

# Influence of Tailored Applied Magnetic Fields on High-Power MPD Thruster Current Transport and Onset-Related Phenomena

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**Abstract:** This work investigated the effects of tailored, externally-applied magnetic fields on current transport and near-anode processes in the plasma discharge of a magnetoplasmadynamic thruster (MPDT). Electrical and plasma diagnostics were used to investigate how localized applied magnetic fields could mitigate the effects of the “onset” phenomena, including large-amplitude terminal voltage fluctuations and high anode fall voltages associated with unstable operation and anode erosion. An MPDT with a multi-channel hollow cathode was developed and tested with quasi-steady pulses of 1 ms duration at power levels of 36 kW (20 V, 1800 A) to 3.3 MW (255 V, 13.1 kA) with argon propellant in three different magnetic configurations: self-field, applied B field tangential to the anode lip near the exit plane, and applied cusp B field. The current pattern and current densities redistributed to follow the applied poloidal magnetic field lines, which created increased conduction paths to the anode. Also, the anode fall voltage was substantially reduced with both applied B field topologies over a large range of currents. For example, at 10.7 kA, the cusp applied magnetic field decreased anode fall voltages from 45–83 V down to 15 V or lower along much of the anode. The amplitude and frequency of the voltage fluctuations were also reduced over a broad range of currents with the applied fields. E.g., the standard deviations of the fluctuations were lowered by 37–49% at 8–9 kA. In addition, decreases in the mean terminal voltages as large as 31% were measured with the applied magnetic fields. These effects are shown to be associated with the increased current conduction along the applied magnetic field lines in the near-anode region. These results also suggest a likely reduction in frequency and intensity of current-concentrating filaments and anode spots, which contribute to erosion. Overall, both applied magnetic field configurations enabled significant reductions in onset-related behaviors relative to self-field operation. These improvements should lead to reduced anode erosion, i.e., improved thruster lifetime, and increased thruster efficiency with the applied fields. The applied fields used in this study differ from both the topologies and relative field strengths typically used in the vast majority of conventional, so-called “applied-field MPD thrusters” (AF-MPDTs). These results suggest a distinctive and more effective approach to influencing the near-anode phenomena and mitigating the deleterious effects of onset with appropriately designed applied magnetic fields.

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## I. Introduction

### A. Background and Motivation

MANY space missions in the future will face increasing challenges on the spacecraft propulsion technologies required to enable them. New missions will place challenging demands on candidate spacecraft propulsion technologies, from high delta-V maneuvers on robotic spacecraft to greater efficiency and faster trip times on human deep-space expeditions.<sup>1-9</sup> In particular, requirements for increasingly higher delta-V (large changes in velocity) over the course of such ambitious missions imply the need for higher specific impulse ( $I_{sp}$ ) but with acceptable trip times (i.e., sufficient thrust to enable reasonable trip time constraints). The demand for higher specific impulse is made obvious when considering the “ideal rocket equation,” given by<sup>10</sup>

$$\frac{M_{initial}}{M_{final}} = \exp\left(\frac{\Delta v}{g_0 I_{sp}}\right) \quad (1)$$

where  $\Delta v$  is the velocity change of the maneuver (delta-V),  $M_{initial}$  is the initial mass,  $M_{final}$  is the final mass after the delta-V maneuver, and  $g_0$  is a constant given by the gravitational acceleration at Earth's surface, 9.807 m/s<sup>2</sup>. This equation shows the strong exponential dependence between mass, the delta-V, and the specific impulse of the propulsion system. A larger total launch mass for a spacecraft and its propellant ultimately translates into higher costs for the size of a launch vehicle, material, engineering labor, etc. Thus, reducing total launch mass by reducing a spacecraft's total propellant load and decreasing the size and mass of the structure to support it has clear cost-saving benefits. For a given mission delta-V requirement, one can lower launch mass exponentially by increasing the  $I_{sp}$  of the propulsion system used.

Electric propulsion (EP) encompasses technologies with the potential to enable future advanced space missions, due in large part to high  $I_{sp}$ . EP is a class of technologies used for spacecraft propulsion in which an electrical power source is used to convert electrical input power into kinetic energy of the exhaust propellant for thrust. Specific impulses for EP thrusters are generally much higher (typically 800–10,000 s or higher  $I_{sp}$ ) than those of conventional chemical propulsion thrusters (175–450 s  $I_{sp}$ ). EP thruster technologies are generally classified into three sub-categories: electrothermal, electrostatic, and electromagnetic. Electrothermal thrusters (e.g., resistojets and arc-jets) use electrical power to heat and add enthalpy to an expanding propellant gas, similar to chemical thrusters but with resistive heating instead of chemical reactions as the power source. Electrostatic thrusters (e.g., ion thrusters) primarily use electric fields to accelerate ions as thrust, and typically use a separate cathode discharge to neutralize the exhaust beam. Electromagnetic thrusters (e.g., magnetoplasmadynamic thrusters) generally use electromagnetic forces to accelerate a bulk plasma flow. Work in the field of EP also includes an array of more general applications, as well. The commercial space industry would benefit from higher-efficiency technologies for use in attitude control systems, station-keeping processes, and orbital transfer spacecraft.<sup>11</sup> Such advances imply lower propellant requirements, which directly translate into lower launch costs or increased payloads. Further, the discoveries made and technologies developed from the associated challenges in plasma physics, thermophysics, controls, and plasma-surface interactions could also have broad applications in manufacture and analysis of materials, thin-film deposition, energy conversion technologies, and other industrial applications.<sup>12</sup> Also, many physical processes in these thrusters have analogies in other areas of plasma science ranging broadly from arc furnaces to astrophysical phenomena.<sup>13</sup>

A magnetoplasmadynamic thruster (MPDT) is a high-power spacecraft electric propulsion device still in the research stage. The thruster is sub-classified as an electromagnetic (EM) accelerator because it incorporates electromagnetic forces to produce bulk acceleration in a plasma predominantly through Lorentz forces,  $\mathbf{j} \times \mathbf{B}$ , where  $\mathbf{j}$  is the current density and  $\mathbf{B}$  is the magnetic field. In these coaxial thrusters, current flows in the plasma between an inner cathode and a surrounding concentric anode in either pulsed (quasi-steady) or steady-state operation. The interaction of the high current discharge and either the self-induced (current-driven) or externally-applied magnetic field accelerates the plasma to produce thrust.

MPD thrusters can achieve one of the highest power-processing and thrust capabilities amongst EP systems, from 100s of kilowatts up to several megawatts and 10s to 100s of Newtons per thruster at high specific impulses ( $I_{sp}$ ) of 1,000–10,000 s and higher.<sup>14</sup> Therefore, like state-of-the-art ion and Hall thrusters, MPDTs are capable of much higher specific impulses than those of conventional chemical thrusters, albeit with much higher thrust and power density for a given size of thruster. For a given total velocity change (delta-V) demanded by a high-power mission, MPDTs result in smaller and fewer thrusters and associated components than required by most other EP technologies. Ion thrusters and other well-established EP devices are not predicted to be scalable to the combined high specific impulse and relatively high thrust per engine of MPDTs due to fundamental limitations in their physical processes and achievable thrust densities. In many potential high-power applications, ion thrusters would not be practical due to the mass and volume of the large number of thrusters that would be required to process all of the necessary input power. Power levels of 100s of kW per thruster or higher may be required for enabling missions to the Moon, Lagrange points, asteroid, and Mars with megawatt-class spacecraft to support the large cargo infrastructure requirements of human space exploration, and piloted missions could demand even higher power levels (multi-MW) to efficiently deliver sufficient mass or decrease trip times.<sup>3–7</sup> Also, high-power MPDTs could support rapid-response orbital transfer and repositioning for future defense and commercial applications. Thus, MPDTs can effectively fill a high-power niche in advanced propulsion for future space missions.

MPDTs surpass ion and Hall thrusters in terms of power processed per thruster, but the state-of-the-art lifetimes, specific impulses, and efficiencies are not nearly as high as currently achievable by ion and Hall thrusters. Presently, observed performance of MPDTs is typically between 30–50% electrical-to-jet power conversion efficiency, and thruster lifetimes much beyond 1000 hours are challenging.<sup>14</sup> Some limited testing with lithium MPDTs has demonstrated promising efficiencies above 60%.<sup>15,16</sup> One could compare such performance metrics to state-of-the-art ion thruster development, where efficiencies as high as 81% and operation for tens of thousands of hours have

been demonstrated for ion thrusters.<sup>17</sup> Consequently, there is a compelling case for pursuing research in MPDTs to achieve improved efficiencies and lifetimes in the higher power and thrust regime not achievable by ion thrusters. The long-term goals are to enable higher specific impulse operation at the increased power levels of interest, significantly increase efficiency, and enhance lifetime perhaps up to an order of magnitude or more.

The steady-state, self-field MPDT and applied-field MPDT (AF-MPDT) are high-power EP technologies with significant potential for increased performance to enable future mission applications. The self-field MPDT operates without magnets, using only its self-generated magnetic field for acceleration. AF-MPDTs use additional externally-applied magnetic fields typically to increase the thrust, particularly at relatively lower powers (10s of kW to order of 200 kW). Many of the performance characteristics of AF-MPDTs with applied magnetic fields that were large relative to their self-generated fields were well summarized by Kodys.<sup>16</sup> Thrusters operating with lithium (Li) as the propellant were found to have the highest efficiencies, in part due to the low first ionization potential of Li.

In an MPD thruster, plasma acceleration is governed by the magnetohydrodynamic (MHD) momentum equation, given by

$$\rho \frac{D\mathbf{u}}{Dt} = \mathbf{j} \times \mathbf{B} - \nabla P + \nu \rho \nabla^2 \mathbf{u} \approx \mathbf{j} \times \mathbf{B} - \nabla P \quad (2)$$

where  $\rho$  is the mass density,  $\frac{D\mathbf{u}}{Dt}$  is the convective derivative of the center-of-mass velocity vector,  $\mathbf{j}$  is the current density vector,  $\mathbf{B}$  is the magnetic field vector, and  $P$  is the pressure. Initially, a viscous damping term is included (cf., Bellan<sup>13</sup>), where  $\nu$  is the kinematic viscosity. The kinematic viscosity is primarily due to ion-ion collisions and neutral collisions and is typically very small for plasma conditions of interest. The Lorentz force term,  $\mathbf{j} \times \mathbf{B}$ , is the electromagnetic contribution to the force densities and becomes significant at high currents. Near the centerline, the pressure gradient term dominates. The electromagnetic accelerating forces are shown in Figure 1 for the geometry of poloidal current densities and their induced azimuthal magnetic field. The “blowing” component of the Lorentz force density,  $f_z$ , is given by

$$f_z = j_r B_\theta \quad (3)$$

and accelerates the plasma axially (resulting in thrust). The “pumping” component of the Lorentz force density,  $f_r$ , is given by

$$f_r = j_z B_\theta \quad (4)$$

and acts to radially constrain the plasma inward. In the radial direction, electromagnetic pumping forces are balanced by the radial kinetic pressure gradient. In the axial direction, electromagnetic blowing forces and the axial kinetic pressure gradient result in acceleration of the plasma.

At high currents, increased electromagnetic forces pinch the discharge toward the centerline, thus “starving” the near-anode region of charge carriers. In addition, the increased magnetic field near the centerline and cathode impede the mobility of electrons across the magnetic field lines.<sup>15</sup> This reduction in electron transport to the anode at increasing discharge currents causes an increased magnitude in the anode fall voltage (potential difference between anode potential and local plasma potential) to accelerate the electrons to accommodate the necessary charge transport. However, the increase in anode fall voltage also results in increased energy flux to the anode. Significant power can be lost to the anode (reducing efficiency), and the high heat fluxes severely affect anode lifetime by inducing evaporation of anode material. In addition, in this “onset” condition (as it is commonly referred to in the literature) anode spots begin to form and vaporize material on the anode surface, resulting in significant anode erosion and large terminal voltage fluctuations,<sup>18</sup> as will be discussed in more detail in Section B. Operation at high currents well into these onset conditions is unstable, inefficient, and ultimately impractical due to severe erosion. Some means to control or mitigate the physical processes driving these deleterious behaviors must be identified to enable higher power, high efficiency, and long lifetime operation. Approaches using tailored and localized applied magnetic fields are explored in this work.

## B. Review of Previous Work

### 1. Onset Phenomena

One of the critical topics that has pervaded much of the MPDT literature has been the issue of “onset” phenomena, as introduced by Malliaris et al.<sup>19</sup> at AVCO Corporation in 1972. It was also referred to as “critical current” or “critical mode” in the Russian literature.<sup>20,21</sup> Onset represents a collection of operating behaviors associated with a transition to large-amplitude terminal voltage fluctuations, transients in plasma properties, and growth in anode fall voltages in the plasma. Operation within onset conditions is associated with unstable operation and anode erosion. For a given thruster geometry and propellant choice, this transition occurs at some particular value of  $J^2/\dot{m}$  (where  $J$  is the total discharge current and  $\dot{m}$  is the mass flow rate) as either current is increased (at fixed mass flow rate) or flow rate is

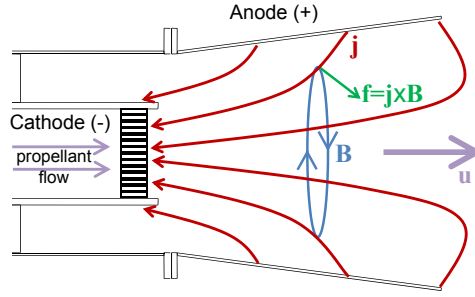


Figure 1. Illustration of MPD thruster current, magnetic field, and acceleration forces.

decreased (at fixed current). Once the transition to onset occurred, the thruster operation was characterized by high-amplitude terminal voltage fluctuations.<sup>22,23</sup> Also, Hugel<sup>24</sup> and Diamant<sup>25,26</sup> identified the formation of “anode spots” during onset. These anode spots are small, discrete points of current concentration on the anode surface and melting of the anode material. An important overall effect was that operation of the thruster in these onset conditions resulted in excessive erosion of the thruster materials, with particularly increased erosion of the anode surface.<sup>27</sup>

Malliaris et al.<sup>19</sup> identified that the critical value of  $J^2/\dot{m}$  where transition to onset occurs scales as  $\sim M_{ion}^{-1/2}$ , where  $M_{ion}$  is the ion mass of the propellant species. Therefore, lower atomic mass propellants (e.g., lithium, hydrogen) should allow stable operation (before onset) at higher currents before transition to onset-related behaviors at a fixed flow rate, which has been consistent with other experimental findings.<sup>15,28</sup> Also, thruster geometries and flow conditions that enabled increased particle densities near the anode were found to increase the condition at which transition to onset occurs. For example, reducing the ratio of the anode to cathode radius ( $r_a/r_c$ ) to increase particle density<sup>19</sup> or increased propellant injection at higher radii (closer to the anode)<sup>29,30</sup> were found to increase the transitional value of  $J^2/\dot{m}$ .

Different models of what effectively causes onset have been proposed. Uribarri<sup>31</sup> classifies these models into two categories: anode starvation models and plasma instability models. In the anode starvation models, current conduction is considered to be a sheath-limited process. Electromagnetic radial pumping forces increase with current and cause what is commonly referred to as “anode starvation” or “anode depletion,” in which a reduction in plasma density near the anode results in fewer charge carriers locally available to support the increasing current.<sup>20,32,33</sup> However, as discussed by Baksht and Shubin,<sup>20,34</sup> the current that can be drawn from the near-anode plasma is limited by the random thermal flux of electrons to the anode surface, which is directly proportional to local number density. This effect is only exacerbated by the presence of a transverse magnetic field (e.g., the azimuthal self-field of the MPDT) that impedes electron diffusion radially outward to replenish the electrons hitting the anode.<sup>35</sup> When the random thermal flux of electrons to the anode is insufficient to supply the requisite current density, electron-attracting anode fall voltages must form across the sheath and near-anode plasma to enable sufficient electron flux.<sup>32,36</sup> Therefore, any mechanisms to either increase local electron number density near the anode or to more generally increase electron transport to the anode (without increasing the anode fall voltage) can potentially mitigate the sheath-limited aspects of onset.

Different plasma instability models have also been proposed as possible causes of onset. Authors have described the conditions for exciting drift instabilities due to the relative motion of the electrons and ions in the plasma currents, as described by Tilley et al.,<sup>37</sup> and various plasma wave excitations have been measured in MPD thrusters in onset conditions (e.g., ion acoustic waves observed by Tikhonov et al.<sup>33</sup>). Onset in such models is typically associated with exceeding some critical value of the drift velocity ( $v_d = \frac{j}{n|q_e|}$ , where  $j$  is the current density,  $n$  is the plasma number density and  $q_e$  is the fundamental electron charge) for the inception of current-driven instabilities.<sup>34,38</sup>

Uribarri<sup>18,31</sup> established a direct link between voltage fluctuations and anode spot formation and extinction, as well as a connection between the anode spots and melting or evaporation of anode material. He established that evaporated anode material effectively “seeds” the discharge with additional plasma to sustain the discharge and overcome current starvation. Uribarri also presented a capacitively-coupled model of how the spots could incite the terminal voltage fluctuations and postulated a current filamentation process as associated with spot formation. Recent work by Giannelli et al.<sup>39,40</sup> identified conditions for the formation of azimuthal instabilities that would lead to such current filamentation in the MPD thruster discharge. Their model linked such filamentation with the current concentration associated with anode spots.

One interesting onset suppression methodology has been to increase the local number density near the anode. Kurtz et al.<sup>23</sup> diverted a fraction (2–10%) of the propellant flow to injection sites on their anode. They were able to increase the current at which onset transition occurred, but only by order of 5% at 10% flow fraction to the anode. The effectiveness of this approach is also further limited by the inefficiencies associated with anode mass injection, namely that the propellant introduced near the anode does not yield as much effective acceleration from the electromagnetic body forces that are higher closer to the thruster centerline (further from the anode). Therefore, introducing increasing fractions of the propellant at the anode would result in decreasing thrust efficiency. Kuriki et al.<sup>41</sup> also studied the effect of injecting propellant near the anode and identified improvements in thrust and efficiency. However, they identified similar increases in performance simply with the inclusion of their anode injection hood even without any additional anode gas flow. As an alternative, one might examine externally-applied magnetic fields as a potentially more effective means of onset mitigation, as will be discussed in the next section.

## 2. Anode Fall Voltages and Applied Magnetic Fields

Another issue in addition to the fluctuations and transients introduced with onset is the formation of large anode fall voltages at high thruster discharge currents. Gallimore<sup>32,36,42</sup> experimentally studied the anode power deposition problem in quasi-steady, self-field MPDTs operated between 150 kW to 7 MW with argon and helium propellants. He found approximately 20–40% of the power to the thruster being lost to the anode, a significant source of inefficiency for the thruster. He identified that the predominant cause of this power deposition was due to a significant anode fall voltage, i.e., the electron-attracting potential difference between the anode potential and the local plasma potential. He measured anode fall voltages of approximately 4–50 V in his tests. He explained the anode fall as due to the formation of large electric fields near the anode to provide sufficient current conduction across the strong azimuthal, self-generated B fields. Gallimore and Choueiri<sup>43</sup> proposed that this conduction was further limited by increased anomalous resistivity resulting from the onset of microturbulence in the plasma. Gallimore identified a connection between the electron Hall parameter ( $\Omega_e$ , the ratio of electron cyclotron frequency to electron collision frequency) and increasing anomalous resistivity (i.e., reduced conductivity). The Hall parameter increased as the azimuthal magnetic field transverse to the anode increased with higher discharge currents. Work by Soulas and Myers<sup>44</sup> around this same time also confirmed the relationship between both increasing anode fall voltage and higher anode power deposition with increasing transverse magnetic field strength. Gallimore tested reducing the electron Hall parameter in the vicinity of the anode by embedding small permanent magnets around the anode to provide a canceling effect on much of the near-anode azimuthal magnetic field. His tests found that this local reduction of the Hall parameter decreased anode fall voltages by 37 to 50%. Gallimore suggested that perhaps using applied B fields intercepting the anode could reduce the anode fall voltage.

Foster<sup>45–47</sup> studied the mechanisms driving the formation of large anode sheath potentials in low pressure argon arc discharges in the presence of a transverse magnetic fields. He identified that the anode fall voltage was highly dependent on the local electron number density. Subsequently, he used an auxiliary electron discharge near the anode to increase local electron number density and found reductions in the anode fall voltage resulted in reduced anode power deposition of 15 to 25%. Also, Foster examined the application of an external magnetic field up to about 40 Gauss coaxial with the anode (intersecting B field), but he found this additional field to have surprisingly little effect on the anode fall voltage (only approximately 1–2 V decrease). However, he suggested that perhaps beneficial reductions in anode fall might only occur if the transverse magnetic field component were significantly decreased relative to the axial field. Also, the lack of a flowing plasma, simplified geometry (not coaxial electrodes), and different directions for the forces on the plasma in his experiments may be expected to yield different results than MPD thrusters.

Hoyt et al.<sup>48,49</sup> examined the use of various applied magnetic fields in high-power coaxial accelerators operating in quasi-steady pulses from 2–12 MW. The constant-radius, narrow-channeled coaxial accelerator geometry used in his study is somewhat different than that used for typical MPDTs, but the Lorentz force-driven acceleration mechanisms are the same. He was able to reduce the anode fall by approximately 30 V with an applied B field relative to self-field operation (no magnets). He used a “magnetic Laval nozzle” shape in his applied B field design, in which the field lines converged and then diverged again as the plasma moved in the downstream direction. This B field shape had field lines intersecting the anode in the region upstream of the exit plane.

At Russia’s Moscow Aviation Institute (MAI), studies were conducted by Tikhonov et al.<sup>50,51</sup> on applied-field lithium-fed thrusters. Their work involved primarily experimental testing and observations on AF-MPDTs at power levels up to 200 kW with specific impulse ( $I_{sp}$ ) of 4500 s and about 48% efficiency. From the results of these (and earlier) tests and some related modeling efforts,<sup>33</sup> Tikhonov suggested that the applied B field and thruster design should be such that the diverging anode surface be contoured to follow the local applied B field lines. This suggestion would result in a transverse-only (non-intersecting) B field near the anode, which is counter to earlier recommendations

by Gallimore, Hoyt, and others. Tikhonov suggested that contouring the B field along the anode in this manner avoids the concentration of anode current attachment to small areas near the anode exit, which results in undesirably high local anode heating and erosion. Localized current attachment downstream on the anode is an expected consequence of advection of the current streamlines downstream by the flowing plasma. He claimed that by contouring the anode with the B field, the current streamlines were shown to spread out over a larger area of attachment, thereby reducing anode current densities.

Also, testing in Russia of a 400–500 kW steady-state, Li-fed MPDT by Ageyev et al.<sup>15</sup> at 8–10 kA demonstrated stable operation and thrust efficiency of 60%. The operating conditions for this thruster should have put it in operation above onset. However, Ageyev used a relatively weak 100 Gauss solenoidal applied B field near the anode exit plane and was able to achieve stable, efficient operation. However, limited details were provided for this empirical demonstration.

In Japan, work was conducted by Tahara, Kagaya, et al.<sup>52–55</sup> on quasi-steady MPD thrusters with externally-applied magnetic fields. Tahara identified that a strong, mostly axial applied magnetic field (mostly parallel to the anode) could suppress the magnitude of voltage fluctuations in their thruster operating conditions. These experiments by Tahara et al.,<sup>53</sup> resulted in higher thrust and somewhat reduced cathode erosion with the applied B fields. However, they used very high applied magnetic fields of order 1000–2000 Gauss. Also, experiments by Kagaya, Tahara, et al.<sup>52,55</sup> showed that their applied magnetic fields (1000–5700 Gauss) notably increased the terminal voltages and anode voltages for the discharge relative to self-field operation.

To date, there is still limited understanding of the processes that govern the overall current conduction in the near-anode region. Additional discussion of related research background can be found in Moeller’s thesis.<sup>56</sup> Gallimore and Foster’s primary motivations and investigations were to reduce anode fall voltage and not characterizing and understanding the current distribution. Further, differences in geometry, plasma flow conditions (or lack thereof), magnetic topologies, and field strengths between Gallimore’s Princeton Benchmark Thruster, Hoyt’s coaxial accelerator, Foster’s quiescent plasma experiment, and Tahara’s thrusters can yield particularly different structure to the current streamlines and conduction in the near-anode region. Moreover, there is a conflict between competing suggestions from authors such as Gallimore and Hoyt (anode-intersecting B fields) and Tikhonov (anode-parallel B fields) for the prospects of using applied B fields to reduce anode falls but also avoid excessive current densities. Further, no details were made available about the current pattern or plasma properties of the promising Ageyev thruster design. Therefore, our research sought to understand the plasma discharge current pattern and conduction in the near-anode region of an MPDT and potential for mitigating onset-related behaviors.

### C. Goals and Approach for This Research

The goal of this research was to investigate the effects of tailored and localized externally-applied magnetic fields at modest field strengths on the plasma discharge and examine the prospect of their use to mitigate the behaviors associated with onset. The motivation is to determine how one can influence the current conduction to mitigate performance losses and anode erosion, thereby leading to potential increases in thruster efficiency, thrust, and lifetime.

For this investigation, the following questions were posed:

1. Can one mitigate behaviors such as the large-amplitude terminal voltage fluctuations and large anode fall voltages with applied magnetic fields primarily focused on the near-anode region?
2. What are the effects of the tailored applied magnetic fields on the plasma properties and current transport in the thruster plasma discharge, particularly in the near-anode region?

There are several components to addressing this goal. The first investigation was to characterize the thruster terminal voltage-current characteristics to identify onset-related transitions and the growth of discharge voltage fluctuations (commonly referred to as “voltage hash”). This characterization was performed with the thruster operating in both self-field mode without magnets and with two distinct, tailored, and localized applied magnetic field topologies. The second study investigated the structure and behavior of the magnetic field, current pattern, and current densities inside the thruster discharge with a new magnetic probe array. To understand the influence on parameters that drive the changes in current conduction to the anode, a new triple Langmuir probe was used to study changes in the local plasma number density, electron temperature, and plasma potential with and without the applied magnetic fields. This progression of investigations allows us to examine the response of the thruster in transition to and above onset first at the system level (electrode voltages and related fluctuations), then in the plasma discharge bulk structure (internal magnetic field and current transport), and finally in the local plasma properties of the near-anode region (Langmuir probe measurements). The subsequent sections of this paper are organized to convey the implementation approach and results of these investigations.

## II. Experimental Setup and Diagnostics

### A. MPD Thruster and Pulse-Forming Network

To provide the fundamental testbed and framework for this research, a new high-power MPD thruster (MPDT) designated as “Odysseus” was designed and fabricated, as shown in Figures 2 and 3. The overall geometric proportions of the new Odysseus MPD thruster were modeled after the Russian Ageyev-type extended anode design with a tungsten multi-channel hollow cathode<sup>15</sup> and scaled up slightly in size while maintaining a similar anode to cathode radius ( $r_a/r_c$ ) of 2.52 based on the anode exit radius and cathode outer radius. The inside of the multi-channel hollow cathode (MCHC) is tightly packed with thoriated tungsten welding rod segments each 0.64 cm diameter by 2.5 cm length. The interstitial spaces between adjacent cathode rods form open areas that effectively act as hollow cathodes, and the resulting increase in emission area allows the MCHC to provide the necessary high discharge currents with lower current densities than with a comparably sized solid rod cathode. Propellant is injected through the multi-channel hollow cathode as in the Ageyev thruster, not in the inter-electrode gap as is typically done in gas-fed MPDTs. The anode is stainless steel, with a straight upstream segment leading to a downstream flared section. A boron nitride backplate is used to insulate between the anode and cathode in the upstream end of the discharge chamber. External cathode and anode-potential surfaces were also covered with insulating Kapton. Aluminum was used for the rest of the mounting and support structure.

