Response of a cylindrical plasma to time-varying external electromagnetic field: numerical simulation studies

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Abstract: **Electric propulsion systems can provide high specific thrust compared to chemical propulsion systems, and are suited to long duration missions such as planetary missions. On the other hand, in case of many of the conventional electric propulsion systems, the performance is limited by electrode wastage. In order to overcome this difficulty, several novel ways to drive plasma via external electromagnetic field have been proposed [Toki et al., 2004], but not much has been known regarding the plasma behavior when it is exposed to time-varying external field. In this presentation, we show our recent results on the plasma response to the time-varying external electromagnetic field using full particle simulation and. Implications of the results to the next generation electric propulsion systems will be discussed.**

Nomenclature

δ	=	skin depth
δ_0	=	skin depth without magnetic field
c	=	speed of light
ω	=	frequency of external electromagnetic field oscillation
k_i	=	imaginary part of wave number
ω_{pe}	=	electron plasma frequency
ν	=	collision frequency
f	=	enhancement factor
B	=	magnetic field
j	=	current excited in plasma
E	=	electric field

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I. Introduction

Electric propulsion systems can provide high specific thrust compared to chemical propulsion systems, and are suited to long duration missions such as planetary missions. On the other hand, in case of many of the conventional electric propulsion systems, the performance is severely limited by electrode wastage. In order to overcome this difficulty, several novel electric propulsion systems have been proposed, such as the Variable Specific Impulse Magnetoplasma Rocket (VASIMR) [Chang Diaz et al., 2000], Mini-Magnetospheric Plasma Propulsion (M2P2) [Ziemba et al., 2001], helicon ion thruster [Shamrai et al, 1997], acceleration of helicon plasma due to rotating electric field (Lissajous acceleration) [Toki et al., 2003] and acceleration of helicon plasma due to internal plasma current induced by time-varying external current [Toki et al., 2004].

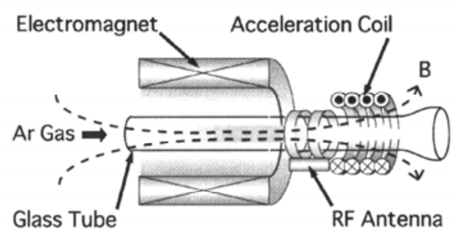

Figure1. Plasma acceleration mechanism proposed by Toki et al.[2004]

The mechanism subsequently proposed by Toki et al (2004) and Shinohara et al (2004) make use of the frequency dependence of the plasma skin depth [Shinohara and Kawai, 1996]. In the proposed system, the magnetic field is given in such a way that it expands radially toward the downstream portion where the acceleration coil is wound. The profile of the coil current is a train of a certain pulse. The azimuth current inside the glass tube plasma couples with the externally applied magnetic field. The skin depth of plasma is determined by characteristic time-scale of the external coil current. If the current is varied slowly, large volume of the plasma is accelerated by the coil current, while if the current is varied rapidly, less plasma is influenced. Therefore, non-zero net acceleration is expected even when the external current is varied periodically in time.

II. Skin depth of the induced field

A. Collisionless plasma

When we apply a time varying external current to the acceleration coil, diamagnetic current is induced in the plasma within the skin depth scale. The direction of the diamagnetic current is to negate the effect of the external current. Figure 2a shows dispersion relation of electromagnetic waves propagating perpendicular to the external magnetic field. The blue and the red curves corresponds respectively to the extraordinary and the ordinary waves. When the waves are evanescent, we can define the skin depth, [$\delta = 1/k_i$], where k_i is the imaginary part of the wave number. The skin depth depends on the wave frequency as seen in Figure 2b, in which the skin depth normalized to the plasma inertial length, [$\delta_0 = c/\omega_{pe}$], is plotted for both of the wave modes.

