thruster jet expansion.

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**Abstract:** It is known that ERT integral parameters depend on the test conditions. This is especially true for the SPT. We know that the same thruster's model demonstrates different performances in different chambers. Unfortunately up to now there are no criteria, which permit the comparison of a thruster operation under different test conditions. In this work we have tried to develop a numerical procedure permitting the calculation of how the vacuum chamber influences SPT jet parameters. In the vacuum chamber, if the thruster operated, the residual gas pressure increases in order. Therefore first of all it was necessary to simulate ion motion in a residual gas atmosphere, which formed as a result of ion flow interaction with the chamber walls. Also with the help of the PIC model the concentration of the particles flew out the thruster and distribution of charge-exchange particles in the chamber volume were calculated. In order to calculate the electric field density in the ion jet, a fluid model was developed. The model was based on the solution of continuity, momentum and mass conservation equations. The results of calculations were compared with the results of calculations carried out for the Centropazio[15] chamber and also with the calculations done with the help of the ISP-2000 software package. (ISP-2000 calculates how Hall thruster jets interact with the spacecraft). Also the charge-exchange ion current was compared with the experimental data obtained with the help of Lengmuir probes located in the thruster back hemisphere. The convergence of calculation was about 20%. The possibility, if it is possible to correct probe performances inaccuracy happening due to residual gas influence was examined with the help of developed software.

## Nomenclature

- $\varepsilon_i$  = ionization potential;
- $\lambda_e$  = electron thermal conductivity;
- $\sigma_i$  = ionization section;
- $\sigma_y$  = elastic event section;
- $\varphi$  = potential;
- $\nu_e$  = frequency of electron-atom or electron-ion interaction;
- $E$  = tensivity;
- $e$  = elementary electron charge;
- $j$  = ions current;
- $k$  = Boltzman constant;
- $m_e$  = electron mass;
- $m_i$  = ion mass;
- $n_a$  = concentration of neutral particles;

- $n_e$  = concentration of electrons;
- $n_i$  = concentration of ions;
- $p_e$  = electron pressure;
- $T_e$  – electron temperature;
- $T_H$  = temperature of neutral atoms;
- $V_e$  = electron velocity

## I. Introduction

This project researches the operation of the Hall thruster in vacuum chamber and the processes which occurred in the chamber.

Preprocessor interface:

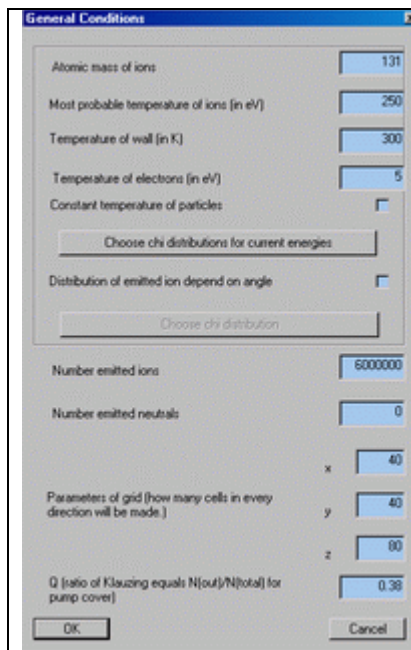


Figure 1. General Conditions dialog

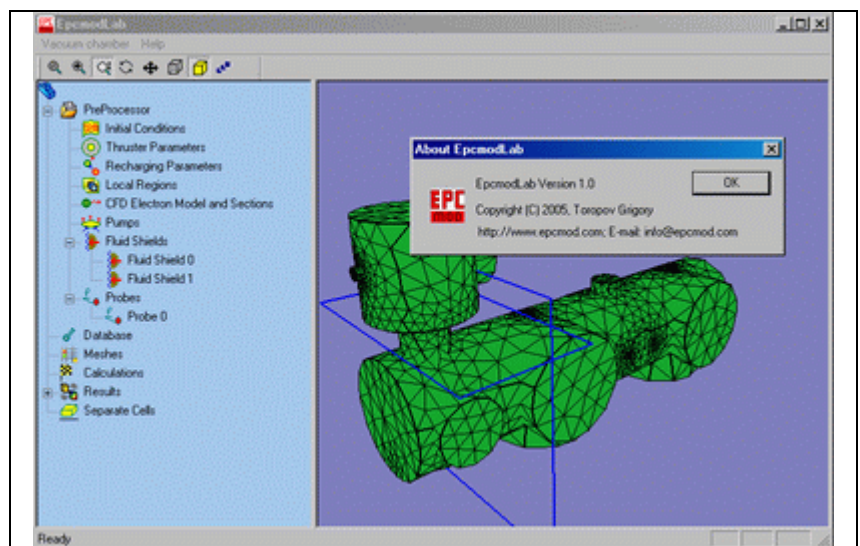


Figure 2. EpcmodLab main window

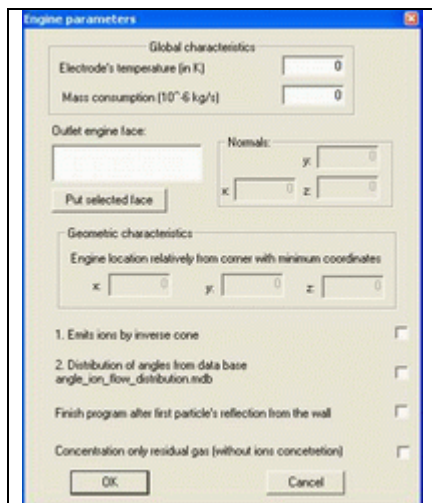


Figure 3. Thruster parameters

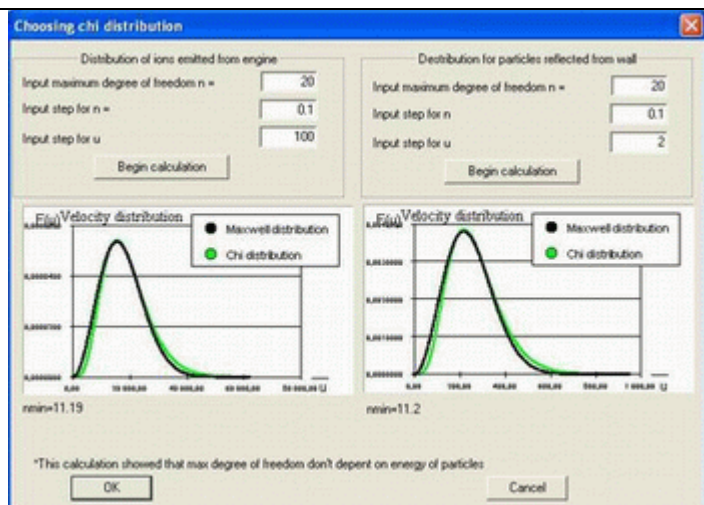


Figure 4. Matched chi distribution for Maxwell's distribution for predefined temperature

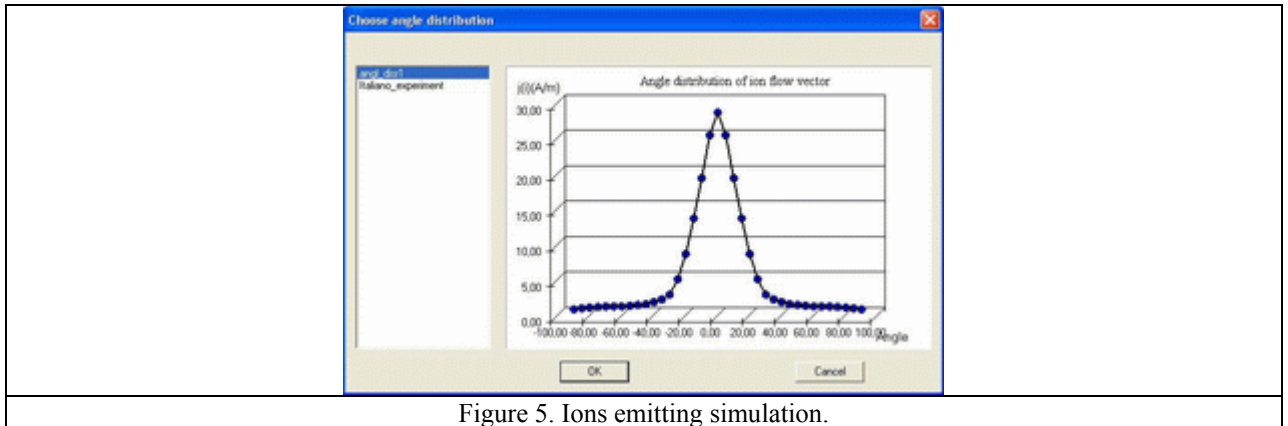


Figure 5. Ions emitting simulation.

The thruster emits a predetermined number of ions and neutral particles. Ions were emitted by predetermined function  $j=f(\alpha)$ , where  $\alpha$  – angle between the direction of the ion and the normal direction to face of the thruster;  $j$  – density of ion's current. Neutrals were emitted in all directions to normal direction of start face with equal probability. Ions of working gas become neutral atoms after interaction with the chamber wall and leave the wall with the velocity taken by Maxwell's distribution for temperature of the wall.

A Particle which touches the wall reflects with equal probability in all directions to normal of the face in the point of touching. All particles move by straight lines (After the first iteration density of calculated electric field was considered). Reflections occurred until particle touched the face of pump.

It must be considered also that for finding out the concentration of neutral atoms, ions, and recharged ions there was used trajectory algorithms for determined number of simulated particles with definite boundary conditions (angle distribution for fast ions flow simulation, determined number of recharged neutral particles in definite cell for recharged particles distribution (the initial values of recharged particles in cell we determined after neutral particles distribution calculation and tracing the fast ion through resonance cross section for current ion energy) , and cosine probability reflection to normal to wall of chamber for particle-wall interaction).

## II. General Guidelines

### 1. Calculation of Klauzing ratio for different channels and comparing them with table values.

For checking the algorithm some tests were made. Different geometries of the tube were taken and the ratio of Klauzing was found which equals  $Q = \text{Out}/N$ ; where  $N$  – total number of emitted particles and  $\text{Out}$  is number of particles left the tube. (If a particle fell back on the start face it was decided that it didn't leave the tube). Then we compared  $Q$  with table values. Changing the length of the extrude and setting emitting surface in the beginning of the coordinate system the ratio of Klauzing was calculated.

