

Instant Start Electride Hollow Cathode

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C12A7 electride is a crystalline ceramic in which electrons clathrated in sub-nanometer sized cages act as a conductive medium. Due to its unique atomic structure and large size, C12A7 electride has a predicted work function of 0.6 eV. A prototype C12A7 electride hollow cathode was fabricated and operated. It has been successfully started at room temperature without a heater. The operation is stable and reproducible, and barrel temperatures below 600°C have been observed. Multiple C12A7 electride cathodes have been fabricated and operated under various conditions, including over 50 hours of operation on a single insert with no signs of degradation. Additionally, C12A7 electride has been shown to be resistant to contamination during handling and fabrication. Experimental results from a C12A7 electride cathode are presented, including operation on xenon and iodine, and a zero-dimensional phenomenological model that gives insight into the operation of the cathode is discussed.

I. Introduction

Hollow cathodes are the primary electron source in many electric propulsion applications, as well as in a number of ground-based devices such as gaseous lasers and plasma processing sources. They are often preferable to filament sources due to their increased robustness and lifetime. Hollow cathodes are cylindrical in shape, and consist of an orificed tube with a low work function material placed inside^{1,2}. The ease with which the electrons are emitted off the insert is related to the work function of the material³⁻⁵. Lower work function indicates equivalent emission can be obtained at lower temperatures, improving the power efficiency because lower temperature cathodes lose less heat. Additionally, a low temperature cathode could be fabricated from less expensive materials instead of refractory metals.

Calcium aluminate $12\text{CaO} \cdot 7\text{Al}_2\text{O}_3$, or C12A7 in ceramic material science vernacular, is a novel material found to have properties suited to electron emission when in the electride form due to its low work function, which is derived from its unique structure. The C12A7 phase is one of several alumina-lime phases found in common cements. C12A7 has a naturally formed nanostructure, in which subnanometer-sized cages form a three-dimensional crystal lattice, as shown in Figure 1. The unit cell consists of twelve cages. Although this cage structure is similar to those found in clathrate phases of ice and in zeolites, there is an important difference in that the unit cell of C12A7 is positively charged. There are four fewer electrons on the atoms that comprise the framework cage of C12A7 than are needed to neutralize the cage. Interestingly, in the non-electride phase, the positive charge is counteracted by two atomic oxygen ions (O^{2-}) that are clathrated

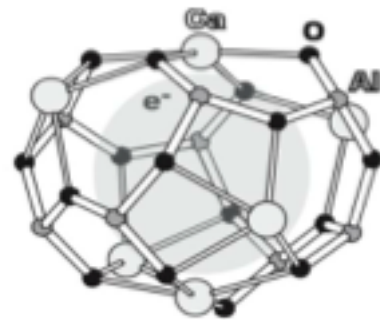


Figure 1. Structure of C12A7 electride in which an electron is clathrated within four of the 12 cages of the positively charged lattice framework²⁶.

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(floating) within two of the twelve subcages. New properties can be imparted to C12A7 if the free oxygen ions are substituted with anions like O^- and H^- , but especially interesting features are possible when four electrons are substituted for the two O^{2-} ions to form C12A7 electride, the first and to date only inorganic electride discovered that is stable at high temperature^{6,7}. The stability of the C12A7 electride is attributable to the cage structure as well as the fully oxidized nature of the lattice.

The work functions of current state-of-the-art hollow cathode insert materials lanthanum hexaboride (LaB6) and cerium hexaboride (CeB6) are near 2.7 eV, while the work function of barium-impregnated porous tungsten (Ba-W) is near 2.1 eV⁸. LaB6 and CeB6 are generally heated to approximately 1900 K to obtain sufficient levels of emission, while Ba-W is heated above 1300 K⁸. These temperatures require well-made heaters and good thermal insulation. Furthermore, Ba-W cathodes, while operating at lower temperatures, are more susceptible to both poisoning and high rates of evaporation if operated at high current⁸. In contrast, the work function of C12A7 electride has been measured in field emission tests to be as low as 0.6 eV, due to its unique charged lattice structure⁹⁻¹¹.