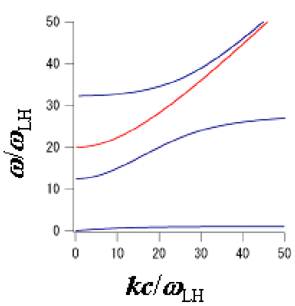

Figure2a. Change of the skin depth with external alternate current frequency

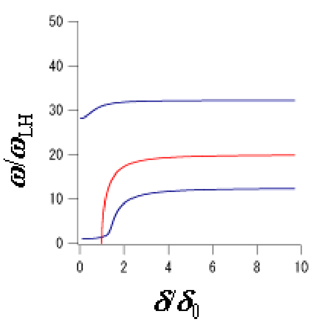

Figure2b. Change of the skin depth with external alternate current frequency

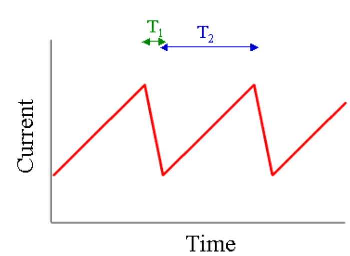

Figure3. Wave profile of external alternate current which has several frequencies

In the proposed propulsion system, the external current is composed of a series of asymmetric triangular-shaped oscillation as shown in Figure 3. The time intervals T_1 and T_2 consist of different set of Fourier modes with different frequencies, thus with different skin depth scales. Therefore, it is expected that during these intervals, different amount of plasma (and thus diamagnetic current) comes under the influence of the external alternate current, and as a result, non-zero diamagnetic current arises after a single cycle of the oscillation.

B. Collision plasma

In the case of collisionless plasma, skin depth is determined essentially by a ratio of the external alternate current frequency to the normalized frequency. If we consider the influence of collision, skin depth is dependent not only on the ratio of the external alternate current frequency to the normalized frequency but also on the collision frequency of the plasma. The skin depth in a collisional plasma was evaluated analytically by Shinohara et al (1996). Figure 4 summarizes the result, in which the skin depth scale normalized to the plasma inertial length is plotted versus the normalized collision frequency for the ordinary waves.

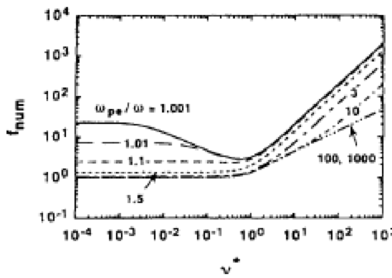

Figure4. Relationship between enhancement factor f of skin depth and normalized collision frequency ν^* for various values of ω_{pe}/ω for ordinary wave

III. Numerical Simulation

Simulation setting

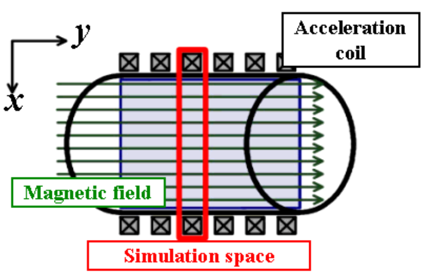

Figure5. Simulation setting

In order to discuss response of the plasma to the external electromagnetic field, we have performed numerical simulations using a one dimensional (1-d in space, 3-d in velocity space) particle-in-cell code. The simulated region is a part of a cross-section of the cylindrical plasma as shown in Fig. 5. In order to study the frequency dependence of the plasma behavior, we apply the external alternate current, taking its frequency and current strength as external parameters. The present study is preliminary in the following points: the simulation system is not the entire cylindrical plasma with three spatial dimensions, but has only 1-d in space; the applied external current is sinusoidal, instead of asymmetric time series as shown in Figure 3; the plasma explicitly includes no effect of collision. These simplifications are made at this stage since we would like to concentrate on basic physics underlying the proposed propulsion system. We plan to relax the above simplifications one by one in our future studies.

