

2D fluid model of a magnetized plasma in a closed chamber

IEPC-2013-139

*Presented at the 33rd International Electric Propulsion Conference,
The George Washington University • Washington, D.C. • USA
October 6 – 10, 2013*

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Abstract: Understanding the behaviour of the magnetized plasma in the thruster ionization chamber is crucial to optimize the design and operation of this kind of thrusters. In particular, in Onera ECR thruster, the walls of the chamber can be dielectric, grounded or floating conductors. Depending on the boundary conditions, swirl currents may appear in the plasma; their effects on the plasma behaviour and overall thruster performances are unknown. Our goal in this work is to develop a model for the magnetized plasma produced in the chamber, using a 1- and a 2-fluid MHD framework. The models are based on the following assumptions: the problem is assumed to have an axial symmetry, and the electrons are supposed to be isothermal in the chamber. The magnetic field is assumed to be static, with a plasma β small enough to neglect induced magnetic fields. In the 1-fluid approach, the plasma is supposed to be quasi-neutral, and to obey local ambipolarity (ion and electron axial and radial velocities are equal). The sheath is not resolved, and the plasma is supposed to reach the boundaries of the vessel with the Bohm velocity. In the 2-fluid approach, electrons and ions are treated separately; they are coupled through the Poisson equation. In this case, the sheath is resolved, and electrostatic boundary conditions are prescribed at the wall, depending on the wall property. The model equations are solved using a finite element technique. Some preliminary computations are shown; they focus on a discharge in a closed chamber with an axial magnetic field. In the 1-fluid model, the evolution of the ionization rate required to sustain the plasma as a function of the plasma magnetization shows three regimes: a fully magnetized case, a non-magnetized case, and a transitional region. In the 2-fluid model the same trend is observed. The development of swirl current in the plasma is also observed.

I. Introduction

A large variety of electric thruster concepts use static magnetic field to confine and accelerate a plasma produced in an ionization chamber. Such devices are the MPD thruster, the Gas Dynamic Mirror, the HEMP thruster, Helicon thruster or ECR thruster. Quite generally, these thrusters are composed of a source region, where the plasma is produced, and an expansion region, where it is accelerated in the static magnetic field.

ONERA has been involved in the characterization of Helicon-type current-free thrusters [1], and currently develops of new kind of ECR thruster on its own [2]. Both types of thruster involve magnetic confinement with permanent magnets. The confinement is increased because the electrons in the plasma are bound to the magnetic field lines. At low pressure, collisions are infrequent, and thus the electrons cannot diffuse easily across the magnetic stream tubes. Thus, the magnetic field decreases the electron loss rate on the walls of the vessel parallel to the magnetic field lines; this in turn decreases the power requirement to maintain the discharge. Additionally, the magnetic field acts as

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virtual walls that can shape the plasma, and acts as a nozzle to direct the plasma expansion. So the magnetic field can be tailored to provide magnetic confinement, as well as plasma acceleration.

These magnetically confined plasma sources used for space propulsions pose several theoretical challenges that have been under scrutiny in the last years. For example, the formation of the so-called double layer in the expansion region of the plasma has been investigated experimentally ([3], [4]) and also theoretically, using a fluid model of the expanding plasma, assuming the presence of a fraction of hot electrons [5]. The acceleration mechanism of the ions and the thrust production mechanism have been investigated experimentally [6], and also using fluid models. In particular, the contribution of the azimuthal current in the acceleration and thrust production has been shown [7], [8], [9]. In the source region, the magnetic confinement has also been addressed using fluid approaches and kinetic models. Finally, the problem of the plasma detachment, which is of paramount importance for the overall efficiency of the thruster, has been investigated using various fluid models [10], [11], [12]. To summarize, there are currently three key issues whose understanding is required for the design of an efficient system: 1 the plasma confinement in the source chamber, 2 the acceleration mechanism and 3 the detachment of the plasma from the magnetic stream tubes.

The plasma confinement is a key problem to the efficient production of the plasma in the ionization region; wall losses decrease the thruster efficiency, induce heating and sputters wall materials. For example, the coaxial ECR thruster developed at ONERA [2] is composed of an open coaxial source firing in a 2m long vacuum tank, as shown in Figure 1. The magnetic field is purely diverging, with a resonance condition met about halfway of the source. A MW source is connected to the coaxial cavity, and produces an ECR plasma in the cavity, that expands downstream in the diverging B field region. In this arrangement, several configurations can be chosen for the cavity walls. The central and outer conductor could have their (DC) voltage floating with respect to the vacuum tank ground. In this case the plasma charges them up to their floating voltage. Typically, the outer conductor can float up to 150 V above the ground, while the central conductor is near ground or negatively biased with respect to the ground. Because the floating potential is lesser than the plasma potential, the ions in the source can reach energies up to 300 eV while undergoing an ambipolar acceleration in the expanding plume. In another configuration, one or both of the conductors can be grounded. When the outer conductor is grounded while the central antenna is left floating, a current of a few tens of milliampere is collected from the outer conductor; the shape of the plume is then modified, while the total current in the plume is halved. If both conductors are sheathed with a dielectric layer, the discharge behaviour is also changed.

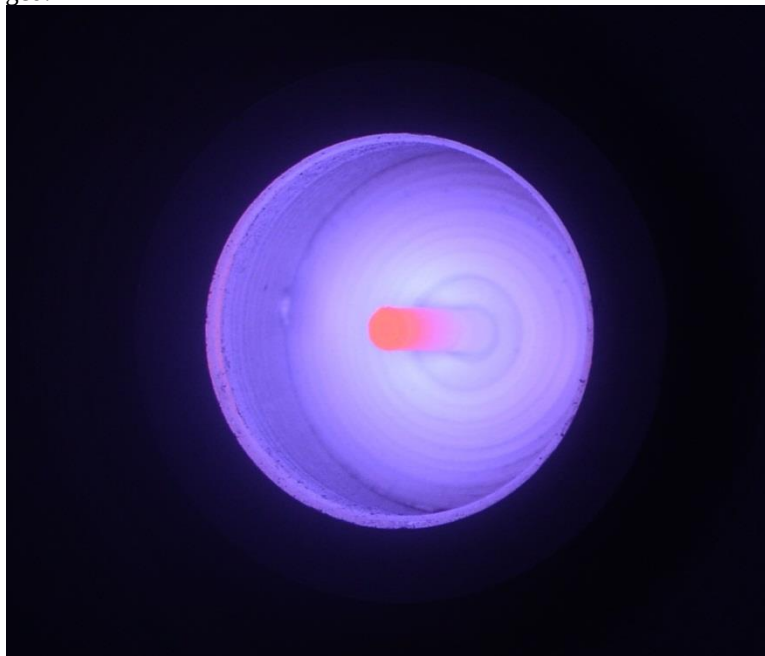


Figure 1 – Front view of the coaxial ECR thruster developed at ONERA. The inner and outer walls are floating conductors, while the endplate is a dielectric wall (BN).

Understanding the behaviour of the ECR source depending on the coaxial cavity properties, as described above (insulated, floating or grounded electrodes), or the magnetic field configuration is a requirement to improve the performances of our ECR thruster. For this purpose, modeling the plasma is helpful to test and refine our understanding of the physics.