This paper will describe the C12A7 electride one-step fabrication process developed at Colorado State University. We will then discuss the process by which we integrated the fabricated electride into a hollow cathode and the various challenges that arise. The operation of the C12A7 electride hollow cathode will be characterized, including its performance at high currents and when running on iodine. A zero-dimensional phenomenological model will be described. Results from the phenomenological model will be combined with experimental results to give insight into the internal plasma conditions and operating temperatures of a C12A7 electride hollow cathode.

II. Cathode Fabrication

The glass form¹² of C12A7 electride was fabricated through a one-step procedure at the CSU CEPPE Laboratory in a vacuum furnace capable of reaching temperatures of 1800°C^{13,14}. The precursors ($CaCO_3$ and Al_2O_3) were mixed in a 12:7 stoichiometric ratio and carefully ground with a mortar and pestle to minimize the crystal size and help facilitate a solid-state reaction. The mixture was then heated to 1700 °C over a several hour period in a crucible fabricated from fine-grained graphite. After heating, the crucible was cooled to below the recrystallization temperature within 30 minutes. The graphite crucible was found to be necessary for the successful formation of C12A7 electride. Although the exact mechanism is unknown, it is believed that the graphite crucible is needed to supply carbon anions to occupy the subcages and permit the formation of the lattice, but which evacuate upon cooling leaving behind their electrons⁹.

The prototype hollow cathode was constructed from a 6.35 mm diameter tantalum tube, with a thoriated tungsten orifice plate. Different orifice sizes were investigated and test results are discussed in Section V. The downstream end of the tantalum barrel was wrapped with 10 layers of radiation shielding, fabricated from 0.0127 mm thick tantalum foil. Unlike traditional hollow cathodes, no heater was incorporated in the design. A single graphite enclosed keeper was placed around the cathode barrel. The keeper orifice was 2.54 mm and the orifice plate was placed 1.27 mm downstream from the cathode barrel orifice plate. In testing configurations in which the temperature was measured, the enclosed keeper was replaced with an external tantalum wire loop for ease of access. In this configuration, a 1.5 mm diameter tantalum wire was bent into a loop approximately 6.3 mm in diameter, which was placed approximately 1.27 mm downstream from the orifice plate. A thermocouple was spot-welded to the barrel as close to the junction between the orifice plate and the barrel as possible.

The C12A7 electride emission surface was made from electride slivers cut from the graphite crucible with a diamond-coated dremel blade. A single sliver was inserted into a small graphite tube that was 2.54 cm long and 5.08 mm in diameter with an orifice of 1.9 mm on the downstream end. The graphite tube prevented the C12A7 electride from converting to its non-conductive eutectic (CA+C3A) form when heated (through natural hollow cathode operation) in tantalum. We believe the graphite provides an anionic template, as it does during the original formation process.

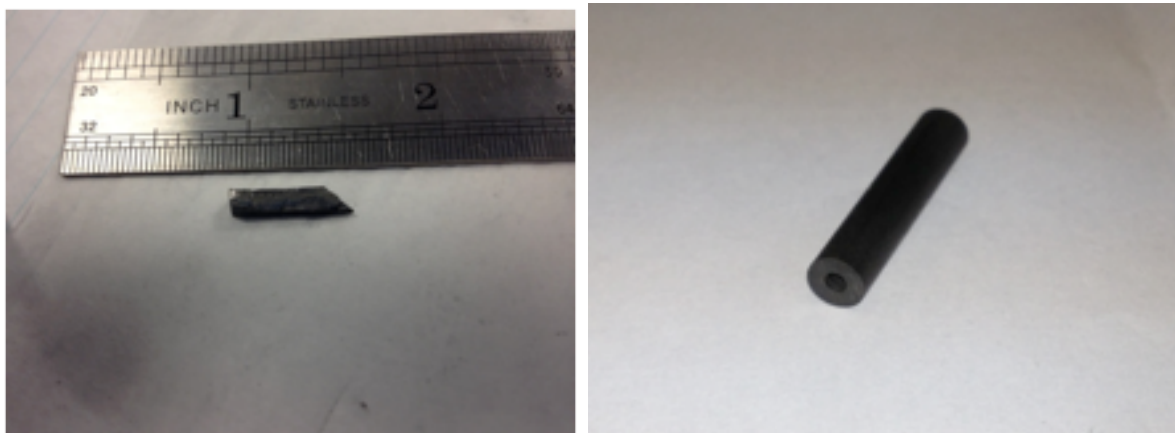


Figure 2. Cut C12A7 electrider sliver (left) and the graphite tube liner (right).

