Test Data Analysis and Simulation Modeling for the Electromagnetic Emission from Stationary Plasma Thrusters

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Abstract: Methods and results of simulation modeling for the non-thermal emission from stationary plasma thrusters that being an interference for a channel of spacecraft communication with ground systems are discussed based on the test results. Comparative analysis is made for the statistical characteristics of stationary plasma thruster emission in the radio range obtained by tests and appropriate characteristics of the developed simulation models of such processes.

Nomenclature

$I(t)$	= in-phase component of the complex envelope
$Q(t)$	= quarter-phase component of the complex envelope
$\varphi(t)$	= complex envelope phase
ω_0	= central frequency

I. Introduction

MULTI-YEAR application of stationary plasma thrusters (SPT) verified their reliability and efficiency, but along with that revealed a number of peculiarities to be taken into account while integrating SPT with spacecraft (SC) and its systems. So, analysis of numerous results of test studies for the SPT self electromagnetic emission showed that in addition to a thermal component such emission contains a wide-band pulsed stochastic process, spectral density of which is from several tens of Hz to several tens of GHz [1, 2]. Such emission can represent an interference for the channel "Earth- SC" of the space communication systems and should be taken into account at the design stage while defining the required power budget of radio line and selection of algorithms for the received signal processing.

It should be noted that currently general approaches based on the analytic description of interference are not applicable to the SPT interference directly, because the latter has a combination of parameters being beyond the limitations of the known analytical models. Thus, development of new analytical models for the SPT emission and the use of alternative methods of simulation modeling are the actual tasks aimed at the interference immunity increase for the space communication systems of SC with electric propulsions.

One of the approaches implying the development of equivalent mathematical models for the SPT emission that are based on the results of test studies in time domain is discussed in this paper.

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Application of the SPT emission models in the problems of analytical designing and simulation modeling for the space communication radio systems is the final aim of such models development.

II. Characteristics of SPT Emission in Time Domain

To develop equivalent mathematical models, it is necessary to know fine structure and statistical characteristics of the EP emission. A number of publications is known currently [1-5], which deal with the study of EP emission characteristics in the time domain. Let's consider typical parameters of interference generated by stationary plasma thrusters by the example of the Hall thruster Aerojet BPT-4000, for which a lot of measurement results in the time domain is available [1].

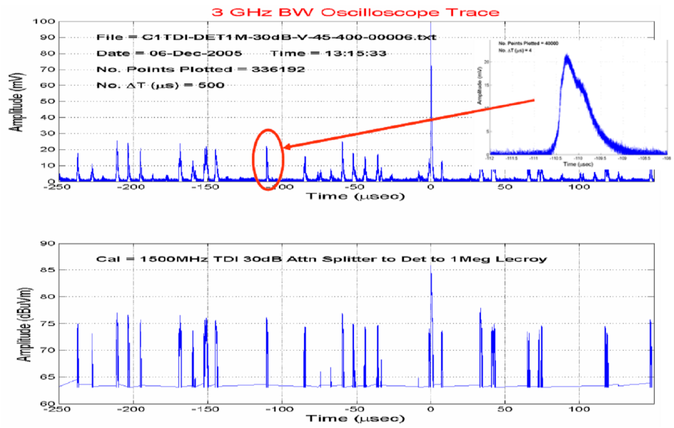


Fig. 1. Representations of amplitude envelope of the Aerojet BPT-4000 emission.

As an example, Fig. 1 shows data obtained for the following mode: power 4.5 kW, discharge voltage 400 V. There are about 25 pulses within the time interval of 400 µs. Absolute values of the electric field intensity converted to the antenna aperture (bottom plot) are from 65 to 85 dBµV/m, the most part of pulses having amplitude of about 75 dBµV/m. One of the short pulses of this sequence is shown in Fig. 1 in enlarged scale. It is shown that at the level of 0.5 of the maximum, the observed pulses have duration from 0.5 µs to 5 µs. It is noted in [3] that for the SPT-100 emission in L-, S- and C-bands the pulse width from 1.5 µs to 2 µs is typical, that being comparable to the width of 0.5-5 µs registered for the thruster BPT-4000, but substantially more than the pulse width of 10–200 ns indicated earlier for the thruster SPT-100 [4].