In low pressure plasmas, the electron and ion mean free path are usually of the same order or greater than the plasma chamber dimension. In the literature, there are two approaches for the modeling of low pressure magnetized plasma sources. The first is the kinetic method, where electrons, ions and neutrals are either described statistically by a population of macro-particles or by a distribution function. The first method yields the so-called Particle In Cell methods [13], [14], [15]. The main advantage of these methods is to derive from first principles; very little macroscopic modeling is required, and only microscopic models are needed. As a consequence, these models accurately capture the non-local effect due to the large electron or ion mean free path. The drawback is that for plasma density high enough, a prohibitively large number of macro-particles is needed to model the plasma. This comes from the fact that PIC method needs, in average, enough particle per mesh cell; as the plasma density rises, the mesh cell size must be decreased to capture the Debye length scale. Hence, a larger number of cells is required, which also means a larger number of macro-particles.

The second method describes the plasma in a fluid framework. In this case, a set of transport equations, coupled to the Maxwell equations, is used to describe the plasma. Appropriate closure models are required to take into account the collisional effect, or if needed the non-local effects [16]. Depending on the set of assumptions made, this fluid description yields the MHD equations (either ideal or collisional), or a multi-fluid description of the plasma. Fluid models can address high-plasma density situation, as well as lower density. For example, they have been extensively used to study low pressure capacitively coupled rf discharges for pressure as low as 10-100 mTorr [17]. Even with local closure, they compare reasonably well with PIC-MCC models [18]. Furthermore, it is possible to include non-local closures taking into account collisionless heating to model more accurately low pressure cases [19].

The advantage of these fluid models is that they can be solved at reasonable computational cost, compared to kinetic model. Even with local closure, they are a convenient tool to study low pressure discharge with reasonable accuracy. When extrapolated at lower pressure, the trends they underline give insight on the particular behaviour of a low pressure discharge.

In this frame, our aim is to model the thruster as a whole to understand the broad factors of influence on the performances. The model must be able to take into account the plasma chamber, and the expansion region, self-consistently. A fluid model is an interesting candidate to address this problem, because it can be solved at reasonable computational cost

This paper describes a fluid model we are developing to tackle the issues related to the development of our ECR source. Our ambition is to have a tool to describe the main parameters controlling the thruster performances, in the chamber and in the acceleration region. As a first step, and because analytical solution exists, we consider a magnetized low-pressure discharge in a closed chamber to test our model.

II. Model description

A. Assumptions

The following assumptions are used to derive the model equations:

- Axial symmetry
- Isothermal electrons: This hypothesis derives from the assumption that the electron thermal conductivity is large, and thus that the electron temperature is constant everywhere. This assumption, while used in recent models ([20]), is currently discussed, in particular in the expanding plume region [21]. Yet there is no clear agreement on this matter. For the sake of simplicity, in this analysis, this assumption of isothermal electrons is retained.
- Constant background neutral density: depletion effects are not considered in this model.

- Volume plasma production: the plasma production is assumed to be due to ionizing collisions in the bulk of the discharge, therefore it is modelled with an ionization coefficient k_i in the ionization chamber. The ionization rate is a steep function of the electron temperature. Because the electron temperature is uniform in the chamber, so is the ionization rate.
- Negligible secondary emission: electron and ion induced secondary emission is neglected as a first step. Accounting for ion induced secondary emission would require to decrease the electron flux collected to the wall; the amount of the decrease being dictated by the secondary emission coefficient.
- Plasma loss mechanism: due to the relatively low pressure in the ionization chamber (a few Pa), volume recombination is unlikely in the chamber and in the expanding plume. The plasma recombines on the walls of the chamber.
- Cold ions : the ions temperature is assumed to be much lower than the electron temperature.

B. Model equations

The base equations of the model are the particle and momentum transport equations, both for ions and electrons. The ions are assumed cold. For numerical consistency, they are supposed to be isothermal, with a temperature much smaller than the electrons.

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x} n_i u_i + \frac{\partial}{\partial y} n_i v_i = -\frac{n_i v_i}{r} + k_i(T_e) n_g n_e \quad (1a)$$

$$\frac{\partial n_i u_i}{\partial t} + \frac{\partial}{\partial y} n_i v_i u_i + \frac{\partial}{\partial x} n_i u_i u_i = -q n_i \frac{\partial \varphi}{\partial x} - \frac{q}{\alpha} n_i w_i B_y^+ - \frac{n u_i v_i}{y} - \frac{1}{M} \frac{\partial p}{\partial x} - \frac{v_i}{M} n_i u_i \quad (1b)$$

$$\frac{\partial n_i v_i}{\partial t} + \frac{\partial}{\partial y} n_i v_i v_i + \frac{\partial}{\partial x} n_i u_i v_i = -\frac{q}{M} n_i \frac{\partial \varphi}{\partial y} + \frac{q}{M} n_i w_i B_x^+ + \frac{n_i w_i^2 - n_i v_i^2}{y} - \frac{1}{M} \frac{\partial p}{\partial y} - \frac{v_i}{M} n_i v_i \quad (1c)$$

$$\frac{\partial n w_i}{\partial t} + \frac{\partial}{\partial y} n v_i w_i + \frac{\partial}{\partial x} n u_i w_i = \frac{q}{M} n_i (u_i B_y^+ - v_i B_x^+) - \frac{2 n_i v_i w_i}{y} - \frac{v_i}{M} n_i w_i \quad (1d)$$

$$\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial x} n_e u_e + \frac{\partial}{\partial y} n_e v_e = -\frac{n_e v_e}{y} + k_i(T_e) n_g n_e \quad (2a)$$

$$\frac{\partial n_e u_e}{\partial t} + \frac{\partial}{\partial y} n_e v_e u_e + \frac{\partial}{\partial x} n_e u_e u_e = -\frac{q}{m} n_e \frac{\partial \varphi}{\partial x} - \frac{q}{m} n_e w_e B_y^+ - \frac{n u_e v_e}{y} - \frac{k T_e}{m} \frac{\partial n_e}{\partial x} - \frac{v_e}{m} n_e u_e \quad (2b)$$

$$\frac{\partial n_e v_e}{\partial t} + \frac{\partial}{\partial y} n_e v_e v_e + \frac{\partial}{\partial x} n_e u_e v_e = -\frac{q}{m} n_e \frac{\partial \varphi}{\partial y} + \frac{q}{m} n_e w_e B_x^+ + \frac{n_e w_e^2 - n_e v_e^2}{y} - \frac{k T_e}{m} \frac{\partial n_e}{\partial y} - \frac{v_e}{m} n_e v_e \quad (2c)$$

$$\frac{\partial n_e w_e}{\partial t} + \frac{\partial}{\partial y} n v_e w_e + \frac{\partial}{\partial x} n_e u_e w_e = \frac{q}{m} n_e (u_e B_y^+ - v_e B_x^+) - \frac{2 n_e v_e w_e}{y} - \frac{v_e}{m} n_e w_e \quad (2d)$$

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{1}{y} \frac{\partial}{\partial y} \left(y \frac{\partial \varphi}{\partial y} \right) = q \frac{n_e - n_i}{\epsilon_0} \quad (3)$$

The electrons being isothermal, a simplified energy balance equation is used. We assume that a suitable power source balance the electron energy losses to the wall and in the bulk of the plasma through inelastic collisions (only ionization here). We set the power P to be $P = k_i(T_e) n_g N_e E_i$, where N_e is the total number of electrons in the discharge chamber and E_i is the ionization potential.

