

# On the anomalous diffusion mechanism in Hall-effect thrusters

IEPC-2005-117

*Presented at the 29<sup>th</sup> International Electric Propulsion Conference, Princeton University,  
October 31 - November 4, 2005*

J.M. Gallardo\* and E. Ahedo<sup>†</sup>  
*Universidad Politécnica de Madrid, Spain*

**This work investigates numerically the development and saturation of azimuthal instabilities, the phenomena that sustain them, and the dependence on several plasma and control parameters. The temporal and azimuthal evolution of a stationary, azimuthally-uniform solution at a given axial section of the thruster is analyzed. The model predicts the development and non-linear saturation of an azimuthal instability, which reduces the azimuthal electron drift by about one order of magnitude. This means an increase of the transverse transport by the same amount, in good agreement with the experimental evidence. The instability features same-order roles of the azimuthal oscillations of electric force and pressure and, related to this, a decisive contribution of temperature azimuthal oscillations.**

## I. Introduction

One of the basic hypothesis in most theoretical models of Hall-effect thrusters (HET) corresponds to the axial symmetry of the problem. In fact most configurations keep this symmetry, providing we neglect the azimuthal distortion in the magnetic field, inhomogeneities in the feeding system, the local influence of the cathode and other considerations which may be reasonably considered as comparatively small perturbations.

Nevertheless, the axial symmetry of the problems does not ensure the axial symmetry of the solution. The presence of strong azimuthal and hybrid longitudinal-azimuthal waves propagating in the plasma is widely reported<sup>1</sup> and affects significantly the discharge.

Certain authors<sup>2,3</sup> consider that these waves contribute dominantly to electron transport phenomena. In fact, several semi-empirical expressions for the 'anomalous diffusion' are included in most numerical models in the literature in order to achieve realistic results of the performances of the analyzed device. Nevertheless, there is no general agreement in the causes and mechanisms of this enhanced transport and many authors relate it to wall interaction effects.<sup>3,4</sup>

In this work we perform a simplified analysis of the growth and propagation of purely azimuthal waves in a plasma in conditions corresponding to the ionization region of a conventional HET. A special attention is paid to the associated electron transport phenomena, so that the influence of these oscillations on the current characteristics of Hall-effect thrusters is described.

The paper is organized as follows. Section II comments on the basis of the mathematical model. Section III presents a linearized analysis of the growth and propagation of azimuthal waves. Section IV describes the numerical model used to perform the non-linear analysis. Section V describes the corresponding non-linear results.

---

\*Ph. D. Candidate, Escuela Técnica Superior de Ingenieros Aeronáuticos(ETSIA), [juan.gallardo@itp.es](mailto:juan.gallardo@itp.es).

<sup>†</sup>Associate Professor, ETSIA, [eduardo.ahedo@upm.es](mailto:eduardo.ahedo@upm.es).

## II. Formulation of the model

The model presented here is closely related to the one-dimensional (1D) model of Ref. 5. This macroscopic model considers the plasma to be composed by three independent fluids: electrons  $e$ , ions  $i$  and neutrals  $n$ . Ions and neutrals are assumed cold, but an energy equation, which includes heat conduction, is considered for the electrons. The magnetic field is considered purely radial and constant in time (i.e., only the externally applied field is considered).

The main differences with the model in Ref 5 are:

- The hypothesis of azimuthal symmetry is relaxed, so that the model is now bi-dimensional in space. A quasiplanar approximation (i.e. no cylindrical effects) is adopted, with  $x$  following the thruster axis and  $y$  in the azimuthal direction.
- Only classical electron-neutral collisions are taken into account in the electron collision frequency  $\nu_e$ .
- Electron inertia terms are kept in the electron momentum equation.
- Quasineutrality is not imposed, so that a finite Debye length  $\lambda_D$  appears.