III. Summary of Previous Experimental Work

The start-up procedure for the C12A7 electrider hollow cathode differs from that of traditional hollow cathodes. In these cathodes, a heater is used to bring the emitter material to a temperature at which it emits enough electrons to sustain a plasma discharge. A voltage usually on the order of several hundred volts is applied to the keeper to initiate the discharge. Once the cathode has started it typically no longer needs supplemental heating. In contrast, the C12A7 electrider cathode is started at room temperature with high keeper voltages or high mass flow rates of propellant. The needed keeper voltage for start-up is inversely related to the mass flow rate. Commonly, a kilovolt is applied to the keeper and the mass flow rate is increased until the discharge is initiated, usually at approximately 30 sccm of xenon. After operation commences, the mass flow rate is lowered to the desired level and the keeper is operated at 0.3 A in a current-limited mode. Ballast resistors are used in the keeper circuit to limit the peak current at start up.¹⁵

To achieve reproducible operation, an electrider insert needs to be operated for approximately 10 hours and be shut-down and re-started several times¹⁶. Reproducible operation is quantified by comparing the barrel temperature and anode voltage at a given set-point of discharge current and flow rate. Figure 3 shows the preparation process for an insert, beginning with the first operation and ending after changes were no longer observed in the barrel temperature or anode voltage.

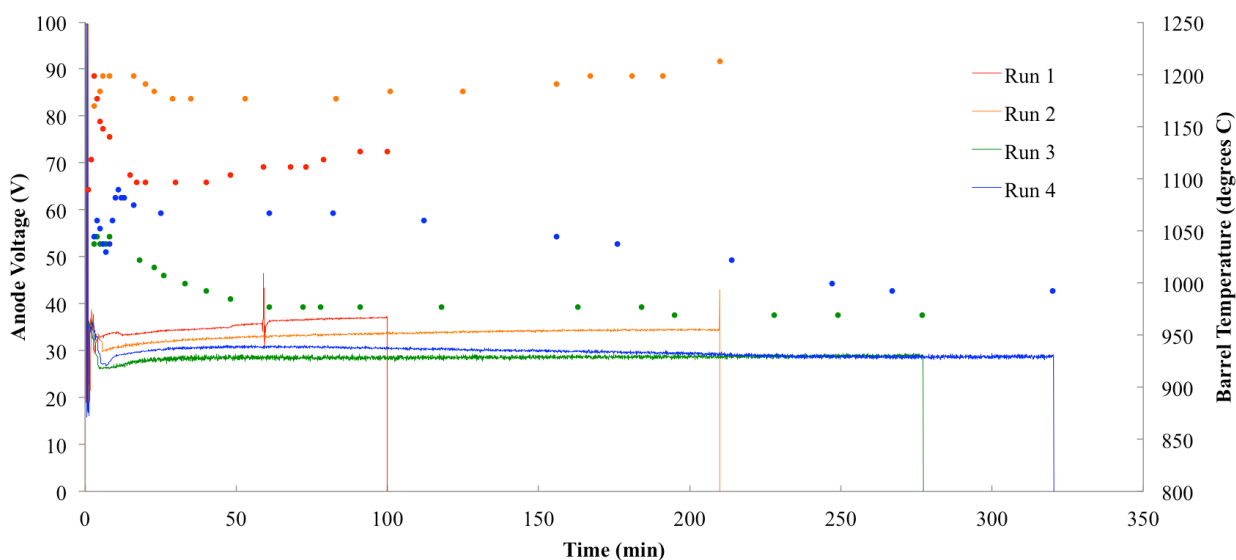


Figure 3. A graph of the anode voltage (primary axis, continuous line) and barrel temperature (secondary axis, discrete points) over the course of four operations at 5 sccm of xenon and 3 A of total emission. The operation was deemed reproducible after the third and fourth runs. A relatively small orifice was used in this test, which lead to a barrel temperature of approximately 1000°C.