All quantities are put in a dimensionless form, using the radius R of the chamber as its reference length. We define x as the axial direction, y as the radial one and z as the azimuthal direction. We set:

$$\begin{aligned} x &= x^+ R, \quad y = y^+ R, \quad t = t^+ \frac{R}{U_0} \quad n_e = n_0 n_e^+, \quad n_i = n_0 n_i^+ \\ u_e &= U_0 u, \quad v_e = U_0 v, \quad w_e = U_0 w, \\ u_i &= U_0 U, \quad v_i = U_0 V, \quad w_i = U_0 W, \\ \phi &= \frac{kT_e}{e} f, \quad p = n_0 k_B T_e p^+, \quad B = B_0 B^+ \end{aligned}$$

We define U_0 as the acoustic ionic sound speed $U_0 = \sqrt{\frac{kT_e}{M}}$. A set of dimensionless parameters naturally appear in the equations:

- $\alpha = \frac{U_0 M_i}{e B_0 R}$, the reduced ion Larmor radius
- $\delta = \frac{m_e}{M_i}$, the electron to ion mass ratio
- $\gamma = \frac{T_i}{T_e}$, the ion to electron temperature ratio
- $\eta = \frac{\lambda_D}{R} = \sqrt{\frac{\epsilon_0 k T_e}{e^2 n_0^2}} \frac{1}{R}$, the reduced Debye length
- $Z = \frac{k_i n_g R}{\sqrt{\frac{k T_e}{M}}}$, the reduced ionization frequency
- $F_e = \frac{\nu_e R}{U_0}, F_i = \frac{\nu_i R}{U_0}$, the reduced collision frequencies of electrons and ions, respectively.

When writing the dimensionless equations for the case of isothermal electrons, it is convenient to use the non-conservative form of the equations. Additionally, we set $h = \ln(n_e^+)$ and $H = \ln(n_i^+)$. Therefore, we get:

For the electrons:

$$\frac{\partial h}{\partial t} + u \frac{\partial}{\partial x} h + v \frac{\partial}{\partial y} h + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{v}{y} = Z \quad (4a)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial}{\partial x} u + v \frac{\partial}{\partial y} u + \frac{1}{\delta} \frac{\partial h}{\partial x} = \frac{\partial f}{\partial x} + \frac{1}{\alpha \delta} w B_y^+ - (F + Z) u \quad (4b)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial}{\partial x} v + v \frac{\partial}{\partial y} v + \frac{1}{\delta} \frac{\partial h}{\partial y} = \frac{\partial f}{\partial y} - \frac{1}{\alpha \delta} w B_x^+ + \frac{w^2}{y} - (F + Z) v \quad (4c)$$

$$\frac{\partial w}{\partial t} + u \frac{\partial}{\partial x} w + v \frac{\partial}{\partial y} w = -\frac{1}{\alpha \delta} (u B_y^+ - v B_x^+) - \frac{v w}{y} - (F + Z) w \quad (4d)$$

For the ions,

$$\frac{\partial H}{\partial t} + U \frac{\partial}{\partial x} H + V \frac{\partial}{\partial y} H + \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{V}{y} = Z \exp(h - H) \quad (5a)$$

$$\frac{\partial U}{\partial t} + U \frac{\partial}{\partial x} U + V \frac{\partial}{\partial y} U + \gamma \frac{\partial H}{\partial x} = -\frac{\partial f}{\partial x} + \frac{1}{\alpha} w B_y^+ - (F + Z \exp(h - H)) U \quad (5b)$$

$$\frac{\partial V}{\partial t} + U \frac{\partial}{\partial x} V + V \frac{\partial}{\partial y} V + \gamma \frac{\partial H}{\partial y} = -\frac{\partial f}{\partial y} - \frac{1}{\alpha} w B_x^+ + \frac{W^2}{y} - (F + Z \exp(h - H)) V \quad (5c)$$

$$\frac{\partial W}{\partial t} + U \frac{\partial}{\partial x} W + V \frac{\partial}{\partial y} W = -\frac{1}{\alpha} (UB_y^+ - VB_x^+) - \frac{VW}{y} - (F + Z \exp(h - H))W \quad (5d)$$

The Poisson equation reads

$$\frac{\partial^2 f}{\partial x^2} + \frac{1}{y} \frac{\partial}{\partial y} \left(y \frac{\partial f}{\partial y} \right) = \frac{\exp(h) - \exp(H)}{\eta^2} \quad (6)$$

In the following, we will focus on a near collisionless description of the plasma. Therefore, we will neglect the contribution of collisions in the momentum transport equations, ($F_e = 0$ and $F_i = 0$).

C. One-fluid model

It can be interesting to use the asymptotic limit $\eta \rightarrow 0$ of vanishingly small Debye length. With this assumption, the Poisson equation becomes, to the zeroth order:

$$n_e^0 = n_i^0$$

This means that the ion and electron number densities are equal everywhere in the domain. Another assumption one can find in the literature in this frame of one-fluid model is the local ambipolarity assumption. Ions and electrons are supposed to have the same velocity in the r-z plane. However, they do not have the same azimuthal velocity. With this assumption, the meridional (x-y plane) current density j is zero everywhere. The ambipolar electric field adjusts such that $j=0$. Note that this hypothesis, assumed in the early works on detachment [22], [23] while still used in some papers [24] is currently debated [8]. In the drift-diffusion limit (ie massless electrons), Bogdanov et al. [25] have shown that local ambipolarity is true as long as $\nabla T_e \times \nabla n = 0$, where n ion-electron number density and T_e is the electron temperature. Thus, for isothermal electrons, in the drift-diffusion limit, this assumption is consistent. For situation with low collision rate, this assumption warrants careful checks.

Using these two assumptions, the electric field can be removed from the axial and radial momentum equation. The particle balance equation of the ions becomes redundant with the electron one. Noting that $\delta \ll 1$ and that $\gamma \ll 1$, we get:

$$\frac{\partial h}{\partial t} + u \frac{\partial}{\partial x} h + v \frac{\partial}{\partial y} h + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{v}{y} = Z \quad (7a)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial}{\partial x} u + v \frac{\partial}{\partial y} u + \frac{\partial h}{\partial x} = \frac{1}{\alpha} w B_y^+ - Z u \quad (7b)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial}{\partial x} v + v \frac{\partial}{\partial y} v + \frac{1}{\delta} \frac{\partial h}{\partial y} = -\frac{1}{\alpha} w B_x^+ + \frac{w^2}{y} - Z v \quad (7c)$$

$$\frac{\partial w}{\partial t} + u \frac{\partial}{\partial x} w + v \frac{\partial}{\partial y} w = -\frac{1}{\alpha \delta} (u B_y^+ - v B_x^+) - \frac{vw}{y} - (F + Z)w \quad (7d)$$

In this case, we have neglected the ion azimuthal velocity W , since $W \ll w$ [22].

This one fluid model cannot resolve the sheath, since $n_e^0 = n_i^0$ in the present quasi-neutral approximation. However, the formation of the sheath requires that the ions enter the sheath have at least the Bohm velocity [26]. This imposes a condition on the normal velocity of the plasma at the boundaries. Therefore, in the dimensionless form adopted here, we require as boundary conditions that $U.n = 1$ at the walls. This kind of boundary condition defines an eigenvalue problem, whose eigenvalue is the reduced ionization coefficient Z . This coefficient Z is searched iteratively, by balancing the steady-state particle balance equation (7a).

Note that letting $\delta \rightarrow 0$, the equations used by Ahedo are recovered [7]. In this case the azimuthal momentum balance equation yields that $u \times B = 0$, meaning that the electrons are bound to the magnetic streamtubes.

D. Two-fluid model

The set of equations (4-6) defines a 2-fluid model of the plasma. The ion and electron balance equations are coupled through the Poisson equation. To complete this model, the boundary conditions on the walls need to be defined, for the electrons, the ions and the potential.