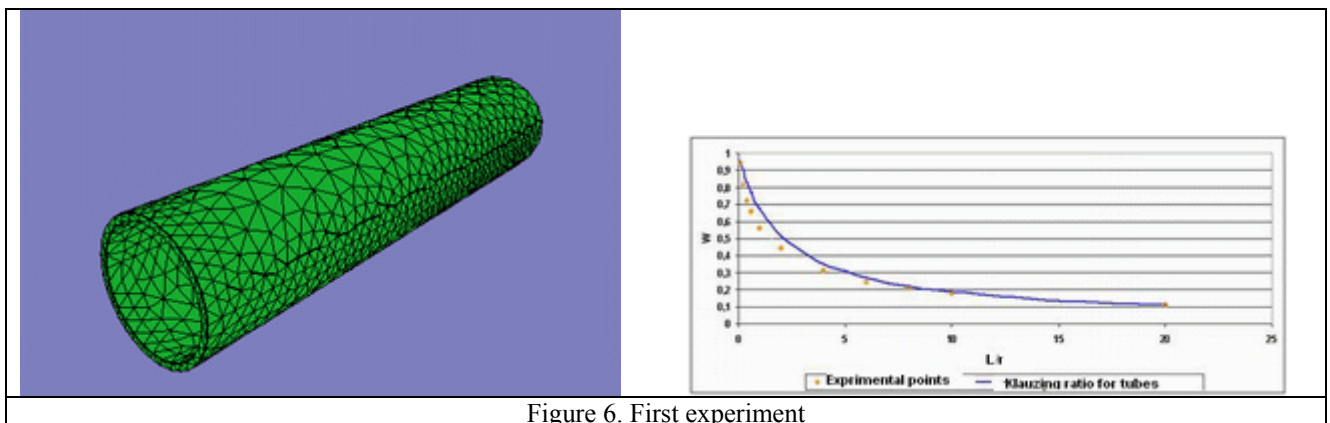


Figure 6. First experiment

The blue line shows the theoretical curve which was taken from “Dynamics of discharged gas” written by Koshmarov J.A. and Ryzhov J.A. Moscow 1977.

The same operations which were carried out in the first experiment were repeated but the shape was ring. Ratio  $r1/r2 = 0.4$  for this experiment.  $R1$  - radius of inner cylinder and  $r2$  – radius of outer cylinder.

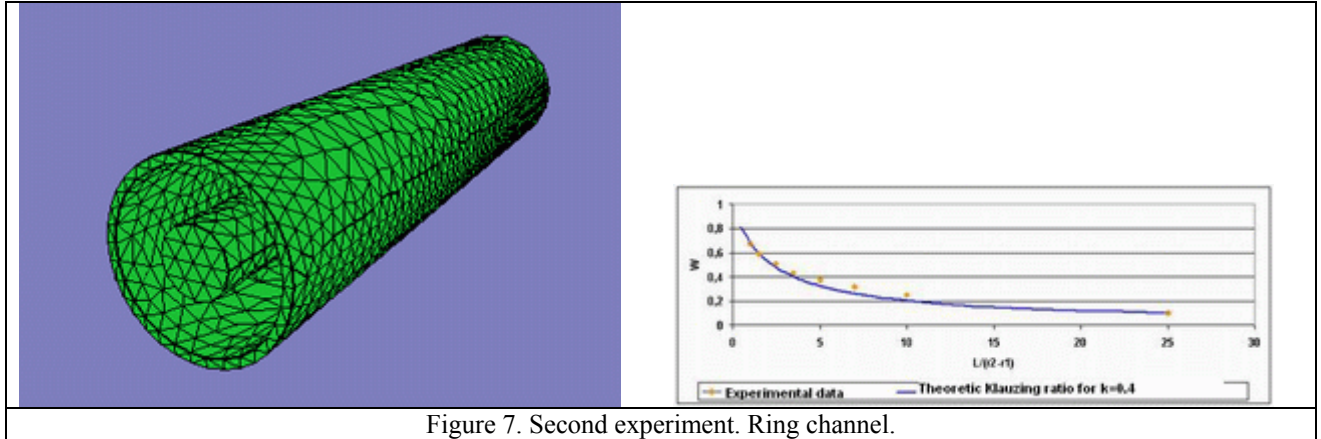


Figure 7. Second experiment. Ring channel.

For convergent cone channel the half angle of top equals 20 degrees. Shown are charts of Klauzing ratio dependence for cone diffuser from  $L = l/ra$  for the half angle of the cone equally 20 degrees;  $ra$  – radius of start face and  $l$  – length of channel.

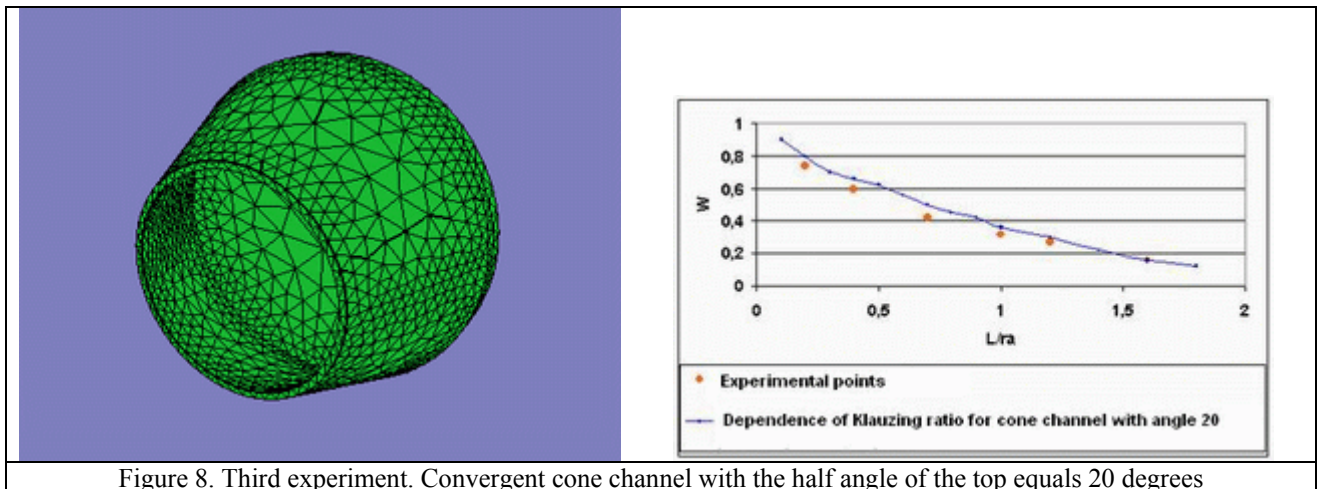
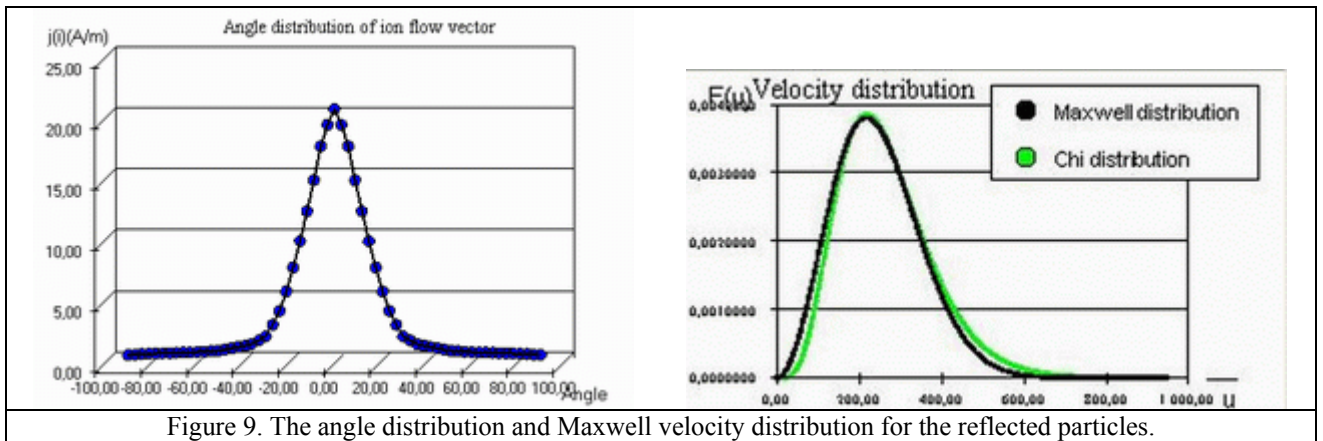


Figure 8. Third experiment. Convergent cone channel with the half angle of the top equals 20 degrees

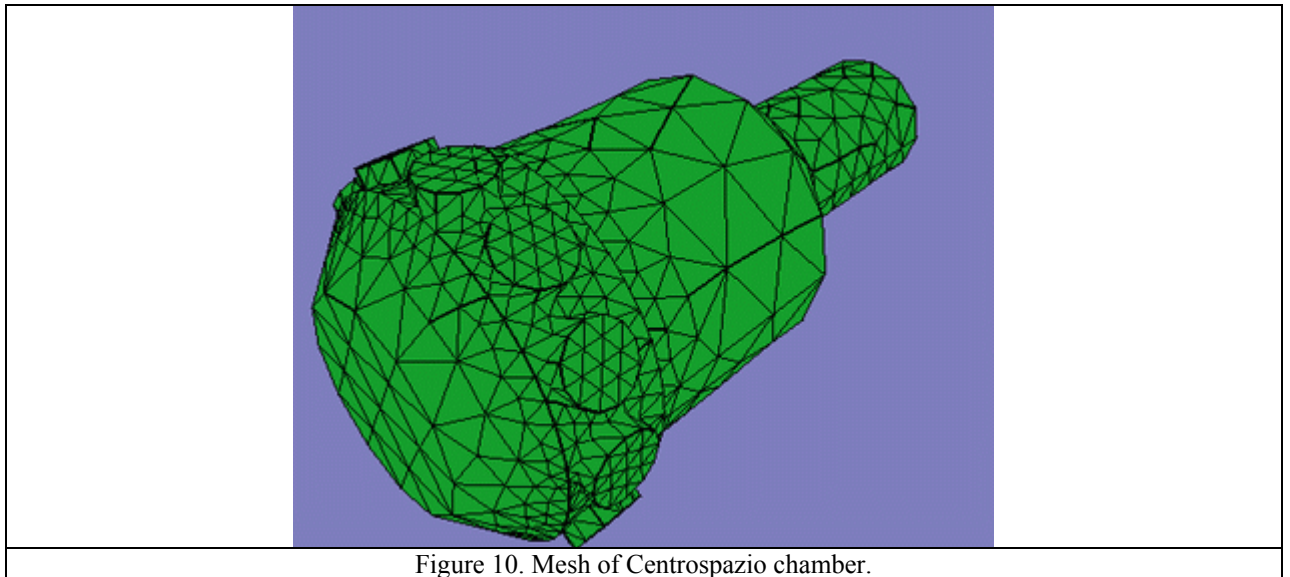
**2. Comparing results of the program with calculations made at Centrospazio for Alta IV-4 chamber. (Recalculated with volume in cells)**

The same experiment as in Centrospazio for chamber Alta IV-4 was duplicated. The 3kW thruster was set in the same place as they did with mass consumption 5.4 mg/s and angle's distribution shown on image 1 which was taken from the article "The new EP test facilities at Centrospazio and Alta" published in IEPC 2003.

