Magnetic System for Hall Thrusters Evaluation and Design

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Abstract

Evaluation of parameters of magnet system for modern Hall thrusters could be made by using methods of conform images. This works the better the less the ratio of acceleration gap to radius of acceleration ring is taken. Simple algebraic formulas make it possible to fulfill express analysis of various configuration of a discharge chamber.

Introduction

Magnet system for Hall thrusters, both SPT and TAL is a key fraction, that specify both thruster operational processes and mass/ size properties, because namely the magnet system takes 80% of the engine mass [1,2].

The Hall thruster magnet field should meet a number of conditions providing its stable and effective operation since due to magnetization of an electron component magnetic lines of force work also as lines of equal potential of en electric field, which provides ion generation and acceleration.

Modern magnet systems are made to meet certain conditions, like that:

- -in an area of discharge grad B>0 should be met, that provides stability of the discharge;
- -the field distribution should provide also correct alignment of ionization and acceleration areas relative to each other and acceleration chamber walls; and
- -a specific configuration of lines of force should be formed in the nearwall area to focus ion flow and provide minimal sputtering of the discharge chamber.

Finally an optimal spatial distribution of the magnetic field occurred to be rather complicated and a complex magnet systems should be evaluated to set it. It could be pointed two typical solutions to form fields of various configurations:

- -multipole magnet systems
- -magnet systems using magnet screens or shunts.

The first solution is based on a superposition of fields induced by set of separate magnet systems, which could be regarded independently, and resulting field is sum of fields induced by each system. In this case a task could be reduced to calculation of a number of simple bipolar magnet systems, which calculation is well known [3,4].

The second way is based on incorporation into area, occupied by the magnetic field, of certain parts made of ferromagnetic materials led to redistribution of the magnet flux and its focus inside the parts. That leads to dramatic changes of the magnetic field configuration in the part vicinity, that constitutes its effective area of impact. That allows to control magnetic field by variation of an parts shape and dimensions.

Both ways mentioned above had been used for evaluation modern Hall thrusters [5,6,7].

To design magnet system an iterational method is usually used because an opposite task - to find a magnet system configuration by given parameters of a magnetic field, could not be solved uniquely. Iterational method conceives that an initial configuration being given, its magnetic field is calculated and result is compared with the magnetic field needed to be formed. After tat a certain changes were put in the initial configuration and the procedure goes until the satisfactory correspondence were obtained. The key position in this procedure lies in calculation of field for given magnet system. But even for simple configurations this problem needs in computer modeling and takes a lot of time to process one version.

The task of magnet system design pursue usually two goals:

- calculation of profiles of magnetic lines in the discharge area for given configuration of the magnet parts;
- estimation of magnet system parameters so as force of magnetization, magnetic circuit cross section, etc.

Contrary to multipole systems, calculation of magnet systems with magnetic screens and shunts encounters a specific difficulties since the field should be found in the area of the screens impact so that task could not be divided on a separate, more simple subtasks.

Specific configuration of the magnetic screens a poles of a modern Hall thrusters magnet system allows, under a certain, quite real assumption to find simple algebraic expressions for distribution of magnetic field and determine all necessary for magnet system evaluation parameters.

Problem formulation

Under certain suggestions, quite reasonable for real configurations of a magnet system, it turns out well to find analytical description of magnetic field into an acceleration chamber of a Hall thruster in form of simple algebraic formulas, that makes it possible to find most substantial features of the magnetic system without resort to tedious numerical methods of solution of the Maxwell equations.

The important occasion here is potentiality of the magnetic field in vacuum. For this case the Maxwell's equations take a form as:

$$\mathrm{rot}H = 0 \qquad (1)$$
$$\mathrm{div}H = 0$$

so that by introduction of a scalar potential function φ so that

$$H = \mathrm{grad}\varphi \qquad (2)$$

the problem of of finding the magnetic field might be reduced to solution of the Laplas equiation for the potential:

$$\Delta\varphi = 0. \qquad (3)$$

Due to large number of magnetic conductivity of a magnetic circuit, that forms a field in the operational gap, the magnetic lines of force go out of the magnetic core virtually in normal direction to its surface, so that with a good accuracy the boundary condition meets

$$\partial\varphi / \partial\tau = 0 \qquad (4)$$

(τ assigns direction of the tangent to surface), another words, the magnet core surface could be considered as an equipotential one.

The typical configuration of the magnet system of a Hall thruster is shown on Fig.1. Since an average radius of the acceleration gap R is much more than its width h, the magnetic field configuration in the zero approximation could be treated as a plain.