Electron thermal velocity (1 dimension)	0.02 [1/c]
Mass ratio	200
Debye length	1.5 [grid]
Temperature ratio (Ion/Electron)	1.0
ω_{pe}/ω_{ce}	1.0

Table1. Plasma parameter

Run1	$\omega=1.1\times\omega_{LH}$	$B_1/B_0=0.01$
Run2	$\omega=1.7\times\omega_{LH}$	$B_1/B_0=0.01$
Run3	$\omega=2.5\times\omega_{LH}$	$B_1/B_0=0.01$
Run4	$\omega=1.1\times\omega_{LH}$	$B_1/B_0=0.48$

Table2. External alternate current frequency

The simulation runs are performed using parameters summarized in Table 1. The main spatial direction is x, and the background magnetic field is in *y*-direction. The simulation system is from x=-3.41*δ_0 to x=10.24*δ_0, but the plasma is located only from x=0 to x=9.33*δ_0, and the rest is assumed to be a vacuum. External alternate current was applied to the *z* direction at x=-2.52*δ_0, so that the extraordinary waves are transmitted into the plasma. The electromagnetic field oscillation is excited around the plasma surface (x~0). This oscillation excites the extra ordinary wave. External current frequencies used in the runs are listed in Table 2, which are all within the evanescent regime for the extraordinary waves.

The current induced in a plasma

Before discussing the simulation results, let us first estimate how much current is expected to be induced in the plasma by the external alternate current, within the linear regime. Assuming the plasma to be cold, the waves should satisfy the dispersion relation

$$\begin{pmatrix} S-n^2\cos^2\theta & -n\cos\theta\sin\theta & -iD \\ iD & 0 & S-n^2 \\ n^2\cos\theta\sin\theta & n^2\sin^2\theta-P & 0 \end{pmatrix}\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}=0 \tag{1}$$

with

$$L\equiv 1-\sum_s\frac{\omega_{ps}^2}{\omega\left(\omega-\omega_{cs}\right)},\quad R\equiv 1-\sum_s\frac{\omega_{ps}^2}{\omega\left(\omega+\omega_{cs}\right)},\quad D=\frac{1}{2}\left(R-L\right),\quad S=\frac{1}{2}\left(R+L\right)$$

with $\theta=\pi/2$.

Now let us write the induced magnetic field by the external current, *By*, as (2).

$$B_y = B_0 + B_1\exp\left(\frac{-x}{\delta}\right)\sin\left(\omega t\right) \tag{2}$$

Then the two components of induced electric field must be represented as,

$$E_x=-\left(B_1\delta\omega\right)\frac{D}{S}\exp\left(\frac{-x}{\delta}\right)\sin\left(\omega t\right) \tag{3}$$

$$E_z = -B_1\delta\omega\ \exp\left(\frac{-x}{\delta}\right)\cos\left(\omega t\right) \tag{4}$$

From these, the current induced in the plasma is given as

$$J_x=\frac{B_1\delta\omega^2}{4\pi}\frac{D}{S}\exp\left(\frac{-x}{\delta}\right)\cos\left(\omega t\right) \tag{5}$$

$$J_z = \frac{-B_1}{4\pi}\left(\frac{1}{\delta} + \delta\omega^2\right)\exp\left(\frac{-x}{\delta}\right)\sin\left(\omega t\right) \tag{6}$$

Integrating over the x direction, we obtain the total induced current

$$J_x^{tot} = \left(\frac{B_1\delta^2\omega^2}{4\pi}\right)\frac{D}{S}\cos\left(\omega t\right) \tag{7}$$

$$J_z^{tot} = \left(\frac{-B_1}{4\pi}\right)\left(1+\delta^2\omega^2\right)\sin\left(\omega t\right) \tag{8}$$