The velocity distribution of emitted particles was set by Maxwell's distribution with temperature of reflected particles equally 300 K.



The article "The new EP test facilities at Centropazio and Alta" also shown the dimensions of the chamber. Therefore the shape of the chamber could be constructed. Nine extrudes are pumps. It looks so:



Shown are results for 600000 emitted ions and 30000 emitted neutrals(5%) and grid 40x40x80.

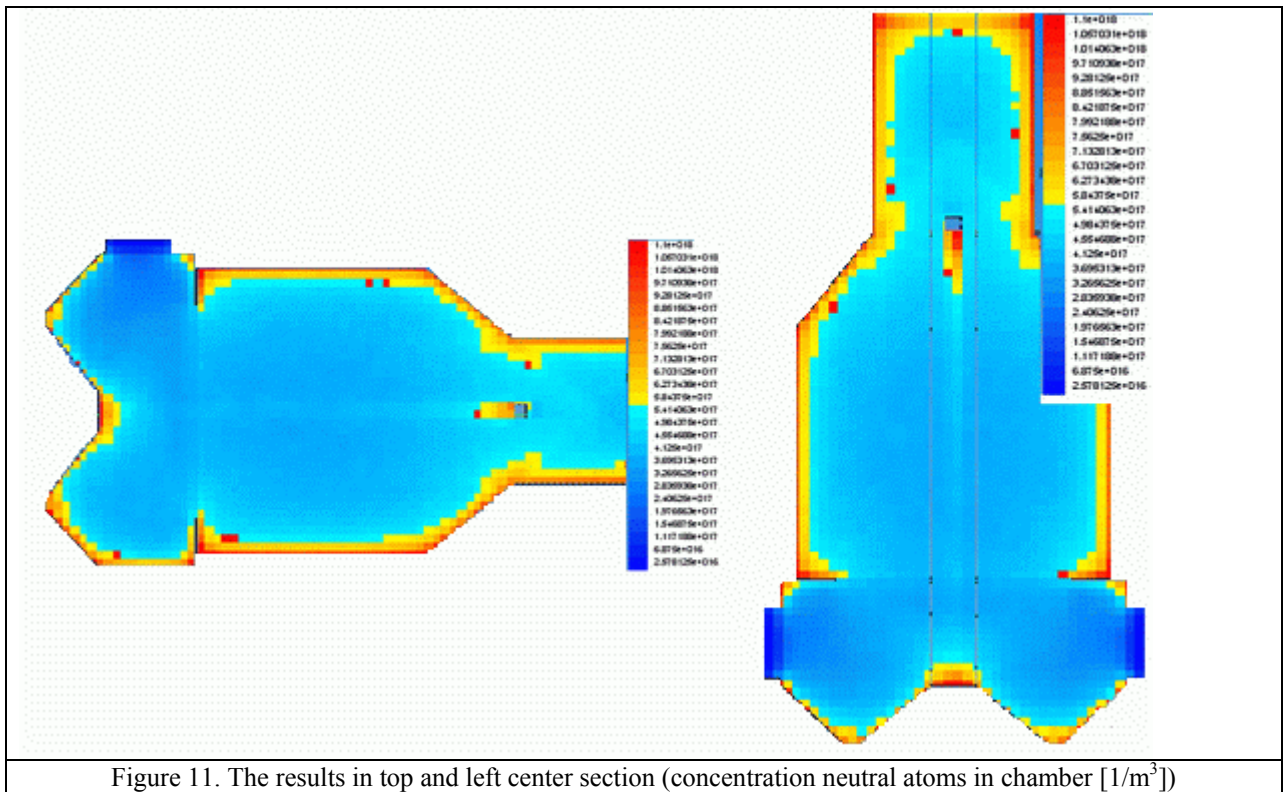


Figure 11. The results in top and left center section (concentration neutral atoms in chamber [ $1/m^3$ ])

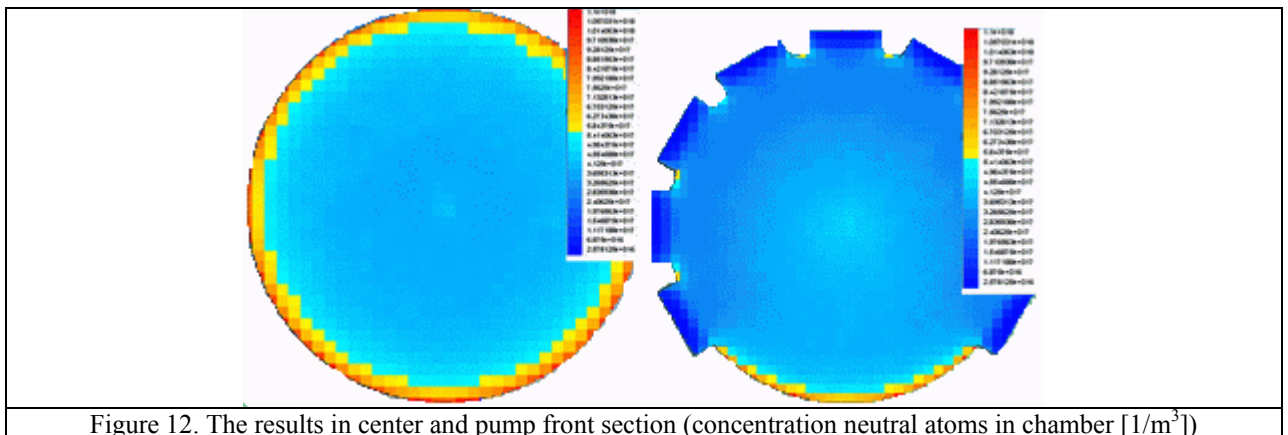


Figure 12. The results in center and pump front section (concentration neutral atoms in chamber [ $1/m^3$ ])

Shown are Italians distribution taken from "Basic issues in electric propulsion testing and need for international standards" in IEPC 2003[13,15]:

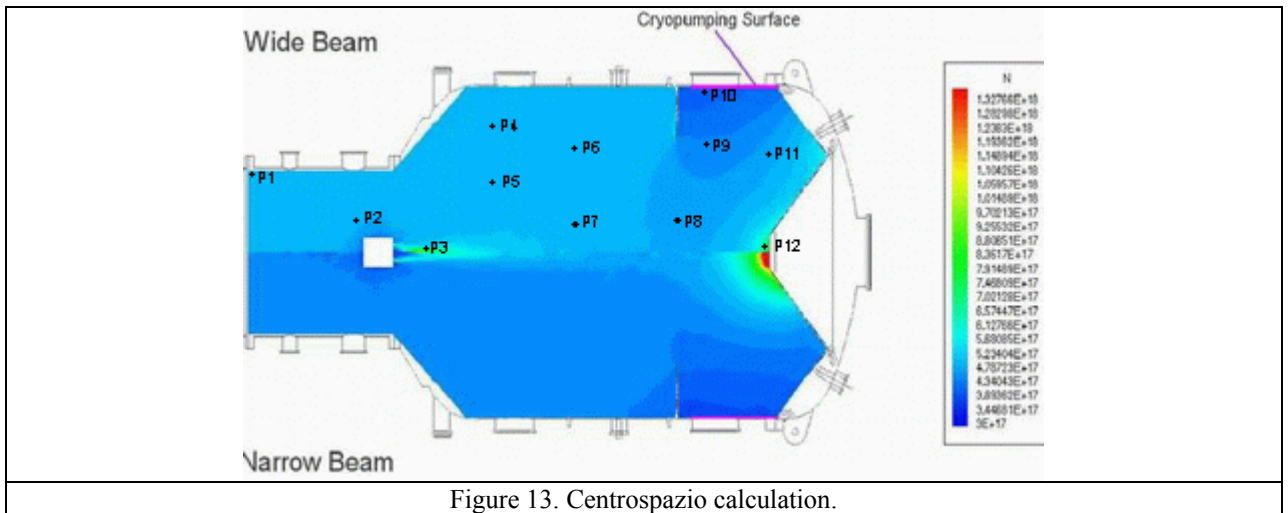


Figure 13. Centrosazio calculation.

The values of concentration in the volume of the chamber had approximately 12% difference in the two models but in our application there was two times increasing of concentration on the edges.

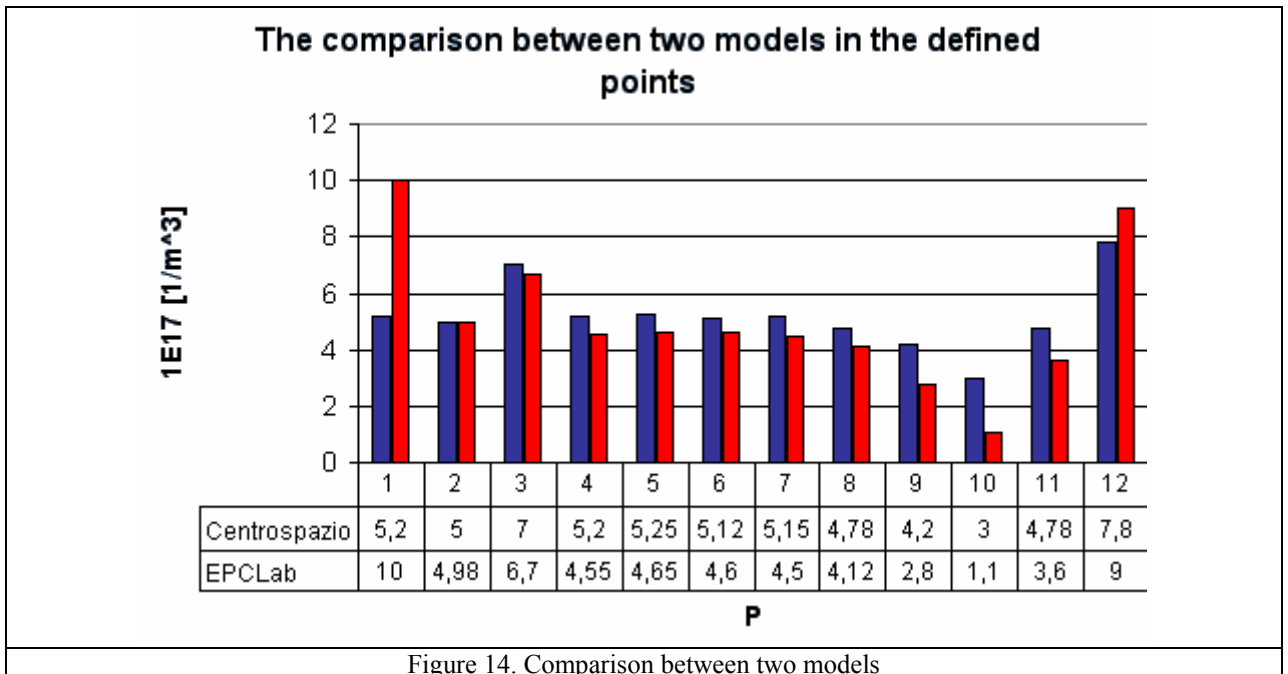


Figure 14. Comparison between two models

**3. Comparing the results of the program with calculations made by the Italians for chamber which stands at MAI.**

We carried out the same experiment but in an other chamber which stands at MAI in 208 department.