Due to their small mass, the electron sound speed is much greater than the acoustic ionic sound speed. The electron macroscopic velocity, somewhat tied to the ion velocity, is smaller. Assuming an isothermal electron population, by analogy with electrostatic probe theories, we can assume that the electron collection rate on the wall is driven by the electron flux J_{ew} impinging the wall [27]:

$$J_{ew} = \frac{1}{4} C_s n_e \quad (8)$$

With $C_s = \sqrt{\frac{8kT_e}{\pi m}}$.

For the ions, depending on the potential of the wall, they can be either collected, with a flux set by the magnitude of the wall potential, or they can be repelled. Therefore, from the model point of view, we require that the ion flux impinging the wall is at least zero, or greater.

$$J_{iw} \geq 0$$

For the potential, as a start in this paper, we assume that the vessel walls are grounded. Thus Dirichlet-type boundary conditions are prescribed for the Poisson equation. It is noteworthy that other situations could be modeled, such as a floating conductor wall. In this case, the potential on the wall is driven by the net charge collected on the wall. Dielectric walls could also be modeled by considering the local current density reaching the wall. This latter case would translate into Neumann-type boundary conditions.

III. Numerical implementation

The set of equation (4) and (5) is solved within a finite element framework. A Galerkin Least-Square method is used to solve the balance equations of the electrons and the ions. A standard Galerkin method is used for the Poisson equation. The Galerkin Least-Square method has been previously used for the compressible Euler equation [28] and the MHD equations [29]. The principle is to minimize the residual of the set of equations. The electron balance equations in XX can be given in a matrix form:

$$\frac{\partial U}{\partial t} + A_x \frac{\partial U}{\partial x} + A_y \frac{\partial U}{\partial y} = S \quad (9)$$

Where the natural variable and the Jacobian matrix are:

$$U = \begin{pmatrix} h \\ u \\ v \\ w \end{pmatrix}, \quad A_x = \begin{pmatrix} u & 1 & 0 & 0 \\ 1 & u & 0 & 0 \\ 0 & 0 & u & 0 \\ 0 & 0 & 0 & u \end{pmatrix}; \quad A_y = \begin{pmatrix} v & 0 & 1 & 0 \\ 0 & v & 0 & 0 \\ 1 & 0 & v & 0 \\ 0 & 0 & 0 & v \end{pmatrix}$$

The source term encloses the effect of the ionization and of the Lorentz and Coulomb forces.

$$S = \begin{pmatrix} Z\sqrt{\delta} - \frac{v}{y} \\ qE_x - \frac{qwB_y}{\sqrt{\delta}\alpha} - (F+Z)\sqrt{\delta}u \\ qE_y + \frac{qwB_x}{\sqrt{\delta}\alpha} + \frac{w^2}{y} - (F+Z)\sqrt{\delta}v \\ \frac{q(uB_y - vB_x)}{\alpha\sqrt{\delta}} - \frac{vw}{y} - (F+Z)\sqrt{\delta}w \end{pmatrix}$$

To obtain the steady state solution of (9), a time-marching method is used. A time step Δt is introduced, and the system (9) is linearized for a small increment ΔU .

$$U^{n+1} = U^n + \Delta U$$

Where the superscript n denotes the time discretization $t = n\Delta t$. The linearization gives:

$$\frac{\Delta U}{\Delta t} + A_x^n \frac{\partial U^n}{\partial x} + A_y^n \frac{\partial U^n}{\partial y} + A_x^n \frac{\partial \Delta U}{\partial x} + A_y^n \frac{\partial \Delta U}{\partial y} + A^n \Delta U = S^n + \frac{\partial S^n}{\partial U} \Delta U \quad (10)$$

Where second order terms have been neglected. If we define :

$$L^n = \bar{A}^n + A_x^n \frac{\partial \bullet}{\partial x} + A_y^n \frac{\partial \bullet}{\partial y} \quad (11)$$

$$f^n = A_x^n \frac{\partial U^n}{\partial x} + A_y^n \frac{\partial U^n}{\partial y} - S^n \quad (12)$$

With

$$\bar{A} = A^n + \frac{1}{\Delta t} I - \frac{\partial S^n}{\partial U} \quad (13)$$

$$\bar{A}^n = \begin{pmatrix} \frac{1}{\Delta t} - Z & \frac{\partial h}{\partial x} & \frac{\partial h}{\partial y} + \frac{1}{y} & 0 \\ 0 & \frac{1}{\Delta t} + \frac{\partial u}{\partial x} + (F + Z) & \frac{\partial u}{\partial y} & \frac{qB_y}{\alpha\delta} \\ 0 & \frac{\partial v}{\partial x} & \frac{1}{\Delta t} + \frac{\partial v}{\partial y} + (F + Z) & -\frac{qB_x}{\alpha\delta} - 2\frac{w}{y} \\ 0 & \frac{\partial w}{\partial x} - \frac{qB_y}{\alpha\delta} & \frac{\partial w}{\partial y} + \frac{w}{y} + \frac{qB_x}{\alpha\delta} & \frac{1}{\Delta t} + \frac{v}{y} + (F + Z) \end{pmatrix} \quad (14)$$

The linearized problem (9) can be written as:

$$L^n \Delta U = -f^n \quad (15)$$

A least-square method is the used to solve this problem (15). We define the functional I as :

$$I(\Delta U) = \int_{\Omega} (L^n \Delta U - f^n)^2 dx dy + \nu \int_{\Gamma} (B \Delta U - g^n) dx dy \quad (16)$$

The goal is to minimize this functional. This can be achieved if, we have:

$$\int_{\Omega} (L^n V)^T (L^n \Delta U) dx dy + \nu \int_{\Gamma} (B \Delta U)^T (BV) dx dy = \int_{\Omega} (L^n V)^T (yf^n) dx dy + \nu \int_{\Gamma} (BV)^T (yg^n) dx dy \quad \forall V \quad (17)$$

This minimization problem translates into a matrix problem :

$$\mathbf{K} \Delta U = R \quad (18)$$

Where

$$\mathbf{K} = \begin{pmatrix} K_{nn} & K_{nx} & K_{ny} & K_{nz} \\ {}^T K_{nx} & K_{xx} & K_{xy} & K_{xz} \\ {}^T K_{ny} & {}^T K_{xy} & K_{yy} & K_{yz} \\ {}^T K_{nz} & {}^T K_{xz} & {}^T K_{yz} & K_{zz} \end{pmatrix}, \text{ and } R = \begin{pmatrix} R_n \\ R_x \\ R_y \\ R_z \end{pmatrix}$$

The matrix K is symmetric definite positive. The solution of (18) is sought using a conjugate gradient algorithm to invert the matrix.

The numerical implementation of this method is based on the finite element software package FreeFem++ [30], developed at the Université Pierre et Marie Curie. The finite elements are P1 function, on a triangle mesh. At each time step, the matrix K is assembled, and inverted. Note that higher order elements could be used, at a cost on the computational time.

IV. Results

A. Computational domain

The domain is 2 by 1 rectangle. In dimensionless unit, the radius is 1, and the axial length is 2, as shown in Figure 2. The side and top boundaries of the domain are set as grounded walls, as the lower boundary is defined as the chamber axis.

In the finite element approximation, the domain is meshed using triangle elements. An iterative mesh adaptation algorithm is used, to automatically refine the mesh where the electron velocity gradients are sharper (close to the walls). The minimum element size is 10^{-3} , while the maximum element is at most 4×10^{-2} .

