

# Modeling Magnetic Nozzle Effects in Different Plasma Radii with an Axisymmetric Particle-in-cell Method

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**Abstract:** An axisymmetric simulation model of magnetic nozzle related to the AF-MPD thruster using Particle-in-cell (PIC) method has been presented. Taking the plasma states on the thrust exit plane as basic hypotheses in simulation, the performance of thruster varies significantly with applied magnetic field as well as geometric conditions. To find the most effective magnetic nozzle in such model, three cases of different plasma ejection locations and radii have been calculated and compared. The more convergent part of magnetic field to be utilized, the better effects will be implemented to the whole system. Moreover, a few interior mechanisms related to hall current and magnetic confinement have been demonstrated in further understanding of the working processes of magnetic nozzle.

## Nomenclature

|              |   |                         |
|--------------|---|-------------------------|
| $B$          | = | magnetic field strength |
| $E$          | = | electric field strength |
| $F$          | = | total thrust            |
| $j$          | = | current density         |
| $\dot{m}$    | = | mass flow rate          |
| $s$          | = | location vector         |
| $v$          | = | velocity                |
| $t$          | = | time variation          |
| $\rho_0$     | = | charge density          |
| $\epsilon_0$ | = | free space permittivity |
| $\mu_0$      | = | free space permeability |
| $\eta$       | = | efficiency              |

### Subscripts

|                |   |                                      |
|----------------|---|--------------------------------------|
| $e, i$         | = | electron, ion                        |
| $z, r, \theta$ | = | components: axial, radial, azimuthal |

## I. Introduction

THE basic concept of the magnetic nozzle involves the passage of a fully-ionized particle bunch through an applied magnetic solenoid. The plasma formed by ionization via helicon waves or electron-atom collisions in the discharge chamber of thruster can be restricted and accelerated by the nozzle-shaped magnetic field. Such effects bring desirable improvement in the performance of electromagnetic thrusters, like the VASIMR<sup>1</sup>, the Helicon thruster<sup>2, 3</sup> and the AF-MPD thruster<sup>4, 5</sup>. However, operating processes of the magnetic nozzle vary from one propulsive unit to another due to different plasma characteristics.

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The system of magnetic nozzle was testified to be both the collimator and transformer<sup>3, 6</sup>, where divergence level of plume effectively decreased and thermal energy of particles was transferred into kinetic energy via restraint of magnetic field lines and conservation of magnetic moment. Physical models in theoretical studies of the magnetic nozzle have been mostly simplified to be with the converging-diverging configuration and produced by a single current ring<sup>7</sup>. According to such modeling procedure, plasma plume was analysed as fluid that streamtubes of electrons usually overlapped those of ions or of magnetic field on the basis of quasi-neutrality condition and strongly-magnetized assumption respectively<sup>8, 9, 10</sup>. Continued progress in numerical scheme treated the two main charged particles separately<sup>11</sup>. Results from this hybrid model agreed well with those from pure fluid codes.

In contrast to the investigation of magnetic nozzle effects mostly done by fluid models and related to the VASIMR and Helicon thruster, some efforts have been made to obtain the PIC simulation results of plasma regime concerning with AF-MPD thruster. By assuming the relative axial position of plasma ejection plane as well as plasma beam radius, the more effective way to utilize magnetic nozzle is summarized.

## II. Numerical Method and Model

### A. Particle-in-cell Method

Considering the general working conditions of an AF-MPD thruster, the PIC method based on particle dynamics turns out to be suitable and competent to capture various characteristics of plasma motion within magnetic nozzle where both the plasma density and collision rate drop rapidly. In the standard PIC procedure, initial particles are assumed discrete in space and moved in the background of electric and magnetic fields. To sketch the electromagnetic field distribution and plasma kinetic movement, Eqs. (1) and (2) are considered to be the most important rules.

$$\left\{ \begin{array}{l} \nabla \cdot \mathbf{E} = \frac{\rho_0}{\epsilon_0} \\ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \times \mathbf{B} = (\mu_0 \epsilon_0) \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{j} \\ \nabla \cdot \mathbf{B} = 0 \end{array} \right. \quad (1)$$

$$\left\{ \begin{array}{l} m \frac{d\mathbf{v}}{dt} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \\ \frac{d\mathbf{s}}{dt} = \mathbf{v} \end{array} \right. \quad (2)$$

The whole PIC simulation is promoted according to electron-ion time-scale so that it requires millions of iteration steps to reach the final steady state. In order to solve the problem of quite large amount of computation, some reasonable accepted acceleration methods can be implemented<sup>12, 13, 14</sup>. There are mainly three aspects related to such methods: (a) use super particles, each representing  $10^6$ - $10^7$  individuals; (b) reduce the relative mass of ion to electron by  $f$  ( $f < 1$ ); (c) increase free space permittivity by  $\gamma^2$  times. The influence of such artificial processing on other parameters are listed in Table I.

