Azimuthal Non-uniformities in Accelerators with Closed Electron Drift

Vladimir Baranov
vladimir_i_baranov@newmail.ru
Yuri Nazarenko
Yuriy_Nazarenko@hotmail.com
Valery Petrosov
Keldysh Research Center
Moscow, Russia
007-095-4564608
kerc@elnet.msk.ru

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From the operation experience of Accelerators With Closed Electron Drift (ACD) it is known, that the presence of azimuthal heterogeneities (AH) usually makes worse the characteristics of the Hall plasma accelerator. Therefore on the development of new models of accelerators they usually aim to remove all azimuthal non-uniformities. The reasons causing of non-uniformities, can be various: irregular supply of a working gas, asymmetrical magnetic system, inhomogeneous dissipation of heat, inaccuracy at manufacturing of the accelerator etc.

Azimuthal non-uniformity can rather strongly change the mode of the Hall thruster operation. For example, if we put the wall hindering a course of an azimuthal Hall current, then the magnetic field cannot execute its main function – to magnetize of electrons and reduce their conductivity. In an outcome the electrons current through the accelerator may increase in many times, and since it will not work normally any more. In this connection it would be rather useful to receive quantitative methods of an evaluation of influence of such heterogeneities on working parameters of the accelerator, and the solving of this task has become object of the given article.

Introduction

The azimuthal heterogeneity (AH) makes influence mainly on the conductivity of electron component of plasma. In this report the generalization of the classical formulas for conductivity in plasma with a cross-sectional magnetic field is carried out for cases, when there are some azimuthal zones with various parameters of plasma. There are obtained the analytical relations for the electron conductivity. These relations in the specific case of homogeneous plasma turn into the well-known classical expressions.

However the consideration only electrons motion does not exhaust the influence of AH on plasma parameters of ACD. As it is marked in the present paper, the azimuthal oscillations are very important for electron conductivity. Probably these oscillations are inherent to all modern Hall type accelerators. The presence of these oscillations can increase the electron conductivity of plasma in a zone of acceleration in several times.

At presence of AH it is very important to take into consideration the motion of ions too. It is shown, that even rather small (~10%) AH in concentration or magnetic field can cause origin of strong electrical fields along azimuth. These fields result in appearance of the significant ion flows along azimuth and to the essential reorganization of plasma discharge.

Currents in magnetized plasma

It is known, that exactly electrons form the structure of the longitudinal electrical field in Hall accelerator [1]. Therefore it is logical to put in a basis of the analysis of AH the equation of electrons movement. In our case it can be written down in the following view

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$$\frac{d\vec{V}}{dt} = \frac{e}{m}(\vec{E} + \frac{\vec{V}}{c} \times \vec{B}) - \nu \cdot \vec{V} \quad , \qquad (1)$$

Where V – the electron velocity; E, B - electrical and magnetic fields; ν - the frequency of electron collisions. In absence of collisions (ν= 0) equations (1) are easily integrated. In the assumption, that the electrical and magnetic fields are mutually perpendicular, that is, **E** = (E, 0,0), **B** = (0,0, B) we receive from (1)

$$\begin{cases} \dot{V}_x = a + \Omega V_y \\ \dot{V}_y = -\Omega V_x \end{cases} \qquad (2)$$

Here the designations a = eE/m, Ω = eB/mc are entered. The system (2) has the obvious decision

$$\begin{cases} V_x = V_0 \cdot \sin(\Omega t + \eta) \\ V_y = -a/\Omega + V_0 \cdot \cos(\Omega t + \eta) \end{cases} \qquad (3)$$

Where V_0 and η - constants of integration. The electron movement consists of rotation with frequency **Ω** around of a magnetic line and drift with constant velocity U (U = a /Ω) in a direction [**E** ×**B**] (along the azimuth for Hall accelerators)

$$\vec{U} = c\frac{\vec{E} \times \vec{B}}{B^2} \qquad (4)$$

Taking into account the influence of electron collisions we receive instead of (3) the following decision of the equations of movement (1)

$$\begin{cases} V_x = V_0 \cdot \exp(-\nu \cdot t) \cdot \sin(\Omega \cdot t + \eta) + \nu \cdot a/(\Omega^2 + \nu^2) \\ V_y = V_0 \cdot \exp(-\nu \cdot t) \cdot \cos(\Omega \cdot t + \eta) - \Omega \cdot a/(\Omega^2 + \nu^2) \end{cases} \qquad (5)$$

In this case we have rotation, fading on amplitude, and in addition the velocity of drift along the axis "x", that is in the direction of **E** field.