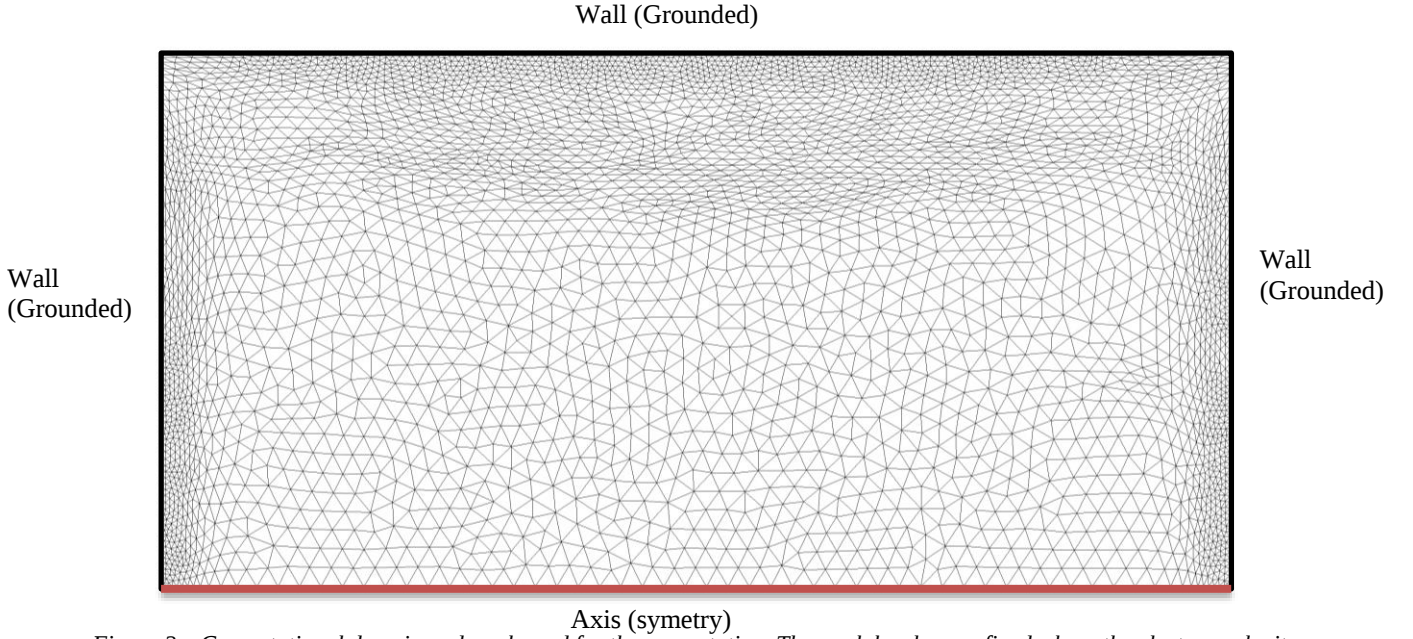


Figure 2 – Computational domain and mesh used for the computation. The mesh has been refined where the electron velocity gradients are sharper.

For the 1-fluid model, the plasma velocity normal to the wall is prescribed to be the Bohm speed. For the 2-fluid model, the electron normal flux is prescribed to be the thermal flux, as given in equation (8), and a Dirichlet boundary condition is used for the potential: $\phi=0$.

B. Computation parameters

The parameters used for the computations are given in the Table 1 below. Note that to speed up the computations, the electron to mass ratio is set to 10^{-4} , higher than the ratio for Argon plasma. This ratio fits approximately an helium discharge. Lower value could be used, but that would require longer computation time, whereas adding little value to the global understanding of the system.

The reduced Debye length is set to 3% of system radius. As previously, smaller values could be used. However, this would require smaller mesh size, and incidentally smaller time step.

Parameter	Value
Reduced ion Larmor radius α	> 1
Ion to electron temperature ratio γ	10^{-3}
Electron to ion mass ratio δ	10^{-4}
Reduced Debye length η	3.1×10^{-2}

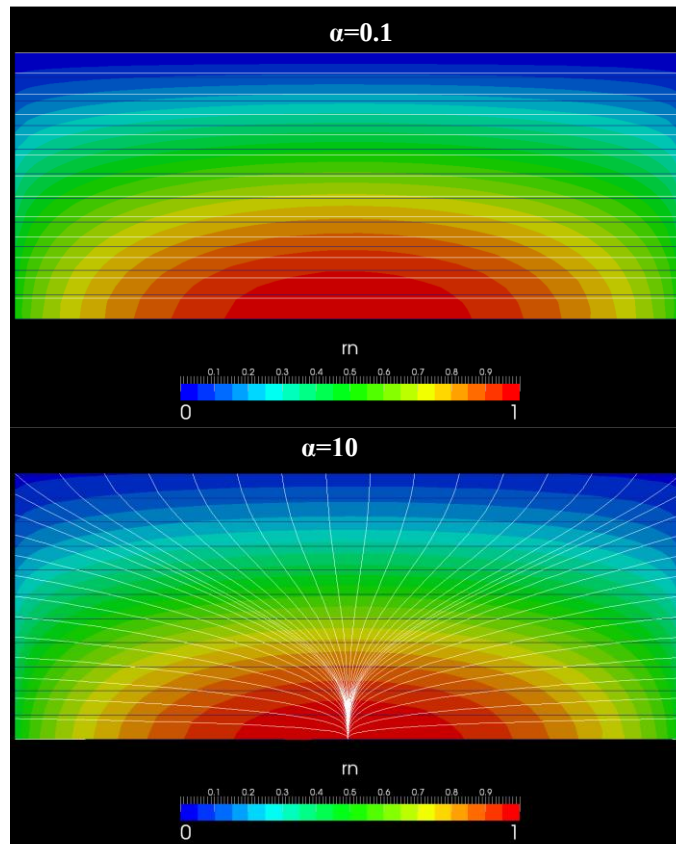
Table 1 – Reduced parameters used in the simulation

C. One-fluid computations

As a simplified model, the case of a closed chamber is first analyzed. This situation is helpful to understand the behavior of the plasma in the case of a purely axial magnetic field, while being simpler to solve. The computational domain is a rectangular box. The lower boundary is the axis of symmetry. The three other boundaries are dielectric walls, where the plasma normal velocity is assumed sonic. A constant background of neutral is assumed in the chamber, which is modeled by a uniform ionization coefficient.

As shown in Figure 3, at large magnetic field ($\alpha=1$), the plasma streamlines and the magnetic lines are coincident. The plasma is produced in the volume of the chamber, and is collected to the chamber lateral walls. As the magnetic field decreases, the plasma streamlines are no longer tied to the magnetic lines. More and more plasma is collected on the top wall, as the magnetic confinement decreases.

This behavior is illustrated in the Figure 4 **Error! Reference source not found.**, where the dimensionless ionization coefficient Z is plotted against the reduced ion Larmor radius. For low α , the magnetic field is strong; the plasma is magnetized and well confined along the magnetic field lines. As a consequence, plasma recombines mostly on the lateral walls. Hence, a lower ionization is required to sustain steady-state plasma in the chamber. As α increases, the magnetic field strength decreases (or equivalently the electron temperature increases), and the confinement is less efficient. Some plasma starts to leak on the top wall. A higher ionization is needed to maintain the plasma density. Finally, for higher value of α , an asymptotic regime is reached, in which the plasma no longer feels the effect of the magnetic confinement.