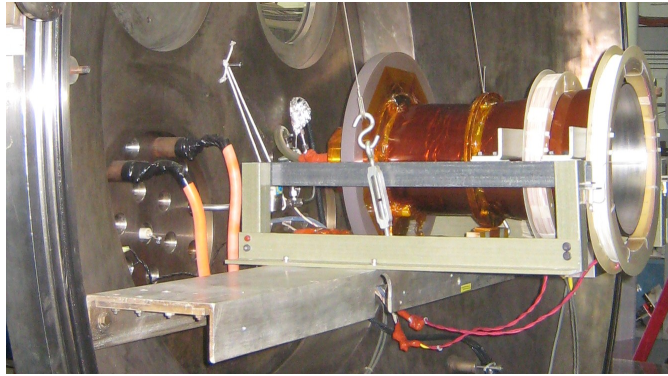


Figure 2. Odysseus MPD thruster with magnets mounted on the vacuum chamber header.

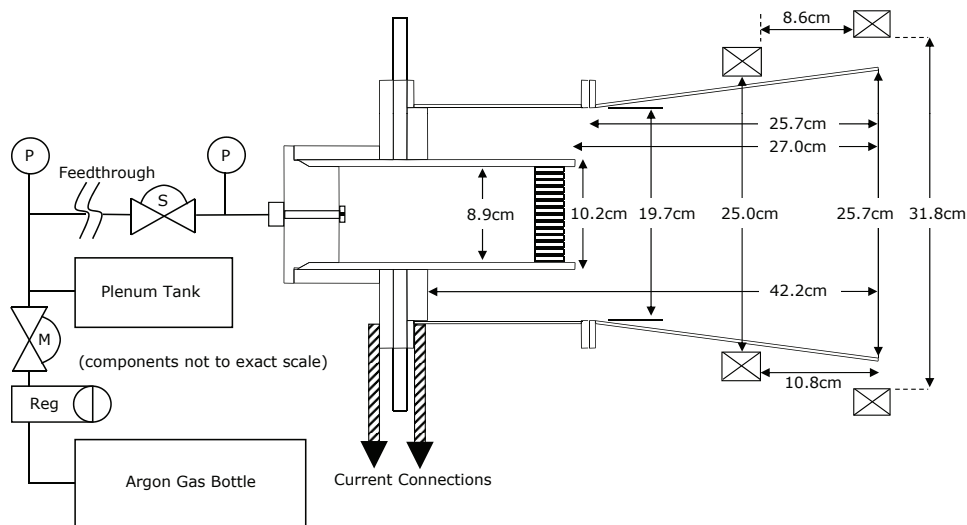


Figure 3. Schematic of the MPD thruster, applied-field magnets, and gas flow system (gas flow system components are not to scale).

The MPDT was operated at quasi-steady power levels from 36 kW (20 V, 1800 A) to 3.3 MW (255 V, 13.1 kA) and 1.0 g/s argon mass flow rate. Due to cost and facility constraints that precluded high power steady-state

operation, the thruster and discharge circuit were designed to operate in a quasi-steady pulsed mode. The quasi-steady portions of the discharge pulses are long enough for the plasma dynamics of interest to reach steady state without requiring refractory materials in all parts except the cathode. The current discharge pulses were generated by two pulse-forming network (PFN) banks consisting of a sequence of capacitors and inductors that were sized to provide a quasi-steady current pulse with a steady segment of 1–2 ms at these power levels. Such high-power, quasi-steady operation has been commonly used in many MPDT investigations to study processes relevant to both pulsed and steady-state operation.<sup>19,52,57</sup>

## B. External Magnets and Applied Magnetic Fields

To investigate the influence of tailored, externally-applied magnetic fields on the thruster discharge, two solenoidal electromagnet coils were designed to mount as rings around the outer body of the anode flared section, as seen in Figures 2 and 3. The magnitudes and polarities of the currents in the two magnets could be driven independently to enable alteration of the shape and magnitude of applied poloidal magnetic field generated. Note that the term “poloidal” denotes vectors with components only in the radial and axial directions (not azimuthal). Power supplies for the magnets were triggered before the thruster to ensure steady currents in the magnets and steady applied fields during thruster firing.

Two distinct, tailored applied B field configurations were used for most of the experimental testing. One “tangential” configuration operated both magnets in the same polarity to generate magnetic field lines mostly tangential to the anode lip near the anode exit plane and anode-intersecting field lines upstream. The other “cusp” configuration operated the magnets in opposing polarities, which generated a cusp structure with magnetic field lines intersecting the anode in the region between the magnets. These two configurations can be seen in Figures 4 and 5. At the specified operating currents, applied magnetic field strengths in the vicinity of the magnets (the downstream section of the anode) were mostly on the order of 50–200 Gauss, with somewhat higher peaks immediately next to the upstream magnet. Note that the cusp B field had lower peak B field magnitudes near the anode. The applied magnetic field magnitudes were much lower near the cathode and centerline. At the location of the cathode tip (downstream front face) and cathode outer radius, the applied magnetic field was 40 Gauss for the tangential field and 11.5 Gauss for the cusp field. At the centerline of the cathode tip, the applied magnetic field was 42 Gauss for the tangential field and 12 Gauss for the cusp field. Also, the steady-state power applied to each magnet only ranged from 160–360 W, which was an insignificantly small fraction of the power to the thruster (36 kW to 3.3 MW).

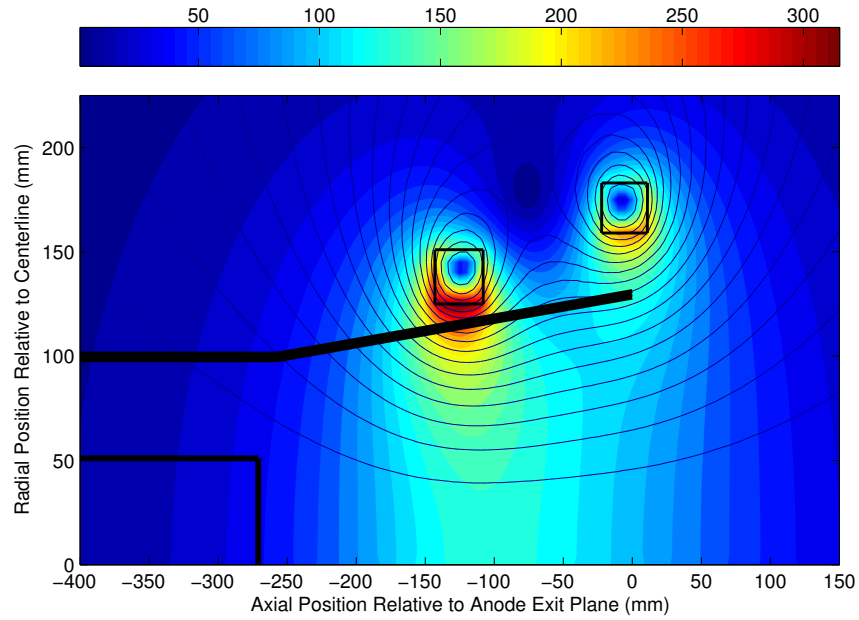
In this investigation, the applied B fields used differ from those typically used in the vast majority of conventional, so-called “applied-field MPD thrusters” (AF-MPDTs).<sup>16</sup> AF-MPDTs generally use much higher applied B field magnitudes relative to the thruster self-field, and their applied B field line structures (magnetic topologies) are generally predominantly axial, with very limited variation in curvature in the inter-electrode region. In our study, the applied B fields were designed to be smaller in relative magnitude to somewhat localize the effects of the applied magnetic fields to address near-anode phenomena. At the higher currents of interest for onset, our applied B fields yielded  $B_{\text{applied}}/B_{\text{selffield}}$  much less than one near the outer radius at the cathode downstream face, and this ratio was only greater than one over the downstream section of the anode. The applied magnetic fields were also designed to allow significant anode-intersecting radial components to the topologies. These anode-intersecting regions were upstream along the anode (but still downstream of the cathode face) for the tangential B field and downstream closer to the anode exit for the cusp B field.

## C. Vacuum Facility and Gas Pulse System

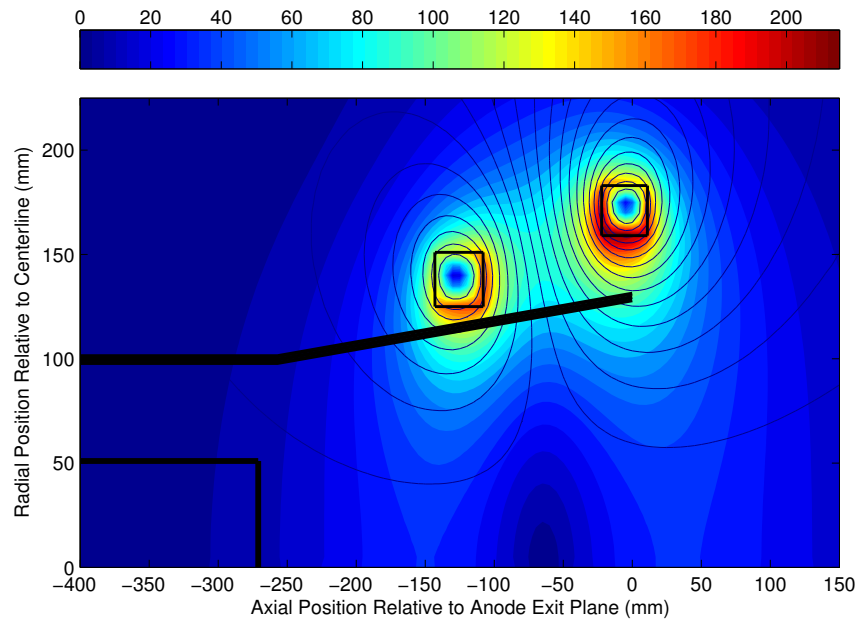
The MPDT was operated in a large 2.3 m diameter by 4.5 m cylindrical vacuum chamber facility at the NASA Jet Propulsion Laboratory (JPL). The MPDT was mounted to a removable header on the vacuum chamber and connected to gas and electrical feedthroughs on the chamber header, as shown in Figure 2. The background pressure was  $1\text{--}3 \times 10^{-6}$  Torr prior to each thruster firing.

An argon gas pulse system and drive circuitry were also developed and integrated into the timing circuit for the PFN. The major components of the gas feed system are shown in Figure 3. The gas system utilizes a 3.5 L plenum tank pre-filled with pressurized argon prior to each firing to match a specified flow rate. A triggered solenoid gas valve would flow argon through the cathode rods in the tube to supply gas to the thruster discharge upon reaching steady flow, not supplied from the surrounding annular region of the discharge chamber as is common in most gas-fed MPDTs.

Further details of these systems and calibration can be found in Moeller’s thesis.<sup>56</sup>



**Figure 4.** Applied magnetic field magnitude (shaded contours) and flux lines for the tangential applied magnetic field. Geometry for the magnets, thruster anode, and cathode are also shown. The top color bar units for the B field are in Gauss.



**Figure 5.** Applied magnetic field magnitude (shaded contours) and flux lines for the cusp applied field. Geometry for the magnets, thruster anode, and cathode are also shown. The top color bar units for the B field are in Gauss.

## D. Diagnostics

### 1. Terminal Voltage and Current Signals

Thruster discharge voltages and currents were measured during thruster firings to enable examination of terminal voltage versus current characteristic profiles and assessment of the growth of voltage fluctuations (“voltage hash”) at higher currents. Thruster terminal voltages were sampled with a 500:1 Tektronix P5205 high-voltage, active differential probe connected across the anode and cathode electrodes. The current was measured via a Pearson Electronics Inc. model 1330 pulse current monitor. These and other diagnostics’ measurement signals were recorded via a high-speed National Instruments digital data acquisition system (DAQ).

### 2. Magnetic Probe Array

Sampling the magnetic field generated inside the thruster provided a fundamental data set for this research. Mapping of the magnetic field structure inside the thruster enables investigation of the influence of the external applied magnetic field on the internal magnetic topology in the discharge region, the current streamlines for the discharge, and current densities along the anode. Also, local magnetic field measurements enable determination of key parameters such as the electron Hall parameter and electron Larmor radius.

A new magnetic probe array (MPA) was constructed for simultaneous sampling of the magnetic field along 15 inductor coils positioned along the probe. Each small inductor performed as a so-called “B-dot” probe (measuring the change in B field with time), enabling measurement of an induced voltage  $V$  across the coil winding due to the time-varying magnetic flux inside the coil given by Faraday’s law:

$$V = -N \frac{d\Phi}{dt} = -NA \frac{dB}{dt} \quad (5)$$

where  $\Phi$  is the magnetic flux bounded by the coil,  $N$  is the number of turns in the coil,  $A$  is the area of the coil, and  $\frac{dB}{dt}$  is the instantaneous time rate of change of the magnetic field penetrating the coil.

Fourteen small commercial inductor chips (Coilcraft Inc., model 1008CS-472XGGB) were used (each chip was nominally 52 turns each, with  $5 \pm 0.1$  microhenries inductance). The dimensions of the inductor chips were 2.8 mm by 2.9 mm by 2.0 mm. This approach was modeled after the work by Romero-Talamas, Bellan, and Hsu.<sup>58</sup> The chips were spaced axially every 20 mm for a total span of 260 mm, and these chips were inserted into a custom-fabricated Delrin plastic fixture made to fit these commercial chips’ dimensions. In addition, a hand-wound inductor coil was made by winding 105 turns of # 40 American Wire Gauge (AWG) magnet wire around a 3.5 mm diameter by 2 mm long core machined from Ultem rod. This hand-wound coil was glued to the tip of the Delrin probe stem with glyptal insulating enamel. While providing an additional 15th measurement, this hand-wound coil also acted as a calibration-correction coil for the smaller inductor chips due to the larger signal-to-noise ratio provided by the increased number of turns and larger coil diameter. All coils and the Delrin stem of the probe array were encased in an 8 mm diameter quartz tube to insulate them from the plasma. The tip of the probe and inductor coils can be seen in Figure 6.

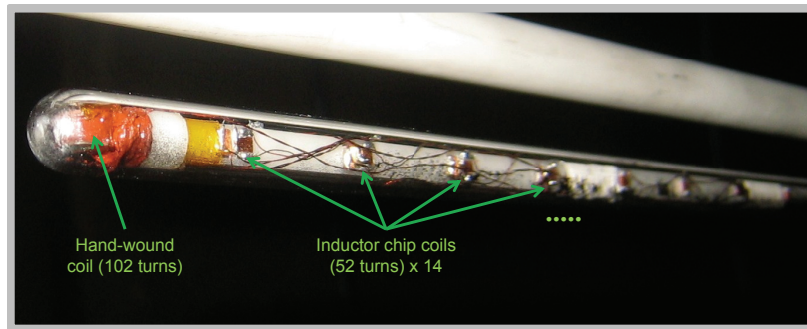


Figure 6. Close-up view of magnetic probe array with B-dot coils.

Downstream, the coil wires transitioned into an insulated stainless steel tube, which provided electrostatic shielding and also additional axial stand-off distance between the probe and positioning stage mounting hardware to minimize the obstruction of the thruster exit plane. Each coil’s output was fed through a simple R-C low-pass filter circuit with cutoff frequency of approximately 90 kHz and terminated at 100 Ohms.

Time integration of the measured B-dot probe's induced voltage signal provided a measure proportional to the absolute magnetic field strength versus time. This time integration was performed numerically on a computer using the digitally recorded voltage trace for each coil. The integrated voltage was converted to magnetic field strength by using a pulsed Helmholtz coil for calibration.

All coils used for the primary measurements were oriented such that they would measure the azimuthal component of the magnetic field inside the thruster. Enclosed current contours of the poloidal (radial and axial) current pattern were obtained by applying Ampere's Law to the magnetic probe array measurements in the interior of the thruster discharge and exploiting azimuthal symmetry. This is discussed in more detail in the references.<sup>56,58</sup>

### 3. Triple Langmuir Probe

A triple Langmuir probe was constructed to enable high-speed measurements simultaneously of electron temperatures, plasma number densities, and plasma potentials. The triple probe method uses a steady applied voltage between two small conducting probes (wire tips in our case), and a third conducting probe acts as a floating potential probe.<sup>59</sup> The triple probe does not require any special high-frequency sweeping of the bias voltage as in a single or double probe apparatus. This makes the triple probe particularly well suited to the noisy, rapidly time-varying discharge environment of the pulsed thruster. Details of the triple probe method and analyses are described in the references.<sup>25,56,59-61</sup> A photo of the sensing end of the triple Langmuir probe is shown in Figure 7.



Figure 7. Triple Langmuir probe and tungsten wire tips alongside magnetic probe.

To create the triple Langmuir probe, three tungsten wires each with 0.254 mm (0.010") diameter and 3 mm long were individually fed through small alumina ceramic tubes with 0.8 mm (1/32") outer diameter (OD) and sealed with ceramabond. Each of these ceramic tubes were glued into three inner holes spaced 1 to 1.5 mm apart within a 4-bore alumina ceramic tube with 4.8 mm (3/16") OD, and this was glued into the end of a stainless steel metal tube to act as an electrostatic shield. An outer alumina ceramic tube with 9.5 mm (3/8" OD) was placed over the metal tube to insulate the conductor from the plasma. This outer ceramic tube extended past the end of the 4-bore ceramic tube by approximately 1.5 cm to act as a "shadow shield" to mitigate against the layering of sputtering deposits that could coat the 3 small alumina tubes and form an electrically conducting layer between probe tips. The small, innermost ceramic tubes holding the tungsten wires extended an additional 1 cm past the outer ceramic tube to avoid any transverse or radial flow obstruction. For measurements near the anode, this geometry allowed measurements of plasma properties with the probe tips within approximately 3.5 to 4.5 mm away from the anode. Inside the metal tube, the thin tungsten wires transitioned to # 22 AWG insulated copper wires. The metal tube extended along the same long axis of the magnetic probe back to the positioning stage mounting structure, continuing outside the chamber to an external circuit box wired to the DAQ inputs.

The battery bias between two probe tips was made by two Eveready model number 732 lantern batteries connected in series for a fixed bias voltage of 25.5 V. Batteries were used instead of an external power supply because of their inherently floating isolation, low noise, and fast time response. The inputs and outputs of isolation amplifiers used for voltage measurements were passively filtered with R-C low-pass filters with cutoff frequencies circa 120 kHz. Measurements from both an isolated current probe (Tektronix TCP312 current probe and TCPA300 amplifier) and voltages from a current-sensing shunt resistor were used to verify ion saturation current measurements independently. Measurement signals were connected to the data acquisition system (DAQ) for high-speed, simultaneous digital recording.

For the triple Langmuir probe, expressions for calculating electron temperature  $T_e$ , electron number density  $n_e$ , and plasma potential  $V_{plasma}$  are derived in Chen and Sekiguchi's original paper<sup>59</sup> and interrogated further for error analyses in Tilley's paper.<sup>61</sup> (Note for clarification that in Tilley's paper, his schematic in Figure 1 shows the voltage

bias  $V_{d3}$  drawn incorrectly with the opposite polarity.) First, electron temperature  $T_e$  can be obtained from the implicit equation

$$\frac{1 - \exp(-\chi_{d2})}{1 - \exp(-\chi_{d3})} = \frac{1}{2} \quad (6)$$

where  $\chi_{d2} = \frac{|q_e|V_{d2}}{kT_e}$  and  $\chi_{d3} = \frac{|q_e|V_{d3}}{kT_e}$  are the non-dimensionalized potential differences,  $k$  is the Boltzmann constant, and  $q_e$  is the fundamental electron charge.

Electron number density,  $n_e$ , can be calculated from

$$n_e = \frac{\exp(1/2)I_{sat}}{|q_e|A_{probe}(\frac{kT_e}{M_i})^{1/2}(\exp(\chi_{d2}) - 1)} \quad (7)$$

where  $I_{sat} = I_3 = -I_1$  is the measured ion saturation current,  $A_{probe}$  is a single probe wire's exposed surface area (assuming equal sized probe tips), and  $M_i$  is the ion mass (argon in our case). Note that it can be shown that the expression  $(\exp(\chi_{d2}) - 1)$  reduces to approximately 1 when the bias voltage  $V_{d3}$  is much greater than the electron temperature.

The plasma potential,  $V_{plasma}$ , is calculated from the electron temperature by

$$V_{plasma} = V_f + \frac{kT_e}{|q_e|} \ln\left(\sqrt{\frac{2M_i}{\pi m_e}}\right) \quad (8)$$

where  $V_f$  is the floating probe potential measured from  $V_2$ , and  $m_e$  is the electron mass. Later, our anode fall voltages,  $V_{fall}$ , will be calculated as

$$V_{fall} \equiv V_{anode} - V_{plasma} \quad (9)$$

where  $V_{fall}$  is calculated as positive for electron-attracting fall potentials, and the anode potential  $V_{anode}$  is measured separately as a reference voltage and averaged during the quasi-steady thruster firings.

Both  $T_e$  and  $n_e$  are calculated with the corrections for the ion current collected at the probe potential relative to the plasma potential as given in the numerical calculations by Laframboise.<sup>62</sup> Laframboise's calculations account for finite sheath thickness and ion temperature for a cylindrical probe in a collisionless and quiescent plasma, but are also applicable to a probe whose axis is aligned with the plasma flow. Measured signals were averaged over the 1 ms quasi-steady time period and across multiple thruster shots to calculate mean values and associated statistics (e.g., standard deviations for use in estimating uncertainties) for each operating condition. Additional sources of error accounting and conditions for validity that were met for the triple Langmuir probe theory were assessed and discussed in detail in Moeller's thesis.<sup>56</sup>

#### 4. High-Speed Video Imaging

A high-speed digital video camera was used to obtain images of the thruster discharge. These videos were taken during early testing with the thruster operating at 9 kA in self-field mode only (without magnets). These measurements used two Vision Research cameras, including model Phantom V710 and model Phantom V7.3, which were particularly useful for providing qualitative insights into transient phenomena in the plasma discharge during quasi-steady operation. The high-speed images used exposure times of 1-5  $\mu$ s and imaging rates of 10,000 frames/s (100  $\mu$ s intervals between frames).

### E. Operating Conditions and Thruster Configurations During Measurements

Thruster discharge voltages and currents were measured during thruster firings to construct voltage versus current characteristics and determine the influence of the applied magnetic fields on the voltage fluctuations ("voltage hash") at higher currents. The magnetic probe array (MPA) was used during thruster firings to acquire data to reconstruct the topology of the magnetic fields and discharge currents in the thruster plasma. The Langmuir probe made local measurements at selected locations in the thruster of key plasma properties, namely the electron temperature ( $T_e$ ), electron number density ( $n_e$ ), and the anode fall voltage measured from the plasma potential ( $V_{fall} = V_{anode} - V_{plasma}$ ). The focus of most Langmuir probe measurements was the near-anode region, particularly in the regions of highest current densities calculated from the previous magnetic probe measurements. The probes were repositioned using a three-axis positioning stage.

The time-varying signals from the diagnostics were averaged over the thruster's quasi-steady time period, generally 0.8 to 1.8 ms after triggering the PFN discharge. These results were averaged over both time and across multiple shots

at each operating condition to generate the means and estimate random uncertainties that are presented in the following sections.

Measurements of thruster voltages, discharge currents, and the MPA were obtained over a range of PFN voltages to span thruster current levels from 1.8 kA to 13.1 kA. This broad range was selected to cover a wide set of operating conditions from well below onset to well above onset and examine whether there were any transitions or trends. The flow rate was held constant at 1.0 g/s. This flow rate was sufficiently low to allow a range of  $J^2/\dot{m}$  values (approximately 4 to 172 kA<sup>2</sup>s/g) well above typical onset values for other thrusters (e.g., beyond 80 to 110 kA<sup>2</sup>s/g in the Princeton Benchmark Thruster<sup>31</sup>), yet still provided sufficient flow in our large thruster geometry for stable, repeatable operation. All measurements were taken over the three primary configurations, including no magnets (self-field operation), the applied tangential B field, and the applied cusp B field.

However, most of the measurements with the MPA were taken at a relatively high current of 10.7 kA, including all of the data at smaller radii in the interior (away from the anode). It is this larger set of data at this higher current that allows for the contour plots of magnetic fields and current streamlines in the thruster interior that are discussed in Section IV-B. This high current is well in the onset regime, as evidenced by the large-magnitude terminal voltage fluctuations shown in Section III. This discharge current value was chosen as a representative and repeatable condition for evaluation of the thruster properties nominally within onset, and, in particular, to observe key spatial variations in the thruster discharge between the self-field operation (no magnets) and the configurations with the tangential and cusp applied magnetic fields.

Measurements with the Langmuir probe were obtained at specific thruster discharge current levels ranging from 8 kA to 10.7 kA, with most data taken at the higher 10.7 kA current (well into onset conditions and paired with the data from the MPA). Additionally, operation at 8 kA was conducted at two locations near the anode where the current densities were found to be highest to examine any relative differences at this more moderate, intermediate current where the thruster was shown to just begin transition to the early effects of onset.

### **III. Influence of the Applied Magnetic Fields on Voltage-Current Characteristics and Voltage Fluctuations**

#### **A. Collection of Data**

The thruster terminal voltage and current traces were measured during each PFN firing. As expected, an increase in the magnitude of the voltage fluctuations is observed with increasing discharge current. To see the structure of the voltage hash, the quasi-steady portion of the time series traces are examined for the three applied B field configurations at three different operating current levels. The voltage traces in Figure 8 show example traces for the case without magnets (self-field operation). A fairly quiescent mode of operation is observed in the voltage response at the lower 5.5 kA. However, an intermediate mode of operation is observed in the 8.0 kA trace wherein the voltage fluctuations become more pronounced, albeit still primarily oscillating equally positive and negative relative to the mean. At the higher 10.7 kA, a substantial growth in the magnitude of the voltage hash is exhibited along with predominantly positively biased spikes in the voltage fluctuations relative to the mean. These results are consistent with the behaviors and three different regimes of voltage hash observed by Uribarri in his own data for the Princeton Benchmark Thruster.<sup>18</sup>

The influence of the applied magnetic fields is examined next. Figure 9 shows example voltage traces with the thruster under the influence of the tangential applied B field at the same operating currents. In this case, there is a reduction in the mean voltage relative to the self-field case, at least for the 5.5 kA and 8.0 kA cases. Overall, a reduction in the magnitude of the voltage fluctuations is also evident. The reduction in the mean voltages and the reduction in the magnitude of the spikes will be analyzed statistically and discussed further in the following section. The three types of regimes of structure to the voltage hash found in the self-field case are also observed with the tangential applied B field. These include a quiescent mode in the 5.5 kA case, an intermediate mode in the 8.0 kA case, and a positively biased spiky hash in the 10.7 kA case.

Figure 10 shows examples of the voltage traces with the applied cusp B field. This case responds similarly to the tangential B field configuration, with reduced mean voltage and a reduction in the amplitude of the voltage fluctuations. Moreover, a somewhat steadier response is observed in the cusp B field voltage time series, exhibiting fewer and smaller-amplitude transitory excursions (drops or rises relative to the moving average) than were observed in the other two configurations. This response is indicative of a slightly steadier and stable behavior due to the cusp B field.