The corresponding set of equations is:

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \vec{v}_e) = \nu_i n_e, \quad (1)$$

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \vec{v}_i) = \nu_i n_e, \quad (2)$$

$$\frac{\partial n_n}{\partial t} + \nabla \cdot (n_n \vec{v}_n) = -\nu_i n_e, \quad (3)$$

$$\frac{\partial m_e n_e \vec{v}_e}{\partial t} + \nabla \cdot (m_e n_e \vec{v}_e \vec{v}_e) = -\nabla (n_e T_e) + e n_e \nabla \phi - e n_e \vec{v}_e \wedge \vec{B} - \nu_e m_e n_e \vec{v}_e, \quad (4)$$

$$\frac{\partial n_i \vec{v}_i}{\partial t} + \nabla \cdot (n_i \vec{v}_i \vec{v}_i) = -\frac{e}{m_i} n_i \nabla \phi + \nu_i n_e \vec{v}_n, \quad (5)$$

$$\frac{\partial n_n \vec{v}_n}{\partial t} + \nabla \cdot (n_n \vec{v}_n \vec{v}_n) = -\nu_i n_e \vec{v}_n, \quad (6)$$

$$\frac{\partial}{\partial t} \left( \frac{3}{2} n_e T_e \right) + \nabla \cdot \left( \frac{5}{2} n_e T_e \vec{v}_e + \vec{q}_e \right) = e n_e \vec{v}_e \cdot \nabla \phi - e n_e \nu_i \alpha_i E_i, \quad (7)$$

$$\frac{5}{2} n_e T_e \nabla T_e + e \vec{q}_e \wedge \vec{B} + m_e \nu_e \vec{q}_e = 0, \quad (8)$$

$$\epsilon_0 \nabla^2 \phi = e(n_e - n_i). \quad (9)$$

The nomenclature can be found in Ref. 5. Notice that wall interaction effects are not taken into account in this work.

Our aim here is to study only purely azimuthal waves. In order to formulate a (1D) azimuthal model, several simplificative assumptions on the axial dependence need to be made. In view of the model we are planning, the most appropriate or justifiable ones are

- The axial electron flux  $n_e v_{ex}$  is known and kept constant.
- Axial gradients are kept constant where needed, so that the uniform solution (taken as initial condition) satisfies the whole set of equations.

By making use of these assumptions we arrive at the following system of equations:

$$\frac{\partial n_e}{\partial t} + \frac{\partial(n_e v_{ey})}{\partial y} = \nu_i n_e - \left[ \frac{\partial(n_e v_{ex})}{\partial x} \right], \quad (10)$$

$$\frac{\partial n_i}{\partial t} + \frac{\partial(n_i v_{iy})}{\partial y} = \nu_i n_e - \left[ \frac{\partial(n_i v_{ix})}{\partial x} \right], \quad (11)$$

$$\frac{\partial n_n}{\partial t} + \frac{\partial(n_n v_{ny})}{\partial y} = -\nu_i n_e - \left[ \frac{\partial(n_n v_{nx})}{\partial x} \right], \quad (12)$$

$$\frac{\partial m_e n_e v_{ey}}{\partial t} + \frac{\partial(m_e n_e v_{ey} v_{ey})}{\partial t} = -\frac{\partial(n_e T_e)}{\partial y} + e n_e \frac{\partial \phi}{\partial y} + e B [n_e v_{ex}] - \nu_e m_e n_e v_{ey}, \quad (13)$$

$$\frac{\partial n_i v_{iy}}{\partial t} + \frac{\partial(n_i v_{iy} v_{iy})}{\partial y} = -\frac{e n_i}{m_i} \frac{\partial \phi}{\partial y} + \nu_i n_e v_{ny} - \left[ \frac{\partial(n_i v_{ix})}{\partial x} \right] v_{iy}, \quad (14)$$

$$\frac{\partial n_n v_{ny}}{\partial t} + \frac{\partial(n_n v_{ny} v_{ny})}{\partial y} = -\nu_i n_e v_{ny} - \left[ \frac{\partial(n_n v_{nx})}{\partial x} \right] v_{ny}, \quad (15)$$

$$\frac{\partial}{\partial t} \left( \frac{3}{2} n_e T_e \right) + \frac{\partial}{\partial y} \left( \frac{5}{2} n_e T_e v_{ey} + q_{ey} \right) = e n_e v_{ey} \frac{\partial \phi}{\partial y} - e n_e \nu_i \alpha_i E_i - \left[ \frac{\partial}{\partial x} \left( \frac{5}{2} n_e T_e v_{ex} + q_{ex} \right) + e n_e v_{ex} E_x \right], \quad (16)$$

$$q_{ey} = -\frac{5}{2} \frac{n_e T_e}{m_e \omega_e^2} \left( \nu_e \frac{\partial T_e}{\partial y} + \omega_e \left[ \frac{\partial T_e}{\partial x} \right] \right), \quad (17)$$