We made numerous test studies for the spectral and time-response characteristics of the SPT self emission [5], [6]. So, with the SPT-100 laboratory model, measurements were made for different values of discharge voltage that was varied from 300 V to 800 V with the step of 100 V, and different values of propellant consumption (from 1.88 mg/s to 2.73 mg/s). Time samples of the EP emission were recorded at the time intervals of 1-10 ms with the sampling frequency of 12.5 ns or 7.1 ns. Each sample constitutes a complex process formed at the output of the bandpass filter with the pass band of 80 MHz or 140 MHz and central frequency, the value of which was varied discretely within the frequency band 100 MHz – 12 GHz with the step of 80 MHz or 140 MHz. Central frequency of the filter was varied either in the "panoramic" mode, i.e. with the constant step within the entire mentioned frequency band, or for the specified frequency bands corresponding to the satellite communication channels "Earth-SC". Special attention was paid to the high-voltage modes of the SPT-100 operation, because pulsed nature of the Hall thruster emission is manifested most characteristically at such modes exactly.

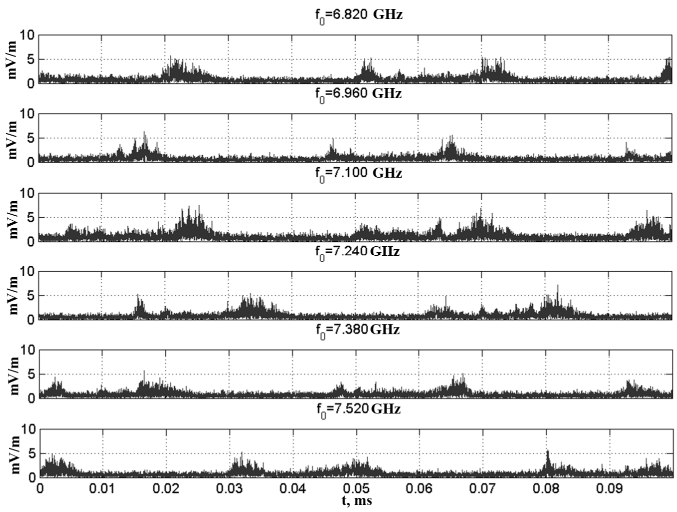
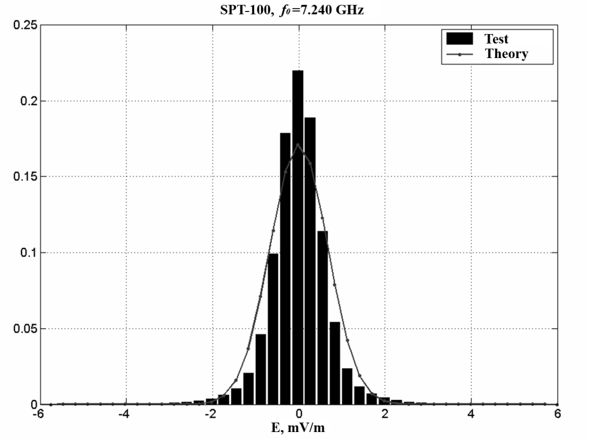


Fig. 2. Representations of amplitude envelope of the SPT-100 emission within the frequency range of 6.82 – 7.52 GHz.

Examples of time representaions of the received signal envelope obtained in the "panoramic" mode for the frequencies 6.82–7.52 GHz with the step of 140 MHz for the mode $U_p = 800\,\text{V}$, $I_p = 2.95\,\text{A}$, $\dot{m} = 2.73\,\text{mg/s}$, are presented in Fig. 2. In general, analysis for the results obtained allows making a conclusion that for the high-voltage modes of the SPT-100 operation its pulsed emission is characterized by the formation of a random train of pulses, duration of which is within 0.2–7 µs, while the repetition period is within 10–30 µs upon the average.

At the same time, it is evident that in some cases the “stacking” of pulses, i.e. emission of a group of closely spaced pulses that may be interpreted as one “long” pulse, is observed. To assess statistical characteristics of the processes under study, an analysis was made for the distribution law of the quadrature representations having pronounced pulsed components. According to the results obtained, that distribution law differed substantially from the Gaussian one. Some analysis results are presented in Fig. 3 for the central frequency of 7.24 GHz coinciding with the frequency range allocated to the deep space communication systems.