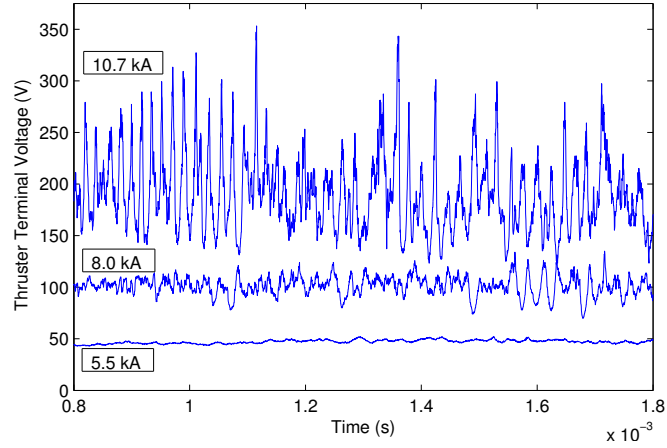


Figure 8. Example voltage traces during quasisteady period (1 ms duration) for the no-magnets case (self-field operation).

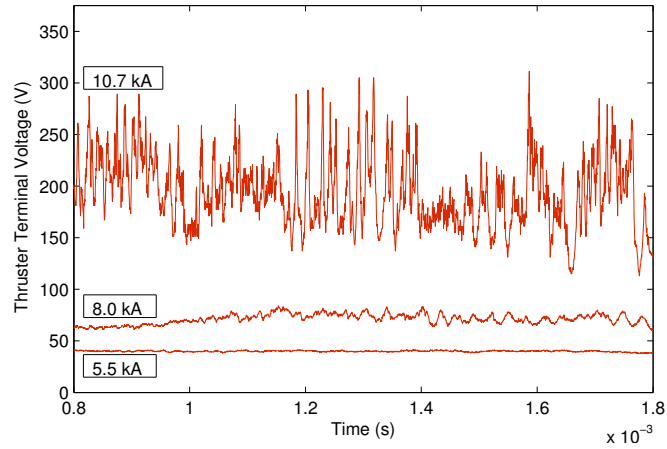


Figure 9. Example voltage traces during quasisteady period (1 ms duration) for the tangential applied B field case.

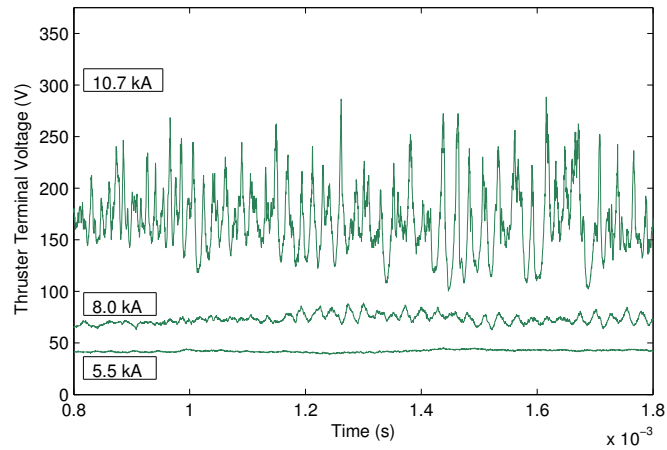


Figure 10. Example voltage traces during quasisteady period (1 ms duration) for the cusp applied B field case.

## B. Reduction of Mean Terminal Voltages and Fluctuations

Examination of the thruster terminal voltage response begins with analyzing the voltage-current characteristics (also commonly referred to as the “V-J curves” in the literature). Each individual voltage time series is analyzed over the 1 ms quasi-steady period from 0.8 ms to 1.8 ms. Additionally, multiple shots of the thruster were taken at each discharge current condition shown and then repeated for each applied B field configuration. These multiple shots were averaged to obtain the data shown in Figure 11. The error bars shown in the plots represent the standard errors of the means, and are dominated by the shot-to-shot variability. As expected, monotonically increasing growth in mean terminal voltage occurs as the current is increased. An initially linear trend for lower currents is observed below approximately 8.5 kA. Above this value, the voltage increases more rapidly with increasing current. This trend is typically associated with a higher growth rate in voltage with increasing current above full ionization conditions.<sup>57</sup>

The V-J curves exhibit one of the most significant effects of the applied magnetic fields. With the applied magnetic fields, a significant decrease in the thruster’s mean terminal voltage occurs for a given current level over a broad range of currents. For example, with the tangential applied magnetic field, the mean voltage is reduced by  $25.7\% \pm 3.9\%$  (95% confidence interval) relative to the self-field at 8.0 kA. A similar reduction of  $24.2\% \pm 4.0\%$  is seen with the cusp applied magnetic field at 8.0 kA. At 9.0 kA, a  $30.8\% \pm 8.5\%$  reduction and a  $30.9\% \pm 8.4\%$  reduction are measured for both the tangential and cusp fields, respectively. As current is increased further, the tangential field case begins to approach the response without magnets. However, the cusp magnetic field configuration still exhibits a relative reduction of  $13.0\% \pm 6.0\%$  at 10.7 kA and a slightly decreased but still notable voltage reduction through the full range of measured currents (13.1 kA). All uncertainties cited above are based on 95% confidence intervals.

This mean voltage reduction is counter to the results of Tahara, Kagaya, et al.,<sup>52,55</sup> who measured increased terminal voltages with their applied B fields relative to self-field operation. However, their applied B fields were much higher magnitude (1000-5700 Gauss), and they were much more axial in shape throughout the thruster discharge (and primarily parallel to the anode).

The standard deviations of the terminal voltage traces are shown in Figure 12. The standard deviations also exhibit a significant reduction in magnitude over a range of currents for the applied B field configurations. This is indicative of a reduction in the magnitude of the voltage hash, i.e., lower-amplitude RMS fluctuations about the mean voltage. Relative to the case without magnets, the tangential applied B field case yields a 36.6% reduction at 8 kA and 45.3% reduction at 9 kA. At 10.7 kA, the tangential B field case exhibits a small 6.3% reduction, but the differences are beginning to return to a similar range of response as the self-field case. For the cusp applied B field, a reduction of 39.5% is observed relative to the self-field case at 8 kA, 48.7% reduction at 9 kA, and still 14.8% reduction at 10.7 kA. Beyond this current level, the cusp B field response also trends towards the same behavior as without the magnets. There is a transition to significantly increased growth in the standard deviations with increasing current that occurs between 7.5 kA to 9 kA. This transition is near the same current range where the thruster voltage transitioned to a stronger increasing dependence on current.

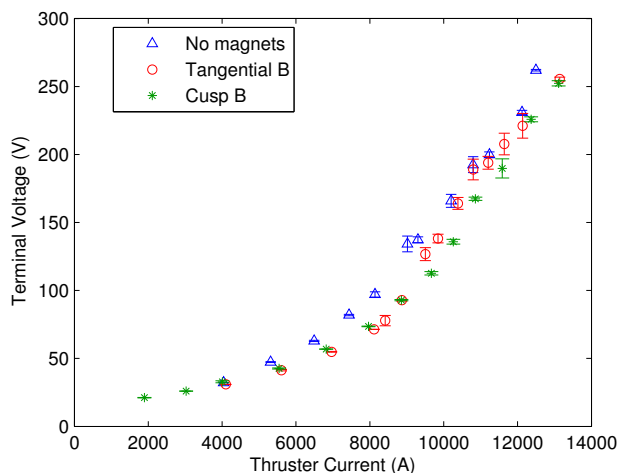


Figure 11. Mean terminal voltage vs. current characteristics comparing cases with no magnets, tangential B field, and cusp B field.

To examine similarities in thruster behavior, compare our results to the data observed by Uribarri for their Princeton Benchmark Thruster in his examination of the voltage signals.<sup>31</sup> Our investigations showed similar trends in the

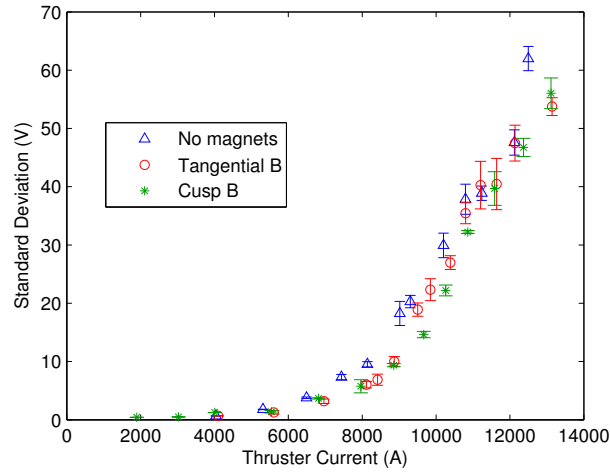


Figure 12. Standard deviation vs. current comparing cases with no magnets, tangential B field, and cusp B field.

growth of the means and standard deviations, including a departure to significantly growing standard deviations (i.e., magnitudes of the fluctuations) approximately 90 to 100 kA<sup>2</sup>/s/g. These results will be discussed further in Section D.

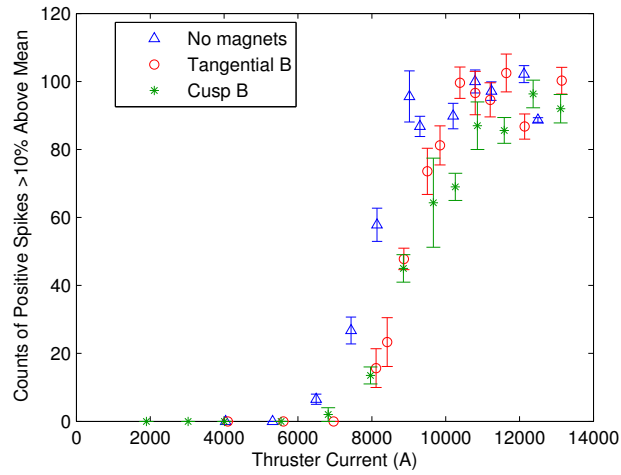
Additional statistical analyses of these voltage traces were described in Moeller's thesis,<sup>56</sup> including higher-order moments of the distribution (skewness and kurtosis).

### C. Analysis of the Transients and Power Spectrum of the Voltage Signal

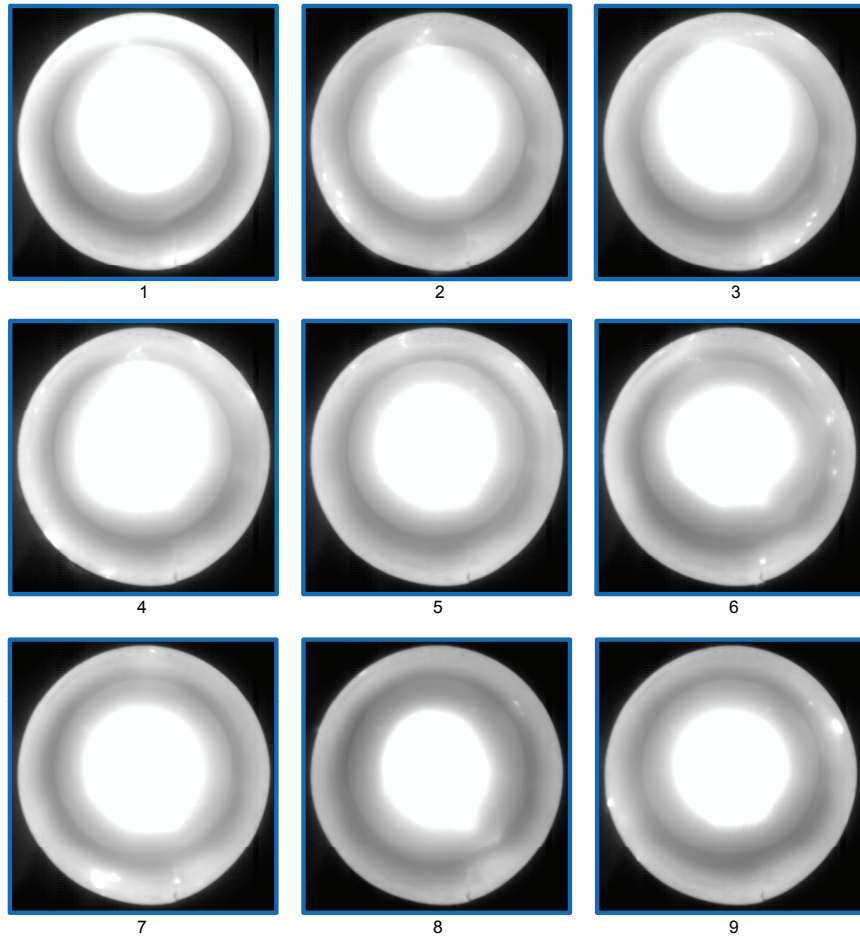
To explore further into the temporal and frequency-dependent response of the voltage signal, various aspects of the transient spikes and power spectra of the voltage signals were analyzed. A threshold was chosen of 10% above the mean thruster voltage for each signal. Then, the positive spikes that exceed this threshold were analyzed, again performing the same averaging over multiple shots and uncertainty analysis for the standard error to characterize shot-to-shot variation.

First, the number of positive spikes were counted that exceed this threshold of 10% above the mean voltage over the 1 ms quasi-steady period, as seen in Figure 13. This effectively represents a sort of "frequency" metric for the spikes. A clear transition to sufficiently high-amplitude spikes occurs beginning at 5.5–6.5 kA for the self-field case, and the number of spikes grows steadily until approximately 9 kA, where the number of spikes approaches a constant value. The initial transition begins at somewhat higher currents, circa 7–8 kA for both applied field cases, and the values increase until approximately 10.7 kA. Overall, the applied magnetic fields increase the current threshold for the rapid growth of large-amplitude spikes by approximately 1–2 kA. The spike-reducing effects of the applied fields can be seen more clearly if the response is examined at fixed current values. For a fixed, specified current in the transition region between approximately 6 to 10.7 kA, the applied magnetic fields result in a significant drop in the number of positive spikes. For example, the applied magnetic fields result in 73–77% reduction in the number of spikes at 8 kA and 47–50% at 9 kA. Because anode spots have been shown to correlate with voltage hash, one can argue that the number of spikes is related to the number of anode spots being formed on the surface of the anode, as indicated in the work by Uribarri.<sup>31</sup>

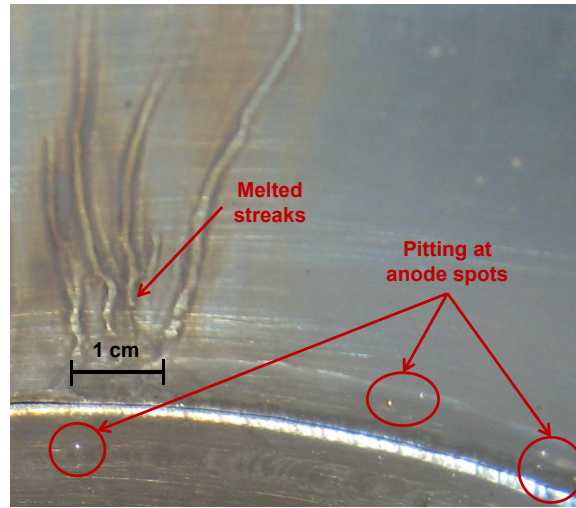
The introduction of anode spotting as a possible mechanism is relevant because evidence of anode spots occurring in our thruster was obtained via high-speed video imaging. Figure 14 shows 9 frames taken with 1  $\mu$ s exposure times and separated by 100  $\mu$ s intervals looking upstream into our thruster during the quasi-steady period. The thruster was firing at 9 kA in self-field mode (no magnets). These images show the higher-luminosity regions around the inner perimeter of the anode lip migrating around the anode over time. Similar to observations by Uribarri<sup>31</sup> and Diamant,<sup>25</sup> these luminous regions are associated with localized, higher-current concentrations along the anode, which have been shown to be associated with anode spotting and current filamentation. Figure 15 also shows photographic evidence of anode damage in our thruster by melting due to localized heating and surface pitting due to anode spots. This relationship between the terminal voltage fluctuations and anode spots will be discussed further in Section VI. Reductions in the number of voltage spikes and potentially in the number of anode spots (and associated erosion) appear to be beneficial effects of the applied magnetic fields.



**Figure 13. Number of positive spikes >10% above mean voltage.**

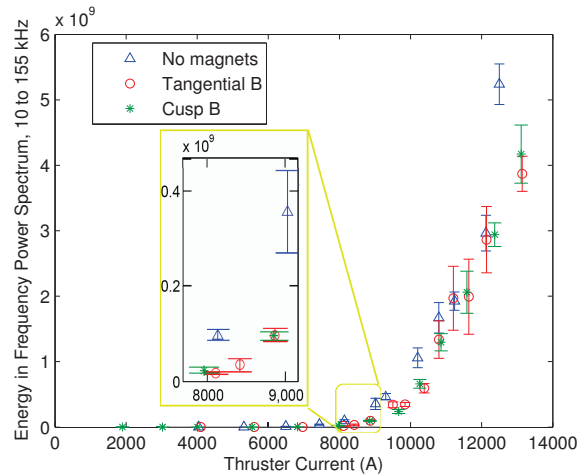


**Figure 14. High-speed video imaging frames showing evidence of anode spots in thruster at 9 kA. Frames represent 1 μs exposure times separated by 100 μs intervals.**



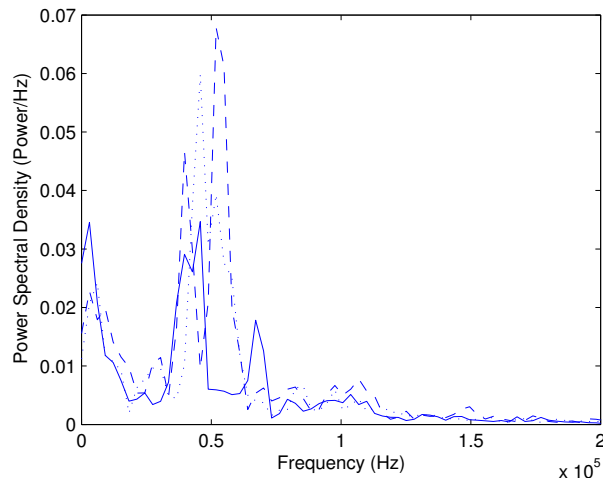
**Figure 15. Evidence of anode erosion in our MPD thruster.**

The energy content in the voltage signal is examined by taking the power spectral density (PSD) in frequency space and integrating over a range of frequencies, as shown in Figure 16. This effectively provides a sort of “intensity” metric. The mean of the signal is subtracted before performing the analysis. The frequency range from 10–155 kHz was chosen for integration. Over 90% of the power content in the PSD is below 100 kHz, so a cutoff of 155 kHz was selected to exclude all of the low-amplitude random noise in the higher frequencies. Further, by ignoring the frequency content below 10 kHz, we avoid very low-frequency shifts in the mean signal over the 1 ms quasi-steady period that are irrelevant to the examination of the voltage hash fluctuations. A transition is observed to much larger rate of growth in the energy at approximately 9 kA without magnets and approximately 9.5 to 9.7 kA with both applied B fields. The applied B fields result in approximately a 73% reduction in the energy at 9 kA and approximately 40% at 10.3 kA. The values begin to statistically overlap at higher currents.

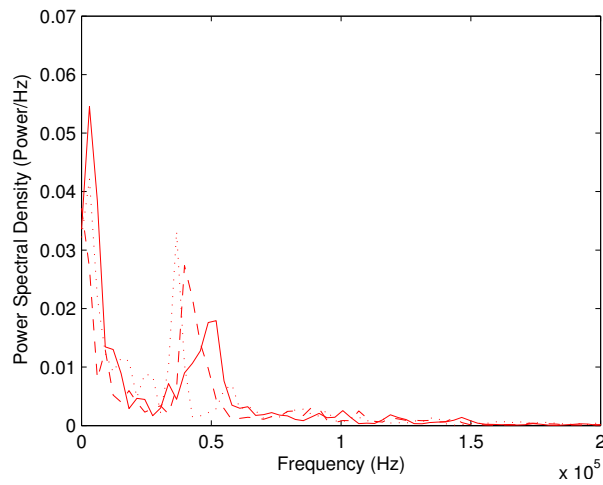


**Figure 16. Total energy in the PSD spectrum, as integrated from 10 to 155 kHz. The inset yellow box shows a zoomed view from 8–9 kA.**

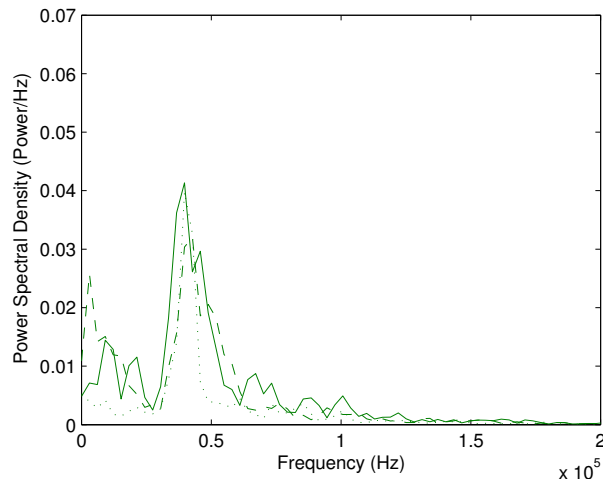
Again, this energy content can be related to the anode spots. Uribarri<sup>31</sup> and Giannelli et al.<sup>39</sup> proposed capacitively-coupled anode sheath models that associated anode spots and current filamentation with fluctuations in the voltages. Based upon these models, the energy in the voltage hash spikes is expected to be related to the magnitude of the current carried in the filaments at the anode spots. Reduced spike energy is observed with the applied-field magnets at currents below approximately 10 kA. Combined with a potential reduction in the number of spots (related previously to the observed decrease in number of spikes), the data suggest what is thought to be an overall decrease in both the intensity and frequency of the anode spots with the applied magnetic fields (discussed further in Section VI).



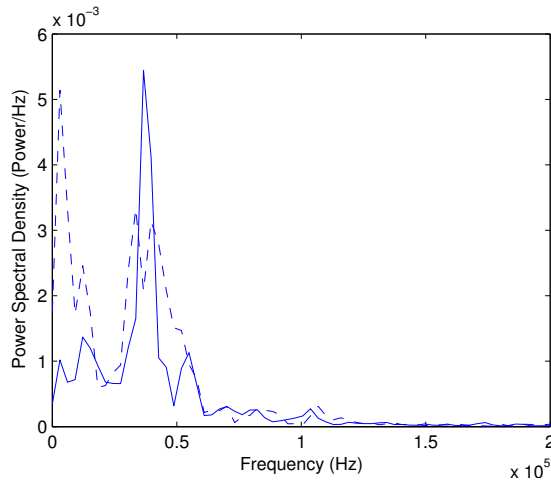
**Figure 17. Power spectral density (PSD) examples for 3 shots without magnets at 10.7 kA.**



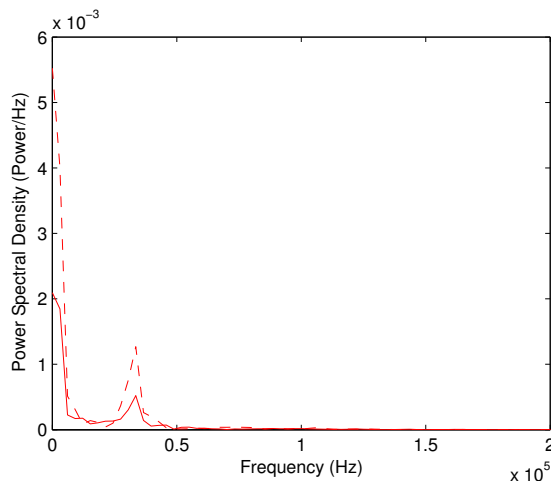
**Figure 18. Power spectral density (PSD) examples for 3 shots with tangential B field at 10.7 kA.**



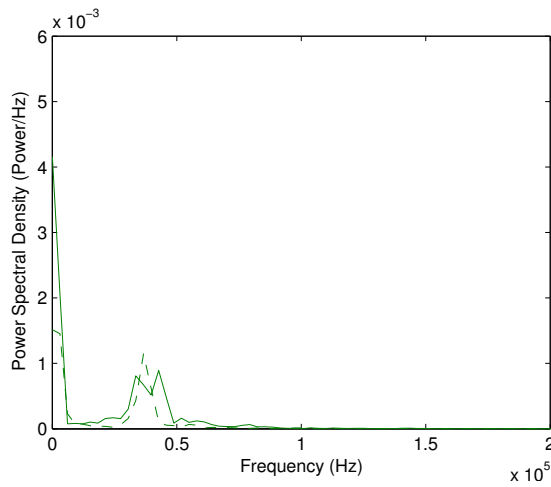
**Figure 19. Power spectral density (PSD) examples for 3 shots with cusp B field at 10.7 kA.**



**Figure 20. Power spectral density (PSD) examples for 3 shots without magnets at 8 kA.**



**Figure 21. Power spectral density (PSD) examples for 3 shots with tangential B field at 8 kA.**



**Figure 22. Power spectral density (PSD) examples for 3 shots with cusp B field at 8 kA.**

Next, the frequency domain content contained in the voltage signals was analyzed by generating power spectral density plots (PSDs). The PSDs were calculated using Welch's method to reduce noise in the power spectra. The mean was subtracted from the 1 ms quasi-steady interval voltage signal time series, then the signal was segmented into 5 time-interval windows with 50% overlap between consecutive windows. A Hamming windowing function was applied and a periodogram was generated for each window interval, and then the resulting periodograms were time-averaged. Figures 17 to 19 show the PSDs of multiple example shots at 10.7 kA in the frequency range below 200 kHz. Peaks can be clearly seen primarily in the range between approximately 40 to 60 kHz. At the same current, a shift to smaller-amplitude peaks at slightly lower frequencies can be seen in the PSDs with the applied B fields relative to the self-field case. Figures 20 to 22 give example PSDs at 8 kA, where characteristic peaks are observed near 40 kHz without magnets and significantly attenuated peaks at just below 40 kHz. Although not shown here, similar peaks were identified at 5.5 kA in the 20–40 kHz range, and the applied B fields were even more effective at suppressing the magnitude of the peaks and shifting them to lower frequencies. These shifts to lower frequencies and lower amplitudes are consistent with the previous analyses of the effective frequency and energy of the spikes.

A more in-depth set of analyses of the voltage signals in the temporal and frequency domains are detailed in Moeller's thesis.<sup>56</sup>

#### D. Summary of Results

**Substantial reductions in the mean voltages over a broad range of operating currents were achieved with the applied magnetic fields.** Relative to the self-field case, both applied B field configurations yielded reductions of approximately 25% at 8 kA and approximately 31% at 9 kA. As current increased, the tangential B field case approached response similar to without the magnets. However, the cusp B field case continued with modest reductions in the mean voltage throughout the entire range of currents. This mean voltage reduction is counter to the results of Tahara, Kagaya, et al.,<sup>52,55</sup> who measured increased terminal voltages with their predominantly axial and higher-magnitude applied B fields. Overall, this implies a significant potential improvement in the total thruster efficiency by achieving the same total discharge current  $J$  (the strongest scaling parameter for high-power MPDT thrust and efficiency) at reduced input power (lower  $P_{in} = VJ$ ). This effect on efficiency will be analyzed later in Section VI-E.