$$\epsilon_0 \frac{\partial^2 \phi}{\partial y^2} = e(n_e - n_i). \quad (18)$$

It is important to highlight that the terms between brackets corresponding to  $\partial/\partial x$  are assumed to be known in advance and not to depend on the azimuthal coordinate  $y$  and time. They balance the ionization-related source term in the energy and particle density equations. Notice that in equation 14 we have chosen to write  $\left[ \frac{\partial(n_i v_{ix})}{\partial x} \right] v_{iy}$  instead of  $\left[ \frac{\partial(n_i v_{ix} v_{iy})}{\partial x} \right]$ ; this is done in order to avoid certain ion drift phenomena which have an unclear physical basis and complicate the analysis. A similar consideration has been done on the momentum equation for the neutrals. The axial temperature gradient  $\partial T_e/\partial x$  has been set equal to 0.

The previous set of equations closes the azimuthal model. The proper boundary conditions are periodic:  $u(y + 2\pi R) = u(y)$ , where  $R$  is the mean radius of the HET.

The effects of the azimuthal oscillations is mainly felt in the Ohm's and Fourier's laws for the axial electric field and axial heat conduction flux. These are

$$\frac{\partial(n_e T_e)}{\partial x} + e n_e E_x \simeq -e B n_e v_{ey} - \nu_e m_e n_e v_{ex}, \quad (19)$$

$$q_{ex} = -\frac{5}{2} \frac{n_e T_e}{m_e \omega_e^2} \left( -\omega_e \frac{\partial T_e}{\partial y} + \nu_e \frac{\partial T_e}{\partial x} \right); \quad (20)$$

axial electron inertia has been neglected in both equations.

### III. Linear stability analysis

The set of equations (10)-(18) admits simple steady solutions. Let us consider the case with  $\frac{\partial}{\partial y} = 0$ ,  $v_{iy} = 0$ ,  $v_{ny} = 0$ ,  $n_{e0} = n_{i0}$ . We will consider  $n_{e0}$ ,  $n_{i0}$ ,  $n_{n0}$ ,  $T_{e0}$  as known in advance. To fulfill the electron momentum equation the azimuthal electron velocity must follow

$$v_{ey0} = \frac{e B [n_e v_{ex}]}{m_e \nu_{e0} n_{e0}}. \quad (21)$$

Here,  $\nu_{e0}$  is the (classical) electron collisionality evaluated at the steady values  $n_{n0}, T_{e0}$ .

The system of equations (10)-(18) may be linearized around the steady solution in a quite straightforward way. If for a given variable  $u(y, t)$  we consider  $u = u_0 + \varepsilon u_1 e^{\lambda t + iky}$  the linearized set of equations may be written as:

$$\lambda A u_1 = B u_1 + C(k) u_1. \quad (22)$$

Here,  $A, B$  and  $C$  are complex matrices and  $C$  depends on  $k$ . Notice that, in order to fulfill the boundary conditions, only wave numbers such that  $Rk = K$  with  $K \in \mathbb{Z}$  are admissible.

Equation (22) defines a complex generalized eigenvalue problem, which may be considered as an implicit formulation of the dispersion relation for the plasma. Unstable modes are characterized by  $\text{Im}(\lambda) > 0$ .

### A. Unstable modes

We consider a base solution with  $n_{e0} = 4.05 \times 10^{17} \text{m}^{-3}$ ,  $n_{n0} = 1.35 \times 10^{19} \text{m}^{-3}$ ,  $v_{ex0} = -594 \text{m/s}$ ,  $v_{ey0} = -1.35 \times 10^5 \text{m/s}$ ,  $T_{e0} = 12.1 \text{eV}$ ,  $R = 28 \text{mm}$ ,  $\lambda_D = 0.9 \text{mm}$ ,  $B = 112 \text{G}$ .

The eigenvalues for  $K = 3$  can be seen in figure 1. The modes with the higher frequency correspond to the electron plasma frequency and are stable. The rest of the modes are not simple, they have significant contributions of most of the variables. Most of them are stable or, at most, marginally unstable, but, in particular, there is an unstable mode with an small oscillatory part which deserves a closer attention.