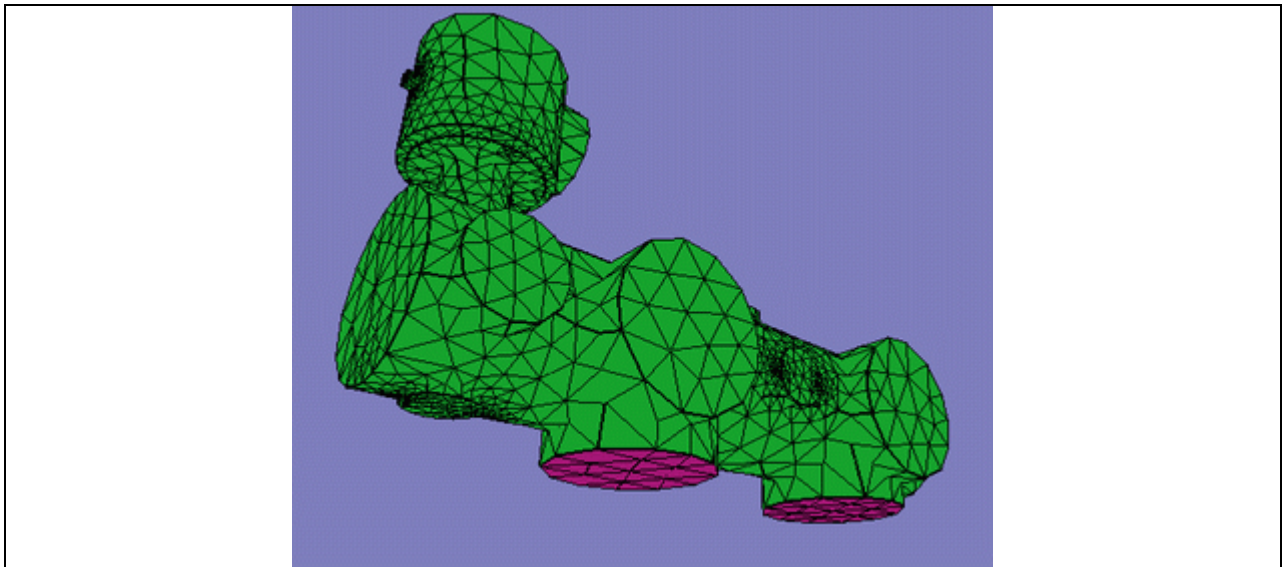


Figure 15. Vacuum chamber at MAI.

Shown are results for grid 40x40x80 and 600000 emitted ions and 30000 neutrals(~5%):

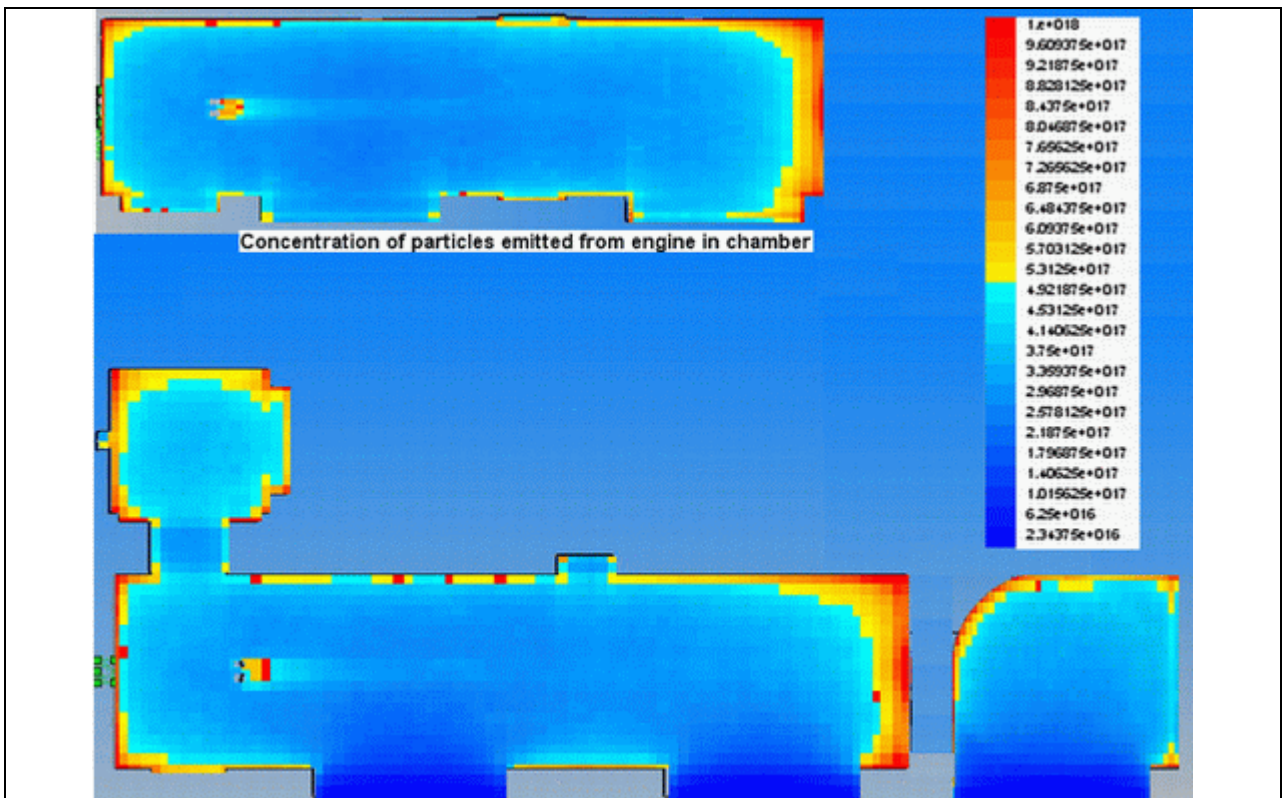


Figure 16. The results in different sections (concentration neutral atoms in chamber [ $1/m^3$ ])

#### 4. Comparing the calculations of recharged ions with experimental data taken from article “Plasma parameter distribution determination in SPT-70 plume” made in RIAME MAI.

In the article “Plasma parameter distribution determination in SPT-70 plume“, published in IEPC 2003 [14] and carried out at RIAME MAI the experiment with estimation of back ion flow of recharged neutral particles was described.



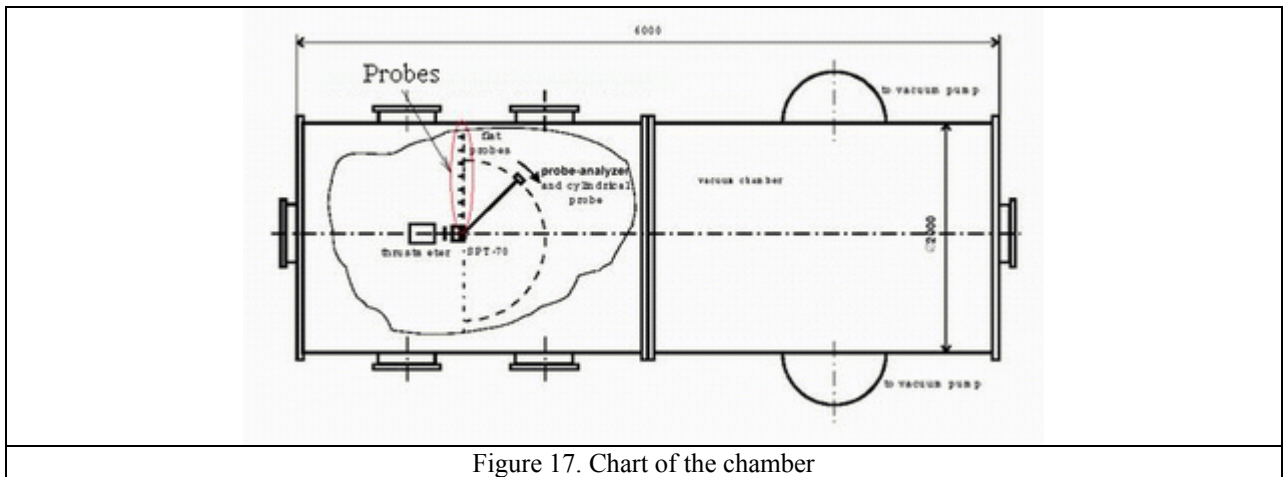


Figure 17. Chart of the chamber

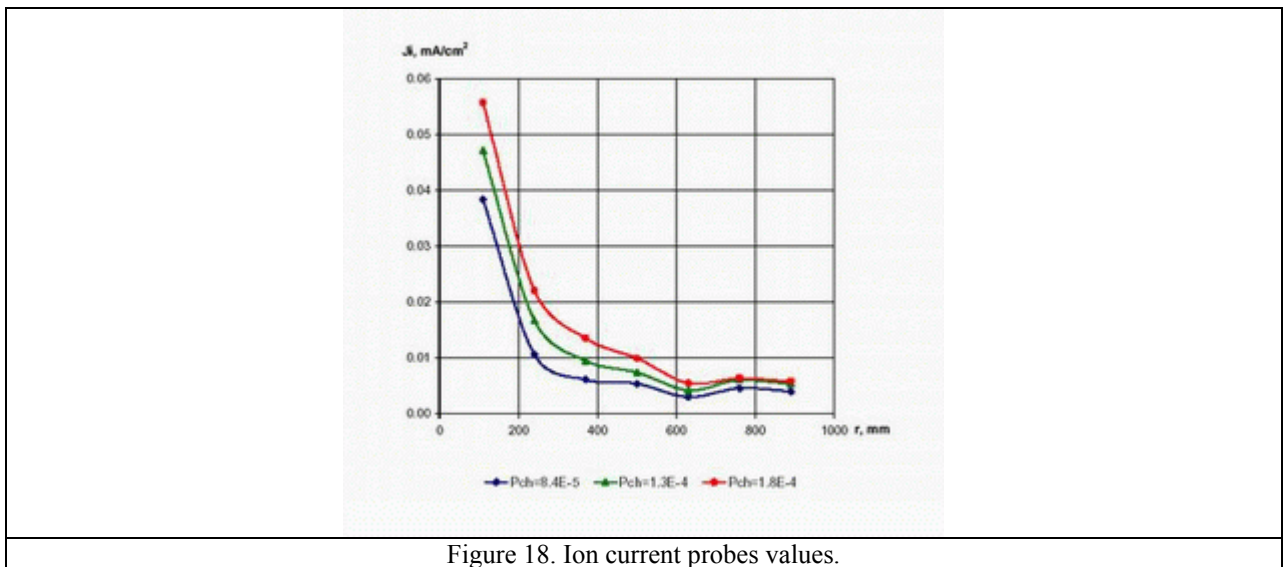


Figure 18. Ion current probes values.