Once an insert performed reproducibly in the insert preparation configuration, it could be used in a finished cathode. The finished cathode had an enclosed graphite keeper, which precluded the use of a thermocouple to measure barrel temperature. Other than the keeper and the mounting bulkhead, the finished cathode had no other changes as compared to the insert preparation configuration.

Using a fully conditioned insert, three different orifice sizes were tested under identical conditions¹⁶. The three orifice diameters were 0.76 mm, 1.42 mm, and 2.03 mm. The insert was operated, removed, and reinstalled twice during testing in the 1.42 mm and 2.03 mm diameter orifice cathodes to ensure that this action did not damage the cathode. During each test, the barrel temperature was measured at several mass flow rates and discharge currents. These tests were done in a test configuration using an external wire keeper, which operated at a constant 0.3 A of current. The flow rate variation results are shown in Figure 4. Additional data is contained in Ref. 16.

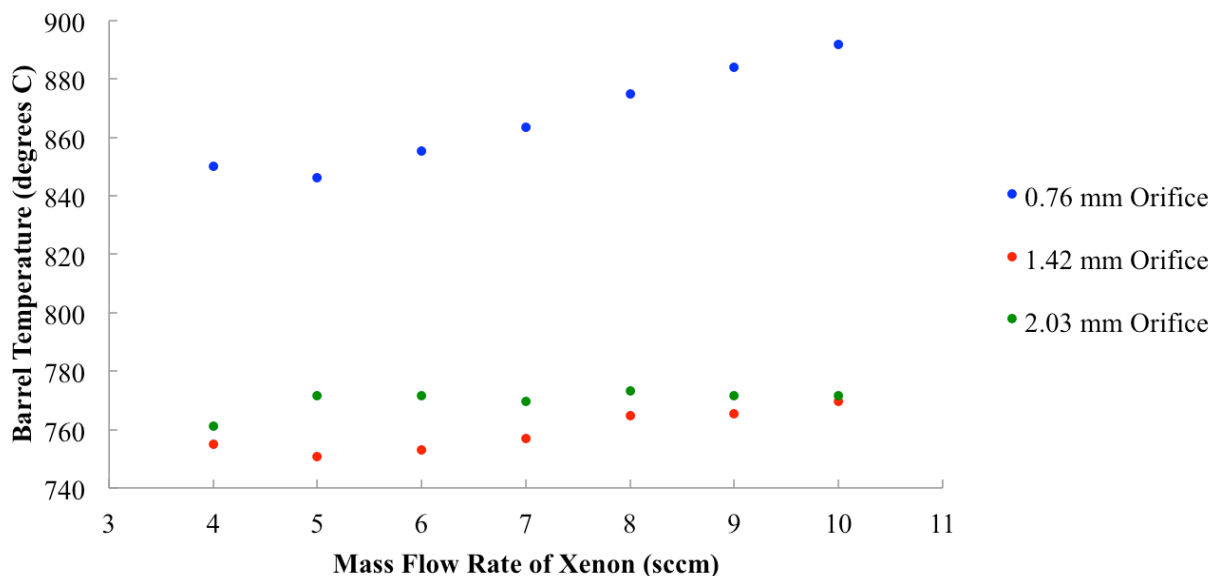


Figure 4. The measured barrel temperature of cathodes with three different orifice sizes as a function of mass flow rate. The discharge current was fixed at 2.8 A.