At transition from the equation of movement for one electron to the equation of movement of the electronic component of plasma we should take into account the following reasons. As velocity V_0 it is necessary to understand the velocity of the chaotic thermal movement of electrons, its value will be determined by the equation of energy for the electronic component. For example in a case when collisions are almost elastic (without lost of energy), electrons will be heated up because of shift along a E field. As a field E it is possible to understand an effective field E^*, where the gradient of electronic pressure enters in addition to E. If we investigate only a macroscopic movement of plasma with the characteristic values L larger, than Larmoure electron radius ($L > V_0/\Omega \sim 1$ mm), the rotation of electrons with frequency **Ω** averages on casual phases η, and we may disregard it. Therefore instead of the equation (1) it is possible to use its simplified variant without the left part. Sometimes this equation in such view is called as the generalized Ohm law.

It is necessary to note that it is impossible to neglect the derivative from velocity in (1) only from reasons of minuteness of the members of the equation rather each other. Because in the case of rare collisions (ν << Ω), the last member of (1) is essentially less than the first one, which we neglect. However formal justification of this simplification is in the fact, that the decision of the equation of movement without derivative

$$\begin{cases} a + \Omega \cdot V_y - \nu \cdot V_x = 0 \\ \Omega \cdot V_x + \nu \cdot V_y = 0 \end{cases} \qquad (6)$$

precisely coincides with (5), if we neglect by the Larmoure rotation in it

$$\begin{cases} V_x = \nu \cdot a/(\Omega^2 + \nu^2) \\ V_y = -\Omega \cdot a/(\Omega^2 + \nu^2) \end{cases} \qquad (7)$$

The formulas (7) or its some other form of record

$$j_x = \sigma E \frac{1}{1 + \Omega^2/\nu^2}, \quad j_y = -\sigma E \frac{\Omega/\nu}{1 + \Omega^2/\nu^2}, \qquad (7.1)$$

are usually used for calculation of plasma conductivity in a magnetic field. The principles considered in this paragraph can be applied to generalization of these formulas for a case, when plasma is non-uniform in azimuth, or, in other words, when some of plasma parameters vary along the direction of electron drift.

Currents in magnetized plasma with the azimuthal heterogeneity

At presence of azimuthal heterogeneity (non-uniformity) the values of electronic currents can considerably change. It is clear already from well known fact, that if to prevent passing of a Hall current, the magnetic field will cease to limit the value of a longitudinal electronic current [1]. Let's carry out consideration of the case, when there are only two azimuthal regions 1 and 2, and within the limits of each region the plasma parameters are constant. Then the equations of movement (6) for region 1 will have the following view

$$\begin{cases} j_{1x} = \sigma E + \gamma j_{1y} \\ j_{1y} = \sigma E_{1y} - \gamma j_{1x} \end{cases} \quad (8)$$

Here instead of velocities v_{1x} and v_{1y} the density of a current are entered

$$j_{1x} = en_1 v_{1x} \ ; \quad j_{1y} = en_1 v_{1y}$$

Value $\sigma = e^2 n_1/m\nu_1$ is the conductivity of plasma in the first region, $\gamma \equiv \Omega_1/\nu_1$. For area 2 similar equations have the following view

$$\begin{cases} j_{2x} = \alpha\sigma E + \delta\gamma j_{2y} \\ j_{2y} = \alpha\sigma E_{2y} - \delta\gamma j_{2x} \end{cases} \quad (9)$$

Here the parameters $\alpha = n_2\nu_1/n_1\nu_2$ and $\delta = \Omega_2\nu_1/\Omega_1\nu_2$ characterize the degree of plasma heterogeneity (at α = 1 and δ = 1 plasma is homogeneous). Let's give some explanations to the equations (8), (9). In them the induced azimuthal components E_{1y} and E_{2y} of an electrical field are entered, and the longitudinal field E is considered identical in both regions 1 and 2. We do so because of it is impossible simply to enter different fields E_{1x} and E_{2x}, and the azimuthal components to leave zero, as in a completely homogeneous case. In such case the condition of potentiality of an electrical field **E** on the border of layers 1 and 2 would not be carried out, that is there would be rot**E** $\neq 0$.