**The applied magnetic fields yield significant reductions in the magnitude and frequency of the voltage fluctuations.** The standard deviations essentially track the magnitude of the RMS fluctuations about the mean voltage, which clearly grow with increasing current, and grow more rapidly above 8–9 kA. The applied B fields result in a 37–49% reduction in the standard deviations at 8–9 kA. The cusp applied B field still exhibits a 15% reduction at 10.7 kA. Both cases trend toward similar magnitudes as without magnets for higher currents. Examination of the transients in the voltage signals showed that the applied magnetic fields resulted in substantial reductions in the number of positive spikes. These reductions were most substantial particularly around the range of 8–9 kA, which seemed to manifest as a recurring transitional region in the response behavior of the voltage transients. In general, relative to without the magnets, the applied B fields resulted in an approximately 1–2 kA higher current threshold at which these transients transition to significantly larger growth with increasing current. High-speed video imaging evidence of current concentrations on the anode was shown in Figure 14, supporting a link between these fluctuations and anode spots. In reducing the frequency and magnitude of the terminal voltage fluctuations, the applied magnetic fields may also yield a decrease in anode spots and associated erosion, which will be discussed further in Section VI.

**Distinct peaks are observed in the PSDs at low frequencies (10s of kHz).** In the frequency-domain analyses, peaks in the power spectral density (PSD) plots in the range of approximately 40–60 kHz were repeatedly observed at higher currents around 10.7 kA and down to approximately 20–40 kHz for lower currents. The applied B fields consistently resulted in significantly lower amplitudes and a shift to slightly lower frequencies for the peaks relative to without the magnets.

### IV. Magnetic Probing of Internal Magnetic Field Topology and Current Profiles

#### A. Collection of Data

The multi-station magnetic probe array (MPA) was used during thruster firings to acquire data to reconstruct the topology of the azimuthal magnetic fields and poloidal discharge currents in the thruster plasma. The MPA was positioned at different locations to obtain measurements throughout the inter-electrode plasma discharge region and near-field plume. The data obtained along the anode was as close as 4–5 mm from the anode surface, as dominated by the 4 mm radius of the probe's outer quartz tube.

## B. Analysis of Magnetic Fields and Current Patterns at High Discharge Current

Examination of the MPA probe data begins with measurements taken at the high current conditions at approximately 10.7 kA. As shown in the previous section, the thruster is operating well within conditions associated with onset (large-amplitude voltage fluctuations at high frequency) at this discharge current. Figure 23 shows a shaded contour plot of the azimuthal B field generated during the thruster firing for the self-field configuration with no magnets. This contour plot shows azimuthal B field in units of Gauss, as specified in the top color bar legend. The contour plot of the thruster half-plane is drawn such that the vertical axis shows radial position with the bottom of the vertical axis at the thruster centerline ( $r = 0$ ), and the horizontal axis is the axial position measured relative to the anode exit plane ( $x = 0$ ). The cathode body is in the lower left side of the plot, with the cathode exit plane at  $x = -270$  mm. The anode inner radius is shown on the top side of the figure. As expected, the magnetic field generally increases in the upstream direction in the thruster. Also, a dip in the contours occurs radially inward just downstream of the cathode face. These effects are due to the increasing enclosed current in the upstream axial direction deeper inside the thruster discharge and the electromagnetic radial pinch on the plasma in front of the cathode.

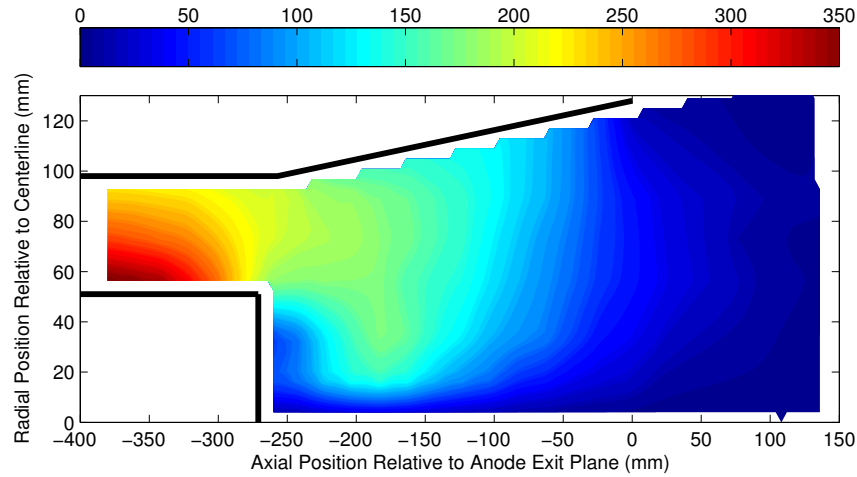


Figure 23. Contour plot of azimuthal B field without magnets at 10.7 kA. Contour values are in Gauss.

The azimuthal B field measurements were used to calculate the enclosed current and current streamlines. To obtain the enclosed current,  $J_{enc}$ , from our azimuthal B field measurements, the pre-Maxwell equation for Ampere's Law is used (where the time-varying term can be ignored for slow, low-frequency phenomena in our thruster):

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} \quad (10)$$

where  $\mathbf{B}$  is the magnetic field vector,  $\mu_0$  is the permeability of vacuum, and  $\mathbf{j}$  is the current density vector. By integrating both sides over a bounded surface of constant radius about the centerline and invoking Stokes' theorem, the right-hand side of the equation becomes proportional to the total enclosed current crossing the surface. The left-hand side of the equation becomes a line integral over the surface boundary's constant radius. By invoking the assumption of azimuthal symmetry, the expression becomes a line integral over constant azimuthal B field over a path length of  $2\pi r$ , which yields:

$$J_{enc} = \frac{2\pi r B_\theta}{\mu_0} \quad (11)$$

where  $J_{enc}$  is the poloidal current enclosed by the contour,  $r$  is the radius, and  $B_\theta$  is the azimuthal magnetic field component.

Figures 24, 25, and 26 show contour plots of the enclosed current fraction for the configurations with no magnets, tangential applied B field, and cusp applied B field, respectively. The contour lines are shown for constant specified values (0.1, 0.2, ..., 0.9) of the fraction of the total discharge current for the thruster firing at approximately 10.7 kA. Thus, these lines represent the current streamlines in the thruster. In all three configurations, we observe a shift toward the centerline of the current streamlines just in front of the cathode due to the electromagnetic forces causing a radial pinch inward. The radial pumping forces are a consequence of the magnetic force densities having a significant radial component. Also, note that the fact that the enclosed current values are still slightly positive at the anode exit plane

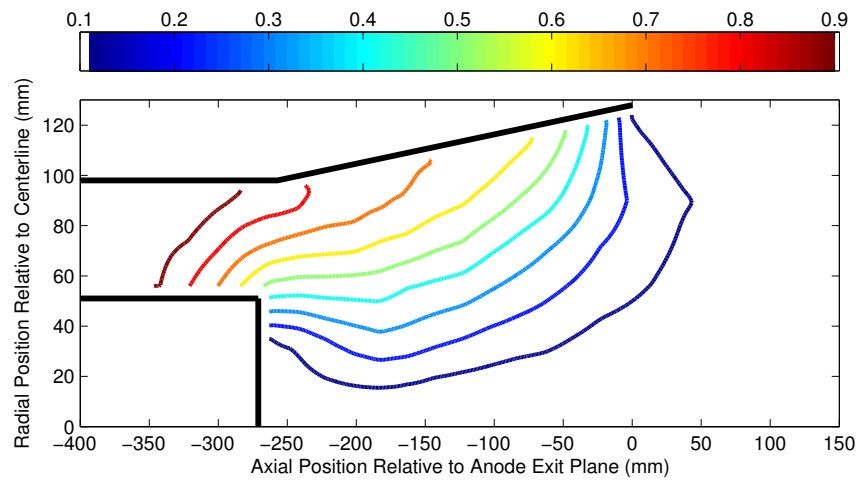


Figure 24. Contour plot of enclosed current fraction without magnets at 10.7 kA. Contours are shown at (0.1, 0.2, ..., 0.9) of total enclosed current.

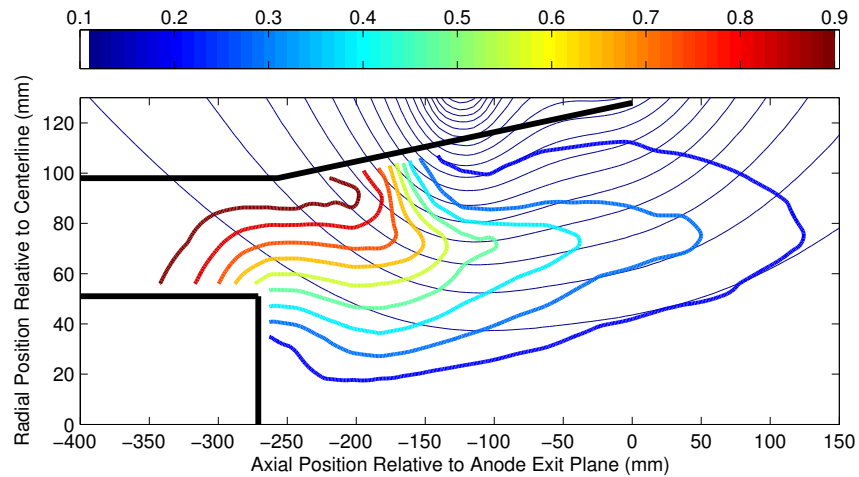


Figure 25. Contour plot of enclosed current fraction for tangential applied B field configuration at 10.7 kA. Applied B field lines are also shown. Contours are shown at (0.1, 0.2, ..., 0.9) of total enclosed current.

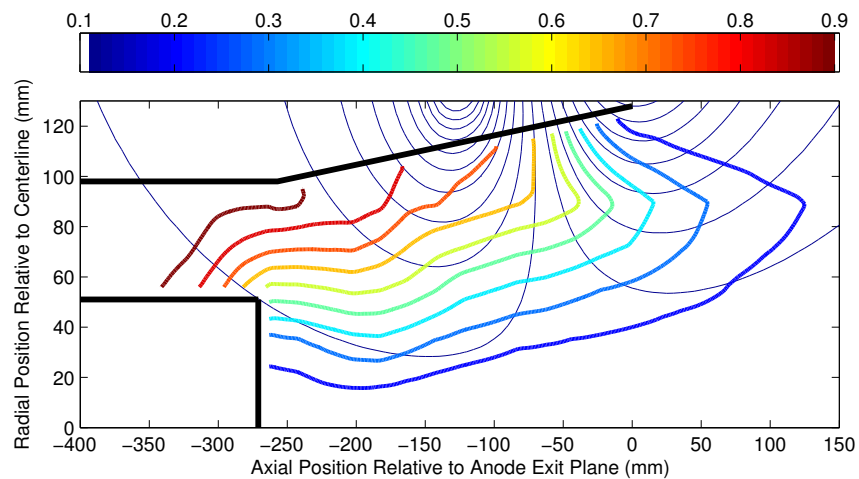


Figure 26. Contour plot of enclosed current fraction for cusp applied B field configuration at 10.7 kA. Applied B field lines are also shown. Contours are shown at (0.1, 0.2, ..., 0.9) of total enclosed current.

( $x=0$ ) and go to zero just slightly downstream of the anode exit implies that the current attachment pattern likely has a small amount of current that loops outside of the anode exit plane to attach along the anode front face and the 10–15 mm wide exposed strip on the anode exterior not covered by the insulating Kapton.

In Figure 24 for the self-field case, we observe the current streamlines attach broadly across most of the anode surface. There is small compression in the current streamlines along the anode lip near the anode exit plane, which is an expected consequence of the convection of the current streamlines downstream due to the flowing plasma.

In Figure 25, the current streamlines exhibit a marked shift due to the tangential applied B field. The current streamlines are convected further downstream in the interior mid-radial region. Most strikingly near the outer radii, the current conduction and attachment are clearly shifted upstream to strong concentration at a more central location along the anode. The current streamlines are convected downstream and then curve back significantly upstream to the central anode region near axial positions  $z = -140$  mm to  $z = -200$  mm. These streamlines follow along the curvature of the applied B field lines at higher radial positions, where the B field lines turn radially toward the anode. The B field lines intersect the anode in this region further upstream from the anode exit. Downstream near the anode lip, the tangential applied B field lines were designed to be near-parallel to the anode. In this downstream region, the current pattern follows the mostly axial applied B field lines, preventing any significant attachment in this region.

The cusp applied B field case shown in Figure 26 exhibits a pattern more similar to the self-field case. Again, there is a convection of the current streamlines further downstream than without the magnets, and the current streamlines turn more sharply back upstream to attach along the anode. The current streamlines near the anode follow along the cusp applied B field lines and attach downstream along where the cusp radial B field lines intersect the anode.

### C. Analysis of Current Densities at High Discharge Current

Next, we examine in more detail the data as profiles along the near-anode region to reveal insights on the current conduction to the anode. The radial current density along the anode is given by<sup>56</sup>

$$j_r = -\frac{1}{2\pi r} \frac{\partial J_{enc}}{\partial z} \quad (12)$$

where  $j_r$  is the radial current density,  $r$  is the radial position, and  $z$  is the axial position along the anode.

Figure 27 shows the profiles of current densities along the anode versus axial position, as calculated from the enclosed current data for the multiple magnetic configurations again at approximately 10.7 kA. The error bars for the current densities again represent uncertainties dominated by the shot-to-shot variability of the enclosed current gradients. These profiles of current densities provide the clearest indication of where the current attachment is concentrating along the anode.

The current density for the self-field case peaks primarily downstream along the anode lip, close to the anode exit plane. There is some modest additional rise in current density further upstream, peaking again near  $x = -260$  mm. The cusp applied B field case exhibits a modest shift upstream of the peak current density by approximately 15–20 mm, shifting to the region of the anode-intersecting cusp B field. As seen in the previous data, the tangential applied B field case results in a notable shift upstream, where the current density peaks strongly in the central anode region where the magnetic field lines begin to turn radially outward and intersect the anode.

Although not shown here, the current densities along the anode for each magnetic configuration also follow a similar functional form over a broad range of thruster currents from 4–13 kA. Only modest shifts downstream occur for the peak current densities with increasing thruster current, as associated with downstream convection of the current streamlines.

We can also look generally at the poloidal current densities throughout the thruster plasma discharge by simply examining the radial component  $j_r$  of equation 10. We use the gradients of our azimuthal magnetic field measurements in cylindrical coordinates to calculate

$$j_r = -\frac{1}{\mu_0} \frac{\partial B_\theta}{\partial z} \quad (13)$$

Figures 28 through 30 show shaded contour plots of the radial current densities generated during the thruster firing for the various applied B field configurations. These contour plots show current densities in units of  $A/mm^2$ , as specified in the top color bar legend, and are drawn with the same position axes as the previous contour plots. Note that the color bar scales differ on all plots (based upon the range of values in each case), and the values change polarity over the range of negative to positive values.

Without the magnets, we can clearly see the regions of high outward radial current densities near the cathode outer radius and downstream near the anode lip, where Figure 27 previously showed a clear a peak in the current density

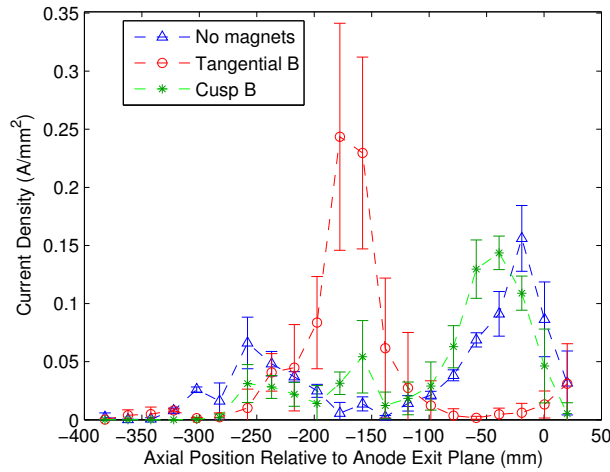


Figure 27. Profiles of current density along the anode vs. axial position at 10.7 kA. All 3 magnetic configurations are shown for comparison.

along the anode. We also observe high inward radial current densities, where the plasma pinches toward the centerline under the radial electromagnetic pinching forces.

The contour plots for the tangential applied B field case exhibit similarly high current densities near the cathode and centerline. In addition, as expected from the regions of high current density along the anode in Figure 27, we observe a region of high outward radial current density upstream in the mid-anode region.

With the cusp applied B field, we again observe the high current densities near the cathode and centerline. Further, high outward radial current densities are seen downstream on the anode in the region where the cusp applied B field lines intersect the anode. Also, we can see a region in the interior of the thruster centered around axial position  $z = -150$  mm with relatively high outward radial current densities that extends outward toward the anode near  $z = -170$  mm.

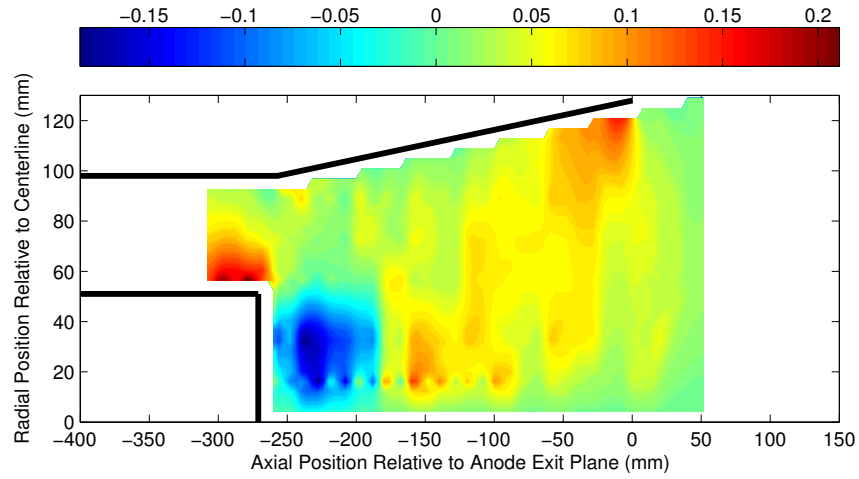
#### D. Summary of Results

**The applied-field magnets caused a clear shift in the current pattern and modified the anode attachment region.**

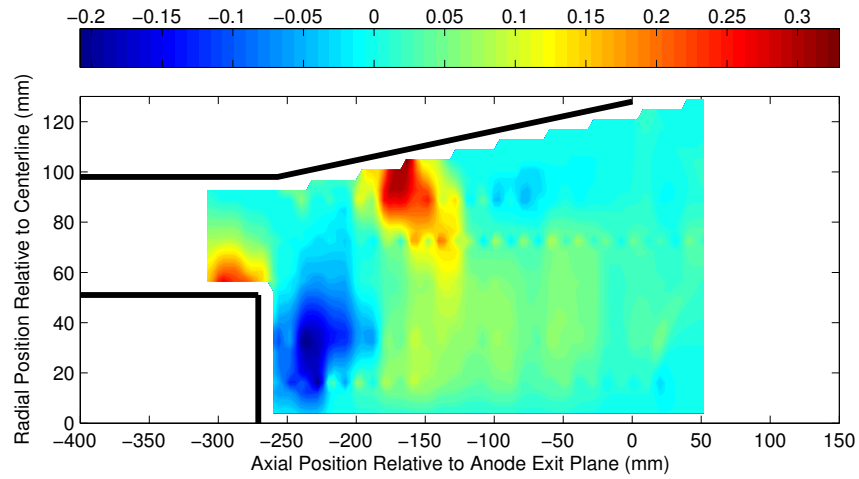
The externally applied magnetic fields result in marked differences in the current pattern. The current streamlines follow the applied B field lines in the near-anode attachment region. The tangential B field causes the current streamlines to shift significantly upstream to follow along where the applied magnetic field lines begin to turn radially outward toward the anode. The applied B field prevents any significant attachment in the downstream region where the applied B field lines are contoured parallel to the anode. The cusp applied B field case exhibits a more subtle but still evident shift in the current streamlines to follow along the cusp field lines near the anode.

**The modified current patterns due to the applied magnetic fields result in related shifts in the regions of high current densities along the anode, and the current attachment pattern clearly follows the applied B field lines near the anode.** The self-field configuration exhibits highest radial current densities along the anode lip near the anode exit plane. The tangential applied B field causes a shift in the concentration of current density well upstream along the anode flared section to where the radial curvature of the applied B field lines begins to intersect the anode surface. This concentration results in higher peak current densities than without magnets, but reduces the current attachment in the downstream anode lip region to negligibly small levels. The cusp applied B field results in a shift of the current density peak region to slightly upstream of the self-field case. Cusp configuration anode current density is slightly broadened and relocated to where the cusp applied B field lines are strongly intersecting the anode. In the cusp applied B field configuration, we also see a region near axial position  $z = -150$  mm of relatively high outward radial current density that extends from the centerline region outward toward the anode. The current densities along the anode also follow a similar functional form over a broad range of discharge currents examined from 4–13 kA.

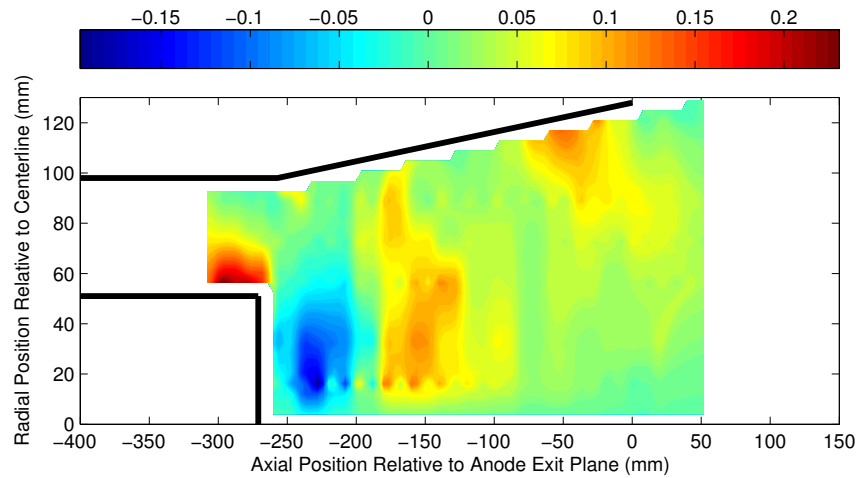
These shifts in the regions of highest current densities are influenced by the shape of the poloidal applied B field lines, effectively resulting from the higher current conduction along the direction parallel to the applied B field than conduction perpendicular to the B field. This effect will be discussed further in Section VI-A and is particularly important in the near-anode region, where the applied B field magnitudes are higher and the ratio of the applied B field to the self-generated azimuthal B field is also increased. Underlying causes are considered in Section VI.



**Figure 28. Contour plot of radial current density at 10.7 kA for the configuration without magnets.**



**Figure 29. Contour plot of radial current density at 10.7 kA for the configuration with tangential applied B field.**



**Figure 30. Contour plot of radial current density at 10.7 kA for the configuration with cusp applied B field.**

## V. Langmuir Probe Measurements of Plasma Properties and Potential

### A. Collection of Data

The Langmuir probe was used at selected locations in the thruster for local measurements of the electron temperature ( $T_e$ ), electron number density ( $n_e$ ), and the anode fall voltage measured from the plasma potential ( $V_{fall} = V_{anode} - V_{plasma}$ ). The focus of most measurements was the near-anode region, particularly in the regions of highest current densities calculated from the magnetic probe measurements. These measurements were low-pass filtered at approximately 120 kHz cutoff frequency.

The Langmuir probe was repositioned over multiple sets of thruster shots to obtain measurements at various locations in the plasma discharge. For the measurements in the near-anode region, the Langmuir probe's long axis was aligned along the anode flared section. The probe tips were radially separated from the anode surface by 3.5–4.5 mm, limited primarily by the outer ceramic tube.

### B. Analysis of Langmuir Probe Measurements Along the Anode at High Current

The previous analyses with the magnetic probe data indicated clear regions of high current density along the anode and a shift in the current patterns with the applied magnetic fields. Thus, our investigation of the local plasma properties begins with measurements circa 10.7 kA along the near-anode region in a strip of points from the anode lip at the exit plane and traversing upstream to the axial position  $z = -179$  mm. Note that the tangential applied B field case exhibited peak current densities circa  $z = -179$  mm, and the cases without magnets and with cusp applied B field showed highest current densities in the region around  $z = -29$  mm.

First, we examine the profiles of electron temperature,  $T_e$ , for each magnetic configuration. Figure 31 shows the electron temperature measurements (in eV) for all three magnetic configurations. Self-field operation exhibits a general trend of increasing electron temperature along the anode in the downstream direction. This generally follows the trend also seen in the increasing current density along the downstream section of the anode, as shown previously in Figure 27. The tangential applied B field case shows an increase in  $T_e$  at  $z = -150$  mm, near where the highest current densities were measured. In addition, higher  $T_e$  is observed in the mid-region of the profile at  $z = -79$  mm,  $-104$  mm, and  $-129$  mm. These values are near the regions of high applied B field intensity, and may be associated with regions of high azimuthal current density induced by the applied B field. The cusp applied B field exhibits a similar increase in  $T_e$  upstream and in the mid-region of the profile, albeit shifted approximately 25 mm downstream. This may be due to the downstream shift in the current pattern relative to the tangential applied B field, and thus a related shift downstream in the regions of higher induced azimuthal current density. This will be shown later in Section VI-B.

Subsequently, we consider the electron number density (i.e., the plasma density, assuming  $n_e = n_i$  for quasineutrality), as calculated from the measured  $T_e$  and ion saturation current. The profiles for  $n_e$  along the anode are shown in Figure 32 for all three cases. Without the magnets, a general trend of increasing number density is observed along the anode in the downstream direction. This trend is likely due to the expansion of the plasma from the cathode emission zone outward towards the anode and accelerating downstream, resulting in higher number densities downstream. With the tangential applied B field, we see a significant increase in the number density upstream at  $z = -179$  mm, near the region of highest current density. In addition to a possible change in the flow field for the tangential applied B field, this region may be experiencing a zone of increased ionization due to the higher current densities. Just downstream of this location, we observe a significant drop off to lower densities along the anode, likely due to the contouring of the strong magnetic field lines in this region, followed by an increase in the number densities again downstream from  $z = -79$  mm to the anode exit plane as the plasma expands toward the anode. Except for the region of highest current attachment at  $z = -179$  mm, the number density decreases relative to the self-field case. In the case of the cusp applied B field, the data show a generally steadier trend to the number density along the anode. Uncertainties in the absolute values of the measurements suggest that the measurements are not statistically significant in their difference relative to self-field operation, except for the increased number density of the tangential B field case upstream at  $z = -179$  mm. However, relative differences do suggest the possibility of small increases in number densities around  $z = -29$  mm and  $z = -154$  mm for the cusp applied B field case relative to without magnets. These are the locations associated with the higher current densities in the cusp configuration, and this may suggest a modestly increased number density to support increased electron random thermal flux to the anode. These are also the regions where the cusp B field lines turn radially toward the anode. It is possible that there is an increased diffusion of the plasma from the interior plasma radially toward the anode in this region of increased radial B field. Such diffusion will be discussed later in Section VI-A.