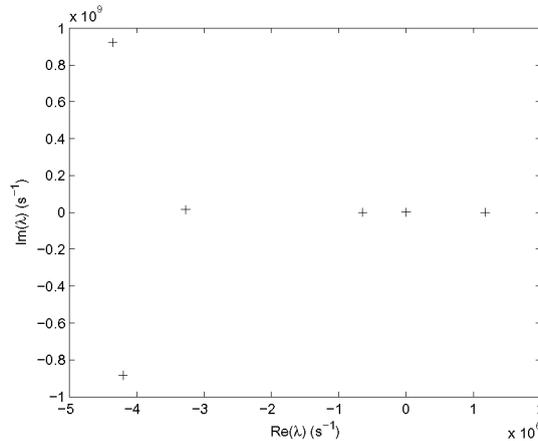


Figure 1. Eigenvalues for the reference case;  $K = 3$

This unstable mode affects significantly most of the variables with the exception of the neutral density, as can be seen in the following table, where the perturbation in the electron density is taken as a reference:

$n_{e1}/n_{e0}$	$n_{i1}/n_{i0}$	$n_{n1}/n_{n0}$	$v_{iy1}/v_{ref}$	$v_{ey1}/v_{ey0}$	$T_{e1}/T_{e0}$	$\phi_1/eT_{e0}$
$1 \angle 0^\circ$	$1.05 \angle -0.5^\circ$	$0.03 \angle -179.7^\circ$	$0.26 \angle -91.9^\circ$	$1 \angle -0.5^\circ$	$0.64 \angle -26.8^\circ$	$1.59 \angle -10.4^\circ$

This unstable mode is nearly quasineutral (ion and electron densities behave nearly in the same way). Notice that the contributions of density, potential and temperature are considerable. The phase angle between the plasma density and the electric potential is *not* 0 and in section V we will see that this fact is of vital importance when studying electron transport.

Unstable modes with very similar characteristics appear for all values of  $K$  between 1 and 13.

### B. Disadvantages of more simplified models

The previous analysis makes clear the importance of phenomena such as the variation of the electron temperature (the unstable modes have a very significant temperature contribution) in the description of these

azimuthal waves. Nevertheless there are two features included in the model that need be justified. These are to keep non-neutrality and the electron inertia.

Let us consider non-neutrality. Figure 2 plots the real part of the most unstable eigenvalue as a function of the wave number  $K$  for different values of  $\lambda_D$ . It can be seen that the number of unstable wave numbers increase as the Debye length decreases. In fact, the stability limit tends to  $\infty$  as  $\lambda_D$  tends to 0. This instability of extremely short wavelengths does not correspond to any physical mechanism but to model limitations. The proper way to correct it is to retain non-neutral effects. Nevertheless, for the description of the most unstable modes, which have moderately low wave numbers, quasineutrality is very reasonably satisfied.

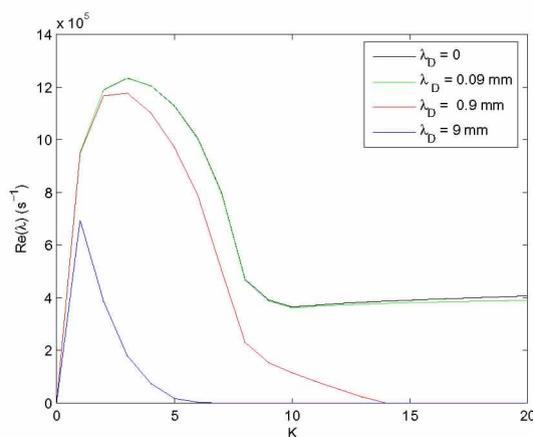


Figure 2. Unstable eigenvalues for different  $\lambda_D$

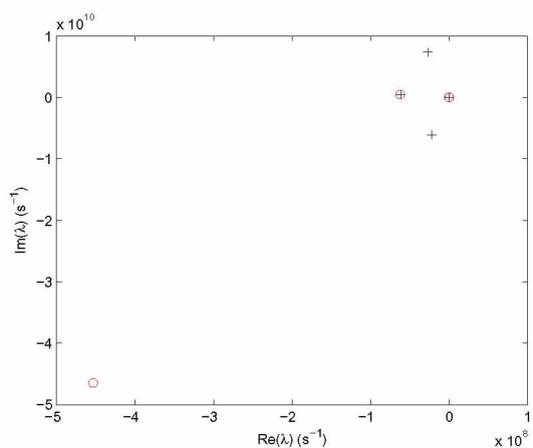


Figure 3. Eigenvalues for  $K = 100$  retaining (black cross) and neglecting (red circle) electron inertia.