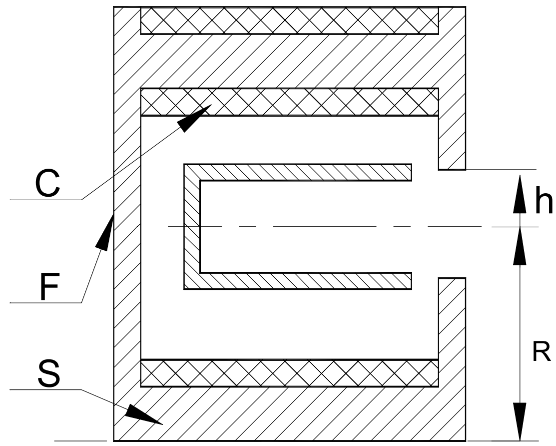

Fig1. Configuration of Hall thruster magnet system.

Hence, there could be formulated two-dimensional plain task of Dirihlet type for the Laplas equation (3) with constant but different values of function φ at the separate sections of the boundary, which consists of magnet core surfaces. For such setting of the problem methods of conform images could be used.

If we take into account that the central magnet core S (see Fig.1), back flange F and coils C are removed off the acceleration gap and their diffuse field feebly effects the field in it, and wall thickness of the magnet poles and shunt being little comparing to the gap width h, than the idealized configuration of the magnet system could be imagined as it shown on the Fig.2. There magnet poles and walls of the shunt, being the parts that form accelerating gap, are depicted as direct lines extended at infinity and treated as slits on a plane of complex variable w = u + iv.

Thereby the parts of magnet system, that were inessential to form magnetic field in the acceleration gap, were removed in infinity and led out of consideration. The boundary includes both sides of the slits, and consists of the external pole $\grave{A}_1\grave{A}_2\grave{A}_3$, internal pole $\grave{A}_7\grave{A}_8\grave{A}_1$ and shunt $\grave{A}_3\grave{A}_4\grave{A}_5\grave{A}_6\grave{A}_7$. Further a case of configuration of the magnetic system, symmetric regarding the axis OO' is taken for consideration, a case of asymmetric one could be considered by the same way but follows much more complicated algebraic calculations. Nearby the points $\grave{A}_2\grave{A}_4\grave{A}_6\grave{A}_8$ of Fig. 2 their complex coordinates indicated. The halfwidth of shunt hole is taken as a scale for all dimensions, and due to that the distance between beam $\grave{A}_3\grave{A}_4$ (or $\grave{A}_6\grave{A}_7$) and axis of symmetry OO', is taken as unit; dimensions a and h of the acceleration chamber are evaluated in units of the hole halfwidth.

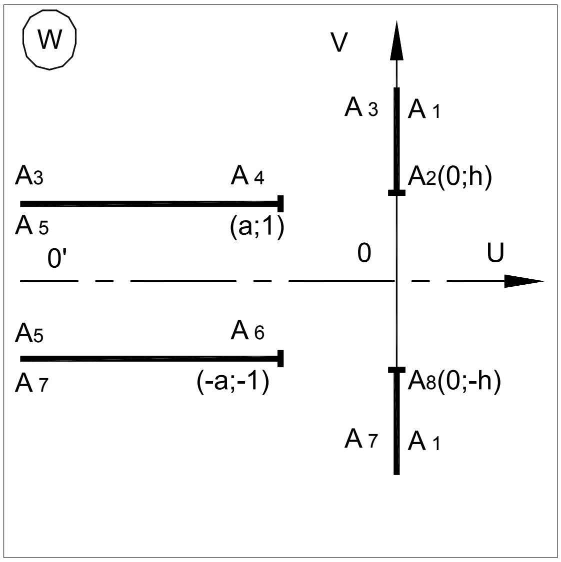

Fig.2 Idealized configuration of magnet systemin in discharge area.