With the measurements of electron temperatures and floating potentials from the triple probe, we can calculate the

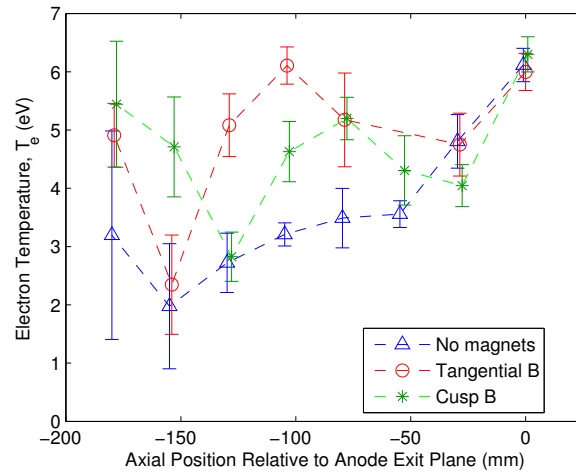


Figure 31. Electron temperature along the anode vs. axial position at 10.7 kA comparing all magnetic configurations. The dashed connecting lines are only as guides for the eye, not an implied functional relationship.

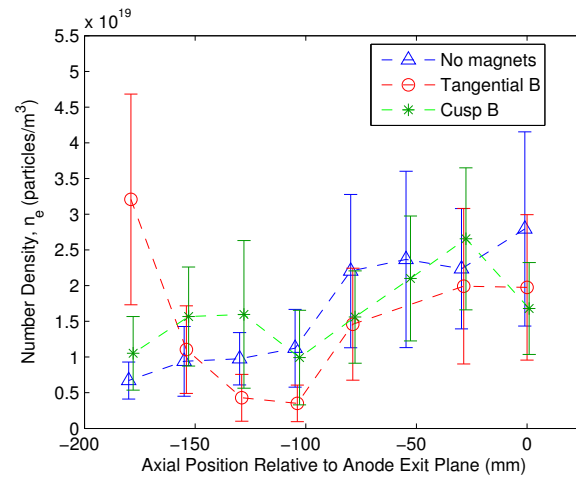


Figure 32. Plasma number density along the anode vs. axial position at 10.7 kA comparing all magnetic configurations. The dashed connecting lines are only as guides for the eye, not an implied functional relationship.

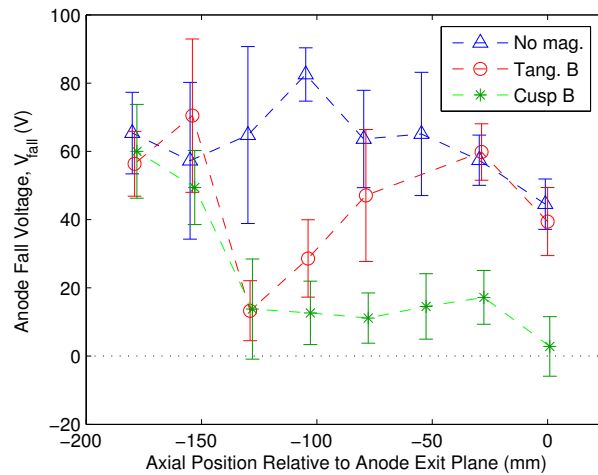


Figure 33. Anode fall voltage along the anode vs. axial position at 10.7 kA comparing all magnetic configurations. The dashed connecting lines are only as guides for the eye, not an implied functional relationship.

plasma potential and cast it as the anode fall voltage,  $V_{fall} = V_{anode} - V_{plasma}$ . Note that the form of this expression implies electron-attracting positive fall potentials and electron-repelling negative fall potentials. Figure 33 shows the measured anode fall voltage for the three configurations. The self-field case shows a generally very high anode fall voltage from approximately 45 to 84 V. Such significant potential drops that will be seen by the electrons in the near-anode plasma will result in high-energy acceleration through this potential well. The associated high-energy electrons will impact the anode surface, causing anode erosion. With the tangential applied B field, a significant drop in the anode fall voltage is seen from  $z = -129$  mm to  $z = -79$  mm. The drop in this region is likely associated with the significant decrease in current density demands in this region under this applied-field configuration. In the case of the cusp applied B field, an impressive reduction in the anode fall voltage occurs over most of the profile from  $z = -129$  mm to the anode exit plane. Here, the average anode fall voltages are reduced by 42 to 70 V and ultimately decreased to only 3 to 14 V. In this case, we suspect that the radially intersecting magnetic field lines result in a significant increase in electron mobility to the anode surface in these regions of measured high current density (e.g., circa  $z = -29$  mm), thus mitigating the need for large anode fall voltages to sustain the required current density. This effect of increased current conduction along the applied B field lines intersecting the anode will be assessed in Section VI-A.

### C. Analysis of Langmuir Probe Measurements Along the Anode at Reduced Current

The plasma properties were also measured at 8 kA thruster discharge current to determine whether the applied magnetic fields at this intermediate current condition caused similar effects on the plasma properties as at higher current. At 8 kA, the beginning effects of transition to onset resulted in much lower magnitude and frequency of voltage fluctuations and transients, and the applied B fields resulted in greater relative reductions in these quantities, as shown in Section III. The terminal voltage reduction was also larger at 8 kA than at 10.7 kA. Figures 34, 35, and 36 show the electron temperature, number density, and anode fall voltage, respectively, for all magnetic configurations at 8 kA. At this 8 kA operating point, only two locations near the anode at axial positions  $z = -179$  mm and  $z = -29$  mm were measured, thus focusing on the regions observed to have the highest current densities. In general, we see a reduction in the electron temperature from the values measured previously at 10.7 kA. However, we still observe a similar increase in the electron temperature at 8 kA with both applied B fields relative to the self-field case. This is again likely due to increased azimuthal currents driven in the plasma with the applied magnetic field, which are discussed later in Section VI.

A statistically significant difference is not observed in number densities at 8 kA relative to the values measured at the higher current of 10.7 kA. At the higher current, increased flow acceleration should cause lower local number densities (from conservation of mass at increased velocity), and higher electromagnetic radial pumping forces should also decrease number densities near the anode. However, at higher current, there may be increased ionization, which would increase local plasma number densities. These two effects may mostly offset each other at these currents, resulting in similar magnitudes for the measured number densities. Nonetheless, relative to without the magnets, the data at 8 kA exhibit a similar pronounced increase in  $n_e$  upstream at  $z = -179$  mm with the tangential applied B field relative to the self-field case. Modest increases in  $n_e$  are also measured with both applied magnetic fields relative to the self-field case downstream at  $z = -29$  mm.

Overall, lower anode fall voltages are observed at 8 kA than were measured at 10.7 kA, which is consistent with the trends measured in other experiments by Gallimore<sup>32</sup> and Soulas et al.<sup>44</sup> However, there is now a clear benefit of reduced anode fall voltage with both applied magnetic fields at 8 kA relative to the self-field case. This is also consistent with the earlier plot of terminal voltage versus current in Figure 11, which demonstrated that the tangential applied B field also had a pronounced reduction in terminal voltage at 8 kA but not significantly at 10.7 kA. At  $z = -29$  mm, the calculated anode fall voltages are negative at this lower current, implying an electron-repelling anode fall. This can physically occur if the electron random thermal flux to the anode exceeds what is necessary to sustain the required current density,<sup>32</sup> an effect which will be discussed in more detail in Section VI.

### D. Analysis of Langmuir Probe Measurements in the Thruster Interior

Additional data were obtained at locations in the interior of the thruster, albeit only in the self-field configuration. At 10.7 kA self-field operation, these data were obtained with the triple Langmuir probe along the anode (as shown previously), at the centerline near the cathode downstream face and anode exit plane, near the cathode downstream radius, and at one location downstream about 20 mm radially inward from the anode. In addition, in earlier tests with a single Langmuir probe, we were able to reliably obtain data at currents up to 9 kA but not above (due to noise issues in the power supply). Nonetheless, the measurements at 9 kA provide insight regarding the general variations and trends within the thruster interior, as these measurements spanned a broad region in the interior of the thruster radially

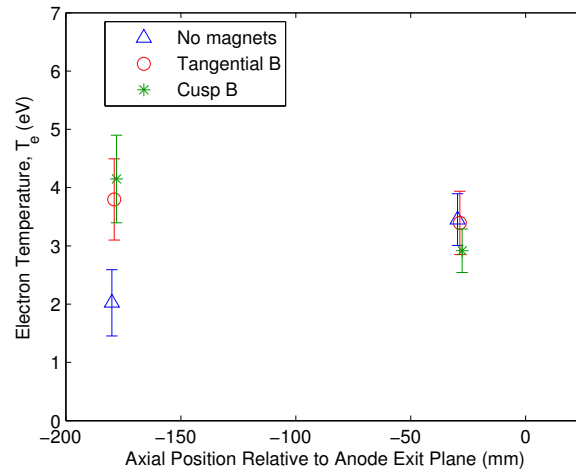


Figure 34. Electron temperature for two axial positions along the anode at 8 kA comparing all magnetic configurations.

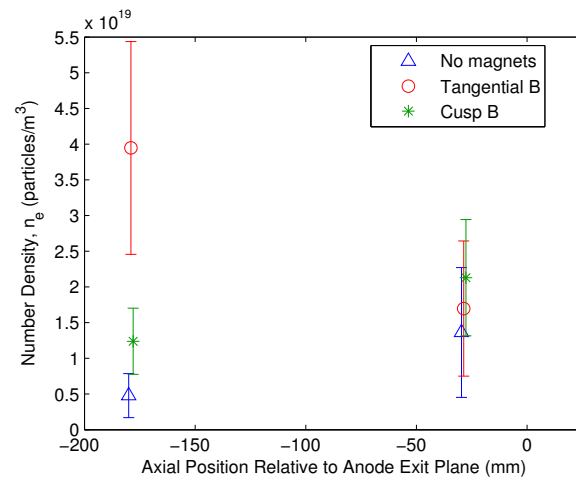


Figure 35. Plasma number density for two axial positions along the anode at 8 kA comparing all magnetic configurations.

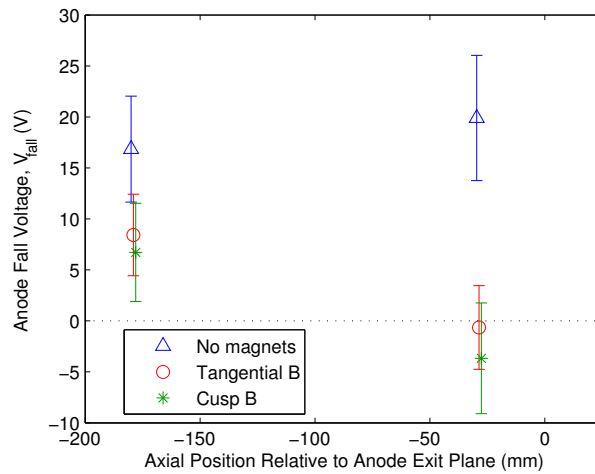


Figure 36. Anode fall voltage for two axial positions along the anode at 8 kA comparing all magnetic configurations.

from the centerline to 24 mm away from the anode and axially from the anode exit plane to 15 mm downstream of the cathode exit plane.

Measurements at these locations were used to calculate the number densities shown in Figure 37 for the self-field case. This figure is a “bubble plot,” wherein the circular “bubbles” are centered at the location of each data point, and the radius of each bubble is allowed to scale with the relative magnitude of the values. The data is shown in red for 10.7 kA and in blue for 9 kA. While the data are too coarse spatially to plot a proper contour plot, this bubble plot approach affords another way to visualize the variations in magnitude between the measurement locations.

Figure 37 shows there are very large gradients in the number densities over the thruster volume. The number densities upstream along the anode are a factor of 42 to 92 times smaller than the values near the cathode downstream face at the centerline. Near the anode exit plane, the number densities near the anode are a factor of 3.8 to 4.4 times lower than at the centerline of the anode exit plane. These results highlight the effect of the radial pumping forces in the thruster that lead to significant reduction of the plasma density near the anode. The radial pumping forces are a consequence of the magnetic force densities having a significant radial component.

Some information about the flow structure can also be inferred from these number densities. Along the centerline, we observe a compression just a short distance in front of the cathode and a trend of decreasing number density with increasing axial position. We also observe a trend of decreasing number density at the mid-radius with increasing axial position. These decreasing trends are likely due to the acceleration and expansion of the plasma flow. This is supported even further by data near the anode, where we see the opposite trend now of increasing number density with increasing axial position. Tikhonov et al.<sup>33</sup> have observed an expanding cathode jet flow field in their MPD thrusters with plasma flowing from a multi-channel hollow cathode of geometry similar to our thruster (as opposed to inter-electrode or backplate gas injection typical of most past gas-fed MPDT studies). This suggests that the plasma in our thruster expands from the cathode front face radially outward (sometimes referred to as a “cathode jet”), following a flow field which is expanding downstream to the near-anode region as a consequence of balancing the magnetic pressure (which is higher upstream) with the kinetic pressure. This type of plasma flow boundary expansion is consistent with the measured number density radial variations and axial profiles along the anode and centerline. Figure 37 also shows an overlay of qualitative flow field lines to illustrate the cathode jet expansion. Figure 37 also includes an overlay on the right side of a high-speed video image (5  $\mu$ s exposure time) of the near-exit plume of the thruster. The radial variations in luminosity support the higher number densities measured closer to the centerline than near the anode radius.

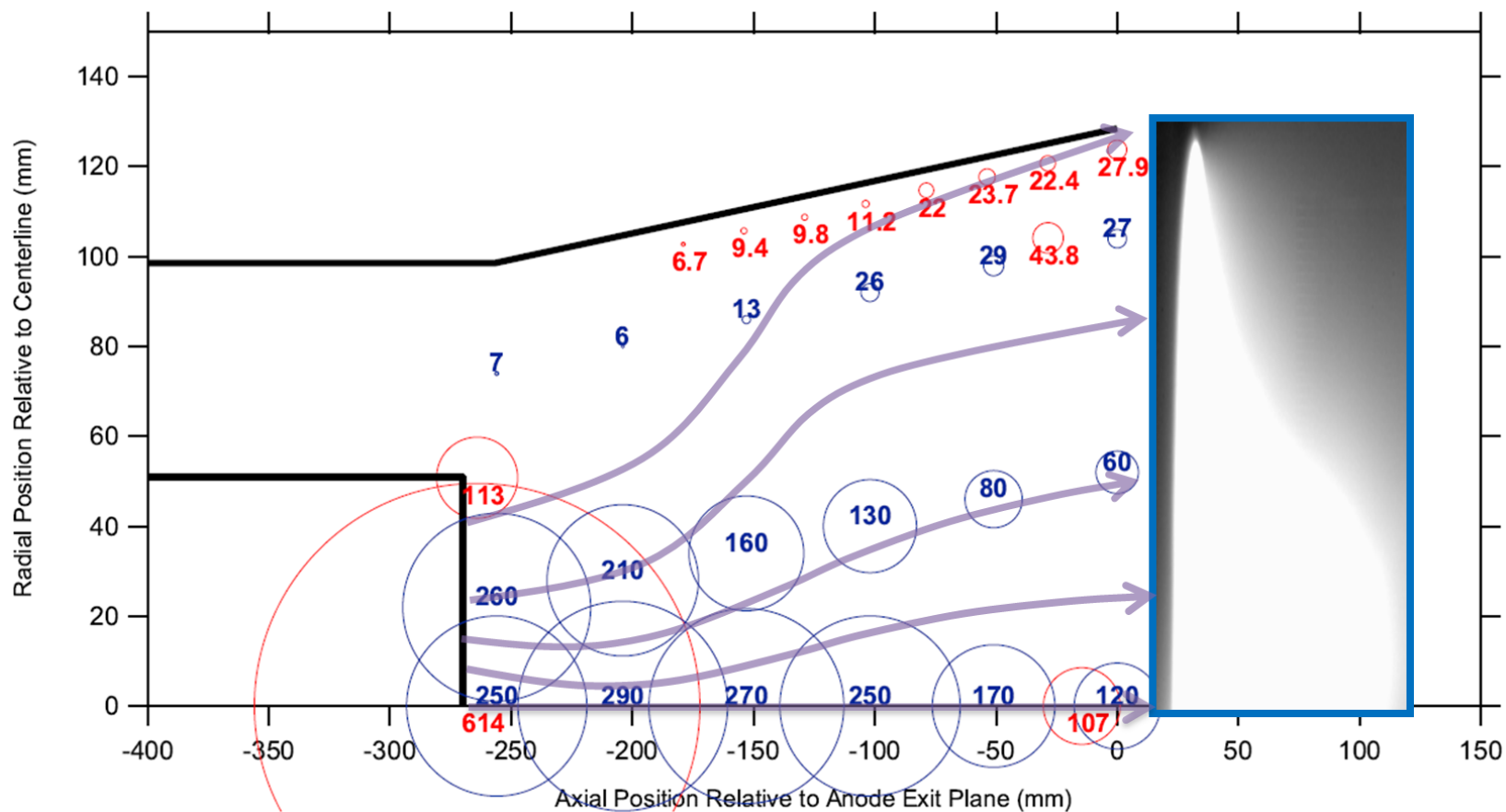
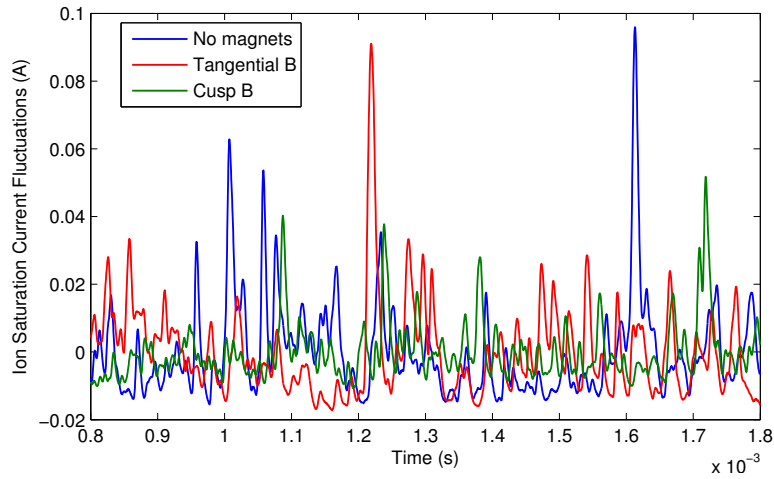


Figure 37. Bubble plot of number density with overlay of qualitative flow field lines for the configuration without magnets at 10.7 kA (red) and 9 kA (blue). Radius of the circular bubbles scale with the relative magnitude of values. Absolute numerical values shown are in units of  $10^{18} \text{ m}^{-3}$ . A high-speed video image of plume luminosity shown on the right side supports the measured radial variations in number densities.

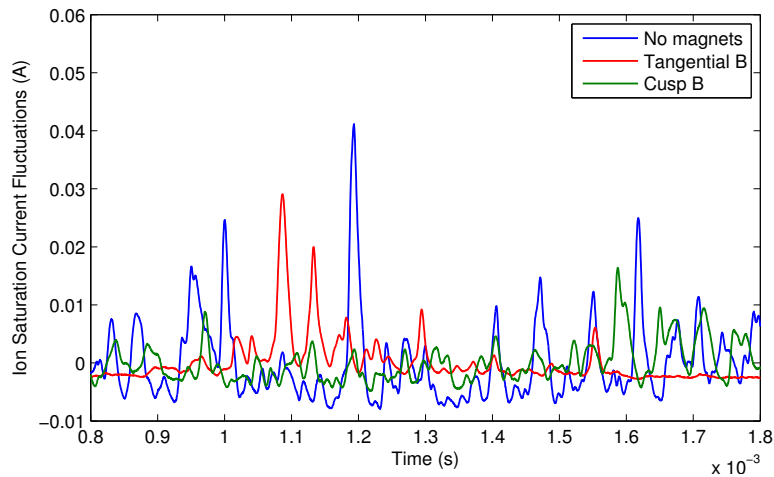
### E. Assessment of Ion Saturation Current Fluctuations

The previously-shown Langmuir probe data represented the plasma properties averaged during the quasi-steady period. Given that various temporal transients were observed in the thruster terminal voltage signals studied in Section III, it is of interest to at least briefly examine and compare the effects of the applied magnetic fields on the ion saturation current fluctuations over time. The ion saturation current collected by the Langmuir probe is directly proportional to the plasma number density. Therefore, fluctuations and transients in the ion saturation current signal represent fluctuations in the local number density. As we will discuss later, the same anode spotting mechanisms that drive terminal voltage fluctuations could drive fluctuations in the near-anode number densities.

Figures 38 and 39 show typical examples of the ion saturation current time-series signals during the quasi-steady period at 10.7 kA and 8 kA discharge currents, respectively. These figures show data taken near the anode at axial position  $z = -29$  mm upstream of the anode exit plane for all three magnetic configurations. These signals were low-pass filtered at 120 kHz and had the means subtracted to more clearly show the major temporal transients relative to the means.



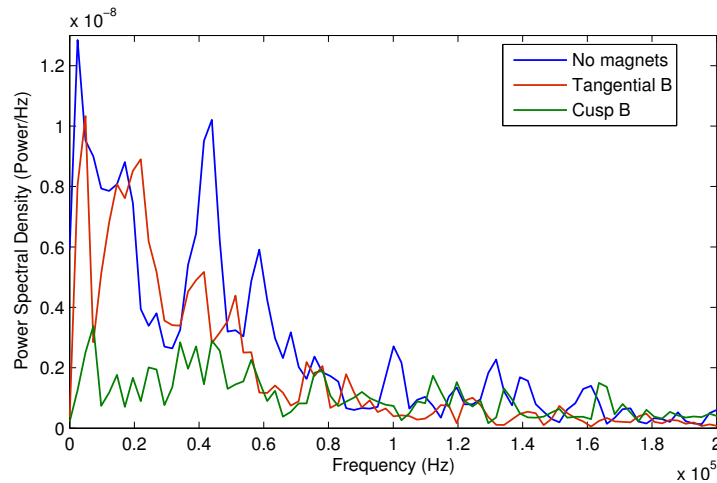
**Figure 38.** Ion saturation current signal fluctuations relative to the mean versus time during quasi-steady period at 10.7 kA and  $z = -29$  mm. Typical examples from all three magnetic configurations are shown for comparison.



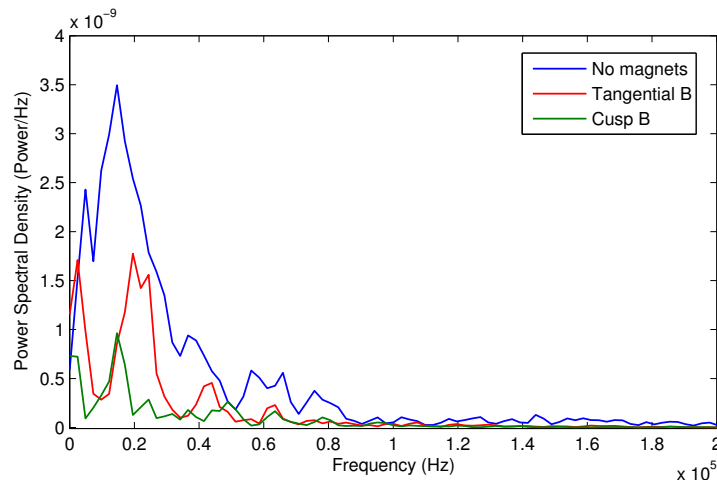
**Figure 39.** Ion saturation current signal fluctuations relative to the mean versus time during quasi-steady period at 8 kA and  $z = -29$  mm. Typical examples from all three magnetic configurations are shown for comparison.

Figures 40 and 41 show the ion saturation current power spectral density (PSD) plots in the frequency domain at 10.7 kA and 8 kA, respectively. These data represent the same signals as shown in Figures 38 and 39 at axial position  $z = -29$  mm. The signals used in the PSD analyses were taken using Langmuir probe data at the full bandwidth (2.5

MHz) of the data acquisition system from the current probe to ensure no lower-frequency attenuation. However, the power in the signals was identified to be clearly dominant in the lower-frequency range, so only values up to 200 kHz are plotted.



**Figure 40.** Ion saturation current power spectral density (PSD) at 10.7 kA and  $z = -29$  mm. Typical examples from all three magnetic configurations are shown for comparison.



**Figure 41.** Ion saturation current power spectral density (PSD) at 8 kA and  $z = -29$  mm. Typical examples from all three magnetic configurations are shown for comparison.

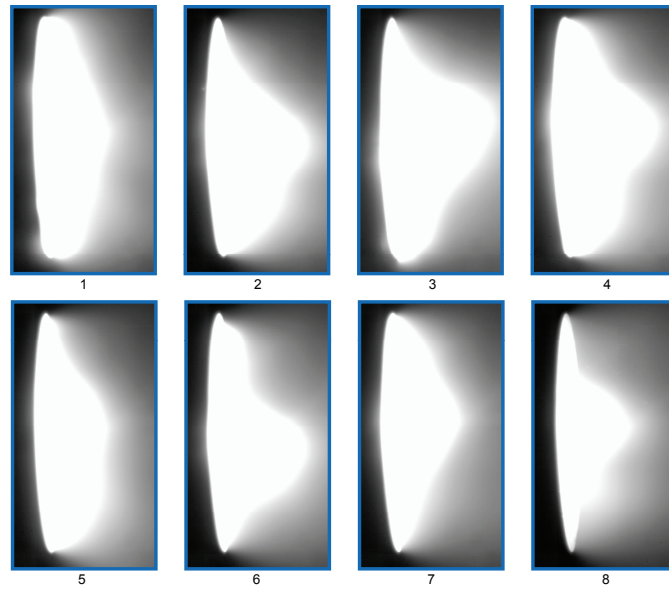
At 10.7 kA, there are clearly large excursions from the means that occur during the time signals. The magnitude and frequency of the large spikes (e.g., greater than 20 mA) in the ion saturation current signal are reduced in the configuration with the cusp applied B field relative to the self-field case. This effect can be seen most clearly in the PSD in Figure 40, where the power over the frequency range is systematically lower for the cusp applied-field case than the self-field case. However, at this high discharge current, the tangential applied B field shows only limited benefit in reducing the power in the fluctuations. These results are consistent with the reduction in magnitude and frequency of thruster terminal voltage fluctuations with the cusp applied B field at this higher 10.7 kA current seen in Section III in Figure 12 of the standard deviations, Figure 13 of the number of large voltage spikes, and Figures 17 to 19 of the PSDs. The peaks that occur at approximately 20–60 kHz in the ion saturation current PSDs are relatively close to the peaks that occur at about 40–60 kHz in the PSDs for the terminal voltage signals.

At 8 kA, large fluctuations from the mean ion saturation current are measured during the time-series signals. However, the amplitudes of the deviations in ion saturation current are smaller in general than at 10.7 kA. Moreover, there are fewer of these large spikes in the ion saturation current at 8 kA than at 10.7 kA. Again, the cusp applied B field greatly reduces the magnitude and frequency of the large spikes (e.g., greater than 15 mA). In addition, we

now see that the tangential applied B field has an effect also on reducing the number and frequency of the large-amplitude fluctuations relative to the self-field case, albeit not as much as the cusp applied B field. The PSD in Figure 41 also shows the effect of the applied B fields on reducing the magnitude of the PSDs relative to the self-field case. Once again, these findings are consistent with the observed reduction in magnitude and frequency of thruster terminal voltage fluctuations with both the tangential and the cusp applied B fields at the lower 8 kA current level seen in Section III in Figure 12 of the standard deviations, Figure 13 of the number of large voltage spikes, and Figures 20 to 22 of the PSDs. At 8 kA, the peaks at approximately 15–45 kHz in the ion saturation current PSDs are also relatively close to the peaks that occur at approximately 30–50 kHz in the PSDs for the terminal voltage signals.