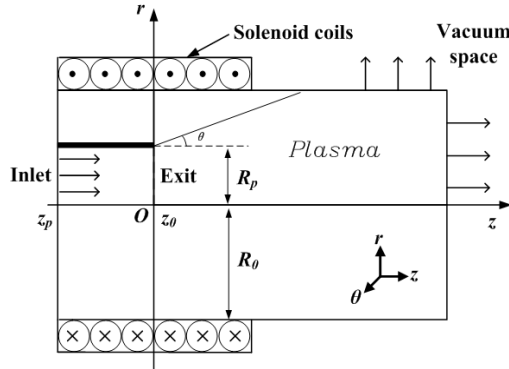
**Table I. Influence of the acceleration method on relevant parameters.**

| <i>Parameter</i>                      | <i>Factor</i>   |
|---------------------------------------|-----------------|
| Super particle weight, $\omega$       | $10^6$ - $10^7$ |
| Free space permittivity, $\epsilon_0$ | $\gamma^2$      |
| Free space permeability, $\mu_0$      | $\gamma^{-2}$   |
| Debye length, $\lambda_{Debye}$       | $\gamma$        |
| Electric field, $E$                   | $\gamma^{-1}$   |
| Induced magnetic field, $B_{induced}$ | $\gamma^{-1}$   |
| Ion mass, $m_i$                       | $f$             |
| Ion velocity, $v_i$                   | $f^{-0.5}$      |

|                                         |           |
|-----------------------------------------|-----------|
| Total magnetic field in ions mover, $B$ | $f^{0.5}$ |
| Electron/Ion Larmor radius, $L_e, L_i$  | 1         |

## B. Simulation Model

Referring to the tested working conditions of 100-kW steady-state AF-MPD thruster reported by NASA Lewis Research Centre<sup>15</sup>, a two-dimensional model with quasi-neutral collisionless fully-ionized plasma passing through an applied diverging magnetic field is established. As shown in Fig. 1, present investigation of magnetic nozzle effects derives from previous modeling for inner processes of AF-MPD thruster<sup>14</sup>. The main structure of thruster is still preserved that it consists of a 76 mm long anode as well as external magnetic coils with 101.5 mm radius and 153 mm length. However, to simplify the physical problem in present model, some modifications are required. The propellant ionization and collision processes inside the discharge chamber are neglected whereas the plasma at the exit of thruster is trying to be remained at its original state.



**Figure 1. Geometry of the simulation model. Origin point lies at the centre of applied solenoid coils.**

During initiation of the simulation procedure, it is assumed that the fully-ionized plasma is formed by single-charged  $\text{Ar}^+$  and electrons. To input these charged particles to the simulation zone, their initial positions are stochastic and velocities yield to half-Maxwellian distribution function with corresponding temperatures. All tracked particles can leave the simulation domain by free expulsion boundaries and will be reflected on the symmetric centre axis. As shown in Table II, several main parameters in present simulation are summarized.

**Table II. Main parameters in simulation.**

| Parameter                              | Value                                   |
|----------------------------------------|-----------------------------------------|
| Super particle weight                  | $3.0 \times 10^6$                       |
| Artificial free space permittivity     | $8.854 \times 10^{-8}$ ( $\gamma=100$ ) |
| Mass ratio of ion to electron          | $\sim 300$ ( $f=0.01$ )                 |
| Applied magnetic field strength, $B_a$ | 0.6 T (max)                             |
| Radius of solenoid coil, $R_0$         | 0.1 m                                   |
| Ion temperature, $T_i$                 | 10 eV                                   |
| Electron temperature, $T_e$            | 1.5 eV                                  |
| Debye length, $\lambda_{Debye}/R_p$    | $\sim 10^{-3}$ - $10^{-2}$ m            |
| Electron Larmor radius, $L_e/R_p$      | $\sim 10^{-4}$ - $10^{-3}$ m            |
| Ion Larmor radius, $L_i/R_p$           | $\sim 10^{-1}$ - $10^0$ m               |

As shown in Fig. 1, ejected plasma radius  $R_p$  is limited by anode and the relative axial location of plasma inlet  $z_p$  to the centre of solenoid coils  $z_0$  is removable. To examine these geometric factors of plasma bunch in the magnetic nozzle effects, three different cases displayed in Table III are calculated and compared in the following section.