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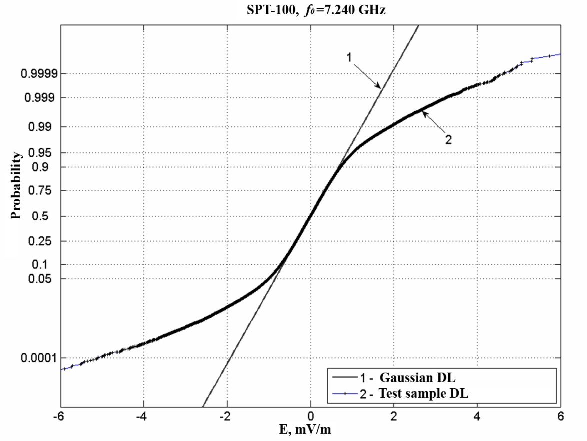

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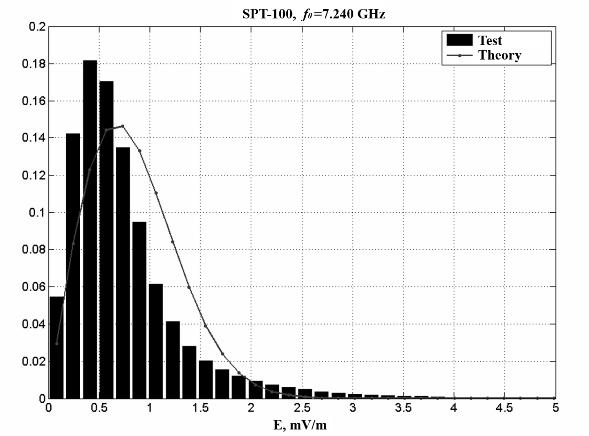

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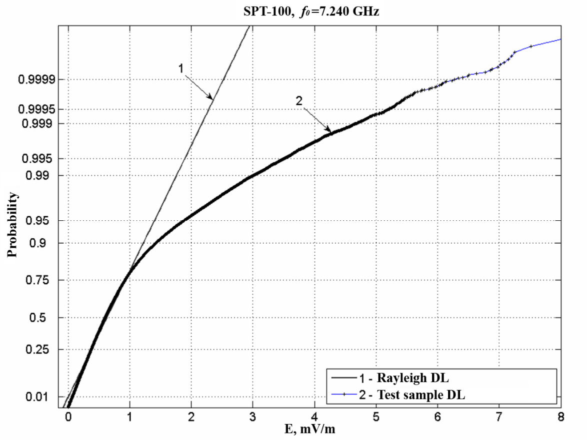

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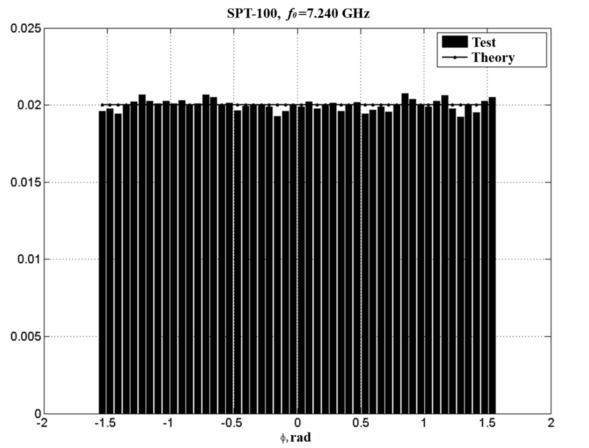