IV. Iodine Testing

Iodine has recently attracted interest as an alternative electric propulsion propellant¹⁷⁻²⁰. Iodine has an atomic mass similar to xenon with slightly larger ionization cross-sections (for both I and I₂). Additionally, it can be stored in low pressure tanks in the solid phase, eliminating the need for the large, high pressure storage solutions mandated by xenon¹⁸. The increased reactivity when compared to xenon was a concern due to the halogenic nature of iodine¹⁹, especially when the susceptibility to contamination of Ba-W hollow cathodes was considered. Given the observed resistance to contamination of the C12A7 electride hollow cathode, it appeared to be an attractive alternative cathode technology.

A C12A7 electride hollow cathode fabricated according to the procedure described in Section II was delivered to Busek Co. Inc. to perform the iodine compatibility testing. Busek has spent several years²⁰ developing an iodine propulsion program, which includes a feed system. The feed system incorporated a heated iodine reservoir with a pressure transducer that could be used to quantify the approximate flow rate. All lines between the reservoir and the cathode were heated to prevent iodine condensation. The

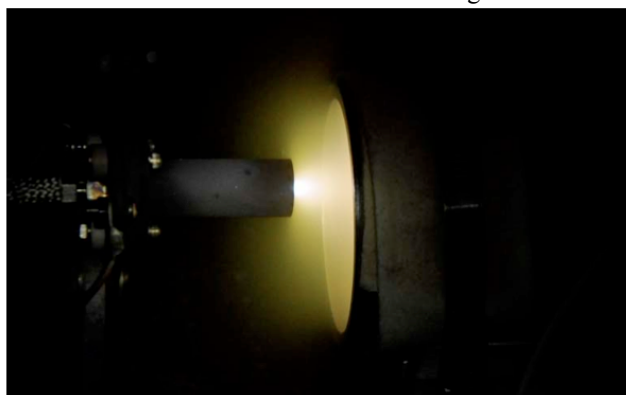


Figure 5. A C12A7 electride hollow cathode operating on iodine.

reservoir was weighed after each day of operation, allowing for the development of a flow rate calibration curve from the measured reservoir pressure. The electrified hollow cathode testing was conducted in a 1.2-m diameter, 1-m long stainless steel diffusion-pumped chamber. The cathode was tested in a diode configuration with a ring anode as shown in Figure 5. The cathode was successfully started on iodine from room temperature with no heater, following the start-up procedure described in Section III. Almost 20 hours of operation with iodine was accumulated on a single C12A7 electrified insert with no signs of degradation or contamination. The 20-hour duration involved multiple restarts from room temperature as well as several exposures to atmosphere, and no difficulty starting and operating the cathode was encountered.

The anode voltage as a function of discharge current was measured at a constant iodine flow rate of approximately 13 sccm. Data were recorded as the current was increased from 3 A to 15 A, and as it was decreased from 15 A back down to 3 A over approximately one hour (Figure 6).