**Table III. Three different cases of the calculated model.**

|        | $z_p$ (relative to $z_0$ ) | $R_p / R_0$ |
|--------|----------------------------|-------------|
| Case 1 | 0                          | 0.5         |

|        |        |     |
|--------|--------|-----|
| Case 2 | - 0.07 | 0.5 |
| Case 3 | - 0.07 | 0.2 |

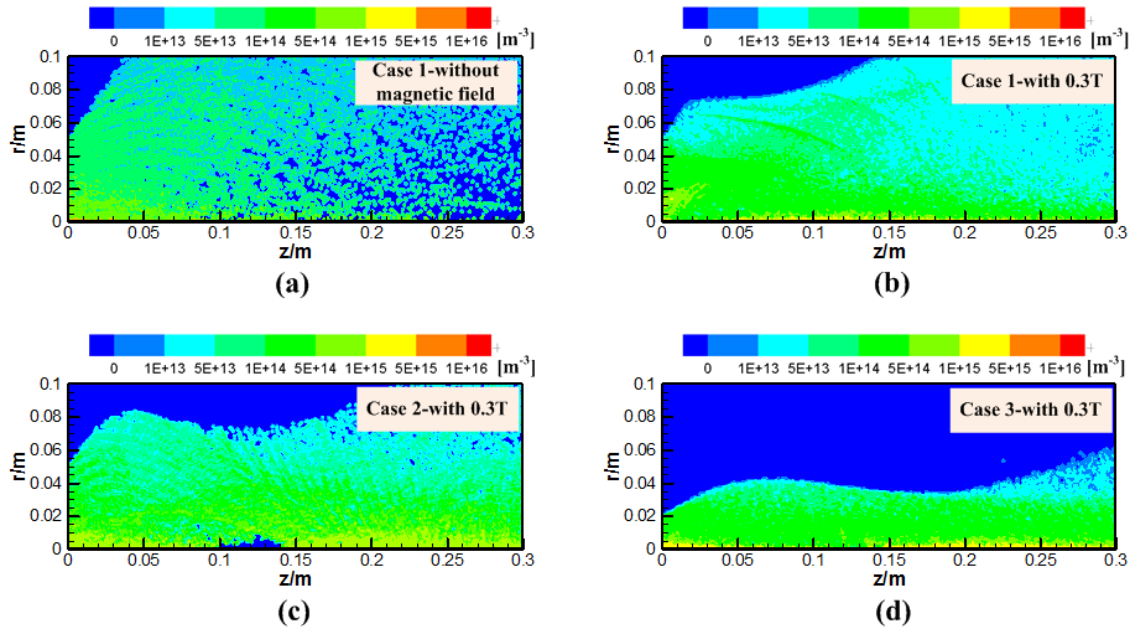
### III. Simulation Results and Discussions

#### A. Plasma Distribution

By applying an external magnetic field, plasma distributions outside the thruster have been significantly altered. In contrast with the case without magnetic field as seen in Fig. 2 (a), ejected plasma plume shows various degree of concentration with magnetic strength from 0 to 0.6 T. A group of contour maps for plasma density distribution in 0.3 T is shown in Fig. 2 (b) – (d). For further specification, the divergence angle of plasma plume and nozzle efficiency defined by Eq. (3) in these cases are presented in Table IV.

$$\eta_{nozzle1} = \frac{1 + \cos \theta}{2} \quad (3)$$

When the plasma inlet plane moves from the centre of magnetic solenoid to its entrance, i. e. from case 1 to case 2, plasma plume of originally free expansion behaves larger trend of gathering by magnetic lines. This obvious change of direction to downstream possibly assists in the reduction of radial plasma energy diffusion. In case 3, plasma of less initial radius is magnetized by less divergent magnetic lines so that its distribution shows stronger restriction.