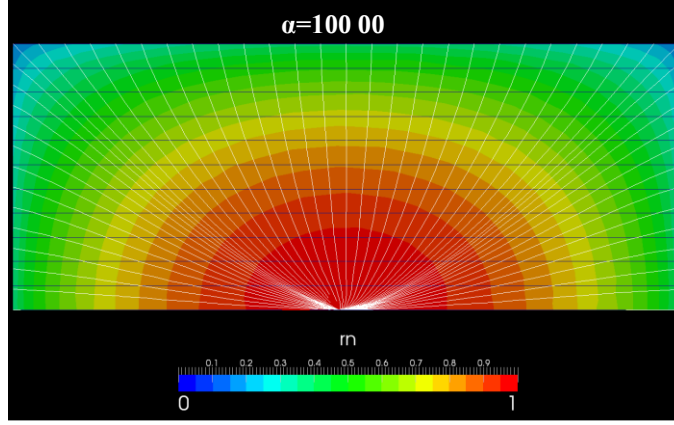


Figure 3 - Normalized plasma density, for $\alpha = 0.1, 10$ and 100.00 . The black lines are the magnetic field lines, the white ones are the plasma streamlines

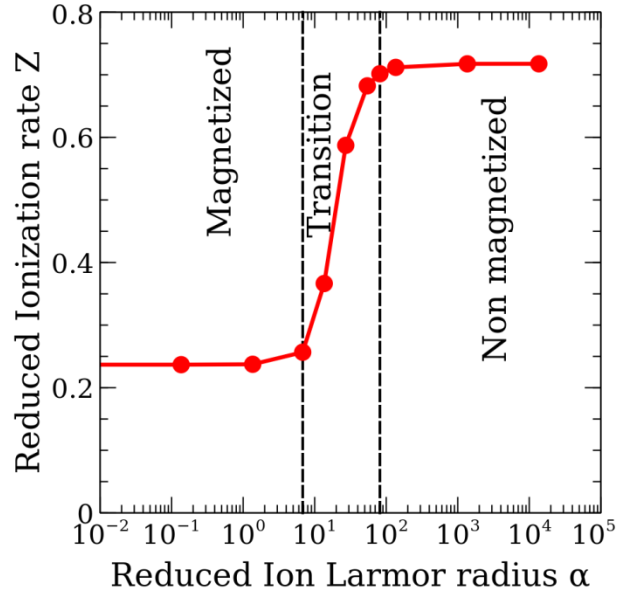


Figure 4 – Reduced ionization rate as a function of the plasma magnetization (reduced ion Larmor radius)

D. Two-fluid computations

Two-fluid computations are performed on the same computational domain. Figure 2 shows the electron number densities and the plasma potential for three magnetizations, $\alpha=10$, $\alpha=100$ and $\alpha=\infty$, and the same input power $P=5$. If we consider the results of the 1-fluid approach shown in Figure 4, a reduced ion Larmor radius $\alpha=10$ falls in the transitional regime, close to the magnetized case. The case $\alpha=100$ corresponds to the beginning of the non-magnetized regime, and the last case $\alpha=\infty$ corresponds to a case with zero magnetic field.

It can be seen that for the same input power, the case $\alpha=10$ reaches a higher electron number density than the two other cases. The improved plasma confinement leads to a decrease of the wall losses. Thus a higher number of electrons can be maintained in the discharge. The two cases at $\alpha=100$ and $\alpha=\infty$ are very similar, as could be expected from the 1-fluid results.

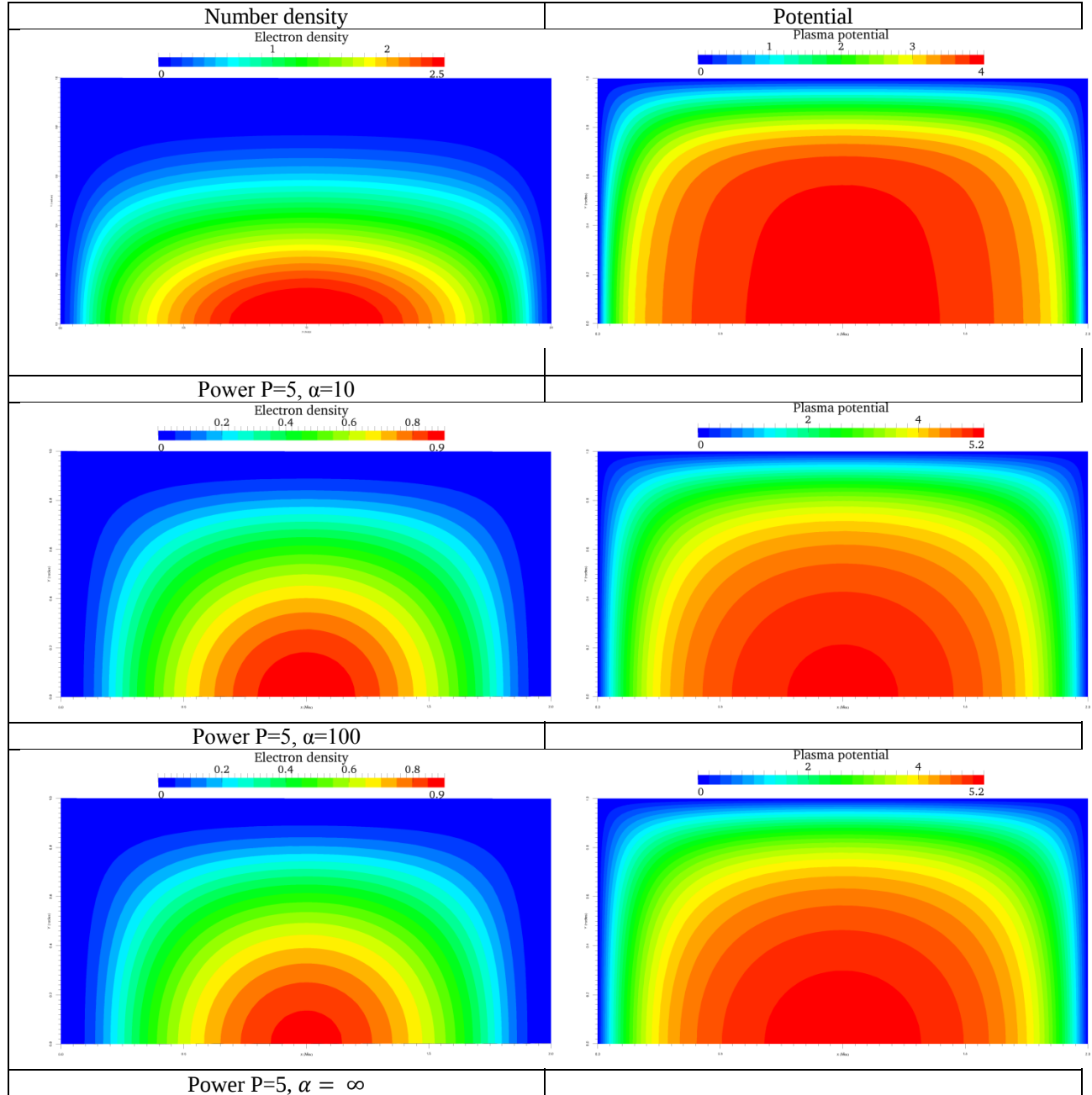


Figure 5 – Electron number densities and potential, for different magnetization α . The case $\alpha=10$ has a similar shape to the one observed in Figure 3.