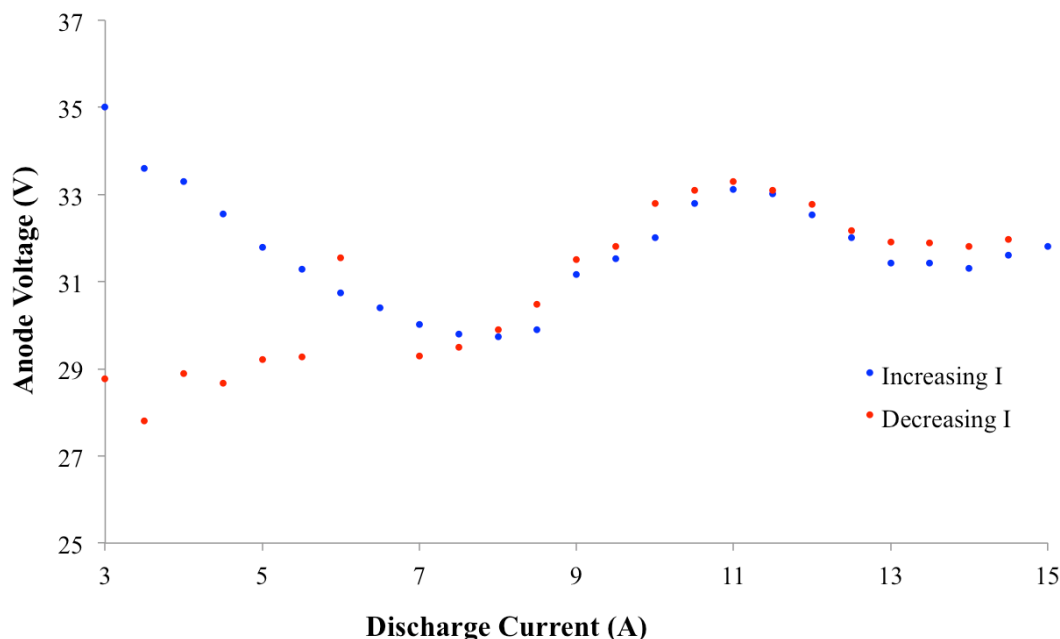


Figure 6. Anode voltage as a function of discharge current for a C12A7 electrified hollow cathode running on iodine with a constant flow rate of 13 sccm and 0.3 A to the keeper.

The cathode performance at lower iodine flow rates was also investigated, as shown in Figure 7. The discharge current was kept constant at 3 A with an additional 0.3 A collected by the keeper. The temperature of the iodine reservoir in the feed system was slowly decreased while the anode voltage was recorded. The anode voltage starts to increase at flow rates below about 5 sccm. It is noted that flow rates shown in Figure 7 were calculated from the pressure transducer mounted on the iodine reservoir and from calibration data that were collected from iodine reservoir weight measurements made after each day of testing. It is not known what the internal pressure of the cathode was, but it is estimated to be approximately one Torr. There is a great deal of uncertainty currently regarding flow rate, and additional testing is required to better calibrate the iodine feed system when operated at low flow rates. It is suspected that the flow rate at which the increase in anode voltage observed is much lower than 5 sccm, because the internal cathode pressure introduces a larger error in the calibration at lower flow rates and because operation on xenon shows an increase at flow rates closer to 1 sccm at an anode current of 3 A.

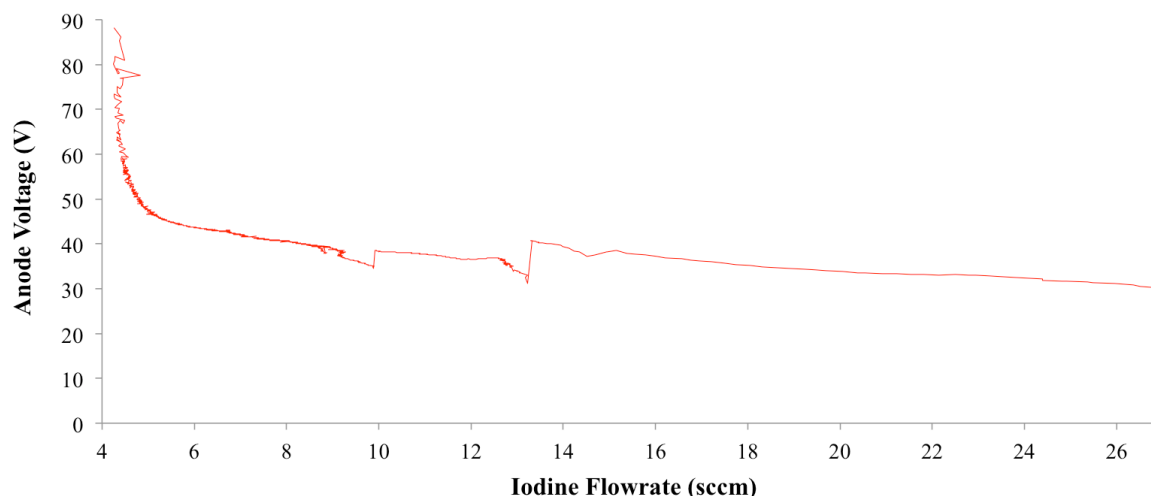


Figure 7. Anode voltage as a function of iodine flow rate for a C12A7 electride hollow cathode.