An attempt was made to carry out this experiment in our application.

1. The shape of chamber was created.

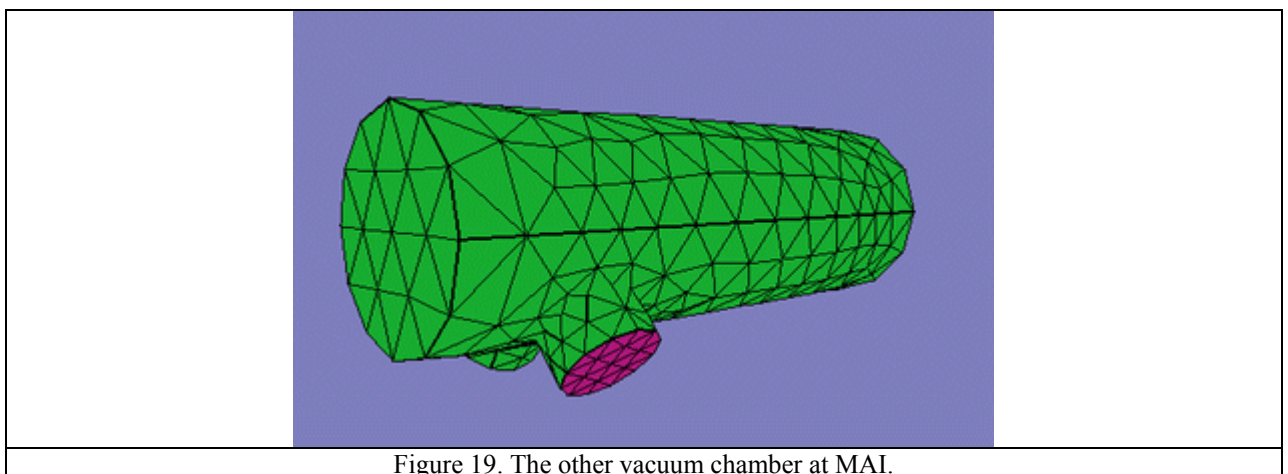
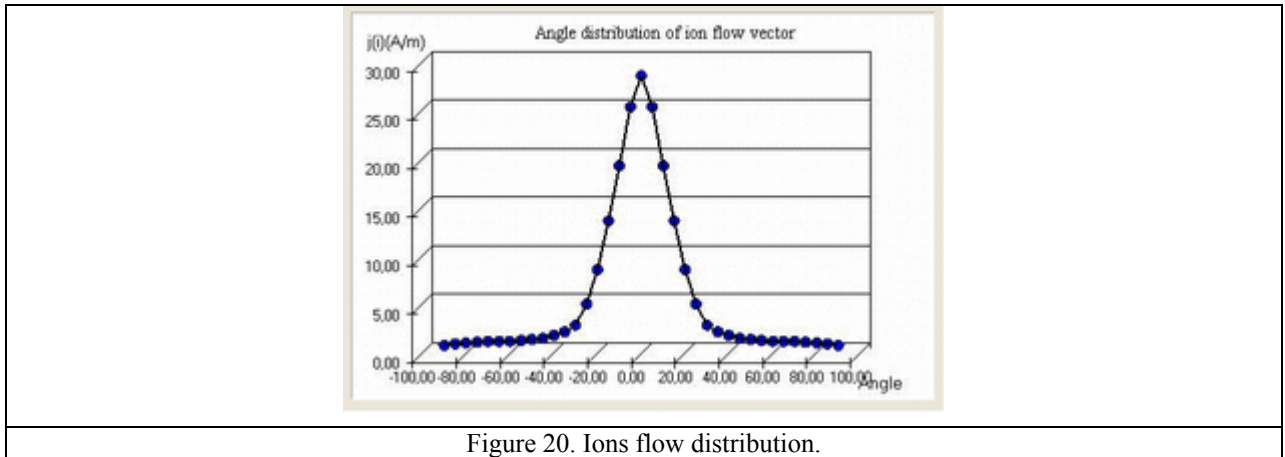
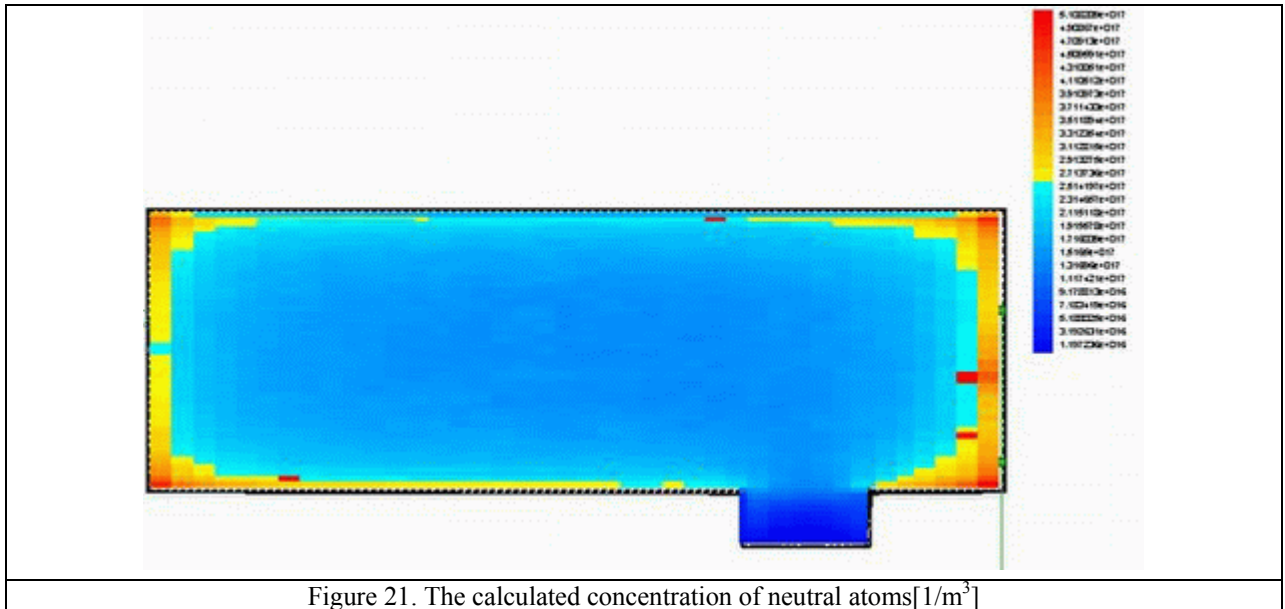


Figure 19. The other vacuum chamber at MAI.

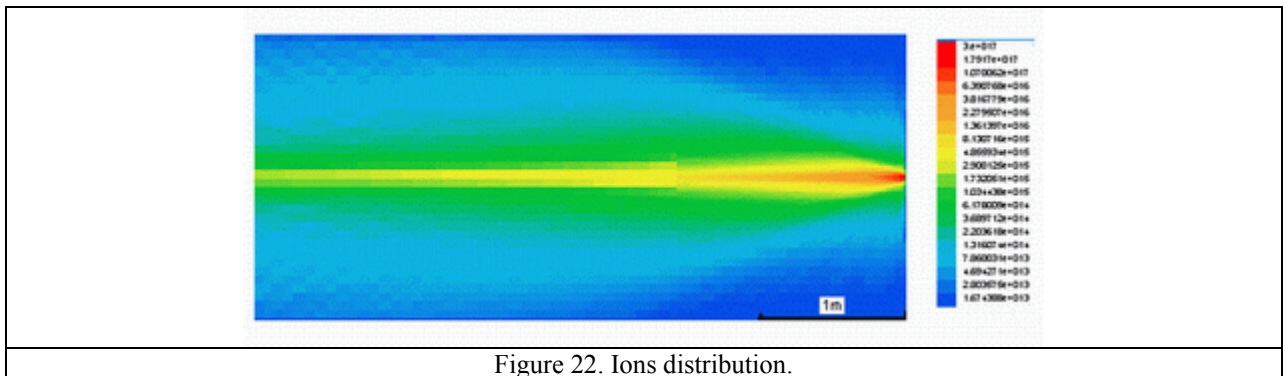
2. The thruster was placed exactly as in initial experiment. Mass consumption equaled 2.35 mg/s and ions current was determined as shown on the chart below:



3. Then neutral particles concentration was calculated for 100000 emitted ions:



4. The distribution of ions emitted from thruster:



5. Shown is the picture of the concentration of recharged particles:

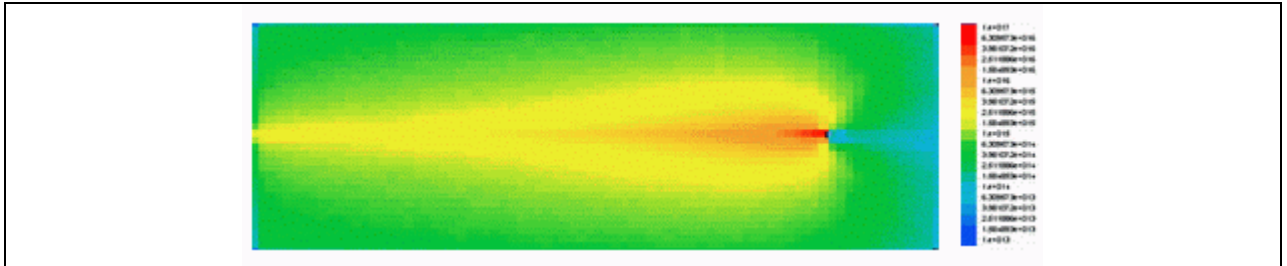


Figure 23. Recharged particles distribution.

6. Ion current equals:

$j = neV$ , where  $e$  – charge of 1 ion;  $n$  - concentration of recharged ions taken from picture;  $V$ - velocity of recharged ions (250 m/s). This is the average velocity for 300 K temperature.

7. A comparison was made between the calculated ion current from the distance from the center of thruster to the probe with experiment curve.