The influence of electron inertia is more subtle and the convenience of its inclusion does not correspond to such a physical motivation, but to numerical convenience. Let us consider figure 3, where the eigenvalues are plotted for a high value of  $K$  with and without neglecting the electron convective term in the momentum equation. Even though these eigenvalues are stable, their modulus are big, and the maximum modulus is nearly one order of magnitude greater for the case without electron inertia. For classical Runge-Kutta algorithms the size of the allowable time-step is inversely proportional to the size of the greatest eigenvalue. This implies that, if we neglect the electron inertia, the computational time increases by nearly one order of magnitude. Notice that the rest of the eigenvalues and eigenvectors nearly do not change and thus the solution is not significantly altered by the inclusion (or not) of the electron inertia.

Finally, the importance of the term of heat conduction is not great for moderate values of  $K$ , and in

practice only increases slightly the stability of the modes with a significant  $T_e$  contribution (this effect increases for short wavelengths). The term is, therefore, marginal when analyzing the electron momentum equation. Nevertheless, electron heat conduction is an important question in the axial problem and it is convenient to keep the term also in this azimuthal model.

#### IV. Non-linear numerical model

The spatial discretization of the non-linear problem has been performed by using a classical second-order scheme of centered finite differences.

The time evolution is a bit more complicated. The system of equations combines classical evolution equations with a purely elliptic equation (Poisson). The system has quite high eigenvalues which impose a severe limitation on the time step, making it necessary to take a very large number of steps in order to perform a representative simulation. On the other hand, the Poisson equation has to be solved at every time step, which is a relatively expensive calculation. Therefore, the total computational cost is surprisingly high and. Although this cost is easily affordable for this 1D problem, finding more efficient algorithms seems very attractive.

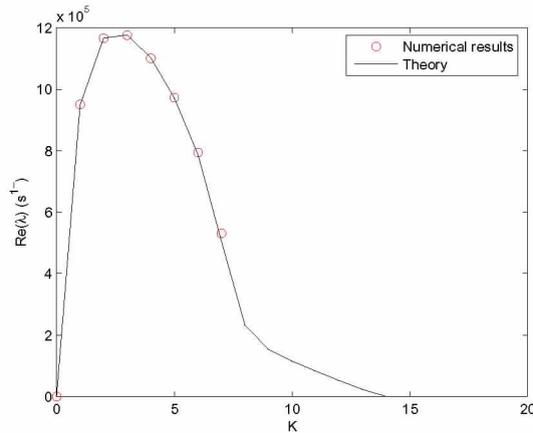


Figure 4. Comparison between numerical results and theoretical calculations for the dominant eigenvalues.

The solution adopted is based on an implicit Crank-Nicholson rule. It does not have the time-step limitations typical of explicit schemes, but forces us to solve a non-linear set of equations at every time step. In order to solve this system a dual time step approach based on a fifth order Runge-Kutta<sup>6</sup> scheme has been adopted to solve the system of equations at every time step. The limitation in the time step applies for each of the sub-steps in the dual time step technique, but there is no need to solve Poisson equation at every sub-step, it is solved only once in the main step. Furthermore, special techniques for the acceleration of the convergence which are not available for unsteady problems, such as multigrid or residual smoothing, may be used.