For closing of the system of equations (8), (9) we need two additional equations. They can be received from the condition of a continuity of the electrons current on the border of regions 1 and 2

$$j_{1y} = j_{2y} \quad (10)$$

and from the condition of potentiality of the induced fields E_{1y} and E_{2y} (the integral along an azimuth $\int Edy$ should be equal to zero)

$$E_{1y} + \beta E_{2y} = 0 \quad (11)$$

Where $\beta = L_2/L_1$ – the relation of lengths along an azimuth of layers 2 and 1. Let's note that the system of equations like (8-11), but for more special case δ = 1, was investigated by Alven [1]. The received system of the equations has the following decision

$$j_{1x} = \sigma E(\alpha + \beta + \gamma^2 \beta\delta(\delta - \alpha)) / \Delta$$

$$j_{2x} = \sigma E\alpha(\alpha + \beta + \gamma^2(\alpha - \delta)) / \Delta$$

$$j_y = \sigma E(-\alpha\gamma(1 + \beta\delta) / \Delta) \quad (12)$$

$$E_{2y} = E\gamma(\alpha\delta - 1 + \gamma^2\delta(\alpha - \delta)) / \Delta; \ E_{1y} = -\beta E_{2y}$$

where $\Delta = \alpha + \beta + \gamma^2(\alpha + \beta\delta^2)$.

As we already marked, in ACD the parameter γ reaches the rather large values ($\gamma \sim 10^2$). At the account of this fact, from (12) follows, that even small azimuthal heterogeneity can considerably increase the longitudinal electron current. In a fig.1 the dependence of the complete longitudinal current Jx from α (i.e. from the relation of density in the region 2 to density in the region 1) is shown for heterogeneity of the B field in 10 % (δ = 1.1, γ = 100, b = 1). As a unit of a current the longitudinal current in a homogeneous case is taken.

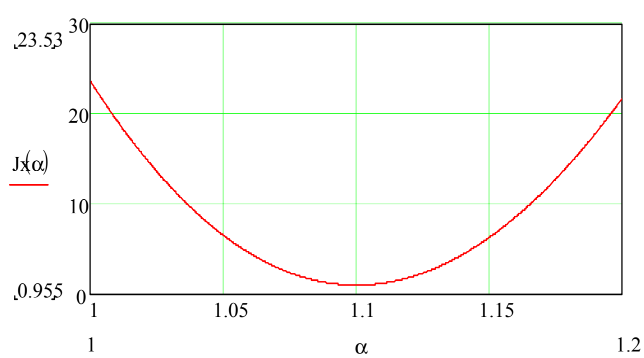

Fig. 1

From this figure we can see, that the complete longitudinal current with AH can exceed a current in a homogeneous case in tens time even at rather small heterogeneity (~ 10 %). Thus, from (12) it becomes clear, that the azimuthal heterogeneity can essentially change the structure of the plasma discharge in ACD.

The received formulas (12) would solve the task with two zones of heterogeneity 1 and 2, if electrons moved with background of the motionless ions, for example, in a solid body. However at research of plasma it is necessary to take into account the movement of ions too, as arising induced azimuthal fields E_{1y} and E_{2y} also can be rather great (see fig. 2), exceeding a longitudinal accelerating field E in some times.

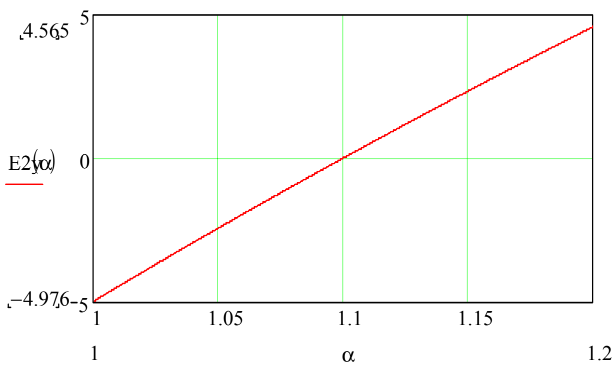

Fig. 2

Here the value of longitudinal E field is taken for a unit, other parameters are taken the same as for a fig. 1. It is clear, that so large azimuthal fields will cause the appropriate ions moving which is necessary to be taken into account on considering of the influence of AH on the discharge parameters. We shall return to this question in the last paragraph. And now we shall look on AH as at the possible reason of fluctuations in plasma.