**Figure 2. Contour maps of ion density distribution. Plasma ejection plane is viewed as the axial zero point  $z=0$ . (Following figures with axial coordinate are in the same case.)**

With various applied magnetic nozzle, the divergence angle of plasma plume changes from  $34.2^\circ$  to  $53.5^\circ$  in case 1 and the minimum value is achieved at 0.3 T. However, moving the initial conditions to case 2 and 3, plume divergent tendency decreases and varies from  $29.4^\circ$  to  $47.1^\circ$  and  $17.7^\circ$  to  $27.5^\circ$  respectively. Meanwhile, the nozzle efficiency increases from 91.3% to 97.6%.

**Table IV. Divergence angle of plasma plume and corresponding nozzle efficiency.**

|        | <i>Divergence angle <math>\theta</math> (<math>^\circ</math>)</i> | <i>Nozzle efficiency <math>\eta_{max}</math> (%)</i> |
|--------|-------------------------------------------------------------------|------------------------------------------------------|
| Case 1 | 34.2 - 53.5                                                       | 91.3                                                 |
| Case 2 | 29.4 - 47.1                                                       | 93.6                                                 |
| Case 3 | 17.7 - 27.5                                                       | 97.6                                                 |

## B. Thrust Variation

As calculated by Eq. (4), thrust variations with different magnitude of applied magnetic nozzle fields are shown in Fig. 3. Assuming the mass flow rate of propulsive plasma is conserved, thrust could be normalized by the value without external magnetic nozzle. It can be seen that as the magnetic field is enhanced, thrust increases to the best degree where plasma is moderately magnetized, then falls rapidly due to stronger circumstances.

$$F_{thrust} \cong \dot{m}_i v_z \quad (4)$$

In the comparative study among three different cases, the highest thrust value is achieved in case 3 with smaller relative plasma radius 0.2. Moving plasma ejection plane from exit of magnetic solenoid to the centre of coils ( $z_p=0$  to  $z_p=-0.07$  m), the maximum thrust value decreases slightly from 1.11 to 1.09 near  $B=0.1$  T. However, in case 2 with stronger magnetic field, thrust drops more slowly than that in case 1 which is exactly the expected outcome.

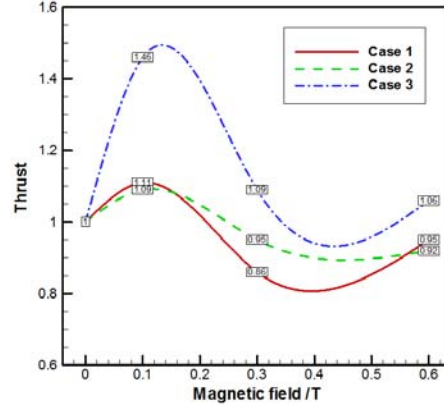


Figure 3. Normalized thrust variations with magnetic field in three cases.

## C. Hall Current

To analyse the physical processes during the magnetic nozzle operating, it is helpful to draw the radial plasma potential curves at different axial locations from thruster exit plane to outer vacuum space. As electrons are quite easy to get magnetized, they are distributing closer to the centre axis at the very beginning, which is exactly the situation seen in Fig. 4 (a) where the radial plasma potentials derived at an axial distance of 0.05 m after ejection behave negative in the range of  $r=0$  to about  $r=0.07$  m. Simultaneously, the concentration trend of electrons drives ions to the axis through their effective interaction. When the plasma plume moves downstream, most highly magnetized electrons still follow the diverging magnetic lines while ions do not so that radial potentials display positive near the symmetric axis as shown in Fig. 4 (b) and (c), that is to say electrons become surrounding ions as a whole.