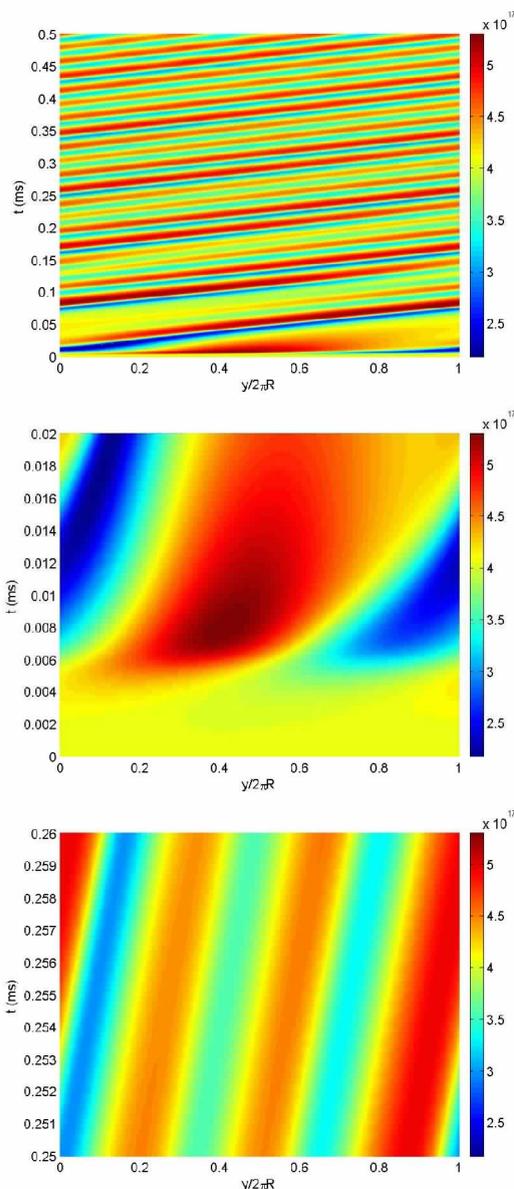
In order to check the accuracy of the method the growth of small perturbations has been analyzed and compared with the results from the linear theory; this comparison is represented in figure 4.

#### V. Results

In this section we expose the numerical results corresponding to the base solution that we have used in the linear stability analysis (with  $\lambda_D = 0.9$  mm). An initial perturbation in  $n_e$  and  $n_i$  with  $K = 1$  has been added in order to facilitate the development of unstable structures.

The time evolution of the electron density  $n_e$  (chosen arbitrarily as a representative variable) is plotted in figure 5. The three sub-figures correspond to different time scales. Notice that the plot on the top shows the complete evolution story. It can be seen that, after a comparatively short time of development, the plasma

structure takes the shape of waves propagating with approximately constant phase velocity and with nearly constant amplitude. The average value for the density does not change significantly during the temporal evolution.



**Figure 5. Time evolution of the electron density.**

The central figure shows the details of the development phase of the unstable waves. Notice that the structure is dominated by the wave number  $K = 1$ , which still corresponds to the initial perturbation. The behavior corresponds quite accurately to the predictions done by the linear analysis.

The figure on the bottom shows a detail of the structure which is obtained once the perturbations are fully developed. The dominant wave number is  $K = 3$ , which corresponds to the most unstable mode. The waves propagate at a phase velocity of approximately 2700 m/s; this is quite close to the ion acoustic velocity. Notice that, for 'long times' this structure behaves as approximately periodical in time.

Since the wave amplitude finally stabilizes, it is clear that non-linear saturation effects play a significant role. A complete analysis is at present being performed but preliminary results indicate that electron energy equation is the key to the understanding of these phenomena.

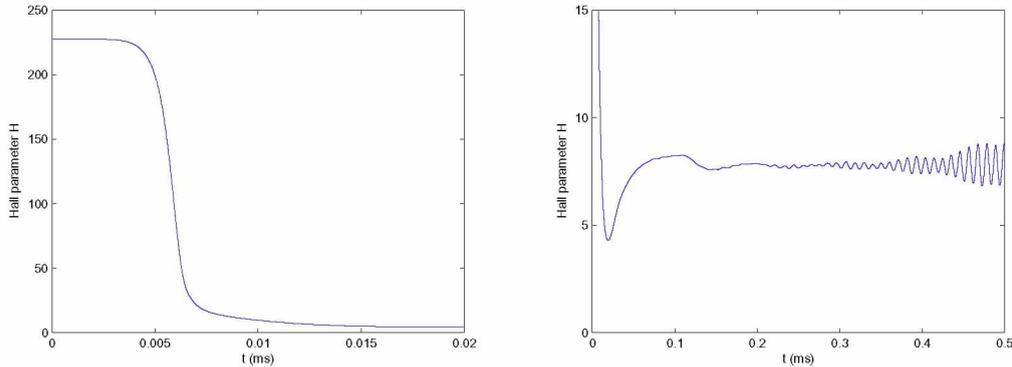
In general, it may be noted that the average values for most plasma variables ( $n_e$ ,  $T_e$ ,  $n_n$ ,  $v_{iy}$ ) do not change significantly due to these phenomena. But very interesting phenomena affecting the electron velocity appear. We will analyze them in the next sub-section.