Azimuthal fluctuations

The joint account both movements of electrons and ions essentially complicates the task for analytical research. Let's begin from consideration of small azimuthal perturbations of plasma. For this purpose we return to the equation (1) and add to it the equations for ions, and also the equation for potential. We shall neglect by the electron collisions because of minuteness of the relation ν/Ω. Then we receive the following system of the equations

$$\begin{aligned}
&\partial n_i/\partial t + \mathrm{div}(n_i\vec{v}_i) = 0;\ \ \partial n_e/\partial t + \mathrm{div}(n_e\vec{v}_e) = 0\\
&\partial \vec{v}_e/\partial t + (\vec{v}_e\vec{\nabla})\vec{v}_e = -(e/m)(\vec{E} + \vec{v}_e\times\vec{B}/c);\\
&\partial \vec{v}_i/\partial t + (\vec{v}_i\vec{\nabla})\vec{v}_i = e\vec{E}/M\\
&\Delta\varphi = -4\pi\pi(n_i - n_e);\ \ \vec{E} = -\vec{\nabla}\varphi
\end{aligned} \qquad (13)$$

Further we believe that system (13) has some stationary decision, and the ions moves along the “x” axis $\mathbf{V}_i$ = {V, 0,0}, and electrons drift along an azimuth (axis “y”) $\mathbf{V}_e$ = {0, U, 0}, the B field is radial **B** = {0,0, B}, and the E field has only longitudinal component **E** = {E, 0,0}. Then we impose on stationary density, velocities and potential the appropriate small disturbances of the same values as sine waves

$$A(\vec{x},t) = a\cdot\exp(-i\omega t + i\vec{k}\vec{x}) \qquad (14)$$

As a result from (13) the dispersion equation for small fluctuations is received [2]

$$\frac{\omega_i^2}{(\omega - k_x V)^2} + \frac{\omega_e^2}{(\omega - k_y U)^2 - \Omega^2} = 1 \qquad (15)$$

where ω_i, ω_e - plasma frequencies for ions and electrons.

From this equation particularly follows that at $(\omega - k_x V) > \omega_i$ or at $(\omega - k_x V) < \omega_i(\Omega/\omega_e)$ the connection between the frequency ω and the wave number k_y always is real (not imaginary), that is the real value of ω correspond to real k_y. According to (14) it means, that the increasing fluctuations in this range of frequencies are not occurred. And, on the contrary, in the range of frequencies

$$\omega_i\Omega/\omega_e < \omega - k_x V < \omega_i \qquad (16)$$

the increasing fluctuations are possible, and to this case the following wave vectors k_y correspond

$$\Omega/U < k_y < \omega_e/U$$

For characteristic values of plasma parameters of ACD ($\omega_i = 10^8\ s^{-1}$, $\omega_e = 5\cdot10^{10} s^{-1}$, $\Omega = 2\cdot10^9\ s^{-1}$) to this range the lengths of waves λ_y~1 mm correspond.

Such self-arising azimuthal fluctuations are inherent in all types of ACD. Obviously, just these fluctuations are responsible for the formation of characteristic erosive structure with the spatial period ~ 1 mm on the channel walls in the region near its exit [2]. This unusual structure has received the name «anomalous erosion».

What will take place with a small azimuthal disturbance in plasma, if its value is essential more than one mm? From (15) follows, that this disturbance will cause harmonic fluctuations (not self-arising, but gradually fading) with characteristic frequency ω

$$\omega = k_x V + \omega_i (\Omega / \omega_e)$$

Such fluctuations can be observed only if somehow artificially to create extended (essentially more than 1 mm) AH in the plasma channel.

For practice the answer to the follows question is of interest: "what disturbance it is possible to consider as small?" For example, if the azimuthal heterogeneity in the density of ions about 10 % has arisen, whether it is possible to consider it as small? From the system (13), particularly, the following ratio between disturbances in potential φ and density of ions n_i can be received

$$n_i = \frac{eN_i}{M} \varphi \frac{k_x^2 + k_y^2}{(\omega - k_x V)^2} \quad (17)$$

With the account (16) and believing $k_x = 0$ (that is, the disturbance has only azimuthal component), we receive from (17)

$$E_y = \frac{n_i}{N_i} \frac{m}{e} \Omega^2 \frac{\lambda_y}{2\pi} \quad (18)$$

where N_i – the stationary density of ions. From this ratio between n_i and E_y follows, that at azimuthal the value of disturbance λ_y ~ 5 cm and (n_i / N_i) ~ 0.1 the induced field reaches value about 300 V/cm, and it in any way cannot already be considered as small, since E_y ~E.