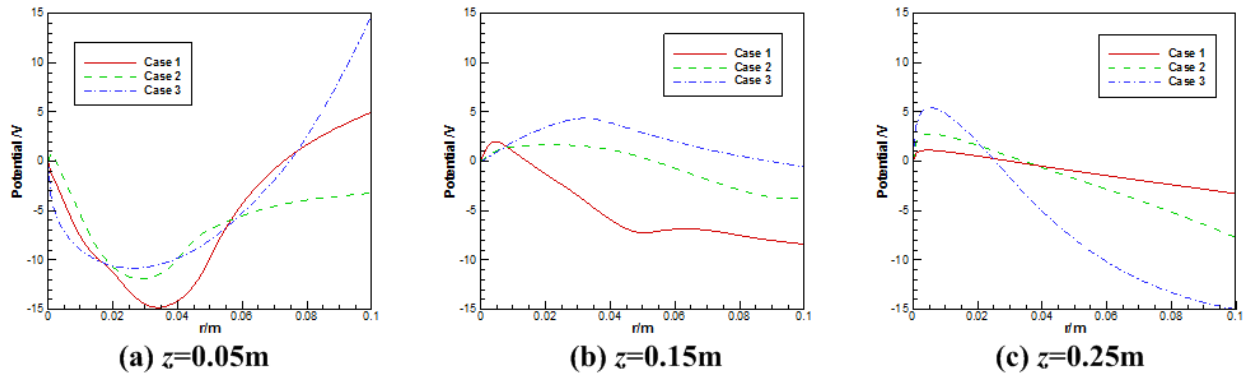


Figure 4. Radial variations of plasma potential at different axial locations. Magnetic field is 0.3 T.

Electrons with spiral motion generate azimuthal currents (i. e. hall currents) inside and on the surface of plasma bunch. It can be concluded from the calculated results shown in Fig. 5 that the case with higher degree of magnetization owns larger hall currents and the direction of such azimuthal currents opposite to those in the magnetic solenoid. Moreover, if two current loops have different conducting directions, there will be both axial and

radial repelling forces between them, as presented in Fig. 6. Therefore, the interaction between solenoid coils currents and plasma hall currents can possibly produce two types of effects, i. e. accelerating and gathering the plasma bunch.

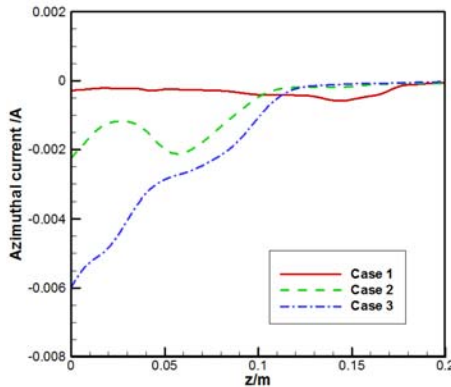


Figure 5. Hall current variations. Applied magnetic field is 0.3 T.

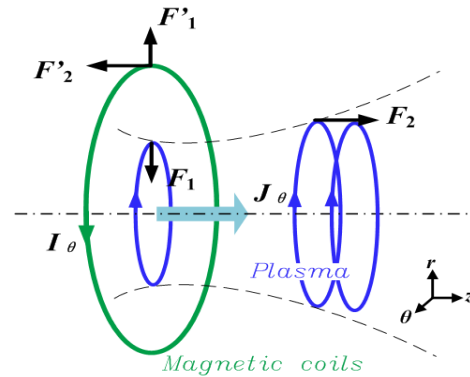


Figure 6. Sketch of interaction between opposed current loops.

#### D. Effective Utilization of Applied Magnetic Field

From the above demonstration of magnetic nozzle effects under different geometric conditions, it might be advanced that to find a more effective way of utilizing the magnetic nozzle, some analyses of plasma movement within the converging-diverging magnetic field should be useful.

As shown in Fig. 7, the simulated magnetic field possesses the highest value near  $z=0$  and less divergent lines near  $r=0$ . The second case in current model has access to larger area of high-intensity magnetic field than case 1, whereas the third case takes more advantage of the less divergent magnetic lines. Plasma ejected into the electromagnetic field may be magnetized to some extent that partial charged particles are travelling around magnetic lines within a certain range. As the magnetic field becomes weaker in the downstream, plasma detachment from magnetic lines occurs. Such processes can be examined by divergence angle variations of magnetic field and particles as seen in Fig. 8. Between  $z=0.1$  m and  $z=0.2$  m magnetized particles follow the curving trend of magnetic lines while after that magnetic lines diverge larger than demagnetized particles. Therefore, the point of detachment becomes quite important in magnetic nozzle operating process that if magnetized plasma does not separate from gradually expanding magnetic field in time, it will lose much axial energy or even travel back turning out no thrust at last.