### A. Anomalous diffusion

The time evolution of the Hall parameter is represented in figure 6. The definition used here is

$$H = \frac{\int_0^{2\pi R} n_e v_{ye} dy}{2\pi R [n_e v_{ex}]} \quad (23)$$

The drop in the electron azimuthal velocity is very significant. Once the unstable waves have grown enough (compare figures 5 and 6) the transport across the magnetic field lines is strongly enhanced; this is what is usually called anomalous diffusion. The average value for this azimuthal velocity is kept roughly constant, but non negligible oscillations at approximately 100 kHz appear as the plasma structure matures.



**Figure 6. Time evolution of the average Hall parameter.**

In order to understand the processes we have to consider the electron momentum equation (13). If we neglect electron inertia and integrate in this equation between 0 and  $2\pi R$  we arrive at

$$0 = - \int_0^{2\pi R} e n_e \frac{\partial \phi}{\partial y} dy + 2\pi R e B [n_e v_{ex}] - \int_0^{2\pi R} m_e \nu_e n_e v_{ey} dy. \quad (24)$$

Notice that, due to the periodic boundary conditions, the pressure term vanishes in the equation.

At the initial instant the second and third terms balance each other, while the first term is 0. Nevertheless, since the second term is kept constant and the third term drops abruptly, the new balance must involve the first term (azimuthal electric field) and the second one (magnetic force). This is possible as long as the electron density perturbation and the potential are not in phase: this would cause the corresponding integral to vanish.

Notice that the previous expression only involves average values. In order to check the relative importance of the different terms, we have plot the r.m.s. values of the terms in the electron momentum equation in figure 7. For the root mean square values we use the definition

$$u_{r.m.s.} = \sqrt{\frac{1}{2\pi R} \int_0^{2\pi R} u^2 dy}. \quad (25)$$

Notice in Fig 7 that the electric field and the electron pressure have nearly the same importance. Since the average value of the electric force balances the magnetic force due to equation (24) we must conclude that the amplitude of the electric force is much greater than its average value.

In order to explain the importance of the electron pressure a simplified analysis of the modes with low wave numbers may be performed. Notice that, for 'long' wavelengths the plasma behaves approximately as quasineutral and electron inertia may be neglected, making it easier to obtain conclusions.

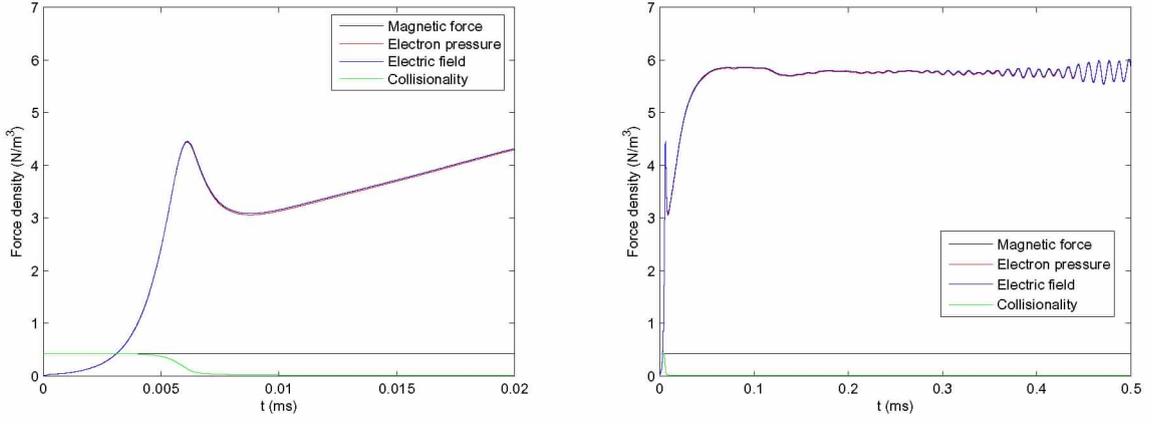


Figure 7. Time evolution of terms in the electron momentum equation (r.m.s.).