These results suggest a relation between the terminal voltage signal fluctuations studied earlier and the ion saturation current fluctuations. The fluctuations or spikes in the ion saturation current represent fluctuations associated with the number density of the local plasma. Diamant<sup>25,26</sup> and Uribarri<sup>18,31</sup> discuss that such number density variations could be associated with erosion and vaporization of anode material that seeds the local plasma in response to anode spots. Thus, these observations may be a direct response to the same anode spotting mechanism in both the terminal voltage fluctuations and number density fluctuations associated with anode erosion.

The reduction in intensity and magnitude of the fluctuations may be due to the rotation of the plasma with the applied B field. The applied B field induces an azimuthal rotational motion to the plasma due to the  $j_r \times B_z$  and  $j_z \times B_r$  terms of the electromagnetic Lorentz force. It is possible that this swirling motion to the plasma could help mitigate some of the local number density fluctuations near the anode and anode spot mode erosion by forcing the plasma attachment to rotate azimuthally around the anode. Localized filamentary current attachment points should be forced to move around the anode surface, spreading out the heating over the anode surface and reducing erosion at local hot spots.



**Figure 42.** Side view high-speed video imaging frames of thruster discharge fluctuations at 9 kA. Frames represent 5  $\mu$ s exposure times separated by 100  $\mu$ s intervals.

Local plasma density fluctuations could also be caused by azimuthal asymmetries in the overall current discharge pattern, particularly in the self-field case. For example, Hoskins<sup>63</sup> observed azimuthal asymmetries in self-field MPDT operation and related these asymmetries to deviations in the radial centroid of the current discharge from the true geometric centerline of the thruster. Evidence of fluctuations in the current discharge of our thruster was obtained via high-speed video imaging. Figure 42 shows 8 frames taken from the side, with 5  $\mu$ s exposure times and separated by 100  $\mu$ s intervals during the quasi-steady thruster firing period at 9 kA in self-field mode (no magnets). These images show fluctuations in the luminosity associated with the denser plasma regions in the plume near the thruster exit. The anode is on the left in these images, and the flow direction is to the right. Coupled with the observations by Hoskins, these oscillations in the luminous discharge regions suggest that asymmetries in the discharge pattern over time may indeed be another mechanism for increased number density fluctuations at higher currents. The induced azimuthal rotational motion with our applied magnetic fields could potentially have a gyroscopic stabilizing effect against such asymmetries, which would act to reduce fluctuations in the local number densities in the near-anode region.

## E. Summary of Results

**A reduction in the anode fall voltage is clearly observed for both applied B fields at 8 kA and for the cusp applied B field at 10.7 kA.** At 10.7 kA, the cusp applied B field significantly reduces average anode fall voltages to only 3–14 V over most much of the near-anode profile. At 8 kA, both applied B field configurations show a substantial reduction in anode fall voltages down to a range from 8 to -4 V, with negative values implying an electron-repelling anode fall. In these cases, a likely cause could be the radial component of the applied magnetic fields leading to increased current conductivity in regions where the applied B field lines intersect the anode (e.g., in regions of high current density). In addition, modest increases in  $T_e$  and  $n_e$  lead to increased random thermal flux of electrons to the anode to support the current densities demanded. These effects combine to increase electron transport to the anode surface in regions of high current density (e.g., circa  $z = -29$  mm), thus mitigating the need for large anode fall voltages. In general, the reductions in anode fall voltages suggest a link to the reduced average thruster terminal voltages previously observed with the magnetic fields. These relationships will be discussed further in Section VI.

**The applied B fields result in increases in electron temperatures near the regions of high applied B field intensity and higher number densities in regions of high radial current density to the anode.** Electron temperatures increase over much of the near-anode region with the applied B fields. These increased temperatures may be associated with heating in regions of high azimuthal current densities induced by the applied B fields. In addition, higher number densities are measured with the applied B fields in the regions where the radial current densities to the anode are highest. The increased number density is particularly pronounced in the case of the tangential B field, where it increases by a factor of roughly 6 to 8 relative to self-field operation. In addition to the applied B field's effect on the flow field, the increase in  $n_e$  upstream for the tangential B field might be due to an increased ionization zone where the current density is higher. Again, these increased number densities support increased electron random thermal flux to the anode.

**Variations of 1–2 orders of magnitude in the number densities between the near-cathode centerline and near-anode region clearly highlight the effects of the radial pumping forces on anode charge carrier depletion without the magnets.** Number densities measured over a broad range of the interior discharge region without the magnets clearly showed significant reductions in the number density between the centerline and the near-anode region. These variations signify charge carrier depletion near the anode, which establishes one of the conditions that lead to onset. Moreover, the expansion of the plasma suggested by the number density variations results in a trend of decreasing number densities in the upstream direction along the anode, thus making it even more difficult to sustain significant current densities further upstream in the self-field configuration.

**The applied B fields show a reduction in the magnitude and frequency of large fluctuations in the ion saturation current time-series signals, potentially implying a relation with the reduced terminal voltage fluctuations and anode spot damage.** At 10.7 kA, we observed a reduction in the intensity and frequency of large spikes in the ion saturation current with the cusp applied B field relative to the self-field case. At 8 kA, we see that the same effect occurs with both the tangential B field and the cusp B field. These results suggest a relation to the earlier demonstrated reduction magnitude and frequency of the terminal voltage signal fluctuations with the applied B fields, as is also supported by a relatively similar range of frequencies in the sets of PSDs. These fluctuations may be associated with anode spots seeding the near-anode plasma with vaporized anode material. It is also possible that the applied magnetic fields could help mitigate current filamentation and anode spot mode damage by forcing the plasma attachment to rotate azimuthally around the anode and thus smooth out heating at localized filamentary current attachment points. This may suggest a possible mechanism for mitigating anode spot damage, as such filamentation into concentrated anode spots would otherwise lead to anode erosion damage through vaporization of anode material in these local hot spots. High-speed video images were also shown highlighting fluctuations in the luminous discharge regions near the thruster exit plane, suggesting unsteady oscillations in the thruster discharge pattern at high currents as another possible mechanism for number density fluctuations.

These effects will be discussed in more detail in Section VI to examine the underlying physics.

## VI. Interpretations and Findings

The data presented in Sections III, IV, and V followed a sequence of increasingly focused investigations. The analyses started with the system-level behavior (e.g., terminal voltage characteristics), continued with investigations of the bulk plasma throughout the interior and near-anode regions, and ultimately focused on studying near-anode plasma properties. This section provides a synthesis of the experimental studies from these different scales, couples the various measurements to calculate relevant new plasma parameters, and relates the observed behaviors to the

driving physics and processes.

### A. Effects on the Current Pattern, Conductivity, and Current Densities

In Section IV, the clear effects of the applied magnetic fields were observed on the redistribution of the current pattern and regions of high current densities in the thruster. The current streamlines follow the applied B field lines in the near-anode attachment region, as seen in Figures 24, 25, and 26. These shifts in the current pattern are associated with similar shifts in the regions of high current densities along the anode, as shown in Figure 27. The tangential applied B field causes the current streamlines and regions of peak current densities to shift significantly upstream, following along the applied magnetic field lines as they begin to turn radially outward toward the anode. In addition to higher current densities upstream mid-anode, current densities are significantly lowered in the downstream region where the applied B field lines are predominantly contoured parallel to the anode. The cusp applied B field results in a more subtle but still clear shift in the current streamlines to follow along the cusp applied B field lines as they intersect the anode surface in the downstream region. The high current density region downstream along the anode lip is slightly broadened and shifted upstream from the self-field case.

These shifts in the current patterns, anode attachment, and regions of highest current densities are evidently influenced by the shape of the poloidal applied B field lines. We can examine these behaviors on the basis of the physics governing the electron mobility. Consider the classical electrical conductivity for the plasma,  $\sigma_0$ , given by

$$\sigma_0 = \frac{n_e q_e^2}{m_e \nu_e} \quad (14)$$

where  $n_e$  is the electron number density,  $q_e$  is the fundamental electron charge, and  $m_e$  is the electron mass.  $\nu_e$  is the electron collision frequency, which can be approximated from the electron-ion collision frequency,  $\nu_{ei}$ , given by the Spitzer model<sup>32,64</sup>

$$\nu_e \approx \nu_{ei} = \frac{q_e^4 n_e \ln \Lambda}{3(2\pi)^{3/2} \sqrt{m_e} \epsilon_0^2 (k_B T_e)^{3/2}} \quad (15)$$

where  $T_e$  is the electron temperature,  $k_B$  is the Boltzmann constant,  $\epsilon_0$  is the permittivity of free space, the plasma parameter,  $\Lambda$ , is

$$\Lambda = 12\pi n_e \lambda_{De}^3 \quad (16)$$

and the electron Debye length,  $\lambda_{De}$ , is

$$\lambda_{De} = \sqrt{\frac{\epsilon_0 k_B T_e}{n_e q_e^2}} \quad (17)$$

Note that electrical conductivity is simply the reciprocal of the resistivity,  $\eta_0$ , i.e.,

$$\eta_0 \equiv \frac{1}{\sigma_0} \quad (18)$$

We assumed that we can ignore collisions with neutrals to express  $n\nu_e \sim n\nu_{ei}$  in equation 15. To establish why this assumption is valid, note that the collision frequency scales as

$$\nu = A_{cs} n v_{rel} \quad (19)$$

where  $A_{cs}$  is the collision cross section,  $n$  is the number density of target particles and  $v_{rel}$  is the (average) relative velocity between the interacting particles. In MPD thrusters, the plasma is highly ionized, and our thruster plasma should be fully ionized at higher current conditions such as 10.7 kA. However, there is still neutralization of ions hitting the anode wall in the near-anode region. The number density of these neutrals will be much lower than the background plasma. In addition, consider the collision cross section,  $A_{cs}$ . For neutrals, where the radius of the atom is of order  $10^{-10}$  m, the collision cross section is of order  $A_{cs} \sim 3 \times 10^{-20}$  m<sup>2</sup>. For electron-ion Coulomb collisions, the collision cross section can be calculated from the Spitzer model equation 15 for  $n\nu_{ei}$  and dividing by the electron number density,  $n_e$ , and  $v_{rel}$  taken as the electron thermal velocity,  $v_{th,e}$ , given by

$$v_{th,e} = \sqrt{\frac{8k_B T_e}{\pi m_e}} \quad (20)$$

For parameters of interest in our experiments, where  $n_e \sim 10^{19}$  m<sup>-3</sup> and  $T_e \sim 5$  eV, we estimate  $A_{cs} \sim 2 \times 10^{-18}$  m<sup>2</sup>. This is two orders of magnitude higher than the neutral cross section, so  $n\nu_e \sim n\nu_{ei}$ . More generally, the electron-ion

Coulomb collisions should occur at much higher frequency than electron-neutral collisions for ionization fractions greater than order of 1%.<sup>65</sup>

The classical electrical conductivity,  $\sigma_0$ , is calculated along the near-anode region for the thruster operating at 10.7 kA without magnets, with tangential applied B field, and with cusp applied B field, as shown in Figure 43. These data were calculated using the Langmuir probe measurements in this region. Note that the dashed lines between data points are merely to guide the eye, not to suggest any specific functional dependence. Since  $\sigma_0$  is a strong function of temperature, we see somewhat increased values for  $\sigma_0$  with the applied B fields over much of the anode, following the results of the electron temperature data seen in the previous section in Figure 31.

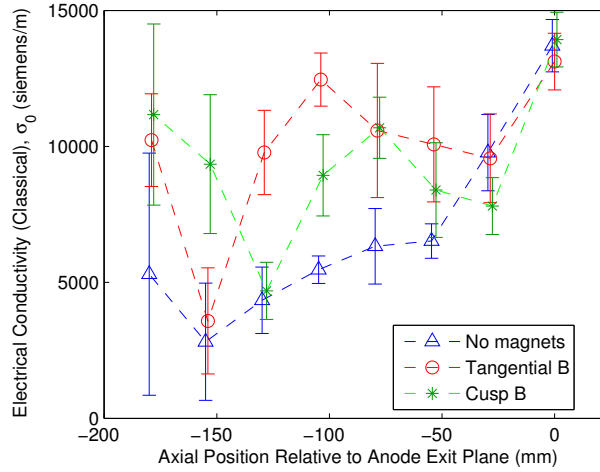


Figure 43. Electrical conductivity (classical, uncorrected) calculated along the near-anode region at 10.7 kA.

The electrons also become tightly bound to their Larmor orbits (i.e., gyro-orbits) around the magnetic field lines as the B field magnitude increases.<sup>13</sup> The electron Larmor radius,  $r_{Le}$ , is given by

$$r_{Le} = \frac{v_{th,e}}{\omega_{ce}} \quad (21)$$

where  $\omega_{ce}$  is the electron cyclotron frequency (i.e., the gyrofrequency), given as a function of the magnetic field intensity,  $B$ , by

$$\omega_{ce} = \frac{|q_e|B}{m_e} \quad (22)$$

A measure of how magnetically bound the electrons are to motion along the B field lines is the electron Hall parameter,  $\Omega_e$ . The electron Hall parameter is defined as the ratio of the electron cyclotron frequency to the collision frequency and can be expressed as

$$\Omega_e \equiv \frac{\omega_{ce}}{\nu_e} \approx \frac{\omega_{ce}}{\nu_{ei}} = \frac{qB}{m_e \nu_{ei}} = \frac{3(2\pi)^{3/2} \epsilon_0^2 (k_B T_e)^{3/2} B}{\sqrt{m_e} q_e^3 n_e \ln \Lambda} \quad (23)$$

This is a measure of how many gyro-orbits around the B field lines the electrons will undergo before they experience an elastic collision. Thus, the higher the Hall parameter, the stronger the electrons will be bound to the magnetic field lines and will be impeded for motion perpendicular to the B field. Figure 44 shows the electron Hall parameter calculated for the thruster at 10.7 kA along the same near-anode region as our conductivity calculations. The data show that the Hall parameter is indeed much greater than 1 over the entire region for all configurations, and the Hall parameter is generally higher with the applied B fields. The applied B field magnitudes are highest in this mid-range to downstream near-anode region, which is in closest proximity to the magnets' coils. Thus, the electrons are indeed strongly bound to the magnetic field lines, which is typically referred to as the electrons being "magnetized." As will be discussed later, the high Hall parameter in the case of the self-field configuration impedes electron conduction radially to the anode, as the magnetic field is purely azimuthal. However, the applied magnetic fields introduce significant radial B components that provide the magnetized electrons a path to the anode.

Let us consider multiple different models and viewpoints for expressing the current density to examine the effects of the Hall parameter and B field on current conduction in the plasma. First, examine the generalized Ohm's law from

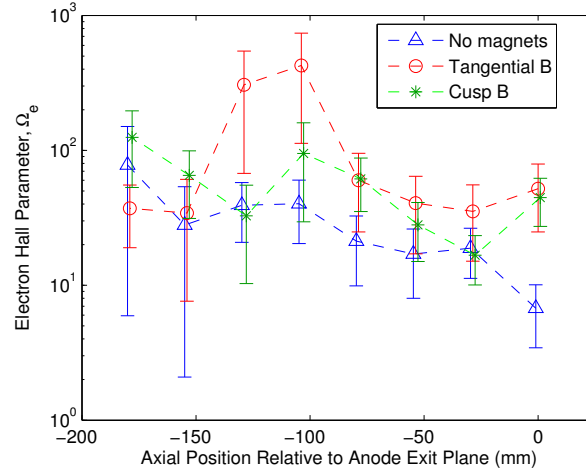


Figure 44. Electron Hall parameter calculated along the near-anode region at 10.7 kA.

MHD given by<sup>13</sup>

$$\mathbf{j} = \sigma_0 \left( \mathbf{E} + \frac{\nabla P_e}{n|q_e|} + \mathbf{u} \times \mathbf{B} - \frac{1}{n|q_e|} \mathbf{j} \times \mathbf{B} \right) \quad (24)$$

where  $\mathbf{u}$  is the streaming velocity of the plasma (bulk velocity), and we assume  $n = n_e = n_i$  in the quasineutral bulk plasma. If we ignore the pressure gradient contribution as small for now, we can write this in tensor form as<sup>32,66</sup>

$$\mathbf{j} = \sigma_0 \begin{bmatrix} \frac{1}{1+\Omega^2} & \frac{\Omega}{1+\Omega^2} & 0 \\ -\frac{\Omega}{1+\Omega^2} & \frac{1}{1+\Omega^2} & 0 \\ 0 & 0 & 1 \end{bmatrix} (\mathbf{E} + \mathbf{u} \times \mathbf{B}) \quad (25)$$

where the coordinate system is in orthogonal coordinates with unit vectors denoted as  $\hat{\mathbf{a}}$ ,  $\hat{\mathbf{b}}$ , and  $\hat{\mathbf{c}}$ , and the component  $\hat{\mathbf{c}}$  defined by the B field direction with  $\mathbf{B} \equiv B_c \hat{\mathbf{c}}$ . For Hall parameter much less than one, the tensor in this equation simply reduces to the unit tensor, resulting in the scalar conductivity solution. However, high Hall parameter clearly has anisotropic effects.

For the self-field case, the B field is purely azimuthal. In this B field geometry, the current density to the anode in the radial direction is given by the  $\hat{\mathbf{b}}$  component as

$$j_b = \sigma_0 \left( \frac{-\Omega}{1+\Omega^2} E_a + \frac{1}{1+\Omega^2} E_b + \frac{-\Omega}{1+\Omega^2} v_b B_c + \frac{-1}{1+\Omega^2} v_a B_c \right) \quad (26)$$

This expression shows how the current density to the anode in the self-field case (in the presence of purely azimuthal B field) is greatly reduced by high Hall parameter due to conduction perpendicular to the B field.

In addition, the case of B field in the radial direction toward the anode can be examined. This is relevant for regions where there is a strong applied B field in the radial direction. For this B field geometry, the  $\hat{\mathbf{c}}$  component of the current density co-aligned with the B field direction gives the current density to the anode (in the same direction) as simply

$$j_c = \sigma_0 E_c \quad (27)$$

where  $E_c$  in this geometry is now just the radial electric field. Therefore, this B field orientation simply results in the classical scalar conductivity times the radial electric field, with no reduction by the Hall parameter. Relative to the self-field case, the current conduction is greatly increased. The current density can either be much larger for a given electric field, or can sustain the same current density with a much lower electric field. We will consider the latter case further in Section C.

Another way to examine the effects of the Hall parameter and B field on current conduction is by analyzing the equation of motion for the electrons, since the electrons are the dominant source of current conduction due to their much smaller mass than the ions. Jahn<sup>65</sup> performed an analysis of AC conductivity for averaged electron motion. If

we take the steady-field DC limit as frequency  $\omega$  goes to zero and examine the case where  $\Omega \gg 1$ , we obtain an expression for the DC electron current density vector in terms of the Hall parameter as

$$\mathbf{j}_{DC} = \frac{\sigma_0}{\Omega^2} \mathbf{E} + \frac{\sigma_0}{\Omega} \frac{\mathbf{E} \times \mathbf{B}_0}{B_0} + \sigma_0 \frac{(\mathbf{E} \cdot \mathbf{B}_0) \mathbf{B}_0}{B_0^2} \quad (28)$$

where  $\mathbf{E}$  is the electric field vector,  $\mathbf{B}_0$  is the magnetic field vector, and  $B_0$  is the magnitude of the magnetic field vector. This expression provides a form for the current density dominated by the electrons, albeit neglecting the pressure gradient term for now (i.e., we assume the effects of the electric and magnetic field terms to dominate the  $\nabla P$  term in this example). This equation shows how high Hall parameter affects the current density and allows a simpler view of how the general form of the magnetic field affects the current density. Again, the conductivity is no longer scalar in nature, as it would be for Hall parameter much less than one. This equation reveals three major contributions. In the first term, the component directly associated with the electric field is reduced by the inverse square of the Hall parameter, making the contribution of that term very small. The second term is associated with the  $\mathbf{E} \times \mathbf{B}_0$  drift motion, which is also reduced by the inverse of the Hall parameter. This Hall current term introduces motion perpendicular to the B field lines. For example, the self-generated azimuthal component  $B_\theta$  of the magnetic field crossed with the radially inward component  $E_r$  of the electric field acts to direct the current downstream. In addition to the electric field, this Hall current term contributes to the current streamlines having such a strong axial component, particularly along the radial distance about half way to the anode, where the B field magnitudes are highest. The third term in the equation is perhaps the most compelling. It is not reduced at all by the Hall parameter, but it essentially scales with the angle between the electric field vector and the magnetic field vector. In the case of the self-field operation, this term is small, as the electric field is nominally oriented poloidally (radially and axially), and the self-generated magnetic field is purely azimuthal, i.e., perpendicular to the electric field. Thus, the third term goes to zero for self-field operation. However, with an applied magnetic field that has a significant component aligned with the electric field (which is primarily radial in the near-anode region), this third term dominates. Classical conductivity along the B field lines supports high current densities compared to cross-field conduction, which is reduced by the high Hall parameter.

This effect is responsible for the change in the current density profile under the influence of the applied B fields in the near-anode region. Current attachment is preferentially shifted to regions where there is strong co-alignment of the electric and magnetic field vectors (predominantly, the radial components). Namely, the highest current densities can occur for the tangential B field upstream on the anode where the magnetic field lines begin to turn radially toward the anode. Highest current densities can occur for the cusp B field in the downstream region where the cusp B field lines intersect the anode. This conduction behavior also partly explains why the high current density anode attachment zone for the self-field configuration occurs far downstream on the anode lip, where the self-generated azimuthal magnetic fields are weakest, yielding lower Hall parameter and higher cross-field current density. This effect is in addition to the overall downstream convection of the current streamlines at high magnetic Reynolds number in the flowing plasma.

The general effect of the B field seen from equation 28 is to significantly decrease the conduction perpendicular to the B field lines but allow for much greater conduction along the directions parallel to the B field. In the self-field configuration, the B field is only azimuthal, as created by its own purely poloidal discharge current pattern. This magnetic field geometry thus requires the current pattern to cross transverse to the self-generated azimuthal B field to reach the anode. However, conduction across these B field lines is impeded by the high Hall parameter. With the applied B fields, conduction to the anode is higher where the applied B field lines turn radially outward toward the anode.

This effect can also be understood from the viewpoint of tracking individual charged particles. Electrons are strongly bound to the B field lines in the regions of high Hall parameter (both through the contributions of the azimuthal self-field and the poloidal applied field), as they are much lower mass than the ions and thus have much smaller Larmor orbits around the B field lines. The much heavier ions have significantly larger gyro-radii and are not “magnetized” in the modest B field magnitudes of our thruster operating conditions. The ions predominantly follow the electric field. The magnetized electrons, however, are driven to follow along the applied B field lines to the anode-intersecting regions of highest current attachment, where the magnitudes of the applied B field are also relatively larger (greater than the local self-field azimuthal B in this near-anode region). This results in the experimentally observed effect of focusing the higher current conduction to the anode in the regions where there is a sufficiently large radial component to the applied B field (relative to the axial and azimuthal components) and significant co-alignment of the local B field vector and E field vector.

Therefore, the anode-intersecting applied B field mitigates the otherwise limiting effect of the Hall parameter on current density to the anode for self-field operation, where the plasma experiences only the self-generated azimuthal B

field and is forced into cross-field transport. This behavior can also be examined in terms of the diffusion process and the effect of the B field on the diffusion coefficient. This modeling viewpoint is relevant if the pressure gradient term is significant. Either assuming isothermal electrons or that the pressure gradient is simply dominated by the gradient in number density,  $\nabla n_e$  (and the temperature gradient is much smaller), this yields a diffusion process for the electrons. The equation for classical diffusion of electrons across a magnetic field can then be written in terms of the electron flux given by Goebel and Katz<sup>67</sup> as

$$\Gamma_e = \mu_{\perp,e} n_e \mathbf{E} - D_{\perp} \nabla n \quad (29)$$

where  $\mu_{\perp,e}$  is the perpendicular electron mobility given by

$$\mu_{\perp,e} = \frac{\mu_{0,e}}{1 + \Omega_e^2} = \frac{1}{1 + \Omega_e^2} \frac{q_e}{m_e \nu_e} \quad (30)$$

and where  $D_{\perp}$  is the perpendicular diffusion coefficient,

$$D_{\perp} = \frac{1}{1 + \Omega_e^2} D_0 = \frac{1}{1 + \Omega_e^2} \frac{v_{th,e}^2}{\nu_e} \quad (31)$$

This equation shows that the diffusion coefficient is strongly dependent on the Hall parameter for cross-field diffusion. Consider the basis of the classical diffusion coefficient,  $D_0 = D_{\parallel}$  (the same as the diffusion coefficient parallel to the B field lines). As discussed by Bellan,<sup>13</sup>  $D$  is a consequence of the random walk motion of particles and collisions, with  $D \sim \frac{(\text{step size})^2}{(\text{time between collisions})}$ .  $D_0$  is thus a function of the mean free path,  $\lambda_{mfp}$ , and collision frequency given by

$$D_0 = D_{\parallel, \text{classical}} = \lambda_{mfp}^2 \nu_e = \left( \frac{v_{th,e}}{\nu_e} \right)^2 \nu_e = \frac{v_{th,e}^2}{\nu_e} \quad (32)$$

Further, we can examine the case of high magnetic fields and high  $\Omega$ , where the diffusion coefficient perpendicular to the B field is reduced because the electrons are orbiting the magnetic field lines in a new length scale associated with the Larmor orbit gyro-radius. This new length scale results in

$$D_{\perp, \text{classical}} \sim r_{Le}^2 \nu_e = \left( \frac{v_{th,e}}{\omega_{ce}} \right)^2 \nu_e = \frac{1}{\Omega^2} \frac{v_{th,e}^2}{\nu_e} = \frac{1}{\Omega^2} D_0 \quad (33)$$

For Hall parameters much greater than one, this is the same as the expression for  $D_{\perp}$  from Goebel and Katz. Thus, the diffusion coefficient,  $D_{\perp}$ , for the diffusion perpendicular to the magnetic field is significantly reduced by the Hall parameter. In our MPD thruster, the electrons are greatly impeded from diffusing toward the anode in the presence of large azimuthal (and axial) B fields. However, the diffusion parallel to the magnetic field lines is unimpeded from the classical diffusion coefficient,  $D_0$ . This effectively allows for much greater electron diffusion radially toward the anode in the regions where there is a substantial  $B_r$  radial component. Again, this is consistent with the measured current pattern and regions of high current densities in our thruster.

## B. Regions of Increased Heating and Induced Azimuthal Current Densities

In Section V, we observed increases in the local electron temperatures at 10.7 kA in the near-anode region as shown in Figure 31 operating at 10.7 kA. These regions of increased temperatures can be shown to relate to the regions of highest current densities in the thruster.