V. Discussion

A. Insert Preparation

It is not surprising that insert preparation is needed if one hypothesizes that the primary function of the preparation process is the elimination of surface contamination. In the context of other low work function emitting surfaces, the C12A7 electride surface is relatively dirty prior to operation. As mentioned in Section II, the slivers for the inserts were cut using a dremel blade, the byproduct of which was a large amount of graphite dust. In addition, x-ray photoelectron spectroscopy taken of the C12A7 electride surface before cutting showed that over 80% of the surface atomic composition was carbon. No special measures were taken when storing the samples prior to their use in a cathode, and they were typically exposed to air for several weeks before they were used. Just prior to use, the samples were wiped with a dry lens cloth, but no solvents were used to avoid disrupting the surface state.

The effects of the naturally occurring plasma cleaning process and the electride quenching that occurs at shutdown may be observed in the data from runs 1 and 2 in Figure 3. Early in the preparation process, higher temperatures and anode voltages and unstable operation are typical. As shown in Figure 3, this unstable behavior ceases through the course of several hours of operation and multiple restarts. The steady-state operation is characterized by constant temperature and anode voltage, both of which are lower than those observed during the preparation phase. However, even after an insert has reached its steady-state operation, slightly higher anode voltages are typically observed in the first 10-30 minutes of a run. While some of this may be due to surface changes, it is unlikely that this is the primary reason because the duration and severity of the burn-in period after a start did not change when the cathode was exposed to atmosphere. It is hypothesized that a heaterless cathode will inherently be more variable at the beginning of a run due to the time needed to reach thermal equilibrium. Unlike traditional hollow cathodes, the C12A7 electride hollow cathode self-heats from room temperature to its operating temperature, all of which occurs after operation has begun.

B. Orifice Sizing

Initial tests of a C12A7 electride hollow cathode with a 0.76 mm orifice showed typical operation temperatures of approximately 1040°C. While this temperature is low when compared to some traditional hollow cathodes, it is not nearly as low as we expected given the predicted work function of 0.6 eV. Three phenomena were hypothesized as causes for this higher temperature.

1. The work function of C12A7 electride is higher than that calculated using ab-initio techniques⁹.
2. The non-radially symmetric nature of the insert causes high work function surfaces (graphite and thoriated tungsten) to be exposed to the interior cathode plasma. Due to their high work function, these surfaces do not self-cool by emitting electrons, as low work function surfaces in a radially symmetric insert would. As a result, the high measured wall temperature is due to plasma bombardment of these surfaces, and it is possible that the C12A7 electride emission surface is cooler than its surroundings.
3. Orifice heating is raising the temperature of the cathode wall, and the cathode wall is hotter than the C12A7 electride needs to be to emit the required discharge current.

In an effort to confirm or eliminate the third possibility, the barrel temperatures of cathodes with three different orifice sizes were measured as a function of mass flow rates (see Figure 4). If orifice heating was the primary factor in the high temperature of the cathode, one would expect to see the temperature decrease significantly with increasing orifice diameter¹. To a certain extent, this is what was observed. As the cathode with the 0.76 mm orifice operated approximately 100°C hotter than the cathodes with the larger orifices. Interestingly, there did not appear to be a dramatic difference in operating temperatures between the cathode with the 1.42 mm orifice and the cathode with the 2.03 orifice. We believe that orifice heating ceased to be the primary contributing factor to the operating temperature at an orifice diameter somewhere between 0.76 mm and 1.42 mm. If orifice heating isn't significant, then it is not surprising that significant temperature changes were not seen between the two larger orificed cathodes. Additionally, at very low discharge currents, all three cathodes were observed to operate at similar low temperatures. It is likely that at this condition, the orifice heating was not the primary heating mechanism due to the low orifice plasma density (as a result of the low discharge current)¹⁵.