e

Fig. 3. SPT-100 emission distribution laws for the frequency 7.24 GHz

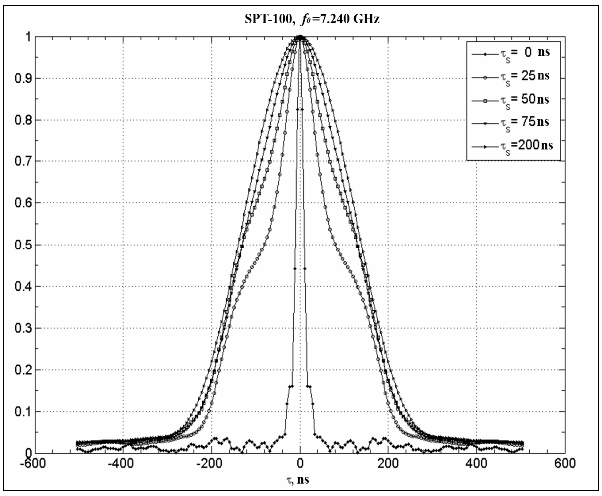

Fig 4. SPT-100 emission correlation coefficient module

Fig. 3a shows a distribution bar chart for the in-phase component of the received signal, to which the ideal Gaussian distribution law corresponding to the representation parameters is superimposed. In Fig. 3b for the same conditions the cumulative distribution law of appropriate representations is plotted on the probability paper allowing visual assessment for the degree of agreement between the certain representation distribution law and the Gaussian distribution law with the mathematical expectation and variance being the same as those of the representation obtained by test (on the probability paper the Gaussian distribution law is represented by an inclined line).

As is obvious from the results obtained for the in-phase (quadrature) component of the received signal complex envelope, its differential (Fig. 3a) and cumulative (Fig. 3.b) distribution laws differ considerably from the Gaussian one. Therefore the differential (Fig. 3c) and cumulative (Fig. 3d) distribution laws for the signal amplitude envelope differ from the Rayleigh distribution law considerably also. With this, in any case the distribution law for the phase of the signal complex envelope is close to the uniform (Fig. 3.e).

Fig. 4 shows the correlation coefficient modules for the complex envelope of the SPT-100 signal for the central frequency of 7.24 GHz. The curves correspond to the initial sequence of values and to the sequences that passed through the smoothing (band-pass) filter with various values of response time (from 25 ns to 200 ns) that allows filtering for the additive noise components. It is obvious that the correlation coefficient contains two components: narrow peak corresponding to the wide-band component of emission related both to the additive thermal noise and the SPT thermal emission, and a part being wider in its base that is defined by the non-thermal (pulsed) emission of SPT. Correlation interval of the latter is about 0.12-0.15 µs that coinciding with the duration of the shortest pulses registered for the SPT emission.

So, electromagnetic emission of stationary plasma thrusters is non-Gaussian, has pronounced component in a form of a random sequence of pulses and a noise component being a sum of the SPT thermal emission and the measurement instrumentation noises.

III. Mathematical Description for the SPT Non-Thermal Emission

According to the analysis of the above test data, within the L-, S- and C-bands the spectrum width for a random pulsed sequence of SPT emission is substantially less than the carrier, that allowing consideration of the discussed processes within the radio receiving system bandpass as being a narrowband one and the use of the complex envelope method [7], [8] for the thruster emission modeling.

In this case the stochastic process $n_\Sigma(t)$ describing the thruster emission may be represented as a sum of the pulsed process (PP) $u_p(t)$ and noise $n(t)$

$$n_\Sigma(t) = u_p(t) + n(t) \tag{1}$$

Or in the complex form:

$$\dot{n}_\Sigma(t) = \dot{u}_p(t) + \dot{n}(t) = \left(\dot{U}_p(t) + \dot{N}(t)\right)\exp\left(j\omega_0 t\right) \tag{2}$$

where ω_0 is the central frequency of spectrum, $\dot{U}_p(t) = U_{op}(t)\exp\left(j\varphi_{op}(t)\right)$ is the pulsed process complex envelope, $U_{op}(t)$ is the pulsed noise amplitude envelope, and $\varphi_{op}(t)$ is the PP total phase characteristic.

The pulsed process complex envelope may be presented in more details as:

$$\dot{U}_p(t) = \sum_{i=1}^{\infty} u_o(t - \sum_{l=1}^{i} T_{pi})\exp\left(-j\left(\varphi_{oi} + \varphi_{opi}(t)\right)\right) \tag{3}$$

where $u_0(t)$ is the function describing the form of a single noise pulse, φ_{oi} is the initial phase shift for each PP pulse, $\varphi_{opi}(t)$ is the phase variation law within the *i*-th noise pulse that can take the following effects into account:

- random variation of carrier frequency from pulse to pulse,
- random variation of phase within each pulse,
- nonprobabilistic variation of phase within each pulse, etc.