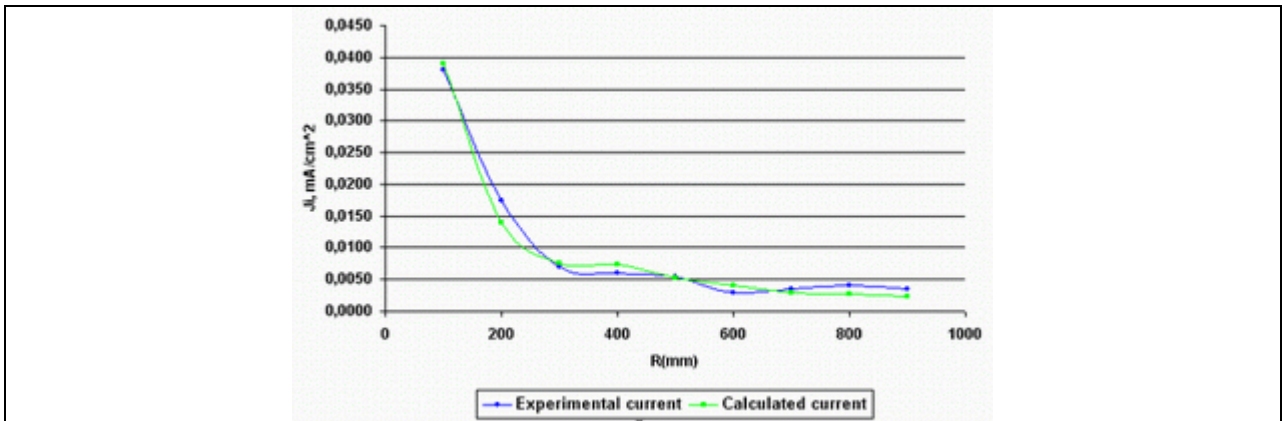


Figure 24. The Comparison of experimental and calculated ion current.

It should be noted that our calculations were made without initial residual gas in the chamber.

**4. Concentration of ions emitted from SPT-70 in the volume of the chamber without charge-exchanging process. Comparison with ISP.**

The first step for calculation of the electric field in the chamber is to find the distribution of the concentration of ions emitted from the thruster in the volume of the vacuum chamber. In our experiment the SPT-70 thruster was taken with angle distribution of ions.

The trajectory of 300000 ions was calculated with energy taken by Maxwell with maximum of probability 250 eV. Ions emitted with equal probability from the start face shown in next image.

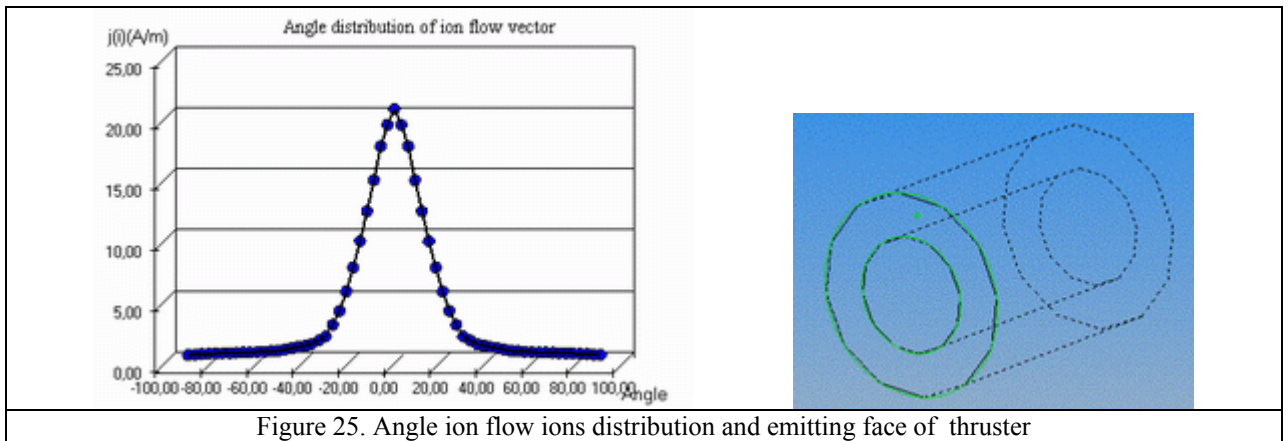


Figure 25. Angle ion flow ions distribution and emitting face of thruster

After the calculation of ions trajectory and knowing their velocity, we can calculate the concentration of ions in the cell. Shown are results.

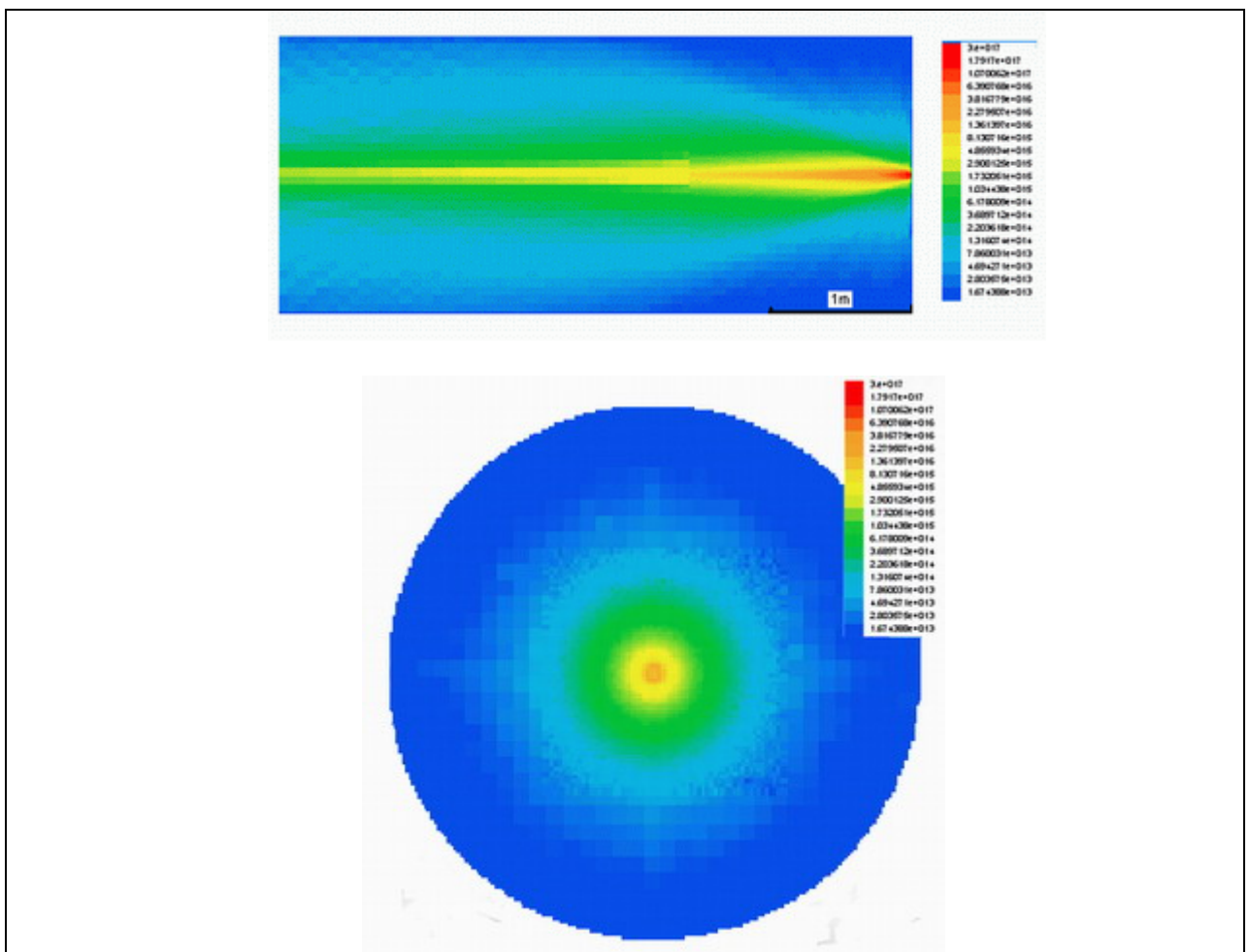


Figure 26. The left and front sections on 740 mm distance from thruster (Concentration of ions  $[1/m^3]$ )

Shown is the distribution of concentration of the same thruster with the same angle distribution but ions were emitted from one point. Calculated in ISP (Program researched for calculation of the same processes in space)[16].

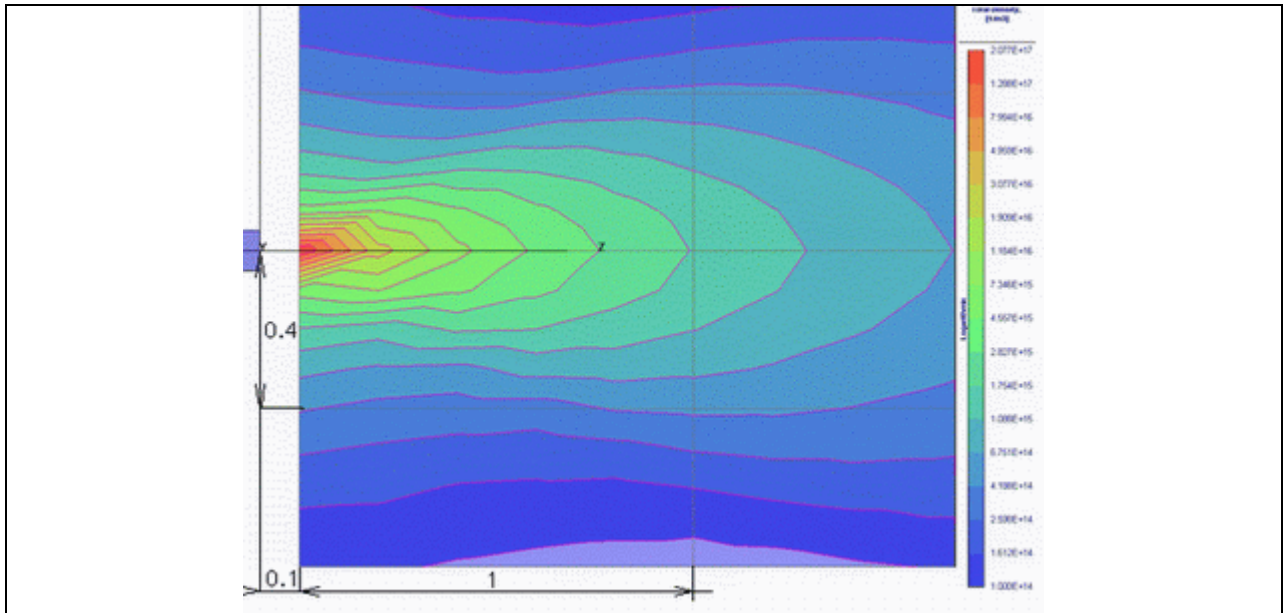


Figure 27. The left section of ions concentration calculated by ISP application (Concentration of ions [1/m<sup>3</sup>])

## 6. Potential distribution calculation.

### I. Introduction.

Usually the Boltzman equation is used in the potential calculations:

$$\varphi - \varphi^* = \frac{kT_e}{e} \ln\left(\frac{n_e}{n_e^*}\right) \quad (1)$$