It is curious, that in the equations (13) we beforehand have neglected by the electron collisions (ν = 0) and, nevertheless, again have received as well as in the previous paragraph, but already from other physical model, that the disturbance of the density only in 10% causes occurrence of the large electrical fields along an azimuth.

It is very important to pay attention as well to the following circumstance. If to apply results of the decision of a task about two azimuthal regions (12) to the increasing fluctuations of a type (16), it turns out, that these fluctuations explain not only "anomalous erosion", but also the abnormal high conductivity of ACD plasma in the acceleration zone. As the abnormal conductivity here we mean that well known fact, that the real electron conductivity in the acceleration zone is much above, than the classic conductivity (7.1) when as ν we take the frequency of electrons collisions with heavy particles of plasma. And the applicability of the mentioned formulas (12) for calculation of the conductivity is quite motivated by the fact that the frequency of fluctuations ω is significant less then frequency Ω, that is, the induced electrical fields can be considered as quasi-stationary. Then it is possible to consider, that the plasma is divided into two type approximately equal areas along an azimuth. In the first type of regions the density on some percents is higher (peaks of waves), than in second (cavities). It is easy to see, that the boundary conditions (10-11) for two areas or for several will be identical (see following paragraph). Thus, even a small difference in the densities in cavities and peaks can excite the arising azimuthal fluctuations (16) which increase the longitudinal electron conductivity of plasma in several times.

This result is especially important for understanding of processes in ACD with metal walls (so-called - thrusters with anode layer). Where the conducting walls of the channel usually are under potential of the cathode, and for an explanation of the abnormal high conductivity it is impossible to involve the near-wall conductivity, since electrons cannot go on the channel walls.

Reorganization of structure of azimuthal non-uniform plasma

It is clear that for an exact prediction of the behavior of ions in the ACD channel at the presence of AH the three-dimensional modeling of the plasma flow is necessary. But now it is difficult even two-dimensional numerical modeling of an azimuthal homogeneous flow reproducing real processes with sufficient accuracy. Therefore for finding out what processes occur in plasma in the channel of ACD at the presence of AH we shall limit ourselves to the simplified physical model as the first approximation.

It is useful to consider a case, when we have a lot of azimuthal layers, but inside each layer the plasma is completely homogeneous. For N layers the equations similar (8-11) can be written as

$$E_1 l_1 + E_2 l_2 + ... + E_N l_N = 0$$
$$j_{y1} = j_{y2} = ... = j_{yN} \equiv j \qquad (19)$$
$$j_n = \sigma_n E + \gamma_n j$$
$$j = \sigma_n E_n - \gamma_n j_n \qquad (20)$$

where the index n = 1, 2, … N corresponds to the number of a layer; j – the common for all layers the azimuthal current; E – the common for all layers the longitudinal electrical field. From (20) we can find the field in the n-th layer

$$E_n = \gamma_n E + j(1+\gamma_n^2)/\sigma_n \qquad (21)$$

And substituting this expression in (19), we receive the formula for the azimuthal current j

$$j = -E\left(\sum_{n=1}^{N}\gamma_n l_n\right)/\left(\sum_{n=1}^{N}(1+\gamma_n^2) l_n/\sigma_n\right) \equiv -E\Sigma \qquad (22)$$

Here the average Hall conductivity is designated as Σ. By the direct substitution it is easy to be convinced, that in the specific case N = 1 we receive the classical expressions for Hall current j and longitudinal current j_1

$$j = -\sigma E\gamma/(1+\gamma^2);\ j_1 = \sigma E/(1+\gamma^2);\ E_1 = 0$$

Taking in account (22) the expressions for fields (21) can be rewritten as

$$E_n = E(\gamma_n - (1+\gamma_n^2)\Sigma/\sigma_n) \qquad (21.1)$$

With the help of last equations it is possible as a first approximation to look after dynamics of ions movement in the azimuthal direction at the presence of AH. Suppose we have only two zones (N = 2) with identical lengths ($l_1 = l_2$), and in first of them the density of plasma is higher on 10 % than in another. Then if $\gamma \sim 10^2$, we have from (21.1) $E_1 \sim 5E$, $E_2 \sim -5E$, that is, on the one border between the layers the ions will be move towards the border, and on another border backwards. Then near the one border the zone with the increased density occurs, and near another one with lowered. Thus, a bit later we will receive the four layers of AH instead two. From the formulas (21.1) follows, that the stronger density deviates from the average value, the more induced azimuthal field E_n. And if the density is less than average one, the field is negative, and vice versa. The layers with the greater density formed owing to ions shift move in the positive direction of an azimuthal axis. Such behavior of ions should result in gradual alignment of the density along an azimuth.