By combining the continuity equations for ions and electrons, the azimuthal current satisfies

$$\frac{\partial}{\partial t}(n_e v_{iy} - n_e v_{ey}) = 0, \quad (26)$$

which may be expressed as

$$n_e(v_{iy} - v_{ey}) = \Gamma(t). \quad (27)$$

Now, we may re-write the azimuthal equation for the electron dynamics,

$$0 = -\frac{\partial(n_e T_e)}{\partial y} + e n_e \frac{\partial \phi}{\partial y} + e B n_e v_{ex} - m_e \nu_e (n_e v_{iy} - \Gamma), \quad (28)$$

and re-arranging terms

$$e \frac{\partial \phi}{\partial y} = \frac{1}{n_e} \frac{\partial(n_e T_e)}{\partial y} - \frac{e B (n_e v_{ex})}{n_e} + \frac{m_e \nu_e}{n_e} (n_e v_{iy} - \Gamma). \quad (29)$$

Since the electric field comes from a potential, one has  $\int_0^{2\pi R} \frac{\partial \phi}{\partial y} dy = 0$ , yielding

$$\Gamma \int_0^{2\pi R} \frac{m_e \nu_e}{n_e} dy = \int_0^{2\pi R} \frac{1}{n_e} \frac{\partial(n_e T_e)}{\partial y} dy - \int_0^{2\pi R} \frac{e B (n_e v_{ex})}{n_e} dy + \int_0^{2\pi R} m_e \nu_e v_{iy} dy. \quad (30)$$

The ion azimuthal velocity  $v_{iy}$  is much smaller than the electron velocity and thus the collisional term plays a very secondary role. Therefore we arrive to

$$\Gamma \simeq \frac{\int_0^{2\pi R} \frac{1}{n_e} \frac{\partial(n_e T_e)}{\partial y} dy - \int_0^{2\pi R} \frac{e B (n_e v_{ex})}{n_e} dy}{\int_0^{2\pi R} \frac{m_e \nu_e}{n_e} dy}. \quad (31)$$

If  $T_e$  were constant, then

$$\int_0^{2\pi R} \frac{1}{n_e} \frac{\partial(n_e T_e)}{\partial y} dy = T_e \int_0^{2\pi R} \frac{1}{n_e} \frac{\partial n_e}{\partial y} dy = 0, \quad (32)$$

and the current equation would *not* be altered by the potential perturbations. This is probably the main reason why the azimuthal gradient of the temperature has a vital importance in order to describe the phenomenon.

## VI. Conclusions

The model presented in this paper predicts the presence of unstable purely azimuthal modes in the discharge of a Hall-effect thruster. Non linear saturation effects limit the growth of the waves and lead to a pseudo-periodic regime where the perturbation amplitude is kept approximately constant.

The azimuthal waves are responsible for non-linear electron transport phenomena which enhance the plasma diffusion by a large factor. The contributions of both the electric field and the electron pressure term are determinant in order to explain these phenomena.

## Acknowledgments

This work was sponsored by the Ministerio de Educación y Ciencia of Spain (Project ESP2004-03093).

## References

- <sup>1</sup>Choueiri, E., "Plasma oscillations in Hall thrusters," *Physics of Plasmas*, Vol. 8, No. 4, 2001, pp. 1411–1426.
- <sup>2</sup>Janes, G. and Lowder, R., " Anomalous electron diffusion and ion acceleration in a low-density plasma ," *Physics of Fluids*, Vol. 9, No. 6, 1966, pp. 1115–1123.
- <sup>3</sup>Morozov, A., Esipchuk, Y., Tilinin, G., Trofimov, A., Sharov, Y., and G.Y.Shchepkin, "Plasma accelerator with closed electron drift and extended acceleration zone," *Soviet Physics-Tech. Physics*, Vol. 17, No. 1, 1972, pp. 38–45.
- <sup>4</sup>Bugrova, A., Morozov, A., and V.K.Kharchevnikov, "Experimental investigation of near wall conductivity," *Sov. J. Plasma Physics*, Vol. 16, No. 12, 1990, pp. 849–856.
- <sup>5</sup>Ahedo, E., Gallardo, J., and Martínez-Sánchez, M., "Model of the plasma discharge in a Hall thruster with heat conduction," *Physics of Plasmas*, Vol. 9, No. 9, 2002, pp. 4061–4070.
- <sup>6</sup>Jameson, A., "Time Dependent Calculations Using Multigrid with Applications to Unsteady Flows past Airfoils and Wings," AIAA Paper 91-1596, American Institute of Aeronautics and Astronautics, Washington, DC, 1991.