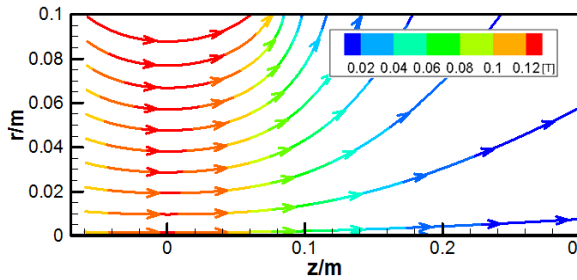


Figure 7. Simulated magnetic field.

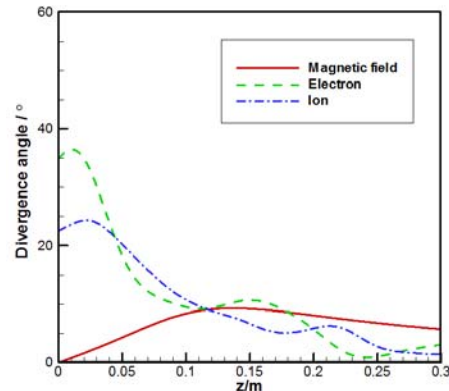
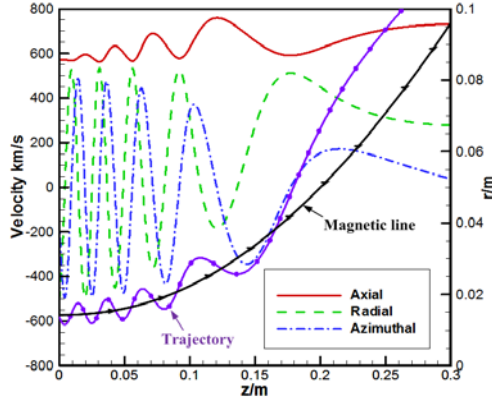


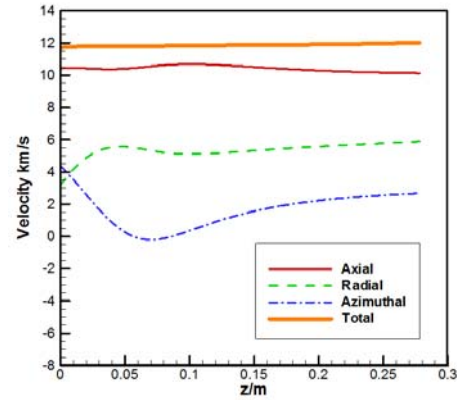
Figure 8. Average divergence angle variations of magnetic field and charged particles.

Considering the special configuration and magnetic moment conservation in slowly-varying magnetic field, velocity components of a magnetized particle transform in the way that Fig. 9 shows. While this particle revolving around the diverging magnetic line, its radial and azimuthal energy gradually grows into axial component. However, if the magnetic field is not strong enough, electrons are fully magnetized but ions are not; influenced by concentrated electrons, non-magnetized ions might have a little azimuthal motion with the same direction as electrons, but their axial and radial velocities seldom change as the results shown in Fig. 10.

Back to the three cases in present simulation model, plasma can be gathered larger and faster in case 3 because less divergent and stronger part of magnetic field is employed. Most charged particles get magnetized in such case and detach at relatively smaller angles compared with other cases. As a result, the non-axial energy of ions has been partially transformed into axial component and the axial energy which directly affects the thrust has been eventually made the best use of.



**Figure 9. Velocity variations of a magnetized single electron showing with its trajectory.**



**Figure 10. Velocity variations of a non-magnetized single ion.**

## Conclusions

The present investigation of magnetic nozzle effects with Particle-in-cell method has been established upon a two-dimensional model with quasi-neutral collisionless fully-ionized plasma passing through a diverging magnetic field. To examine the way that magnetic nozzle works, three specific cases with different initial geometric situations of plasma have been simulated. From the results, it may be concluded that when the position of magnetic field is suitable and relative plasma radius is small, the total thrust of AF-MPD thruster can be enhanced by 50% at most ( $B=0.1$  T). Plasma bunch performs less divergent tendency and higher nozzle efficiency as the applied magnetic field is better utilized. Moreover, to find the interior physical processes of these acceleration and confinement effects, radial potential distribution and hall currents interaction with solenoid currents have also been demonstrated. As the spirally moving electrons induce the azimuthal currents both inside and on the surface of plasma within a certain axial range after ejection, repelling forces along axial and radial coordinates contribute to gathering and speeding up the whole plasma.

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