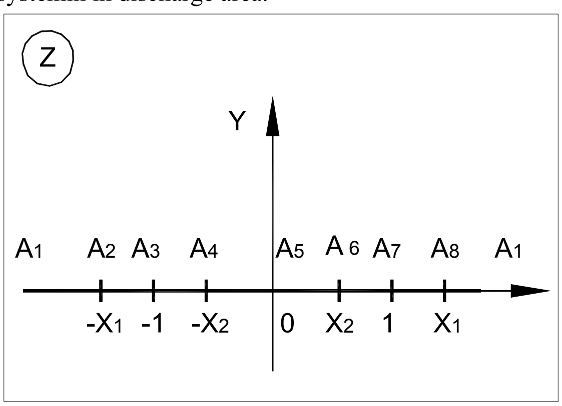

Fig.3 vertexes disposition on planes/

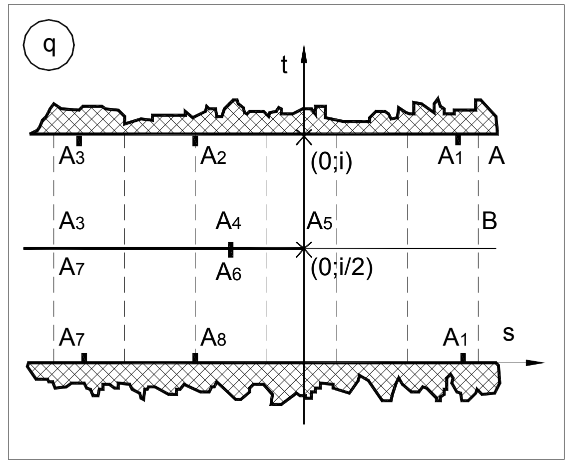

Fig.4 Topological simulation of magnet system with shunt by capacitor with slit.

To find of conform images of a magnetic or electric field produced by two equipotential electrodes of given configuration by conventional procedure, one needs to define function of complex variable realizing transformation of the boundary, created by electrodes, into a horizontal stripe. And conformity between points of boundary should be taken so that upper edge of the stripe to correspond to one electrode and the lower one to another electrode. If that's the case, the stripe to simulate a plane capacitor, its edges to be taken as electrodes, and a voltage drop to be applied between them. Then parallel to the stripe edges direct lines could be treated as a lines of equal potential, and normal to them segments of direct lines, set against the stripe edges, as lines of force and the function found makes their transfer into system of equipotential lines and lines of force of the desired task. But a typical configuration of magnet system of a Hall thruster contains three electrodes, so that «magnetic potential difference» being applied between two poles and the shunt to acquire a floating potential that should be found.

Mathematical procedure

According to theory of conform images, the configuration of Fig.2 could be taken as a polygon of eight vertexes, and four of them being disposed in the infinity. Beams shown on the Fig.2 form the polygon boundary, each of them should be passed around both sides: first path $\grave{A}_1\grave{A}_2\grave{A}_3$ is made, than in the vicinity of the infinite point $\grave{A}_3$ transfer at the other beam occurs and than path $\grave{A}_3\grave{A}_4\grave{A}_5$ is fulfilled, and so on.

To define function realizing transformation of a polygon to half plane a Cristoffel-Shwartz integral could be used, which, as a matter of fact, fulfills an inverse transformation, half plane to polygon, but owing to one-oneness of conform images, both transformations could be taken as existing if one were found. This integral allows to pass boundary of the polygon into the boundary of the half plane, i.e. into real axis x. After that a transformation of the half plane into the stripe, as it usually is going on for the case of two electrodes, presents no difficulty. In our case so transformation would tun out inefficient, and another configuration should be found, that would be topologically equivalent to three electrodes and have a known and simple system of equipotential curves.

A mostly simple so configuration could be simulated by a stripe with a slit, as it is shown on the Fig4. , so that upper and lower edges of the stripe to simulate magnet poles, between which the "magnet potential difference" to be applied (it is taken further as unit) and the slit corresponds to the shunt, which potential, due to symmetry of the configuration, is taken as 1/2.The idea of that simulation lies in known property of plane capacitor that infinitely thin plane metallic plate being put into plane capacitor in parallel to its electrodes does not disturb the potential distribution in it. Another words, the force lines do not change their configuration and stay the same as for ordinary plane capacitor.

A Cristiffel-Shwartz integral is given by

$w = \int (z-x_1)^{\alpha_1 -1} (z-x_2)^{\alpha_2 -1} \ldots (z-x_n)^{\alpha_n -1}\, dz$

where n – is a number of polygon vertexes

$x_1 \ldots x_n$ – coordinates of dots of the real axis of the plane z, which the polygon vertexes pass to;

$\alpha_1 \ldots \alpha_n$ – angular factors of the vertexes, defined as $\alpha_n = \theta_n / \pi$ where θ_n - is vertex angle(by radians).