To implement the complex envelope method, let's in the subsequent discussions use the quarter-phase components of the narrow-band process $I(t)$ and $Q(t)$ being the real and imaginary components of (2).

The statistical characteristics of processes included in (2) and (3), which can be obtained based on test sequences, are needed for such model application. They include:

- differential distribution law (DDL) for the quarter-phase components $I(t)$ and $Q(t)$;
- DDL for the amplitude envelope $\sqrt{I^2(t) + Q^2(t)}$; DDL for phase $arctg\left(Q(t)/I(t)\right)$;
- power spectral density (energy distribution) for the process;

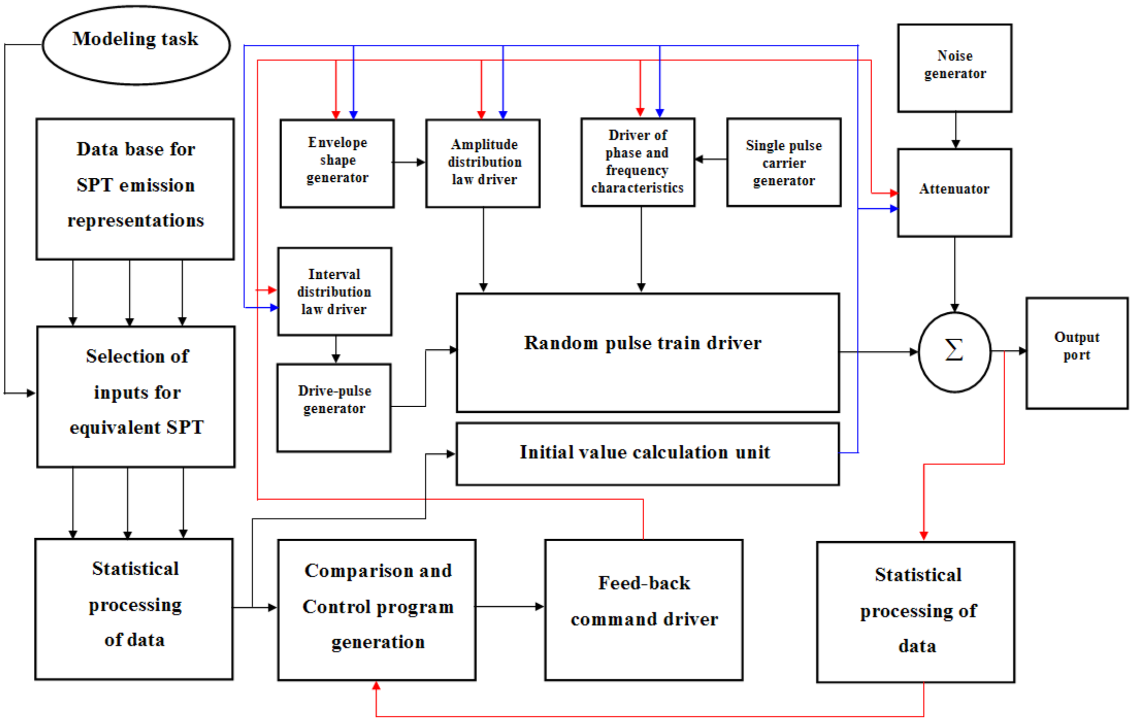

Fig. 5. Block diagram of the software package

- DDL for the pulse duration; DDL for the pulse repetition interval;
- autocorrelation function of time representations of the process.

As to the noise $n(t)$, it may be described by different mathematical models. It should be noted that the noise level in (1) influences substantially the resultant distribution laws and relation of components in (1) that being a subject of independent study.

IV. Computer Modeling for the SPT Emission in Radio-Frequency Range

A special software package was developed in the MATLAB/Simulink environment for the simulation modeling of processes of the SPT self emission in radio-frequency rage. This package allows to generate stochastic processes with the given statistical characteristics, which have properties of the SPT emission. General block diagram of the package is presented in Fig. 5.