Figure 6 gives different axial profiles in the discharge. The symmetry of the discharge appears clearly, as well as the fact the bulk of the discharge is quasi-neutral $n_e \approx n_i$. Quasi-neutrality is breached on the side sheaths. If we consider the sheath entrance to be the position where the ion velocity reaches the Bohm speed ($U=1$ here), the sheath width is approximately 5 Debye length (0.16). In the sheath, ions accelerate, and reach the side walls with 2.5 times the Bohm speed.

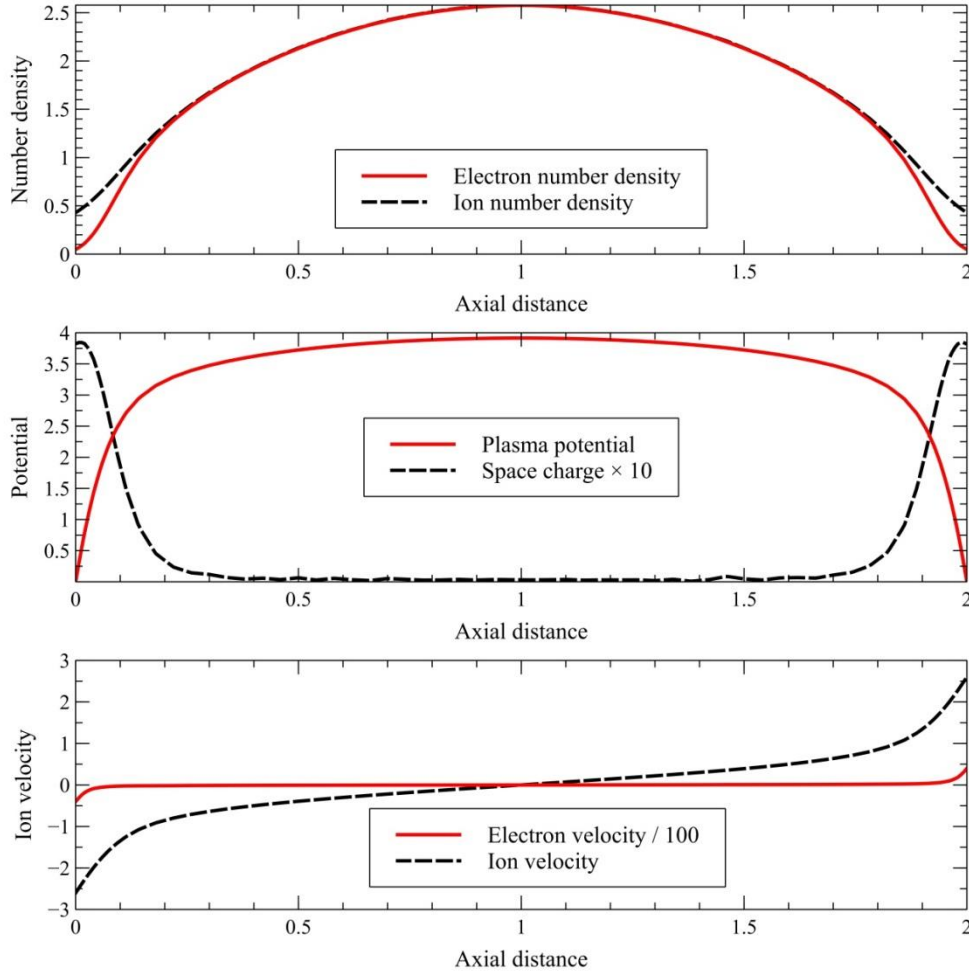


Figure 6 –Axial profiles in the chamber, $\alpha=10$, $P=5$. The ion and electron number densities are equal in the bulk of the discharge. Sheaths form close to the wall where a steep voltage drop appears.

Figure 7 gives the radial profiles for the same discharge conditions. Note that the number density is has a Bessel-like shape, except close to the sheath. The electron radial velocity is close to zero in the bulk of the discharge, while their azimuthal velocity increases radially. In the conditions considered here, the electron Larmor radius is given by

$$r_{Le}/R = \alpha\sqrt{\eta} = 0.1 \quad (19)$$

Therefore, the electrons are confined by the axial magnetic field, and acquire a significant radial velocity in the sheath only, where the azimuthal current becomes significant.

The plasma current density can be plotted, to monitor the current flow in the plasma. Overall, the ion and electron current reaching the walls are equal. But, because we assume grounded conductor on the domain boundaries, the current need not to be equal everywhere on the wall (contrary to the case of the 1-fluid model). In steady state, the divergence of the plasma current vanishes, and thus we get:

$$\int_{\Gamma} \vec{J}_e \cdot \vec{n} d\Gamma = \int_{\Gamma} \vec{J}_i \cdot \vec{n} d\Gamma \quad (20)$$

Figure 8 displays the plasma current in the discharge. The current enter the discharge through the side walls, and then leave by the top wall. This result confirms that the local ambipolarity hypothesis is not valid in this particular condition with grounded boundaries. Further computations are needed to assess this hypothesis with dielectric boundaries.

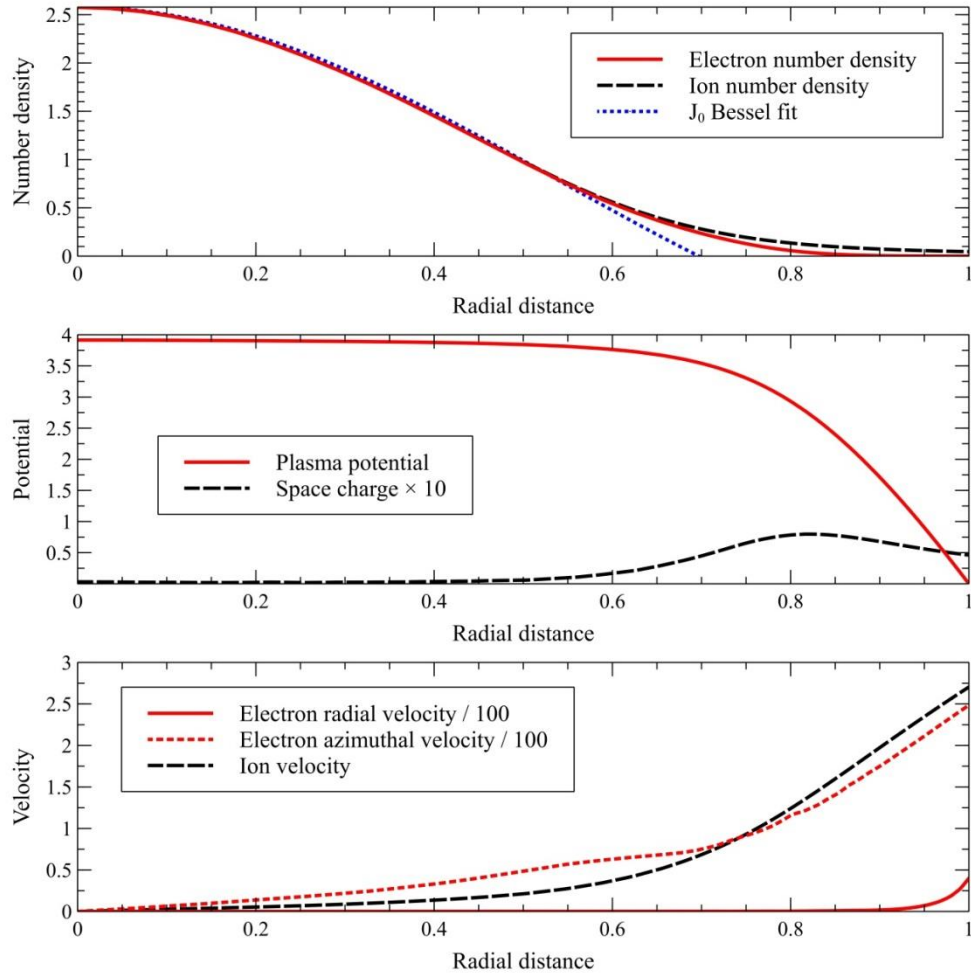


Figure 7 –Radial profiles in the chamber, $\alpha=10$, $P=5$.