The induced azimuthal current density,  $j_{\theta}$ , caused by the externally applied magnetic fields may be estimated using the vector form of the generalized Ohm's law given in equation 24. The azimuthal current density,  $j_{\theta}$ , is obtained from the azimuthal component of this equation as

$$j_{\theta} = \sigma_0 \left( E_{\theta} + \frac{1}{n|q_e|} \nabla_{\theta} P_e + u_z B_r - u_r B_z - \frac{1}{n|q_e|} (j_z B_r - j_r B_z) \right) \quad (34)$$

where subscripts  $r$ ,  $\theta$ , and  $z$  represent the radial, azimuthal, and axial components, respectively. However, we can make some simplifying assumptions that are generally true for MPD thruster acceleration processes,<sup>32</sup> particularly near the anode, such as  $u_z \gg u_r$ ,  $\mathbf{E}$  is mostly radial, and  $E_{\theta}$  and  $\nabla_{\theta} P$  go to zero (or the gradient is vanishingly small) due to azimuthal symmetry in steady state. With these assumptions, we get the following expression for the azimuthal current density

$$j_{\theta} = \sigma_0 \left( u_z B_r - \frac{1}{n|q_e|} (j_z B_r - j_r B_z) \right) \quad (35)$$

We use this expression to then calculate an estimate of  $j_\theta$  in the near-anode region given the applied poloidal B field and our measured  $n_e$ ,  $j_r$ , and  $j_z$ . The calculated  $j_\theta$  values for the tangential and cusp applied B field configurations are shown in Figure 45. For  $u_z$ , we assume an average from the estimated thruster exhaust velocity obtained from the  $bJ^2$  thrust and  $I_{sp}$  model described separately in Section E. A sensitivity analysis showed that the results were not significantly sensitive to the expected range of  $u_z$  for our parameters. The classical electrical conductivity,  $\sigma_0$ , given by equation 14 was assumed for the calculation. We observe substantial azimuthal current densities in the regions where the thruster experiences high applied B field crossed with the current density vector. Therefore, the azimuthal current densities are moderated somewhat in the regions of highest current attachment, as these locations happen to also occur where the current streamlines are mostly parallel to the local applied B field lines. Also, note that the reversal in polarity of  $j_\theta$  for the cusp applied B field case is associated with the reversal in the direction of the B field vector across the cusp region caused by the opposing currents in the two magnets.

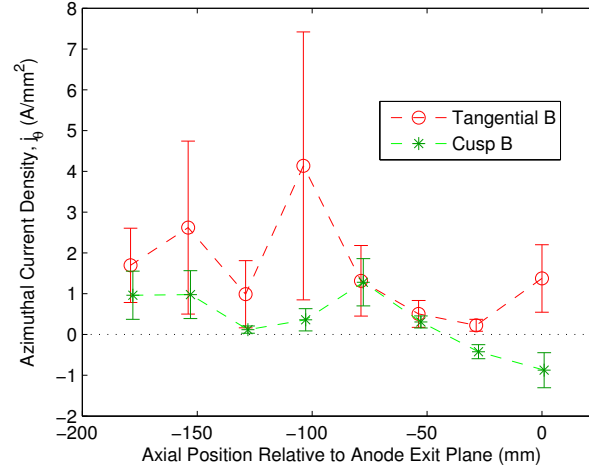


Figure 45. Azimuthal current density estimated along the near-anode region at 10.7 kA.

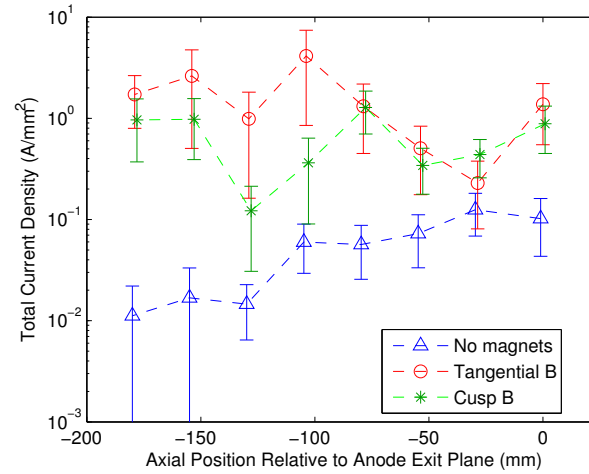


Figure 46. Total current density estimated along the near-anode region at 10.7 kA.

The magnitude of the total current density,  $|j_{total}| = (j_r^2 + j_\theta^2 + j_z^2)^{1/2}$ , is calculated using the estimated  $j_\theta$  and shown in Figure 46. For the self-field configuration,  $j_{total}$  generally increases in the downstream direction, as do the electron temperatures. The contribution of the calculated  $j_\theta$  component greatly increases the total current density for the tangential and cusp applied B field configurations. For the applied B field cases, the regions of increased and decreased electron temperatures in Figure 31 also roughly follow the trends seen in the total current densities.

The increased temperatures are likely due to Ohmic heating of the plasma in the high current density regions. Ohmic heating scales with the resistivity (and thus with the inverse of the conductivity) as  $\sim \eta j_{total}^2 = \frac{j_{total}^2}{\sigma}$ .<sup>65</sup> This

is consistent with our observation that the regions of increased temperatures seem to follow the local trends in higher current density (and thus regions of greatest heating). The applied B fields lead to higher temperatures (relative to the self-field case) in regions of high azimuthal current densities.

In addition to the immediate vicinity of the anode, regions of high current densities can be clearly identified in the interior of the thruster discharge. All operating configurations exhibited high current densities around the downstream perimeter and in front of the cathode, as associated with regions of high current attachment to the cathode outer body and front face. Additionally, the regions of high radial current density identified along the anode extend into the interior of the discharge, generally following along the enclosed current streamlines. This is as expected since we observe a compression of the current streamlines as they converge close the anode in the regions of highest anode current densities.

Heat generated in these interior regions can thermally conduct to the anode, which is another possible cause of increased electron temperatures. The plasma thermal conductivity is not scalar in the presence of the magnetic field. As shown by Woods,<sup>68</sup> the electron contribution to the thermal conductivity in the direction parallel to the B field,  $\kappa_{\parallel, the}$ , scales as

$$\kappa_{\parallel, the} = \kappa_{0, the} = \frac{5}{2} \frac{k_B}{m_e} \frac{P_e}{\nu_e} = \frac{5}{2} \frac{k_B^2 n_e T_e}{m_e \nu_e} \quad (36)$$

which is the same as the classical thermal conductivity value, whereas the thermal conductivity in the direction perpendicular to the B field,  $\kappa_{\perp, the}$ , scales as

$$\kappa_{\perp, the} = \frac{1}{1 + \Omega_e^2} \kappa_{\parallel, the} \quad (37)$$

The thermal conductivity is not scalar and scales as the inverse square of the Hall parameter perpendicular to the B field, much like the electrical conductivity, as discussed in Section A. Therefore, thermal conduction from the interior plasma radially outward to the anode region is higher in regions of substantial radial applied B field, but the thermal conductivity is decreased in the direction perpendicular to the B field. In the regions where the applied magnetic field has a high axial component but little to no radial component, this results in a decrease of the thermal conduction from the plasma radially outward toward the anode. This occurs, for example, in the region close to the axial positions of the applied-field magnets, which helps explain the relatively lower electron temperatures with the applied-field magnets that occur upstream around axial positions  $z = -129$  mm and  $z = -154$  mm and at the downstream location  $z = -29$  mm.

We have thus far considered only the classical electrical conductivity from equation 14. However, studies have shown (cf., Choueiri and Caldo<sup>43, 69</sup>) that microturbulence instabilities in MPD thrusters can trigger so-called “anomalous resistivity,” wherein the resistivity can be much higher (i.e., the electrical conductivity can be much lower) than the estimate of classical resistivity and electrical conductivity. Caldo<sup>69</sup> showed that the threshold at which this would begin to occur was of order  $v_d \sim 1.5v_{th, ion}$ , where  $v_d$  is the drift velocity given by

$$v_d = \frac{j}{n_e |q_e|} \quad (38)$$

and  $v_{th, ion}$  is the ion thermal velocity,

$$v_{th, ion} = \sqrt{\frac{8k_B T_i}{\pi m_i}} \quad (39)$$

where  $T_i$  is the ion temperature (assumed to be of similar order to  $T_e$  for our plasma) and  $m_i$  is the ion mass (argon in our plasma). Gallimore<sup>32</sup> measured electrical conductivities in his Princeton Benchmark Thruster that were up to 30 times lower than his estimates for the classical conductivity. In Figure 47, we plot the calculated ratio of  $v_d/v_{th, ion}$  for our thruster configurations. For the applied B field cases, this ratio is estimated to be of order 10 to 100 based upon our calculated values of  $j_\theta$ . Therefore, anomalous resistivity is likely to be present, and our estimates of the electrical conductivity,  $j_\theta$ , and  $j_{total}$  are likely to be overestimated. This makes it impossible to quantitatively estimate the absolute heating rates. Nonetheless, the trends in functional forms for the conductivity,  $j_\theta$  and  $j_{total}$ , suggest a relationship between the induced azimuthal current densities and the measured increases in electron temperature.

### C. Reduction of Anode Fall Voltages

The applied magnetic fields in these experiments led to three major beneficial effects related to the voltages: (1) reduced anode fall voltages, (2) lower mean terminal voltages (thus, reduced input power for the same discharge current), and (3) reduced intensity and frequency of the large fluctuations and spikes in the thruster terminal voltage.

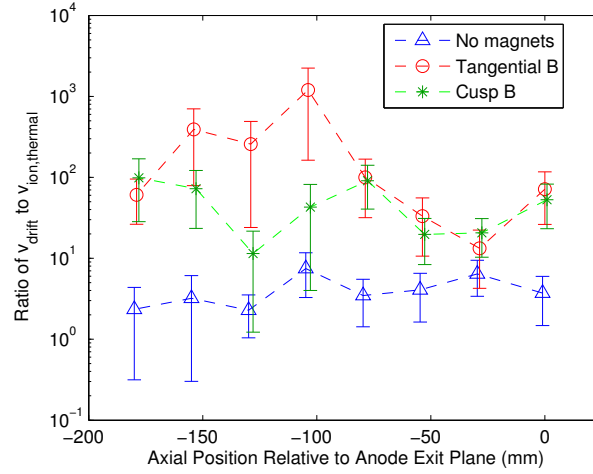


Figure 47. Ratio of drift velocity to ion thermal velocity along the near-anode region at 10.7 kA.

Let us examine the anode fall voltage reduction in some detail first, then consider how it may relate to the reduction in mean terminal voltage and voltage fluctuations.

As seen in Figure 33 at 10.7 kA and Figure 36 at 8 kA, the cusp applied B field caused a very significant reduction in the anode fall voltage,  $V_{fall}$ , at 10.7 kA, and both applied B fields resulted in significant reductions in  $V_{fall}$  at 8 kA relative to the self-field case. The applied B fields can result in greatly reduced electric fields near the anode as a consequence of the increased current conduction to the anode discussed in Section A. Equation 28 related the vectors for the steady-state (DC) current density, electric field, and magnetic field in the high Hall parameter limit. From this equation, the radial component of the current density near the anode surface is given by

$$j_r = \frac{\sigma_0}{\Omega^2} E_r + \frac{\sigma_0 (E_\theta B_z - E_z B_\theta)}{B_0} + \sigma_0 E_0 \cos(\psi) \hat{\mathbf{B}} \cdot \hat{\mathbf{r}} \quad (40)$$

where  $B_0$  is the total magnitude of the B field,  $E_0$  is the total magnitude of the E field,  $\psi$  is the angle between the B field and E field vectors,  $\hat{\mathbf{B}}$  is the unit vector in the B field direction, and  $\hat{\mathbf{r}}$  is the radial unit vector. In the case of self-field operation, the B field is purely azimuthal, and the E field components are purely poloidal. Thus, the third term vanishes. In MPD thrusters near the anode, we also expect the electric field to be predominantly radial, so  $E_r$  will be much larger than  $E_z$  and  $E_\theta$ .<sup>32</sup> In the simplified case of only radial E field, the current density equation reduces to the first term for the self-field configuration, given by

$$j_{r, self\ field} \sim \frac{\sigma_0}{\Omega^2} E_r \quad (41)$$

$$E_{r, self\ field} \sim \frac{\Omega^2}{\sigma_0} j_{r, self\ field} \quad (42)$$

Thus, to sustain a given radial current density, the electric field  $E_r$  must grow, scaling with the square of the Hall parameter. However, with the anode-intersecting applied B fields, there are regions where the E field and B field are significantly co-aligned radially ( $E_0 \sim E_r$  and  $\cos(\psi) \sim 1$ ). This gives an expression for the radial current density and radial E field component in terms of the Hall parameter,

$$j_{r, applied\ field} \sim \frac{\sigma_0}{\Omega^2} E_r + \sigma_0 E_0 = \sigma_0 \left( \frac{1}{\Omega^2} + 1 \right) E_r \approx \sigma_0 E_r \quad (43)$$

$$E_{r, applied\ field} \sim \frac{1}{\sigma_0} j_{r, applied\ field} \quad (44)$$

For the applied B field configurations, in regions where there is significant radial B field, the electric field  $E_r$  can thus be much smaller to sustain a similar order of radial current density. This further supports the discussion in Section A showing highest current densities were associated with regions of significant radial B field and much lower current densities where the B field was only tangential to the anode. Moreover, the result above suggests that the much smaller

electric field required to sustain a high current density could be obtained with a much smaller anode fall voltage, as the steady-state electric field near the anode scales as

$$\mathbf{E} = -\nabla V \quad (45)$$

$$E_r \sim -\nabla_r V_{fall} \quad (46)$$

Therefore, we suggest the increased conduction along the applied B field lines radially intercepting the anode and the associated reduction in E field as prevailing causes of the reduced anode fall voltage.

Another way to examine the cause of large anode fall voltages is to consider the random thermal flux of electrons to the anode. When the electron random thermal flux to the anode surface is insufficient to provide the current density required at the anode, the potential in the anode fall region must reverse from an electron-repelling potential to an electron-attracting anode fall voltage. This must form to accelerate the electrons beyond the random thermal flux alone and sustain the higher current densities with increasing total thruster discharge current. This condition where the requisite anode current density exceeds the “thermal” current density from random thermal flux to the anode surface has been presented as a condition for the inception of so-called “onset” and “critical current” regimes in MPD thrusters by many authors.<sup>20, 21, 31, 32, 34</sup> Thus, we can examine the conditions in our thruster to determine whether we exceed this criteria and how this was affected by the applied magnetic fields.

First, we consider the case assumed by the previous authors, wherein the random thermal electron current density to the anode surface is simply given by the electron charge times the classical kinetic theory expression for electron random thermal flux to the anode surface (not accounting for B field effects),

$$j_e = q_e \Gamma_{0,e} = q_e \frac{1}{4} n_e v_{th,e} \exp\left(\frac{-q_e V_d}{k_B T_e}\right) \quad (47)$$

where  $v_{th,e}$  is the electron average thermal velocity,

$$v_{th,e} = \sqrt{\frac{8k_B T_e}{\pi m_e}} \quad (48)$$

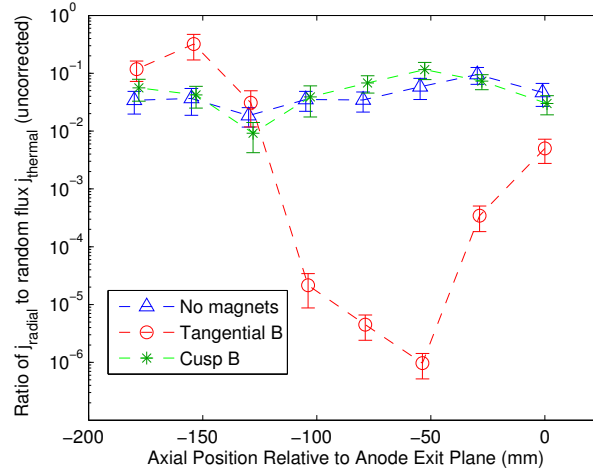
and  $V_d$  is the potential difference between the anode wall and the local ambient plasma. Taking the limit of  $V_d$  going to zero gives an expression for the maximum “saturation” current density before an electron-attracting sheath must form to draw any more current, given by

$$j_e = q_e \frac{1}{4} n_e v_{th,e} \quad (49)$$

Figure 48 shows the ratio of the measured radial current density to this electron thermal current (flux) along the near-anode region. These data are calculated from the measurements in this region with the thruster operating at 10.7 kA with the three different magnetic configurations. This figure shows that the measured current density never exceeds the values calculated for the random thermal current density in this region.

However, we suspect that this expression is likely to be overestimating the actual random thermal current density to the anode due to impeded electron motion in the presence of the magnetic fields. For the conditions in our thruster, the electron Larmor radius is large (order of mm) compared to the electron Debye length (order of micrometers). Thus, we expect that the anode sheath does not extend far enough (order of several Debye lengths) to prevent the motion of the electrons just outside of the sheath from entering the sheath and impacting the anode within the scale of their Larmor radius orbits. However, the electrons that are effectively lost to the anode surface must be replenished by diffusion of electrons from the plasma further away, in which electrons must transport across the B field.

As a rough order-of-magnitude correction to the electron random thermal current in the presence of the B fields, we consider scaling arguments based on diffusion and the diffusion coefficient. As discussed in Section A, the diffusion coefficient  $D$  is a consequence of the random walk motion of particles and collisions, with  $D \sim \frac{(\text{step size})^2}{(\text{time between collisions})}$ . We showed in equations 33 and 32 that the diffusion coefficient parallel to the B field is effectively unchanged, while the diffusion coefficient perpendicular to the B field is greatly reduced by the Hall parameter. In the direction perpendicular to the B field, electron motion is associated with step sizes on the scale of the Larmor radius rather than the mean free path, resulting in this reduction in the diffusion coefficient. Therefore, we might argue that the electron flux perpendicular to the B field lines will be impeded in a similar way. This matters because our number density estimates with the triple Langmuir probe were obtained roughly 4 mm away from the anode surface. However, local Larmor orbit length scales for the electrons were smaller than this distance, typically in a range of 0.4–1.2 mm for our measurements at 10.7 kA.



**Figure 48. Ratio of measured radial current density to electron random thermal current (flux) along the near-anode region at 10.7 kA. Electron thermal current is calculated using classical flux uncorrected for B field effects.**

For our rough order-of-magnitude estimate of the corrected electron flux, we first account for diffusion along the B field lines using  $D_{\parallel}$  and flux perpendicular to the B field lines using  $D_{\perp}$  and then assume that an effective diffusion coefficient can be written as

$$D_{effective} = \alpha D_{\parallel} + (1 - \alpha) D_{\perp} = \left(\alpha + \frac{1 - \alpha}{\Omega^2}\right) D_0 = \gamma D_0 \quad (50)$$

$$\gamma \equiv \left(\alpha + \frac{1 - \alpha}{\Omega^2}\right) \quad (51)$$

where  $\alpha$  is a ratio to account for what fraction of the total B field is in the radial direction (equivalently, the sine of the angle at which the B field vector intercepts the anode surface). This ratio  $\alpha$  is given by

$$\alpha = \frac{|B_r|}{\sqrt{B_r^2 + B_{\theta}^2 + B_z^2}} \quad (52)$$

where  $B_r$ ,  $B_{\theta}$ , and  $B_z$  are the radial, azimuthal, and axial components of the B field, respectively. Given that the flux of electrons is proportional to the diffusion coefficient, we first assume that the flux of electrons to the surface in the direction parallel to the B field is unaffected and is simply the classical electron flux  $\Gamma_0$  given by

$$\Gamma_0 = \Gamma_{\parallel} = \frac{1}{4} n_e v_{th,e} \quad (53)$$

Again, we assumed here that the potential difference  $V_d$  between the anode surface and the local plasma goes to zero to find the electron “saturation” limit (before the reversal of potential to electron-attracting voltage falls is required to draw more current). This  $\Gamma_0$  is appropriate for motion parallel to the B field.

Perpendicular to the B field, we then make a rough order-of-magnitude estimate for the “corrected” electron random thermal flux to the anode surface in a similar manner to the diffusion coefficient by scaling with the same Hall parameter-dependent factor to obtain

$$\Gamma_{\perp} = \frac{1}{\Omega^2} \Gamma_0 \quad (54)$$

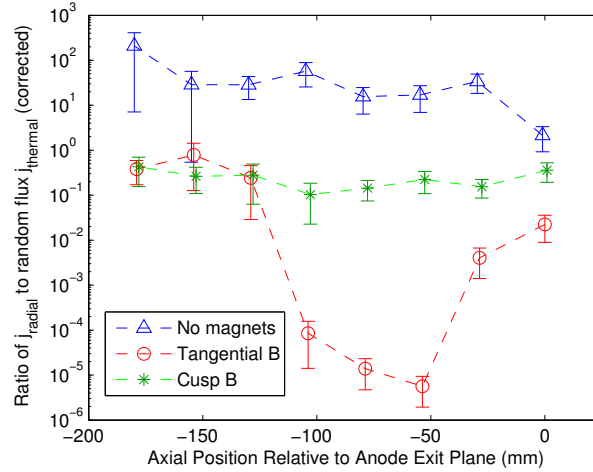
and thus estimate an effective overall flux for the components parallel and perpendicular to the B field as

$$\Gamma_{effective} = \gamma \Gamma_0 \quad (55)$$

Multiplying the flux by the electron charge gives a rough-order model for the corrected estimate for the radial current density from electron random thermal flux to the anode surface in the presence of the B field

$$j_r = q_e \Gamma_{effective} \quad (56)$$

Figure 49 shows the ratio of measured radial current density to the radial thermal current density values calculated assuming this corrected model for the influence of the B field in the manner described above by taking  $\Gamma = \Gamma_{\text{effective}}$ . This corrected thermal flux results in a ratio of measured radial current density to calculated thermal current density that is always less than 1 with the applied B fields but always greater than 1 for the self-field case. In the self-field case, the purely azimuthal B field acts only to reduce the effective flux to the anode. The presence of radial B field components with the applied-field configurations greatly increases the effective flux to the anode surface. Given that these radial B fields also arise in the regions of highest current densities, the discharge is able to sustain the increase in current density (i.e., increased demand) via the increased flux along the B field lines (i.e., increased supply of electron flux). Further, in regions where there is little to no radial component of the applied B field, the measured current densities demanded by the discharge are also so much lower such that the limited supply of electron thermal flux across the transverse B field lines is sufficient.



**Figure 49. Ratio of measured radial current density to electron random thermal current (flux) along the near-anode region at 10.7 kA. Here, a correction to electron thermal current is calculated for B field effects using the Hall parameter.**

This corrected model of electron flux to the anode in the presence of the B field is most likely an excessive, conservative reduction resulting in an underestimate of the actual electron flux, as discussed in Moeller's thesis.<sup>56</sup> However, even in a relative sense, this simple corrected model suggests that anode-intersecting applied B fields can increase the local current density that can be supplied by the electron random thermal motion relative to the self-field configuration. As the ratio of local current density demand to electron thermal flux supply is reduced by the applied B fields, there is a reduction in the required anode fall voltages to augment electron random thermal current to the anode. This result supports the observed reduction in the anode fall voltages with the applied B fields.

#### D. Reduced Terminal Voltages, Fluctuations, Anode Heating, and Anode Spotting

Given the findings in Section C, we can attempt to relate the anode fall reduction to the other observed effects. A significant reduction in mean terminal voltages was observed over a large range of currents, as seen in Figure 11. The average input power is

$$P_{in} = VJ \quad (57)$$

where  $V$  is the average voltage and  $J$  is the average total discharge current. By achieving the same discharge current at lower operating voltages with the thruster, the input power is reduced for a given current with the applied B fields. As discussed later in Section E, this leads to an increase in thruster efficiency.

First, let us focus on why the mean terminal voltage decreases. Consider the terms of the thruster voltage decomposed as<sup>70</sup>

$$V = V_{\text{back EMF}} + V_{\text{ionization}} + V_{\text{thermal}} + V_{\text{fall}} \quad (58)$$

where  $V_{\text{back EMF}}$  is the electromagnetic induced electromotive force ("back-EMF") term,  $V_{\text{ionization}}$  is the power going into ionization divided by the current  $J$ ,  $V_{\text{thermal}}$  is the thermal contribution to the heating power dissipation divided by  $J$ , and  $V_{\text{fall}}$  is the anode fall voltage.

These experiments clearly showed a significant reduction in the anode fall voltage over much of the anode with the applied B fields. This effect was pronounced for the cusp B field at 10.7 kA and for both magnetic field configurations

at 8 kA. If we compare this result to the measured reduction in mean terminal voltages in Figure 11, we can see that both applied B fields resulted in substantially lower terminal voltages at 8 kA and a significant reduction only in the case of the cusp applied B field at 10.7 kA. Therefore, we suggest that the reductions observed in the anode fall voltage directly lower the mean terminal voltages by reducing the  $V_{fall}$  contribution to the total discharge voltage.

Further, the power lost to the anode includes energy deposition from electron impacts, ion impacts, thermionic emission, radiation, and convection.<sup>44</sup> Gallimore showed that 65-95% of the total power deposition to the anode came from the current-carrying electrons. Power lost to the anode from electron heating due to  $V_{fall}$  is reduced with the applied B fields, particularly with the cusp configuration in general and the tangential B field at lower current. The power lost to anode heating via electron bombardment is related to the radial current density as<sup>44</sup>

$$P = \int \Gamma_{r,e} q_e (V_{fall} + \frac{5k_B T_e}{2|q_e|}) dA = \int j_r (V_{fall} + \frac{5k_B T_e}{2|q_e|}) dA \quad (59)$$

where  $\Gamma_{r,e}$  is the radial electron flux and the integral is taken over the entire anode surface. In general, the anode fall voltages in the regions of highest current density contribute the most to this integral. This assumes that all of the power deposited in the anode fall region is deposited to the anode. The cusp applied B field case results in a substantial reduction in  $V_{fall}$  at similar radial current densities as the self-field configuration. This would therefore result in an appreciable reduction in the overall power lost to the anode.

Next, we consider the measured reduction in intensity and frequency of the terminal voltage fluctuations. Where the applied B fields radially intercept the anode, we proposed expressions in Section C showing a markedly higher electron flux than the self-field case, as given in equation 55. We also observed how this could result in conditions for the measured radial current density to be sufficiently supplied by the random thermal current flux with the applied B field configurations, as shown in Figure 49. Several authors<sup>20,25,32,34</sup> have attributed the transition to where the ratio of  $j_r$  to  $j_{thermal,e}$  is greater than 1 as a condition for the inception of significant voltage fluctuations observed in the terminal voltage. Therefore, increasing the thermal electron flux to the anode with the applied B fields should act to mitigate transition to this condition.

Further, Uribarri<sup>18</sup> and Diamant<sup>26</sup> have directly attributed the terminal voltage fluctuations to anode spot formation on the surface of the anode. High-speed video imaging evidence that anode spots occur in our thruster was shown in Figure 14 for self-field operation at 9 kA, and Figure 15 shows evidence of anode erosion due to melting and spotting. The concentration and extinction of these anode spots were shown by Uribarri to result in the release of anode material vapor to the near-anode plasma. Measurements of local ion saturation currents and spectroscopy near the anode by these authors also showed a direct link between fluctuations in local measured plasma density and the release of anode material into the plasma flow. Such spotting behavior was strongly associated with anode erosion. Uribarri<sup>31</sup> proposed a capacitively-coupled model for the interactions in the anode sheath that showed how the formation and extinction of anode spots can be related to the creation of voltage fluctuations (“voltage hash”).

Additionally, both Uribarri<sup>31</sup> and Giannelli et al.<sup>39,40</sup> described the formation of a current filamentation instability and linked this to near-anode current concentration and formation of anode spots. Giannelli also described a similar capacitively-coupled voltage model wherein he linked the formation and extinction of current filaments to voltage fluctuations.

These models and experiments by Uribarri and Giannelli et al. established a relationship between current filamentation, anode spots, and the voltage fluctuations. Assuming these models are correct in their interpretation relating these phenomena, our observed reductions in the frequency and intensity of the terminal voltage fluctuations with the applied B fields strongly suggest reductions in the frequency of anode spotting events and the average current being driven to concentrated anode spots (intensity). The reductions in anode spots with the applied magnetic fields would likely be associated with notably less anode erosion over time in quasi-steady and steady-state MPD thrusters operating at ranges of high current conditions (i.e., high  $J^2/\dot{m}$ ) similar to our thruster operating conditions.