Let's consider general principle of its operation. General modeling task is stated at the initial stage. Typical SPT model is defined (SPT-70, SPT-100 or SPT-140, for example), for which it is necessary to made simulation modeling of the processes of self emission in the radio-frequency range. Frequency band and the thruster operation mode (propellant consumption, discharge voltage, for example) are specified. Time representations of the equivalent SPT model emission obtained earlier are taken from the data base formed by us. If close results are absent, additional measurements are made for the certain SPT model, which are used for the available data base completion.

The resulting process was modeled by (1) – (3), with that the application-specific tools of MATLAB/Simulink environment were used to form random components of the process with the given statistical characteristics.

According to the block diagram (Fig. 5), a sequence representing a sum of noise with the given characteristics and the pulsed stochastic process with radiofrequency stuffing comes to the output port of the modeling complex.

With that the random pulse train generator (Fig. 5) during its operation uses representations of processes of three generators (of the carrier frequency, envelope shape and drive pulses). To implement distribution laws for stochastic processes included in (1) – (3), each generator is provided with appropriate drivers (of the phase and frequency characteristics, amplitude distribution law, interval distribution law).

It should be noted that the modeling procedure is a multi-step one. So, at the first step the test data statistical processing is carried out for the selected model of equivalent SPT, and differential distribution laws are calculated for basic components mentioned in part III of this paper. Initial values of distribution parameters used by the units securing operation of the random pulse train driver are defined based on the test data initial processing by the initial value calculation unit. This relates to the attenuator also that specifies initial relation between the noise level and the intensity of the pulsed stochastic process.

Statistical processing of data and determination of distribution law for the simulative representation formed at the previous step were made at each subsequent modeling step. Based on the results of comparison for statistical characteristics of test and simulative representations and identification of disagreement exceeding specified modeling accuracy the control programs are initiated, which secure operation of the feed-back command driver. Back-feed commands provide variation of parameters of all units in accordance with the specified priorities in order to reduce disagreement of test and simulative representations. The number of iterations is defined by the specified modeling accuracy. Parameters of all units are fixed at this moment, and the modeling complex switches to the mode of generation of simulative sequence of SPT emission in the radio-frequency range.

To demonstrate capabilities of the modeling complex, Fig. (6)-(8) show the results of modeling for the simulative emission of SPT-100 model [5] in the radio-frequency range (7.19 GHz). The following initial data were used for modeling:

- mean peak value of the PP envelope - 1 mV/m; average PP duration - 1.0 μs; average PP pulse period-to-pulse duration ratio - 10; modeling step - 100 ns; shape of the pulse envelope - Gaussian; Poisson distribution law for the interval between pulses; a model of white Gaussian noise (WGN) was used.

Fig. 6 shows the examples of random sequences of pulses with Gaussian envelope for two scales of observation time representation (6.1 – 1 ms, 6.2 – 0.1 ms). In these plots, the following functions are indicated along the vertical axes in the measurement units of electric field intensity (mV/m): a – pulsed process envelope; b – PP and WGN mixture envelope; c – real component of the complex amplitude of the PP and WGN mixture, d – imaginary component of the complex amplitude of the PP and WGN mixture.