Accordingly, near the other border of the initial AH the cavity in the density occurs, which will become deeper, and its width less. However in a reality the formation of such indefinitely narrow zone with very small density will be limited to other physical mechanisms, which are not taken into account in our model, as there will be no also too sharp borders between the azimuthal layers. It follows from the equation of movement for plasma as a whole (instead of for ion and electron components separately), which includes, besides the force of Ampere [j×B], also the pressure gradient ∇p

$$\rho\frac{d\vec{V}}{dt} = \frac{1}{c}\vec{j}\times\vec{B} - \vec{\nabla}p,$$

which, obviously, will prevent the formation of too sharp differences between the density values. Thus, the resulting joint actions of Ampere and pressure forces will lead to a fast alignment of the originally non-uniform plasma density.

If we have AH of the magnetic field H, instead of the density, then the similar consideration of forces acting on plasma shows, that the ions movement in the azimuthal direction will lead to the compensation of the perturbing action of the B field. Obviously, that the full compensation is achieved at the constant ratio (n/B) along the azimuth, since just in this case the continuity of the azimuthal current (j = encE/B) is kept constant without the occurrence of the additional azimuthal electrical fields.

It is possible to make a conclusion, that even the small AH in the ACD channel lead to raising of the large azimuthal electrical fields, which in turn result in such changes in the discharge structure (basically, in the ions density and velocity) that decrease the induced azimuthal E fields. In full correspondence with the general thermodynamic principle (Le-Shatelye principle [3]), the external influence to the system stimulates in it the processes, which tend to weaken this external influence.

However the geometry of the typical modern ACD is those, that the characteristic size of plasma along an azimuth considerably exceed the longitudinal size of the discharge. Therefore during movement of the working body through the discharge interval the complete disappearance of induced azimuthal fields may not occur, because of the lack of time for the azimuthal ions shift. At such variant the longitudinal electronic current can exceed essentially its nominal value, and the overall performance of the accelerator will be low.

For estimations of the velocity of compensation of AH in plasma for the ACD in each concrete case it is important to take into account also the following circumstance. All estimations of azimuthal fields were carried out here through the value of a longitudinal electrical field E. But it is important to remember, that it is necessary to understand this field E as an effective field E^*, in which the gradient of electronic pressure is taken into account also. As is known, in a zone of ionization the electronic pressure can be significant more, than the longitudinal electrical field E. Moreover, the last can sometimes decrease in a zero or even become negative. And then in case of occurrence of the azimuthal heterogeneity the conditions for its full compensation in the ionization zone will be, obviously, considerably greatly better, than in the acceleration zone.

Conclusions

In this work the attempt is undertaken to analyze the possible consequences at occurrence of the azimuthal heterogeneity (AH) in plasma parameters or in magnetic field in the ACD channel. It is established, that even the small heterogeneity (~ 10 %) can cause the occurrence of the strong induced azimuthal electrical fields, which may in some times exceed the intensity of the axial electrical field. Thus the longitudinal electron current through the channel also grows in some times in comparison with a homogeneous case. The analytical expressions for the estimations of the additional conductivity of plasma which occur in it because of the azimuthal fluctuations are received. And this conductivity can be many times more than the usual classical conductivity caused by electron collisions with heavy particles of plasma. It is shown, that the presence of AH in the ACD channel causes also the strong azimuthal movement of ions, that results in such reorganization of structure of the plasma discharge to reduce azimuthal electrical fields up to a minimum. In particular, the presence of AH in the magnetic field results in the appropriate occurrence of AH in the density of plasma. This feature of the ACD plasma behavior can be used for design of the new perspective types of the thrusters, in which the turn of the thrust vector by means of the change of the azimuthal distribution of the magnetic field or the flow rate becomes possible.

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