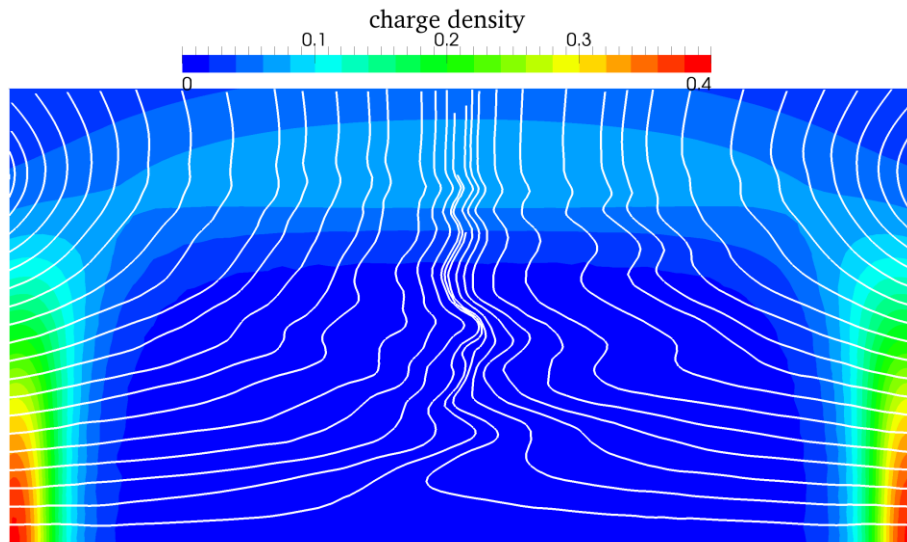


Figure 8 – Charge density (color coded) and current streamlines, $\alpha=10$, $P=5$. Although the total current on the walls is zero, a current flows in the plasma. Magnetized electrons are mostly collected on the side walls, while the ions can reach the top wall

significant proportion. As a result, a current is seen to enter the plasma from the side walls, to exit on the top wall. Because the side and top walls are grounded, the current can flow in the walls to close the current loop.

The radial profiles of the three different cases computed here are compared in Figure 9. As discussed above, there are little differences in the number density and potential profiles between the non-magnetized case and the case with $\alpha=100$. In this latter case $r_{Le}/R = 1$. In the case $\alpha=100$, we observe a large azimuthal current, maximum at the location of the sharpened electron number density gradient. This current induces a centripetal Lorentz force that balances the pressure gradient and the centrifugal force due to the azimuthal velocity.

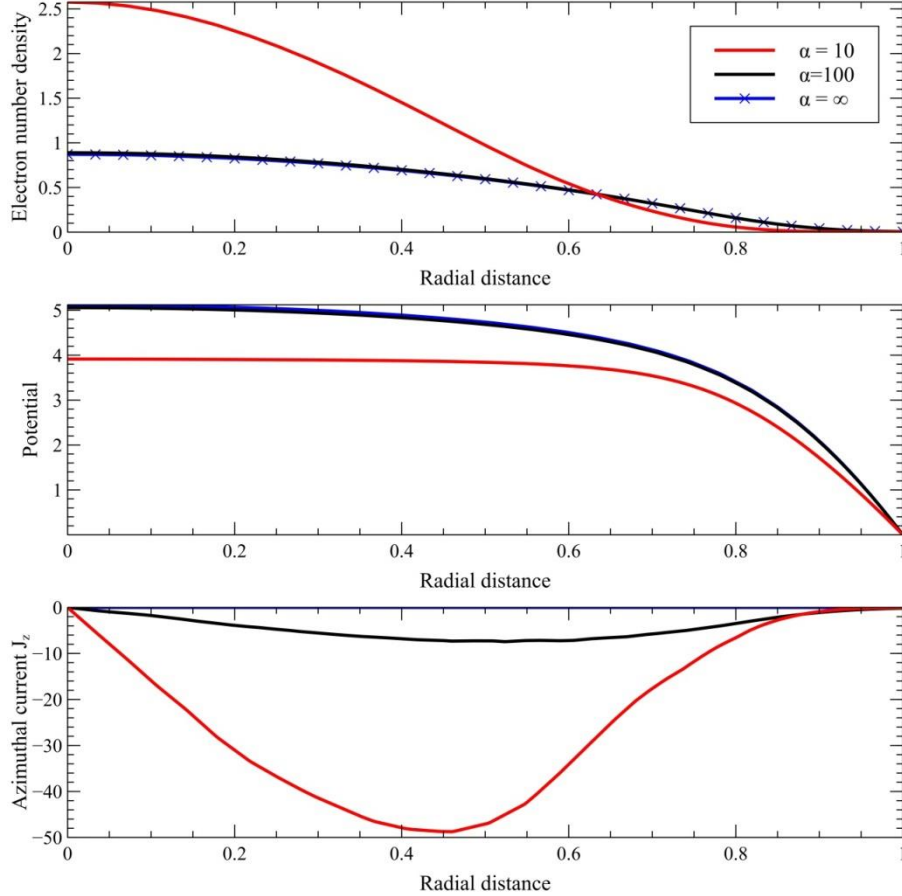


Figure 9 – Comparison of the radial profiles, for $\alpha=10$, $\alpha=100$ and $\alpha=\infty$. Top : Electron number density, Middle – Plasma potential, Down – electron azimuthal current.

V. Conclusion

The purpose of this paper is to present a one-fluid and two-fluid model under development. The base assumptions and constitutive equations have been detailed, as well as the strategy to solve them. Some preliminary results have been exposed. The case of a cylindrical vessel with an axial magnetic field has been considered. The 1-fluid model shows that the confining effect of the magnetic field induces a decrease in the ionization rate required to maintain the discharge. The two-fluid case, while dealing with a vessel with grounded boundaries, gives similar results.

These models are regarded as tools to help the understanding of the magnetized source used in electrodeless current-free thruster, with a particular focus on the ECR coaxial thruster under development at ONERA. Being fluid, they are not able yet to deal with situation where non-locality is paramount; implementing non-local closures would be a way to improve this situation. However, they can provide a wealth of information for the intermediate pressure

ranges (10-100 mTorr) which are of interest in these kinds of thruster, and can help us conceptually to understand the behavior of lower pressure sources. They could be useful to study several phenomena:

- Current loop formation in the plasma , and the role of conductive boundaries (Simon effect);
- Inertial detachment in expanding B field;
- Differential charging in coaxial chamber;

As a roadmap for the development of this study, first we aim at drawing a careful comparison between the 1-fluid and 2-fluid case and other models (see. [20], [31] for example). For this purpose, implementing dielectric boundary conditions is needed. Second, the case of the coaxial ECR thruster will be considered, in particular to understand the self-charging process at work in this discharge.

Further improvements of the model are also considered. For example, neutral depletion, or using non-local closure (this would require to add an energy equations). On the numerical side, going toward a parallel implementation of the model could be useful to deal with larger computational model.

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