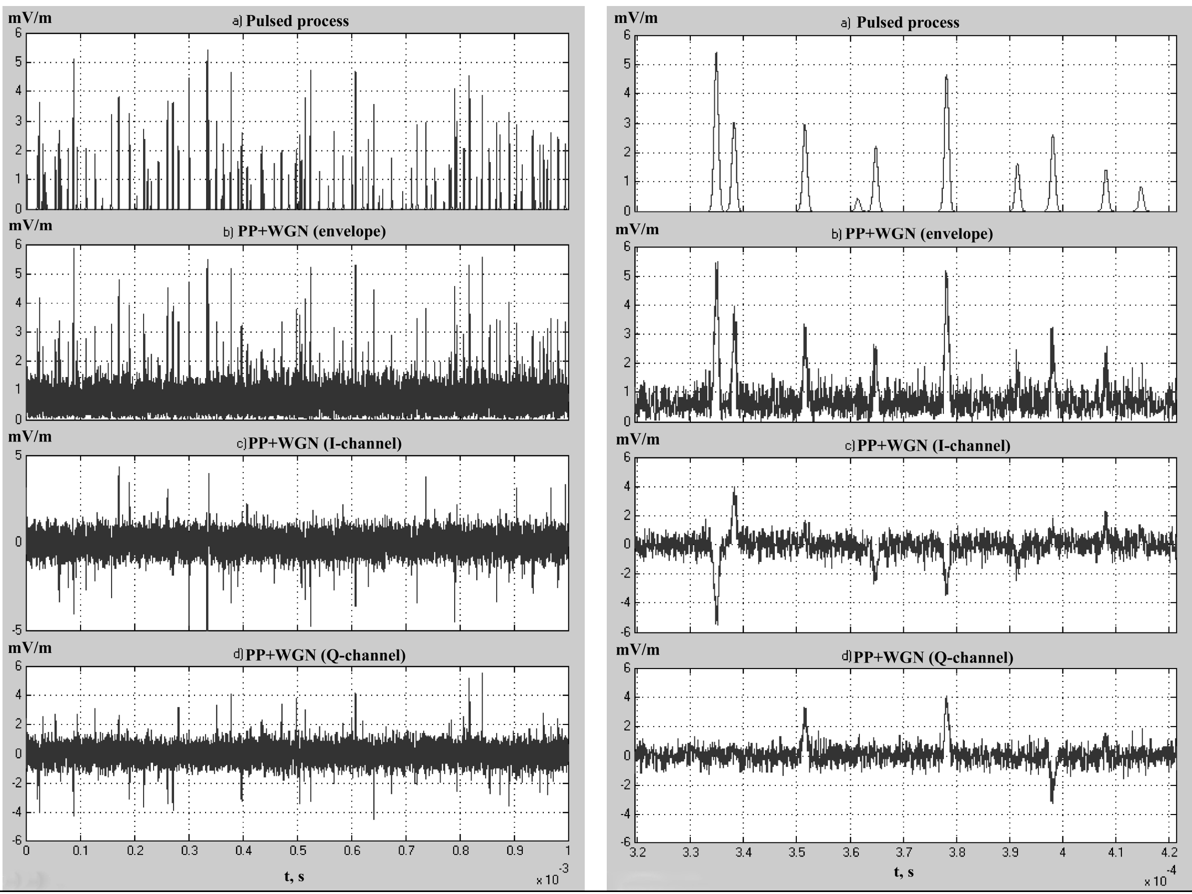

Fig. 6. Pulsed process representations obtained by modeling

Differential and cumulative distribution laws for the quarter-phase components and the PP amplitude envelope were studied to assess statistical properties of the PP representations obtained by modeling, and corresponding mean-square deviation of noise was introduced for each of the mentioned processes ($\sigma = 0.5$ mV/m - for the quarter-phase components and $\sigma = 0.7$ mV/m - for the amplitude envelope). Differential distribution law of the Gaussian process with $\sigma = 0.5$ mV/m obtained analytically and by simulation modeling is presented in Fig. 7 for the noise modeling accuracy control. And in Fig. 8 the cumulative distribution law for the Rayleigh stochastic process with $\sigma = 0.7$ mV/m obtained analytically and by simulation modeling is presented. In Fig. 7, the electric field intensity E (mV/m) is indicated along X-axis and probability density function (PDF) is indicated along Y-axis. In Fig. 8, the electric field intensity in logarithmic scale (dBmV/m) is indicated along X-axis, and the probability of that the resultant process is higher than the appropriate value along X-axis is indicated along Y-axis. It is obvious that the results of noise modeling are in good agreement with the theory.

Examples of the DDL comparison for one of the quarter-phase components and CDL for the amplitude envelope of test and modeling representations (pulsed process + noise) are presented in Fig. 7 and Fig. 8 for the mentioned mean-square deviations of the WGN ($\sigma = 0.5$ mV/m and $\sigma = 0.7$ mV/m). It should be noted that in these examples both test results and results of simulation modeling for the SPT emission in the radio-frequency range are the substantially non-Gaussian stochastic processes. Iterative procedure of the selection of mutual relation for the intensities of PP and WGN that was assessed by the weight coefficient K (K is the PP average energy relation to the WGN spectral power density) was implemented to reach maximum agreement with test representations. It is shown that for each frequency of spectrum there is its own optimum K value; for example, for the frequency of 7.19 GHz $K = 12\ dB$ that corresponding to Fig. 7, 8.