Simulation results

. Magnetic field oscillation at the plasma surface

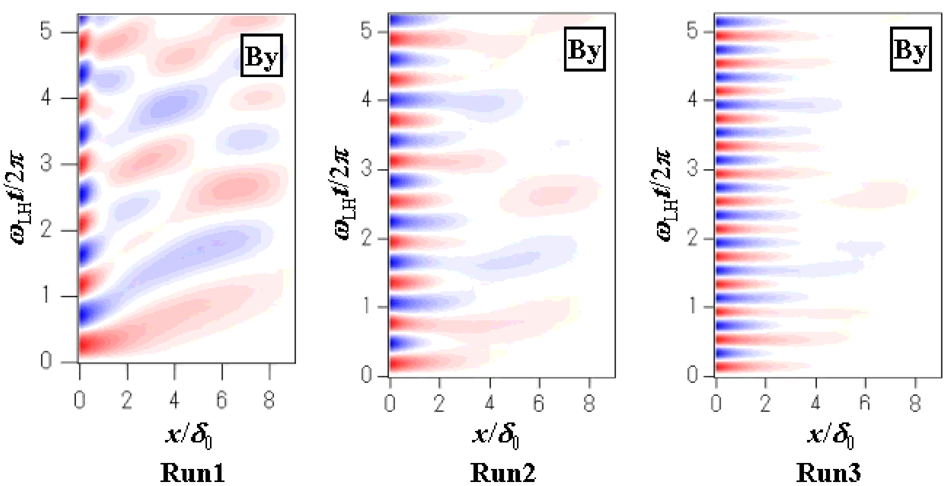

Figure6. Space-time diagram of oscillation of the magnetic field

Figure 6 shows the time evolution of the magnetic field plotted in the space-time phase space. Results of three different runs with different frequencies of the external current are compared. In each run, the magnetic field oscillations are exited in a region near the plasma surface. These oscillations are in principle in the extraordinary wave mode, as confirmed from the measurement of the wavelength and the phase velocity.

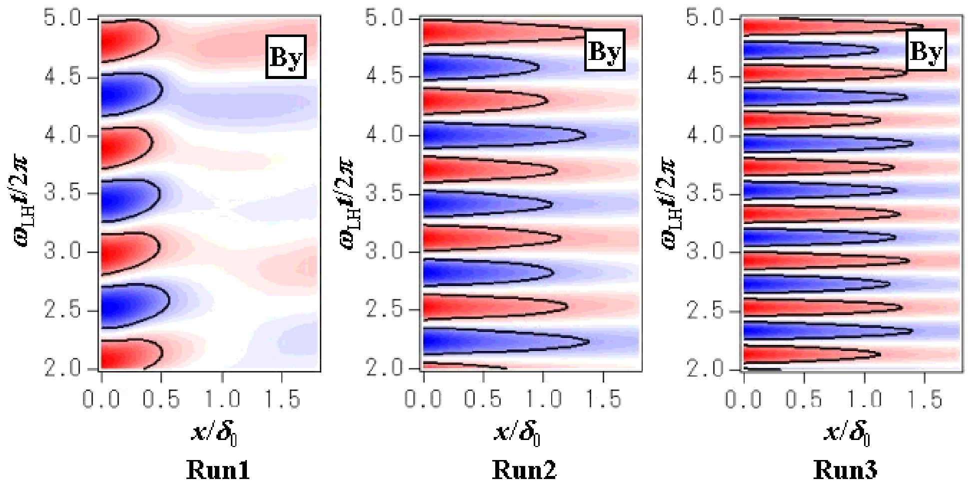

Figure7. Change of skin depth with external alternate current frequency (δ=0.59*δ_0, δ=1.14*δ_0, δ=1.31*δ_0)

Figure 7 is a blow-up of Figure 6. The black line corresponds to the location where *By* is reduced to 1/e of its value at *x*=0. The skin depth determined from the simulation runs match quite well with theoretical predictions.