Because at a Cristoffel-Shwartz conversion transfer of any three dots could be taken arbitrary, let demand polygon vertex À$_1$ to pass to infinite place of the axis z; vertex À$_5$ to pass to zero; and vertex À$_3$ –to pass to 1; then on account of symmetry, the vertex À$_7$ should pass to –1. Coordinates of other vertexes will be defined later, denote them for now as $\pm x_1$ and $\pm x_2$.

All vertexes coordinates and angular factors are thrown together in the Table 1. The Fig3 shows mutual disposition at the plane z of the dots, that correspond to polygon vertexes at the conversion. Note, that axis of symmetry OO' passes to the imaginary axis of the plane z.

Table 1.

Vertex [1]	Coordinates at plane w	Angular factor	Coordinates at plane z
À1	∞	-1	∞
À2	ih	2	$-x_1$
A3	∞	-1/2	-1
A4	-a+i	2	$-x_2$
A5	∞	0	0
A6	-a-i	2	x_2
A7	∞	-1/2	1
A8	-ih	2	x_1

Subject to data of Table1, Cristoffel - Shwartz integral realized transformation of the half plane z to area w could be written as :

$$w = C_1 \int (z^2 - x_1^2)(z^2 - x_2^2)(z^2 - 1)^{-3/2} z^{-1} dz + C_2 =$$
$$= C_1 \int \frac{(z^2 - x_1^2)(z^2 - x_2^2)}{z(z^2 - 1)\sqrt{(z^2 - 1)}} dz + C_2 \qquad (5)$$

This integral could be fully taken

$$w = C_1 \left[\sqrt{1-z^2} + \frac{(1-x_1^2)(1-x_2^2)}{\sqrt{1-z^2}} + x_1^2 x_2^2 \ln \frac{z}{1+\sqrt{1-z^2}} \right] + C_2 \qquad (6)$$

Motion of a test point along real axis x of the plane z follows motion of its image at the plane w along the polygon boundary. To find arbitrary number C_1 the increment of function w should be calculated when the point z = 0 to be gone around over an infinitely little semicircle, on the plane w that follows pass from the beam À$_4$À$_5$ to the beam À$_5$À$_6$ in the infinity and function w to gain increment -2i, that equal to distance between the beams. Assuming $z=re^{i\varphi}$ and taking into account that increment arises only due to term containing the logarithm, we have:

$$C_1 (x_1x_2)^2 \mathbf{ln}\, z \big|_{\pi}^{0} = C_1 (x_1x_2)^2 \mathbf{ln}(re^{i\varphi}) \big|_{\pi}^{0} = C_1 (x_1x_2)^2 (\mathbf{ln}\, r+i\varphi) \big|_{\pi}^{0} = -C_1 (x_1x_2)^2 i\pi$$

Subject to mentioned above $-2i= -C_1(x_1x_2)^2 I\pi$, that follows

$C_1 = 2/(x_1x_2)^2 \pi$

To define second arbitrary number C_2 note that variable z at the beam À$_3$À$_4$À$_5$ is real to vary in limits $1 \geq x \geq -1$, so that w could be written as

$w= C_1 [f(x)+i\pi(x_1x_2)^2] +C_2.$

where f(x) is some real function of real variable <u>x</u>. On the other side, it is obvious that imaginary part of the function w(z) at the beam is equal to unit, that gives equation for imaginary part:

$C_1 i\pi(x_1x_2)^2 + C_2 = I$

hence

$C_2 = -i$

And finally

$$w = \frac{2}{\pi x_1^2 x_2^2}\left[\sqrt{1-z^2} + \frac{(1-x_1^2)(1-x_2^2)}{\sqrt{1-z^2}} + x_1^2x_2^2 \ln\frac{z}{1+\sqrt{1-z^2}}\right] - i \quad (7)$$

Coordinates x_1 and x_2 could be found due to correspondence of vertexes at <u>w</u> and <u>z</u> planes:

$À_2$: $w = ih$; $z = x_1$

A_4 : $w = -a+i$; $z = x_2$

After substitution of these data in (7) the equations for x_1 and x_2 could be derived:

$$h = \frac{2}{\pi}\left[\frac{x_1^2 - 1 - (1-x_1^2)(1-x_2^2)}{x_1^2x_2^2\sqrt{x_1^2-1}} - arctg\sqrt{x_1^2-1}\right] + 1 \quad (8)$$

$$a = \frac{2}{\pi}\left[\frac{x_2^2 - 1 - (1-x_1^2)(1-x_2^2)}{x_1^2x_2^2\sqrt{1-x_2^2}} - \ln\frac{x_2}{1+\sqrt{1-x_2^2}}\right] \quad (9)$$