According to the studies made, in general the simulation model for the PP and WGN mixture secures good enough agreement with test results and can be used for assessing interference immunity of data transmission systems operating under the influence of EP self emission. Accuracy of such simulation model may be increased by additional particularization for the process constituents, in particular more precise specification for the pulse envelope shape as applied to the specific SPT type.

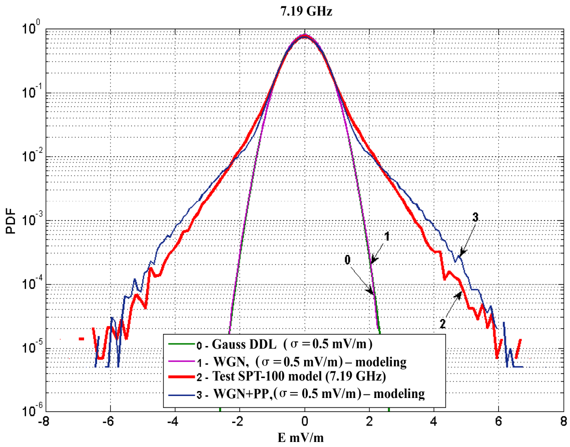

Fig 7. Differential distribution law for the quarter-phase component

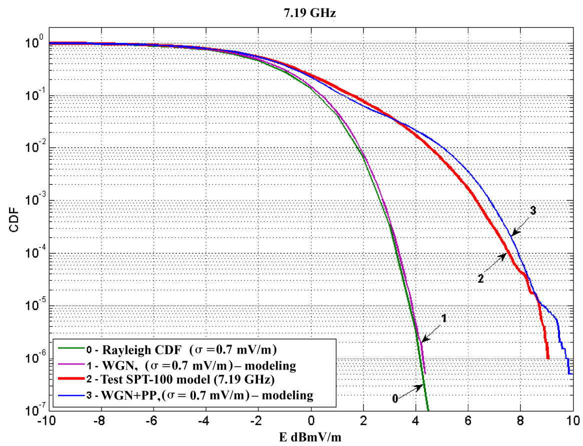

Fig. 8. Cumulative distribution law for amplitude envelope

Thus, the software package developed for simulation modeling of the SPT emission in the radio-frequency range allows generating a mixture of pulsed stochastic process (with the given envelope shape, amplitude characteristic distribution law, repetition interval, pulse duration, and carrier frequency) and noise with the given characteristics.

V. Conclusions

1. Analysis was made for the test results of the SPT self emission study in the radio-frequency range. It was shown that electromagnetic emission of stationary plasma thrusters is non-Gaussian one, has pronounced component in the form of a random sequence of radio pulses and a noise component representing a sum of the SPT thermal emission and the measuring equipment noises.

2. Mathematical description was developed for the SPT non-thermal emission in the radio-frequency range within the pass band of radio-receiving systems that is represented as an additive sum of the pulsed stochastic process and a noise and is based on the complex envelope method. Statistical characteristics of that model use differential distribution law (DDL) for the quarter-phase components, DDL for the amplitude envelope, phase DDL,

pulse duration DDL, and pulse repetition interval DDL. Mentioned DDLs are based on the analysis of test representations of the SPT self emission in the radio-frequency range.

3. Application-specific software package in MATLAB/Simulink environment was developed for the simulation modeling of processes of the SPT self emission in the radio-frequency range. The software package is based on the use of the mathematical description for the SPT non-thermal emission developed by the authors. The principle of operation of such package envisages a multi-step modeling procedure based on the dynamical step-by-step control for the statistical characteristics of the modeling blocks in order to reduce disagreement between the test and model representations of processes and reach specified modeling accuracy.

4. Modeling of parameters corresponding to the results of the SPT emission measurements in different frequency ranges that was carried out using the developed software package revealed good agreement (2% - 5%) of the modeling results and data obtained by test.

5. The developed SPT emission simulation models and the application-specific software package revealed principal possibility for the SPT self emission simulation modeling in the radio-frequency range and can be recommended for use while modeling and prototyping digital algorithms of signal processing in the space communication systems.

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