In addition to measured reductions in the voltage fluctuations, we observed a reduction in the magnitude and frequency of spikes in ion saturation currents measured near the anode with the applied B fields. Examples of these ion saturation current measurements and associated PSDs were shown in Section V-E, and these were found to exhibit similar behavior to the terminal voltage fluctuations (relatively similar range of frequencies and attenuation of fluctuations with the applied B fields). Since ion saturation current is directly proportional to the local plasma number density, these spikes are associated with transient increases in the local number density, as was observed by Diamant<sup>25,26</sup> and Uribarri<sup>31</sup> and linked to anode material vaporization due to anode spots and filamentary current concentrations.

The azimuthal rotational motion of the plasma induced by the applied magnetic fields could be the cause of the reduced magnitude of the ion saturation current fluctuations. The poloidal current densities interact with the applied poloidal B field to create electromagnetic Lorentz forces  $j_r B_z$  and  $j_z B_r$  in the azimuthal direction. Azimuthal rotational swirling motion should cause azimuthal migration of any current-concentrating filaments formed. Motion of

these filaments should also lead to similar migration of local attachment points at anode spots, which would result in reduced residence time for a given anode spot at a fixed point on the anode. Reduced residence time at a particular location would decrease heating at that particular location from the shorter-lifetime spot. This would decrease the energy going into vaporizing anode surface material in a given, fixed local area. Even if the same amount of energy were going into the spots, it would effectively be spread out over a larger area. More of the anode would be heated, but less concentration of that heat at specific spot locations would yield less vaporization of anode material. Reduced vaporization of anode material seeding the near-anode plasma would be another possible explanation for the observed reduction of number density fluctuations. Given the link between terminal voltage fluctuations, anode spots, current filamentation, and anode erosion established by other studies,<sup>25,31,39</sup> a reduction in residence times of local anode spot concentrations is expected to result in less erosion (shorter time for concentrated heating at a fixed point) and decreased voltage fluctuations. Coupled with the decreased anode heating from the reduced anode fall voltage, anode erosion could potentially be reduced even further with the applied B fields.

In addition, we speculate that the spikes in the number density could be associated with measurement of local transients due to other general deviations from azimuthal symmetry in the current attachment pattern, which would also be expected to cause concentrated spikes in the local number density. It is possible that the intensity and frequency of these number density fluctuations could also be reduced because of the induced swirling azimuthal motion of the plasma in response to the applied poloidal magnetic field. The induced azimuthal rotation of the plasma may act to smooth out unsteady asymmetries in the discharge pattern. For example, Hoskins<sup>63</sup> observed azimuthal asymmetries in self-field MPDT operation and related these asymmetries to deviations in the radial centroid of the current discharge from the true geometric centerline of the thruster. Such asymmetries are supported by evidence from high-speed video images of our thruster in self-field operation in Figure 42. These images show fluctuations in the luminous discharge regions near the thruster exit plane, suggesting unsteady oscillations in the thruster discharge pattern at high currents as another possible mechanism for number density fluctuations. The induced rotational motion with our applied magnetic fields could potentially have a gyroscopic stabilizing effect against such asymmetries, which would act to reduce fluctuations in the local number densities in the near-anode region.

Lastly, distinct peaks were observed typically in the range of 30–60 kHz in the power spectral density (PSD) plots of the terminal voltage signals and approximately 15–60 kHz in the PSDs of the ion saturation current fluctuations at 10.7 kA and 8 kA. In addition to the relationship to the frequency of formation and extinction of anode spots, these frequency-domain peaks could be associated with excitation of wave oscillations in the plasma near the low-frequency ion acoustic or Alfvén modes. Tikhonov et al.<sup>33</sup> measured ion acoustic instability driven waves in conditions typically associated with onset. Tilley et al.<sup>37</sup> and Mikhailovskii et al.<sup>71</sup> also described a drift cyclotron instability (DCI) that occurs at frequencies near the harmonics of the ion cyclotron frequency. Since the ion cyclotron frequency is on the order of 10–100 kHz in our plasma conditions, such wave instabilities could be a possible source of the observed peaks in the PSDs. Alternatively, these peaks could simply be associated with resonance at the characteristic frequencies of the L-R-C (inductance-resistance-capacitance) circuit effectively formed by the plasma discharge and sheath themselves. Such a view is a simple extension of the R-C model of capacitively-coupled voltage fluctuations created in the near-anode region in response to spot formation and extinction proposed by Uribarri<sup>31</sup> and Giannelli.<sup>39</sup> Nonetheless, the identification of a similar range of frequencies of ion saturation current fluctuations and terminal voltage fluctuations suggests that the mechanism driving the terminal voltage fluctuations is also likely a process that can cause local plasma density fluctuations (e.g., anode spotting, unsteady plasma current pattern fluctuations, and plasma waves).

## E. Effects on Thrust and Efficiency

Finally, we consider the voltage-current response of the thruster in the three magnetic configurations and attempt a first-order relative comparison of performance parameters. Namely, we estimate the thrust, specific impulse, and thrust efficiency for our three configurations.

The thrust,  $T_{total}$ , will include multiple components and can be expressed as<sup>16</sup>

$$T_{total} = T_{self\ field} + T_{Hall} + T_{swirl} + T_{thermal} \quad (60)$$

$T_{self\ field}$  is the contribution from the poloidal current densities crossed with the self-generated azimuthal B field, which creates radially pinching ( $j_z B_\theta$ ) and axially blowing ( $j_r B_\theta$ ) force densities. This is the only component of thrust that is significant in the self-field case (but is also present in the applied B field configurations). With the applied B fields, three additional terms arise.  $T_{Hall}$  is the thrust associated with the azimuthal current crossed with the applied poloidal B field, which creates pinching ( $j_\theta B_z$ ) and blowing ( $j_\theta B_r$ ) force density components.  $T_{swirl}$  is the thrust associated

with the conversion of the azimuthal rotational momentum into axial kinetic energy through expansion in the magnetic field as the plasma exits the thruster.  $T_{swirl}$  also results in inertial centrifugal forces on the plasma radially outward, counteracting the direction of the electromagnetic pinching forces.  $T_{thermal}$  is the thermal component of the thrust associated with resistive heating and expansion of the plasma through a nozzle (physical or magnetic), but is generally only significant for higher flow rates, where the MPD thruster behaves more like an electrothermal arcjet.

The applied-field thrust terms are complicated to model for our particular thruster conditions. While others have done so for the case of applied-field MPD thrusters with significantly higher relative B field magnitudes and with much lower curvature (mostly strong axial B fields) than we have used in our thruster, such models would not be appropriate for our thruster with more localized B fields with high curvature to the B field lines. For simplicity, we will only calculate the thrust from the self-field term for comparison. This will give a lower bound estimate for the thrust of our applied-field cases, as the thrust will likely be even higher than the self-field term alone with the applied magnetic fields (as evidenced by measurements of applied-field MPD thruster performance by various authors<sup>16,53–55,72</sup>).

We calculate a rough estimate of the self-field thrust given by the modified form for the Maecker formula derived by Jahn,<sup>65</sup> given by

$$T = bJ^2 \quad (61)$$

where  $T$  is the thrust,  $\mu_0$  is the permeability of free space,  $r_a$  is the anode radius,  $r_c$  is the cathode radius,  $J$  is the total thruster discharge current, and  $b$  is a constant defined as

$$b \equiv \frac{\mu_0}{4\pi} \left( \ln\left(\frac{r_a}{r_c}\right) + \frac{3}{4} \right) \quad (62)$$

This formula for the thrust accounts for the self-field electromagnetic forces and pressure over the thruster discharge volume and also accounts for current attachment over the front face of the cathode in the expression for  $b$ .

The specific impulse can be calculated directly from the average effective exhaust velocity,  $u_{ex}$ , given the definition

$$I_{sp} \equiv \frac{u_{ex}}{g_0} = \frac{1}{g_0} \frac{T}{\dot{m}} \quad (63)$$

where  $\dot{m}$  is the mass flow rate and  $g_0$  is a constant given by the gravitational acceleration at Earth's surface, 9.807 m/s<sup>2</sup>.

The thrust efficiency can then be calculated as

$$\eta = \frac{P_{jet}}{P_{in}} = \frac{\frac{1}{2}T u_{ex}}{P_{in}} = \frac{T^2}{2\dot{m}VJ} = \frac{b^2 J^3}{2\dot{m}V} \quad (64)$$

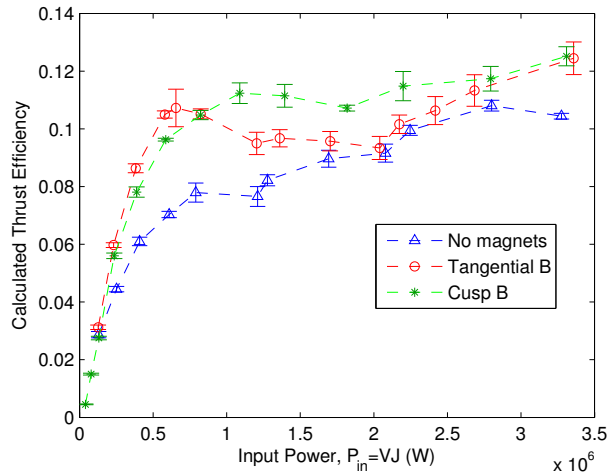
where  $P_{jet}$  is the power in the thrust jet,  $P_{in} = VJ$  is the electrical input power, and  $V$  is the terminal discharge voltage.

Figures 50, 51, and 52 show the calculated estimates of efficiency, thrust, and specific impulse, respectively, versus input power for our three magnetic configurations. The uncertainties shown only account for propagated errors from the measured data. The error bars do not include any uncertainty for the model itself. The dashed lines between data points are merely to guide the eye, not to suggest any specific functional dependence.

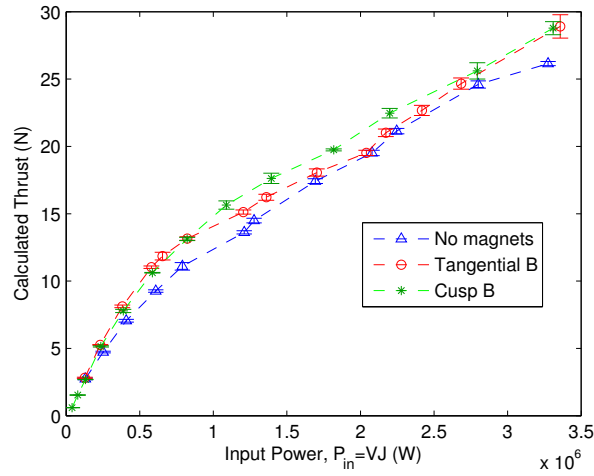
Note that the overall magnitudes of the efficiencies are rather low for two reasons. First, we only included the  $T_{self\ field}$  contribution of the thrust in our calculations. Also, our operating tests were conducted with argon, which is a particularly inefficient propellant (ionization losses, high particle mass, energy lost to multiply-ionized species, etc.). For example, in the self-field Princeton Benchmark Thruster, Choueri and Ziemer<sup>28</sup> measured a range of approximately 7–14% efficiency with argon at 1 g/s over a range of  $I_{sp}$  from 800 s to just over 3000 s. Notably higher efficiencies were obtained with argon at higher flow rates.

For our thruster, consider the relative efficiencies, whose general form and trends should also be seen at much higher efficiencies with other propellants such as lithium. Overall, we see a substantial increase in the efficiency with the applied B field configurations than with the self-field configuration. Also, we see modest improvements in thrust and specific impulse with the applied B fields. These results are directly associated with the improvements from the applied B fields' effect on reducing the mean terminal discharge voltages over a wide range of currents, as previously seen in Figure 11. Again, these results should only improve if we were able to take into account the additional components of the thrust introduced by the applied B field.

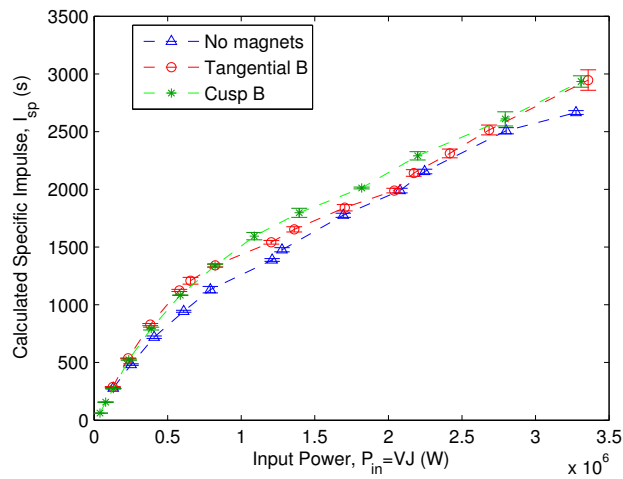
For example, relative to the self-field case, we estimate a 41% relative increase in efficiency with the tangential applied B field and 28% increase with the cusp applied B field at 400 kW. We also calculate a 35% increase in efficiency with both applied B fields at 800 kW. The cusp applied B field has a particularly pronounced increase in efficiency



**Figure 50.** Calculated thrust efficiency for the three magnetic configurations. Note the relatively low efficiencies are due to first-order consideration of only the self-field thrust contribution and testing with argon propellant. Higher efficiencies would be expected with lithium or hydrogen.



**Figure 51.** Calculated thrust (N) for the three magnetic configurations.



**Figure 52.** Calculated specific impulse (s) for the three magnetic configurations.

over a large range of power levels. The cusp applied B field still improves efficiency by 17% even at 2.2 MW. The tangential B field case maintains its beneficial increase in efficiency at least to approximately 1.7 MW.

Overall, we observe an impressive relative improvement in the estimated efficiencies and modest increases in thrust and specific impulse at a given power for the thruster with the applied B field configurations, and this result is a direct consequence of the beneficial terminal voltage reduction effect of the applied B fields. The calculated thrust, specific impulse, and efficiency that we present here for the applied magnetic field cases should only increase further if the applied-field thrust components were included. Ideally, these parameters should be directly measured in the future, e.g., through thrust stand and exhaust plume velocity measurements.

## VII. Conclusions

### A. Summary

The goal of this work was to investigate the effects of externally-applied magnetic fields at modest field strengths on the plasma discharge and examine the prospect of their use for mitigating the behaviors associated with onset. In particular, we posed the following questions:

1. Can one mitigate behaviors such as the large-amplitude terminal voltage fluctuations and large anode fall voltages with applied magnetic fields primarily focused on the near-anode region?
2. What are the effects of the applied magnetic fields on the plasma properties and current transport in the thruster plasma discharge, particularly in the near-anode region?

The issues of onset voltage fluctuations and large anode fall voltages are linked to anode spotting and anode erosion (a lifetime issue), as well as power lost to the anode (an issue for efficiency). Therefore, the attempt to mitigate these onset behaviors is ultimately related to the desire to improve efficiencies and lifetimes in future MPD thrusters. We developed a new MPD thruster, applied-field magnets, associated driving circuitry, and a new set of plasma diagnostic probes to address these questions. The thruster was operated in 1 ms quasi-steady pulses at 1 g/s mass flow rate with argon over a range of power levels and currents from 36 kW (20 V, 1800 A) to 3.3 MW (255 V, 13.1 kA) in configurations without magnets (self-field), with applied tangential B field, and with applied cusp B field.

**This investigation identified significant beneficial reductions in onset-related behaviors with the applied magnetic fields relative to self-field operation. Indeed, over a broad range of currents, the amplitude and frequency of the voltage fluctuations were reduced, the anode fall voltages were greatly lowered, and the mean terminal voltages were decreased.** These results imply substantial improvements in efficiency and lifetime are likely to be obtained through the use of appropriately designed and tailored applied magnetic fields to locally influence near-anode phenomena that drive onset.

Consider each of these interrelated findings. A primary finding was that the current pattern and current densities redistributed to follow the applied poloidal magnetic field lines, which created increased conduction paths to the anode. This led to shifts in the current pattern inside the plasma discharge region and the current densities along the anode. This shift was most pronounced for the tangential applied B field configuration, for which the current pattern moved notably upstream along the anode to attach where the applied B field lines curved to intersect the anode. This shift was shown to be driven by conduction that was increased along the directions parallel to the B field and reduced in directions perpendicular to the B field. Also, increased electron temperatures were measured in regions identified as having high azimuthal current densities induced by the applied B fields.

A second major finding was that the anode fall voltage is substantially reduced with both applied magnetic field topologies over a large range of currents. At 8 kA, the 20 V anode fall measured in the self-field configuration was completely eliminated in both applied-field configurations. At 10.7 kA, the tangential B field had little benefit, but the cusp applied B field decreased anode fall from 45–83 V down to 15 V or lower along much of the anode. We interpreted this result as also caused by the increased conduction to the anode along the anode-intersecting applied B field lines, which results in a substantial reduction in the local electric field required to sustain the radial current densities at the anode.

The amplitude and frequency of the voltage fluctuations were also reduced over a broad range of currents with the applied fields. The standard deviations of the fluctuations were lowered by 37–49% at 8–9 kA with both applied B fields, and the cusp applied field still exhibited a 15% decrease at 10.7 kA. The self-field case transitioned to voltage spikes greater than 10% of the mean voltage circa 6 kA. The current threshold at which the rapid increase in the magnitude and frequency of the voltage spikes occurred was typically increased by 1–2 kA with the applied fields. Nonetheless, for a given operating current up to approximately 10.7 kA, the applied fields consistently reduced the

magnitude of the voltage fluctuations. Above 10.7 kA, the voltage fluctuations for the applied-field cases generally approached similar values as without the magnets.

Enhanced electron mobility to the anode along the anode-intercepting applied B field lines results in an increase in the electron random thermal flux to the anode. The associated reduction in anode fall voltage means there is a decreased electron-attracting potential for accelerating electrons to the anode. This results in reduced electron kinetic energy deposited to the anode, which decreases power lost to the anode in general (a benefit for overall efficiency) but also reduces the local power deposition at anode spot concentrations. This should directly result in less energy going into vaporization of anode material, and thus less anode vapor to seed the near-anode plasma. This is consistent with measured decreases in the ion saturation current spikes (i.e., number density spikes) with the applied magnetic fields, and the similar range of frequencies observed in the PSDs for both the terminal voltage fluctuations and ion saturation current spikes. The applied fields also induce azimuthal rotation of the plasma, which may reduce fluctuations in local number densities in multiple ways (cf., discussion in Section VI-D). In particular, the applied B fields should cause azimuthal rotational migration of any current-concentrating filaments formed. Motion of these filaments should also lead to similar migration of local attachment points at anode spots, which would result in reduced residence time for a given anode spot at a fixed point on the anode. High-speed video imaging evidence of anode spots occurring in our thruster in self-field operation at 9 kA was shown in Figure 14. Given the link between anode spots, current filamentation, and anode erosion established by other studies,<sup>25,31,39</sup> then a reduction in residence times of local anode spot concentrations is expected to result in less erosion (shorter time for concentrated heating at a fixed point) and decreased voltage fluctuations.

A final relation back to the mean terminal voltages and efficiency can be made. The applied B fields resulted in a significant reduction in the mean terminal voltages, with decreases as large as 31% at 9 kA. For the cusp applied B field, terminal voltages were moderately reduced over the entire range of discharge currents. The tangential B field lowered terminal voltages below 10.7 kA. Because the anode fall voltage is a direct contributor to the terms that comprise the total thruster discharge voltage, the anode fall reduction is proposed as the primary cause of the measured decrease in terminal voltage. Given that the electromagnetic thrust scales with the square of the current,  $J^2$ , these voltage decreases were shown to generate notable increases in the estimated thruster efficiency by up to 41% relative to self-field operation (cf., Section VI-E). Moreover, these results should only improve further if we were able to take into account the additional components of the thrust introduced by our particular applied B field configurations, for which existing applied magnetic field thrust models are not directly applicable.

**Overall, these onset-mitigating improvements with the applied magnetic fields should lead to reduced anode erosion, i.e., improved thruster lifetime, and increased thruster efficiency relative to self-field operation.** The measured benefits of the applied B fields were shown to be effective over a broad range of current levels at least to 10.7 kA ( $J^2/\dot{m} = 115 \text{ kA}^2\text{s/g}$ ). Additionally, the cusp applied B field lowered terminal voltages over the entire range of currents (up to the maximum value of 13.1 kA and  $J^2/\dot{m} = 172 \text{ kA}^2\text{s/g}$  studied and potentially higher). Given that  $J^2/\dot{m}$  has been shown to be one of the characteristic scaling parameters for MPD thrusters,<sup>14,57,70</sup> we would expect that these relative advantages should translate to other MPD thrusters operating at a similar range of values. This also implies that if stable thruster operation can be achieved at lower mass flow rates in this thruster or other similar MPD thrusters (e.g., perhaps physically smaller geometry to increase local number densities for stable arc initiation), these beneficial effects could be observed at even higher discharge currents. Such higher-current operation should directly result in higher thrust and efficiencies, as shown in Section VI-E. In addition, because others have identified that the critical value of  $J^2/\dot{m}$  where transition to onset begins scales as  $\sim M_{ion}^{-1/2}$ ,<sup>19</sup> lower atomic mass propellants (e.g., lithium and hydrogen) should allow the extension of our work's findings to operation at even higher currents.

Applied magnetic fields analogous to those examined in this study should strongly be considered for application to future MPD thruster designs, experiments, and simulation efforts. The applied B fields used in this study differ from both the topologies and relative B field strengths typically used in the vast majority of conventional, so-called "applied-field MPD thrusters" (AF-MPDTs).<sup>16</sup> Such AF-MPDTs generally use much higher applied B field magnitudes relative to the thruster self-field. Also, the applied B field geometries of AF-MPDTs are generally extremely axial in form, with limited variation in curvature in the inter-electrode region. Our investigation's applied B fields were much lower in relative magnitude in the interest of localizing the applied magnetic fields to address near-anode phenomena. At the higher currents of interest for onset, our applied B fields yielded  $B_{applied}/B_{selffield}$  much less than one near the outer radius at the cathode downstream face, and this ratio was only greater than one over the downstream section of the anode. Further, our applied B field configurations introduced significant anode-intersecting radial components to the topologies, which was ultimately key in providing the improved electron conduction to the anode that led to many of the observed advantages. Our results suggest a distinctive and more effective approach to influencing the near-anode phenomena and mitigating the deleterious effects of onset with appropriately designed applied magnetic fields.

The results of this investigation suggest that the cusp applied B field is the preferred configuration over the tangential applied B field, given the broader range of currents at which the cusp case improved the onset-related phenomena. The cusp configuration gives a convenient path for the electrons to the downstream section of the anode. At increasingly higher discharge currents, the tangential applied B field likely suffers more than the cusp case from the downstream shift in the accelerating flow field. As discharge current increases, the higher acceleration will cause number densities in general to decrease by conservation of mass in the flow. Electromagnetic radial pinching forces will also be increased with higher current and will be greater in the upstream region (higher  $B_\theta$  upstream, yielding higher  $j_z B_\theta$  in the radial direction) than in the downstream region, further reducing local number densities upstream. However, the tangential applied B field topology directs the higher current conduction path along applied B field lines that only turn radially outward toward the anode in this upstream region, where the number densities are more significantly reduced. Conversely, the cusp applied B field configuration directs the high current conduction path to the downstream region, where local number densities will be reduced relatively less. A simple solution to operating more efficiently at higher currents with the tangential applied B field could be to create a similar applied B field topology but shift it downstream (e.g., by shifting the magnets farther downstream).

The results of this investigation should be extensible to both quasi-steady and steady-state MPD thrusters, as well as different propellants. The overall efficiency, in particular, should greatly improve for lower-atomic mass propellants such as lithium and hydrogen. Lithium, for example, would be a desirable propellant choice due to its low energy for the first ionization potential but high energy for the excited state and second ionization state, resulting in much less energy lost to energy sinks that would not contribute to useful thrust.

## B. Recommendations for Future Work

The results of this study provide a number of interesting directions for potential future work. Further exploration of the trade space for applied-field topologies and relative B field strengths could be examined in future studies to identify optimal (or at least improved) operating configurations. One question to investigate is can one further extend the effective operating range of  $J^2/\dot{m}$  values over which the identified onset-mitigating improvements occur? One approach to investigating this question could be to increase the magnitude of the applied B field with increasing thruster input power, e.g., scaling such that the ratio of the magnitude of the applied B field to the self-generated B field is held roughly constant. Another study could hold a fixed power level (constant discharge current) and examine the effect of varying the applied B field magnitude, i.e., essentially modifying the radial intercept angle of the total B field vector with respect to the anode surface.

Additionally, could a topology be identified that helps to distribute the current pattern over a wider area and thus reduce peak anode current densities, while still maintaining similarly increased performance? Lower current densities would contribute to further improvements in lifetime-limiting anode erosion due to heating. Such spreading of the anode current attachment could potentially be enabled by anode-intersecting B field lines spread over a broader region of the anode. Broadening of this attachment region could be explored both with a cusp-like applied B field with wider separation between the magnets and a tangential-like applied B field shifted further downstream.

Such questions and effects could initially be explored through modeling and simulation, followed by experiments to validate the simulated response. A detailed numerical MHD model for the bulk plasma dynamics should be coupled with an appropriate near-anode physics model of the electron transport and sheath effects. Further, to better understand the nature of the spotting events and instabilities that led to the voltage fluctuations and ultimately anode damage, a near-anode plasma-surface interactions model could be studied. The dynamics and conditions for inception of current filamentation and breakdown of azimuthal symmetry in the diffuse current pattern could be studied with a full 3-D model, as opposed to quasi-2-D models (assuming azimuthal symmetry). Including applied B fields in the model would provide insight into the azimuthal rotation or other mechanisms associated with reducing the fluctuations in the terminal voltages and near-anode number densities. In particular, the nature and source of the low-frequency (10s of kHz) peaks in the PSDs for the terminal voltage signals and ion saturation currents could be studied from the perspectives of both induced instabilities in the plasma and a possible inductive component to the anode spotting and filamentation models proposed by Uribarri<sup>31</sup> and Giannelli et al.<sup>39</sup>

Also, direct measurements of performance (thrust, efficiency, and specific impulse) should be made to identify the absolute magnitude of the performance benefits of the applied B fields. We invoked very simplified thrust and efficiency models, which are likely conservative and underestimating performance with the applied B fields. Future studies should consider directly measuring the thrust with a thrust stand appropriate for pulsed, quasi-steady operation (e.g., a swinging gate thrust stand<sup>28</sup> or other method). Detailed modeling could further inform thrust and efficiency improvements. For example, identifying topologies that result in larger radial components (relative to the axial components) for the current streamlines would produce higher electromagnetic thrust. However, modeling and testing would

be needed to identify whether such configurations indeed increase overall thrust or simply increase electromagnetic thrust at larger radii at the expense of significant thrust contributions closer to the centerline.

Lastly, if facilities and resources would permit, future experiments with applied magnetic fields at these high power levels (100s of kW to several MW) should examine operation at flight-like conditions. For example, steady-state operation would validate that the observed improvements in quasi-steady operation also manifest in steady-state conditions relevant to high-power, long-duration missions. However, this would introduce challenges with steady-state, high-power supplies, increased vacuum chamber pumping requirements, and thermal challenges for cooling the electrodes and magnets. In addition, operation with propellants such as lithium would address performance questions with a propellant that is most favorable to high-efficiency operation. Lithium has some challenges and complexities for safe handling and operation. However, facilities for operating at steady-state high power with lithium as a condensable metal propellant do exist, for example, at JPL.

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