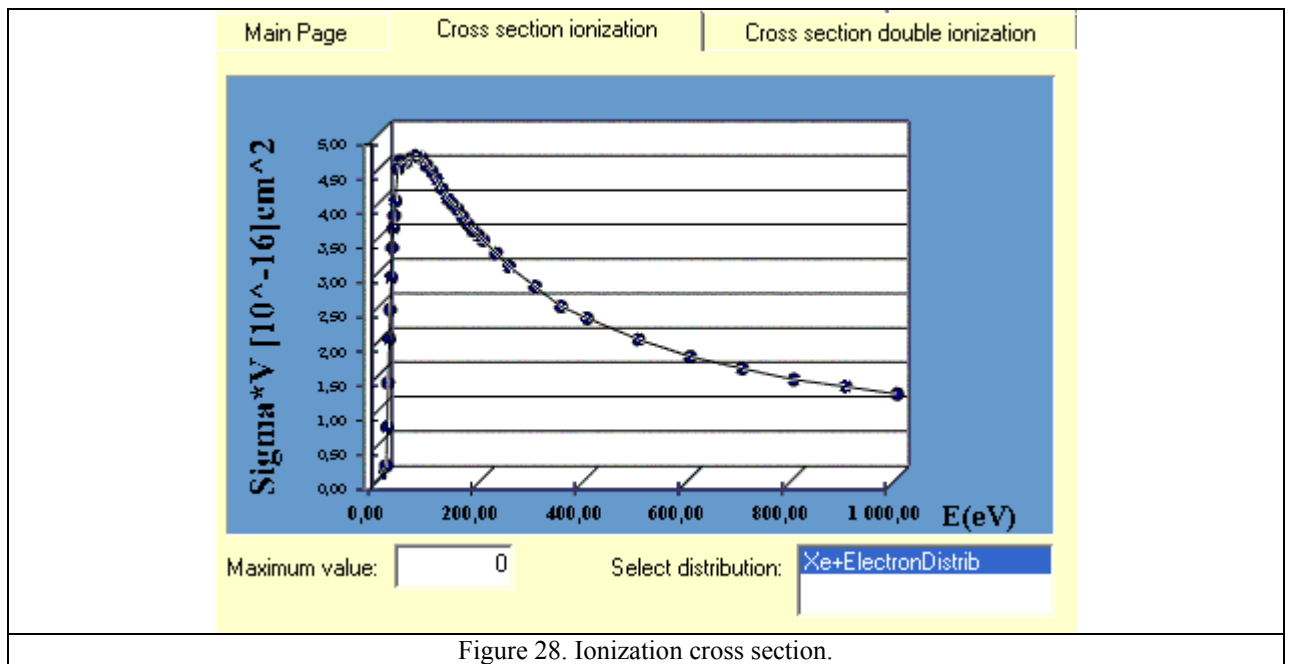
However, when the gradient of electron concentration is high this equation gives us a smaller decreasing of potential. Therefore, we will use a more detailed calculation based on solving 3 equations: from electron continuity equation the decreasing of velocities is received, then from the electron momentum equation the generalized Ohm's law is received which gives us the potential distribution. Because the generalized Ohm's law has a temperature member we must also solve the electron energy equation.

For the solving of these equations the finite difference method was used, putting as initial condition the constant temperature and potential in the entire chamber volume, (except the wall) and then by iterations the real distribution was received.

### II. Equations

*1-st equation (electron continuity equation):*

$$\frac{\partial}{\partial t} n_e + \nabla(n_e \cdot V_e) = n_e n_a \cdot \langle \sigma_i \cdot V_e \rangle \quad (2)$$



2-nd equation (electron momentum equation):

$$\frac{\partial}{\partial t}(m_e n_e V_e) + m_e n_e (V_e \nabla) V_e = -en_e E - \nabla p_e + R \quad (3)$$

$$p_e = n_e k T_e \quad (4)$$

$$R = \frac{en_e j}{\sigma} \quad (5)$$

$$\sigma = \frac{e^2 n_e}{m_e v_e} \quad (6)$$

$$v_e = \langle \sigma_y V_e \rangle \quad (7)$$

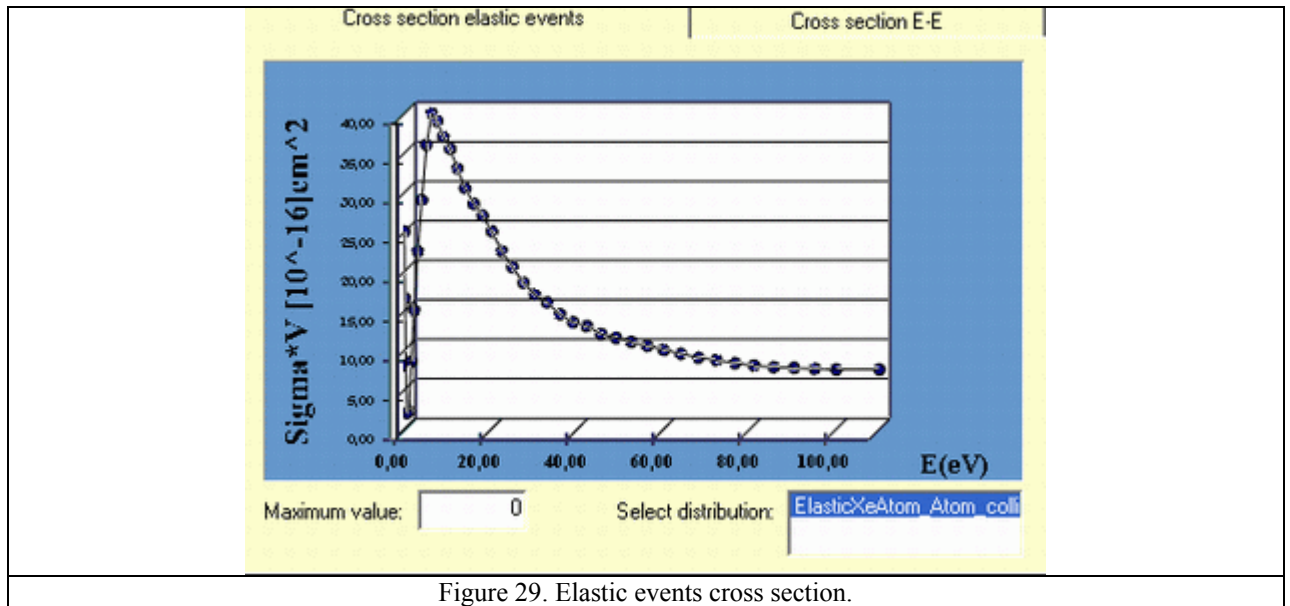


Figure 29. Elastic events cross section.

Assuming a steady state, neglecting the initial term on the left hand side of equation (3) and introducing plasma potential  $-\nabla\phi = E$ , a generalized Ohm's law is obtained:

$$j = \sigma \left[ -\nabla\phi + \frac{1}{en_e} \nabla(n_e kT_e) \right] \quad (8)$$

For given  $n_e$ ,  $V_e$  and  $T_e$  the charge continuity condition:

$$\nabla j = 0 \quad (9)$$

**3-d equation (electron energy equation):**

$$\begin{aligned} \frac{\partial}{\partial t} \left( \frac{3}{2} n_e kT_e \right) + \frac{3}{2} n_e (V_e \nabla) kT_e + p_e \nabla V_e = \\ \nabla \lambda_e \nabla T_e + jE - 3 \frac{m_e}{m_i} v_e n_e k(T_e - T_H) - n_e n_a \langle \sigma_i V_e \rangle \varepsilon_i \end{aligned} \quad (10)$$

Electron thermal conductivity calculated from the condition:

$$\lambda_e = \frac{2.4}{1 + \frac{v_{ei}}{\sqrt{2}v_e}} \cdot \frac{k^2 n_e T_e}{m_e v_e} \quad (11)$$

The concentration of recharged ions is higher because their temperature equals the temperature of the wall and the velocities in two degrees smaller than ions which is emitted from the thruster. It increased only by the electric field, which calculation is published below.

### III. Initial data for calculations

Concentration of neutral atoms:

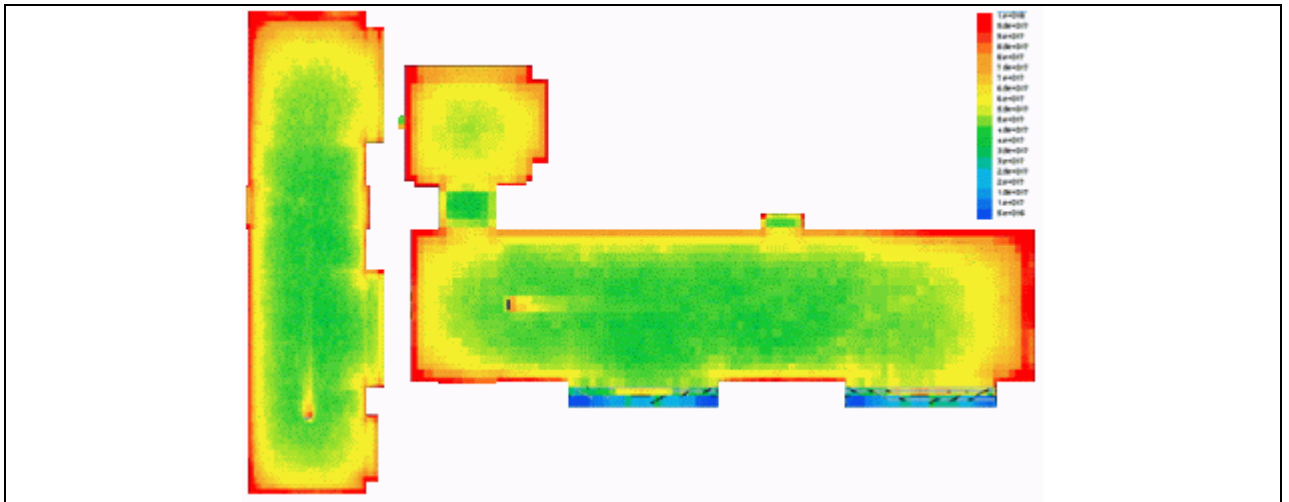


Figure 30. Neutral atoms concentration[1/m<sup>3</sup>]

Fast ions concentration:

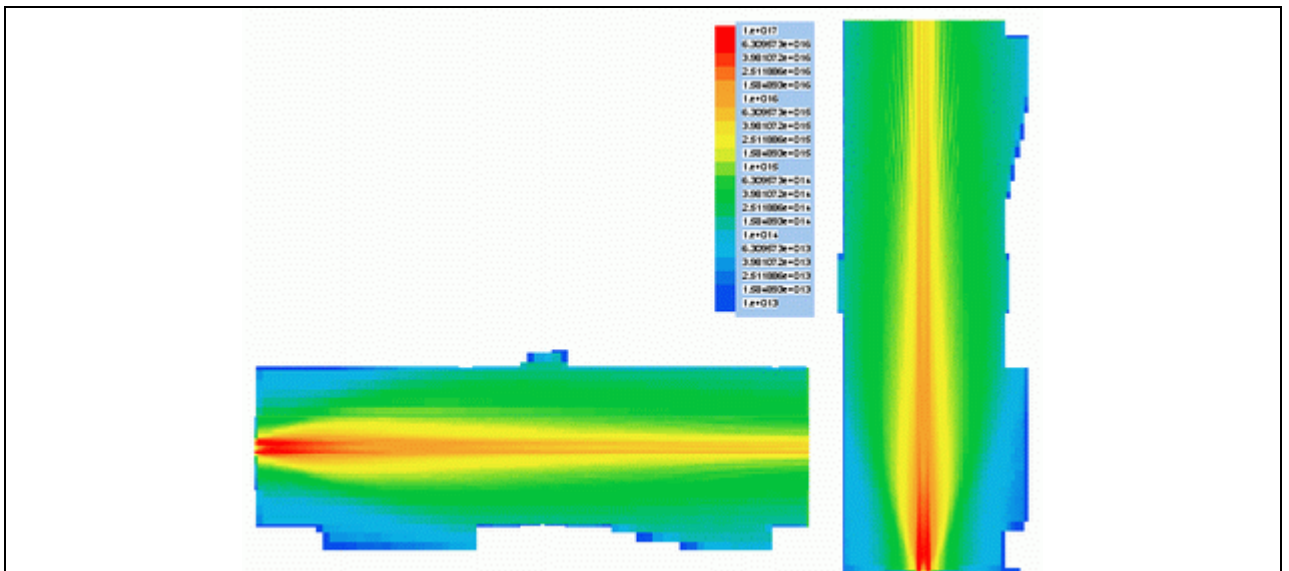


Figure 31. Ions concentration[1/m<sup>3</sup>]

Recharged ions concentration:



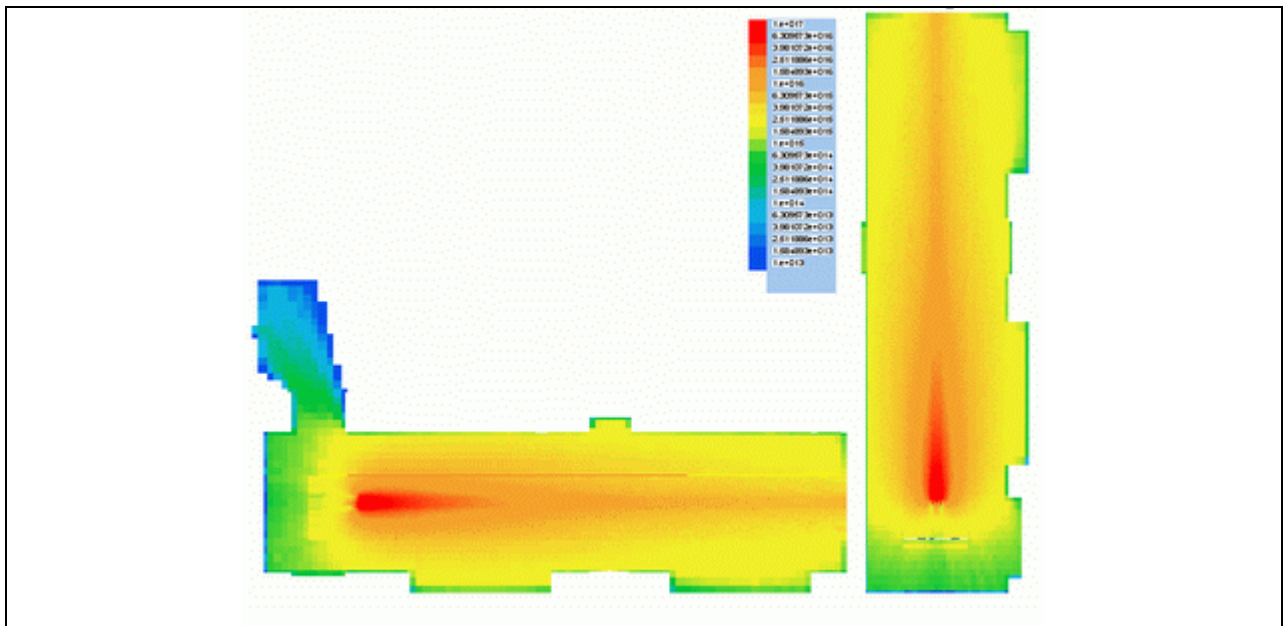


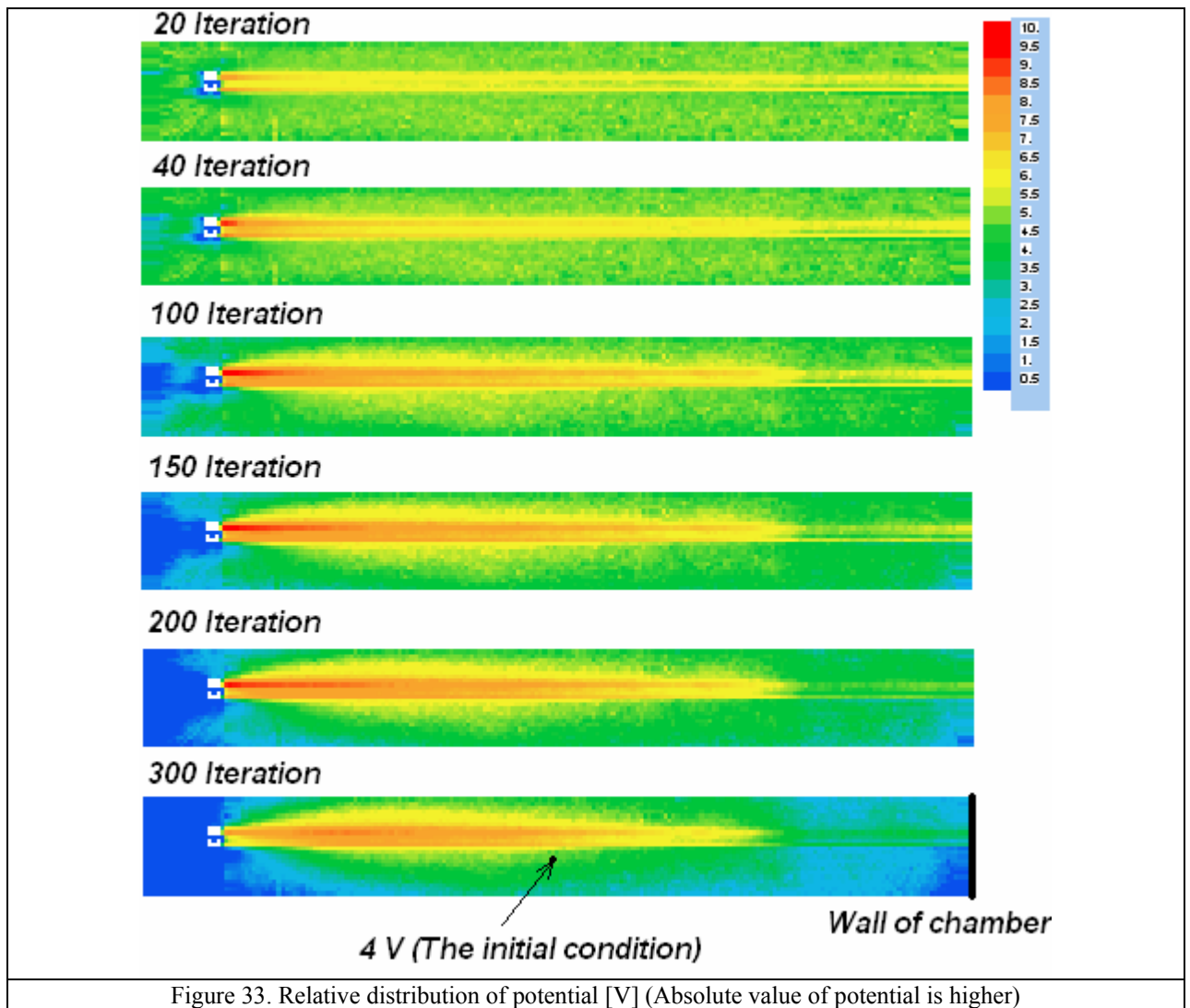
Figure 32. Charge-exchange particles [ $1/m^3$ ]

#### IV. Boundary conditions.

For boundary condition we measured the potential and temperature of electrons in one point of the chamber with Langmuir's probe. The potential of wall was put at zero.

#### V. Pre-Results.

The initial potential was 4V. It's not real value. Just to get the relative distribution.



### III. Conclusion

To create the mesh we need only step file of the shape of the chamber, which could be created in almost any CAD system. The prices on our calculations, validation, components used in developer loop, and community information are on <http://www.epcm.com>

### Acknowledgments

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Books:

- <sup>1</sup> T. Kormen, *Algorithms: building and analyses*, Moscow, 2002.
- <sup>2</sup> T. Pavlovskaya *C/C++ Developing of high standard*, St.Petersburg.
- <sup>3</sup> B. Alekseev, *Probe's method of plasma diagnostics*, Moscow, 1988.
- <sup>4</sup> Y. Shishkin, *Curves and surfaces on the computer display*, Moscow, 1996.
- <sup>5</sup> E. Trelson *COM and ATL 3.0 application*, Moscow, 2000.
- <sup>6</sup> Papas K., *Debugging in C++*, Moscow, 2001.
- <sup>7</sup> Koshmarov, *Dynamics of discharged gas*, Moscow, 1977.
- <sup>8</sup> *Applications developing on Microsoft Visual C++ 6.0*, Microsoft Press, 2001.
- <sup>9</sup> Sekunov N., *Visual C++ 6*, Moscow, 1999.
- <sup>10</sup> Artsimovich, *Physics of plasma*, 1974.

<sup>11</sup> Shaphranov, *Physics of plasma* , 1984.

<sup>12</sup> P. Sendgvik, *Fundamental algorithms on C* , Moscow, 2003.

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<sup>13</sup> *Basic issues in electric propulsion testing and the need for international standards*, IEPC-2003.

<sup>14</sup> *Plasma parameter distribution determination in SPT-70 plume*, IEPC-2003.

<sup>15</sup> *The new EP test facilities at Centropazio an Alta*, IEPC-2003

<sup>16</sup> Perrin V., Metois P., Khartov S., Nadiradze A. *Simulation tools for the plasma propulsion and satellite environment // 52nd International Astronautical Congress Toulouse, France, October 1-5, 2001.*

#### *Reports, Theses, and Individual Papers*

<sup>17</sup> *Numerical analysis of particles distribution in beam of ions source*, 9 Students international science conference , MEI, 2003.

<sup>18</sup> *Hall thruster simulation in vacuum chamber*, Final state science conference «All-Russian competition on best science students projects », M.:MIEM,2004.

<sup>19</sup> *Hall thruster beam simulation*, 11 Students international science conference , MEI, 2005.

#### *Electronic Publications*

<sup>20</sup> <http://www.epcmmod.com>

#### *Computer Software*

<sup>21</sup> MSDN 2003.