Second transformation, of the stripe of the plane <u>q</u> (see Fig.4) in half plane <u>z</u> also could be made through Cristoffel - Shwartz integral. The configuration shown on Fig.4 could be taken as polygon of four vertexes of three of them to situate in infinity and fourth one is placed in point (0;i/2). Following to principle of correspondence the transformation should be made so that points À1; À3 and À7 of <u>z</u> plane to pass to infinite point of the plane <u>q</u>, that should provide correct correspondence of the segments of the boundary to electrodes of the magnet system. The correspondence of the boundary points is shown on the Fig.4. After execution of all procedure related with Cristiffel - Shwartz integral implementation and described above, we have:

$$q = \ln(z^2 - 1) / 2\pi + i \qquad (10)$$

using that and eliminating z in (7) we find equation to transform of the plane q to plane w:

$$w = \frac{2}{\pi}\left[i\frac{e^{\pi q} - (1-x_1^2)(1-x_2^2)e^{-\pi q}}{x_1^2x_2^2} + \frac{1}{2}\ln\frac{1-ie^{\pi q}}{1+ie^{\pi q}}\right] + i \quad (11)$$

Magnet system parameters description

The found above complex function w(q) and, correspondingly its components, allow to fully learn structure of the magnetic field in Hall thruster and define its essential properties.

Images of the magnetic force lines at the plane q are shown at Fig.4 as dotted lines, normal to the stripe edges; they are stated by equations s = const (if <u>q</u> were taken as q = s+it, so that <u>s</u> were real component). After separation of real and imaginary components in equation (11) variouse lines of force could be found in parametric form (with <u>t</u> as parameter) by varying value of <u>s</u>. Fig.5 shows the typical configuration on the force lines for a Hall thruster engine.

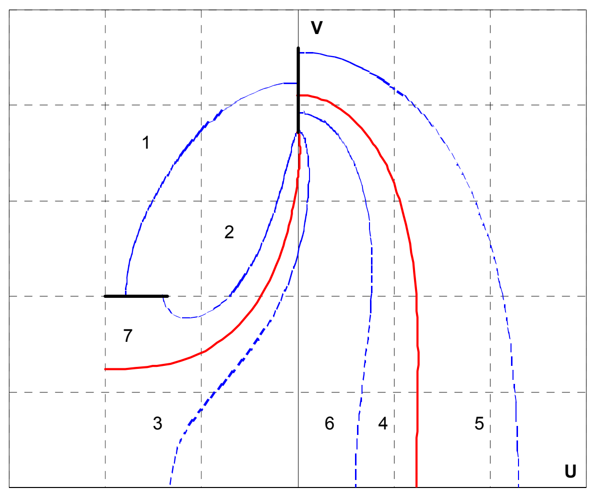

Fig.5. Typical configuration of lines of force for modern Hall thruster magnet system.

It easy to learn from Fig 4 and Fig5 that there are two types of force lines, one of them, starting on one magnet pole ends on the other one (poles are corresponding to upper and lower edges of the stripe of Fig4) ; another lines rest on the shunt (slit of Fig4). Their separation provides line (line 7 on Fig.5) corresponding to value s = 0, that is seated against the endpoint of the slit. That line is defined by equations:

$$u = -\frac{2}{\pi x_1^2 x_2^2}\left[1 + (1 - x_1^2)(1 - x_2^2)\right]\sin \pi t +$$

$$+\frac{1}{2\pi}\ln\frac{1+\sin \pi t}{1-\sin \pi t} \quad (12)$$

$$v = \frac{2}{\pi x_1^2 x_2^2}\left[1 - (1 - x_1^2)(1 - x_2^2)\right]\cos \pi t + \frac{\pi}{2} \quad (13)$$

To axis of symmetry OO' of Fig2 corresponds line À$_5$B of Fig 4, that could be found if values of parameters t = 1/2 and 0<s<∞ were taken.

$$u = \frac{2}{\pi x_1^2 x_2^2}\left[e^{\pi s} + (1 - x_1^2)(1 - x_2^2)e^{-\pi s}\right] -$$

$$-\frac{1}{\pi}\ln\frac{e^{\pi s}+1}{e^{\pi s}-1} \quad (14)$$

Among lines of force, that rest on magnet poles, two configurations of different shape could be chosen. One of them, as line 7 on Fig. 5, is convex everywhere, and derivative du/dv does not change sing along it reducing to zero at the axis of symmetry. The other configuration as if sags down into the shunt, it is concave nearby the axis of symmetry and has the point of inflection where the concavity change its sign (lines 3 and 6 of Fig.5). These configurations are separated by line 4, with mentioned above specific points being merged on the line of symmetry.