. Currents excited in the plasma

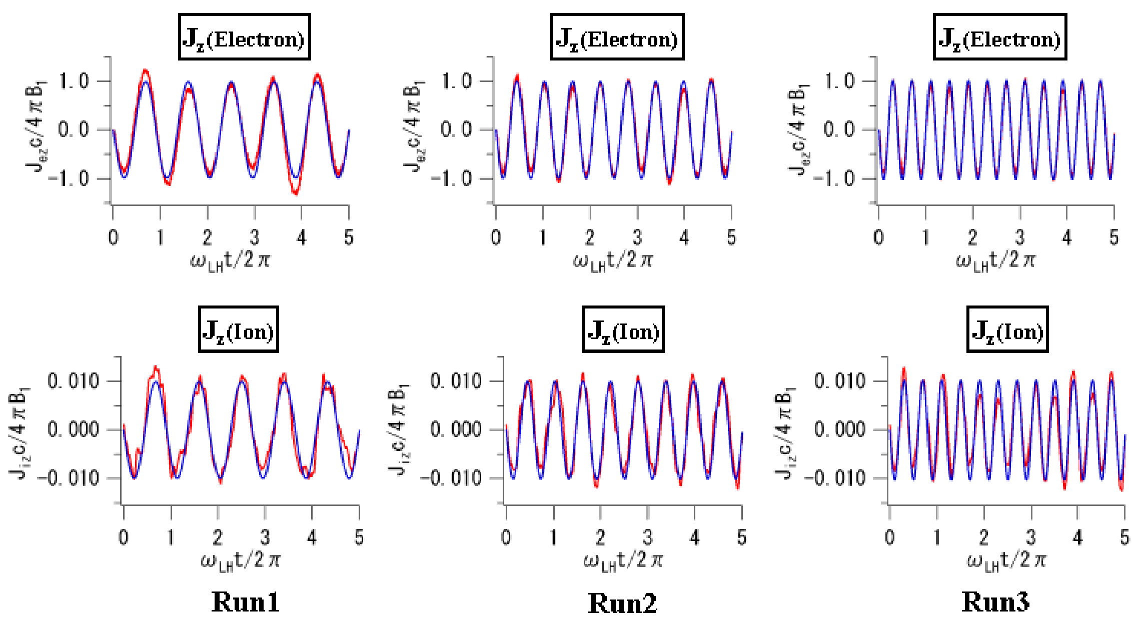

Figure8. Total current excited in plasma

In Figure 9 we plot total current excited in the plasma as a function of time. Red line is the result of the numerical simulations, and blue line is the estimate from the model. They are in good agreement.

. Oscillation of Large amplitude waves

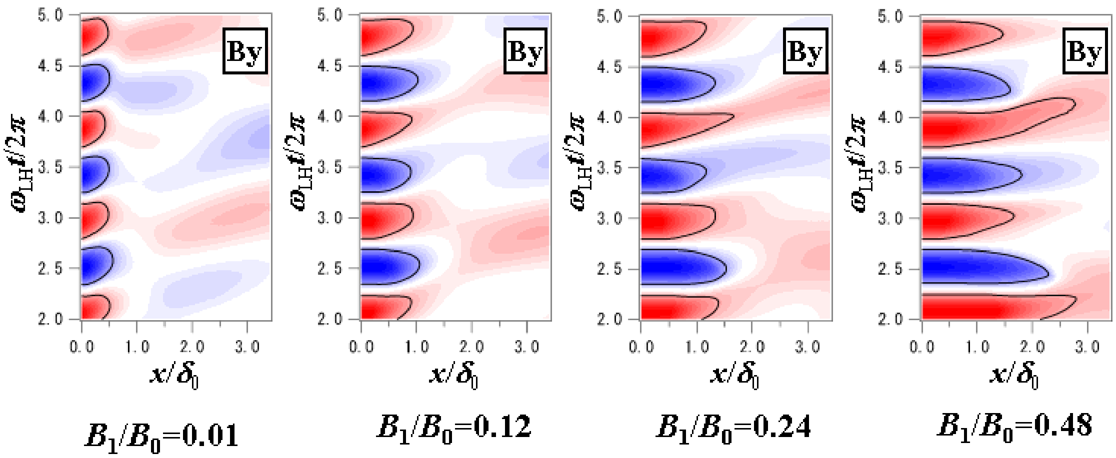

Figure9. Change of skin depth with amplitude. (ω=1.1*ω_{LH})

Figure 9 shows evolution of By when the parameters are the same as in Run 1, except that the external current strength is 12 times, 24 times, 48 times stronger than that used in Run 1. It is found that skin depth (as defined as the e-folding length measured from x=0) is enhanced compared with Run 1, and the oscillation profile becomes complex. This is because the strong electromagnetic waves can now modify the background plasma, and the plasma is pushed toward the positive x direction.

IV. Conclusion

We have examined the frequency dependence of the skin depth by performing 1-dimensional PIC simulations. The results are compared with a simple linear theory. The agreement is satisfactory as long as the amplitude of the external oscillation is small, but as the amplitude is increased, the simulation results start to deviate from the theory, because the strong electromagnetic waves modify the background plasma. Further considerations about this nonlinear regime, as well as extensions of the present simulation settings to more realistic electronic propulsion system is necessary.

V. References

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