Magnetic field on the plane w with components H_u ; H_v could be found as derivative of transformation (q)→(w), i.e.

$$H = H_u + iH_v = dq/dw,$$

for which purpose q as function of w should be known. At this case we should have expressions describing field in variables (u;v) of w plane. But found before relation (11) defines the inverse function w(q). In this connection the expression for field could be taken as:

$$H = 1/ (dw/dq), \quad (15)$$

which defines field through variables (s;t) of q plane. Than relation (11) defines point of w plane for given couple (s;t), and expression (15) gives components of the field.

Making differentiation of (14) and using (15) we find complex function, which real and imaginary parts give two components of magnetic field:

$$H = i\frac{x_1^2 x_2^2}{2}\frac{e^{\pi q}(1+e^{2\pi q})}{(e^{2\pi q}+1-x_1^2)(e^{2\pi q}+1-x_2^2)} \quad (16)$$

For the axis of symmetry q =s+i/2 and s>o. After substitution in (16) we find function which describes variation of magnetic field along the axis (of course, its v-component, because the other one becomes zero on the axis of symmetry).

$$H = \frac{x_1^2 x_2^2}{2}\frac{e^{\pi s}(e^{2\pi s}-1)}{(e^{2\pi s}-1+x_1^2)(e^{2\pi s}-1+x_2^2)} \quad (17)$$

This formula prescribes field magnitude and (14) gives its coordinate on the axis of symmetry.

Fig.6 shows magnetic field variation along the axis of symmetry for typical configuration of discharge chamber of D-110 engine, both calculated by (17) and measured ones.

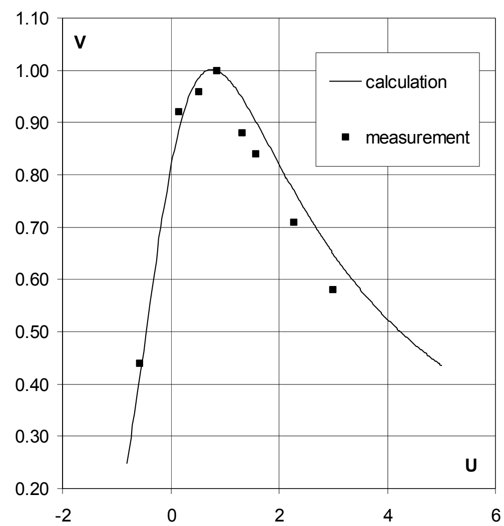

Fig.6 Comparison of calculated and measured values of magnetic field along axis of symmetry.

Obtained relations could be used for quick analysis and evaluation of general parameters of magnet system at a stage of preliminary design. So, to estimate attenuation of the magnetic field strength depending on position of shunt, measurements of a field strength for different depths of discharge chamber usually used. Procedure of measurement is not easy and not so precise one.

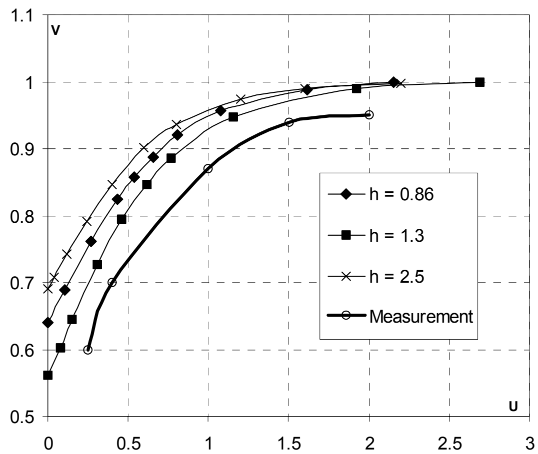

Fig.7 Magnetic field strength variation via discharge chamber depth.

Fig. 7 shows measured curve comparing to calculated one by formulas, found above, for different distances between magnet poles. The plots show that a level of attenuation depends little on the distance between poles. That proves correctness of use the same measured data for different configurations of discharge chamber.

Conclusion

Procedure developed on a base of functions of complex variable, which assume a flat approximation for magnet system configuration, provides satisfactory compliance with experimental data and could be used for express evaluation of Hall thruster magnet system.

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