Another piece of evidence is that a significant increase in the measured temperature with increasing mass flow rate was only observed in the cathode with the 0.76 mm orifice. An increase in mass flow rate only contributes to an increase in temperature through orifice heating and due to the decrease in emission length at high internal pressures²¹. If orifice heating is the primary contributor to the operating temperature, the direct relationship between mass flow rate and operating temperature is expected. If orifice heating were not a contributing factor, however, there would be no significant expected increase in temperature with mass flow rate. As seen in Figure 4, there is no significant increase in operating temperature with mass flow rate of the two larger orificed cathodes, which supports this inference. It is noted that operating temperatures below 600°C were observed in cathodes with the two larger orifice sizes operating at low discharge currents of approximately 1.25 A¹⁵.

C. Phenomenological Model

A zero-dimensional (0-D) energy balance model was used to gain additional insight into the operation of a cathode with a low work function, non-radially symmetric insert. The model was exercised to answer two primary questions:

1. How do the interior plasma conditions of a C12A7 electride hollow cathode compare with those of traditional hollow cathodes?
2. How does the C12A7 electride emission surface temperature compare with the operating temperature of the cathode?

The model was based on work done by Siegfried and Wilbur²¹, Goebel and Katz¹, and Katz et al²². In previous work, a control volume around the insert surface was utilized in conjunction with the conservation of energy and current. Assuming steady-state conditions, the energy leaving the insert must be equal to the energy deposited in the insert from plasma bombardment processes. The heat loss can be quantified with the use of a thermal model²³ and the plasma properties can then be deduced through the conservation of energy and current, as described in Appendix A.

There are several challenges in the application of a 0-D model to a C12A7 electride hollow cathode. One such challenge is that in a traditional hollow cathode, the primary surface exposed to plasma bombardment is the low work function insert. In a C12A7 electride cathode, most of the plasma-bombarded surface area is graphite or tungsten, which have comparatively high work functions. Additionally, the geometry of a traditional hollow cathode is radially symmetric. The sliver-insert of a C12A7 electride cathode leads to radial non-symmetry, which adds complexity. Lastly, the work functions of the materials commonly used to fabricate inserts in traditional cathodes are well known, which is not the case for electride. It is believed that processes in a C12A7 electride cathode greatly improve the work function of the emission surface (as evidenced by the enhanced operation after the insert preparation process described in Section III). As a result, it is difficult to quantify the work function of a C12A7 electride emitter. To address these challenges, a two-step approach was utilized in the development of a C12A7 electride hollow cathode 0-D model. The first step involved a control surface around the graphite tube that accounted for the majority of the inner surface area. The experimentally measured temperature from the thermocouple placed at the downstream end of the tantalum barrel was used to quantify the operating temperature of the cathode. It was assumed that this temperature also represented the temperature of the inner graphite barrel²⁴. A preliminary thermal model was used to calculate the amount of heat flowing from the graphite to the cathode barrel and subsequently lost to the surroundings. In the results presented in this paper, only conduction down the barrel to the room-temperature bulkhead was considered. Future work will include radiation lost from the orifice plate as well as an approximation of orifice heating. A second assumption made was no electron cooling occurred from the graphite. The assumption is well justified because the emitted current from the graphite as predicted by the

Schottky-enhanced Richardson-Dushman equation^{4,25} is on the order of 1×10^{-11} A, which would have no cooling effect.

After making the assumptions described above, plasma conditions inside the cathode could be calculated. The experimentally measured operation temperature was 775°C for a C12A7 electrde cathode emitting a discharge current of 3 A at 4 sccm of xenon with a 1.42 mm diameter orifice. These conditions lead to an interior plasma electron temperature of 2.5 eV, an average plasma density of $3.6 \times 10^{19} \text{ m}^{-3}$, and a sheath potential of 8.34 V. The electron temperature is higher when compared to an NSTAR cathode operating at the TH15 throttle point with 13.1 A of discharge current¹. This is likely due, in part, to the larger orifice diameter of the C12A7 electrde hollow cathode, which leads to a lower internal neutral pressure and thus a higher electron temperature. Additionally, the discrepancy in electron temperatures could be a result of the difference in emitted current. It should be noted that the C12A7 electrde hollow cathode is capable of emitting up to 15 A of discharge current, but the model has thus far only been utilized at the 3 A operating condition.

The larger orifice and lower discharge current (3 A) of the C12A7 electrde hollow cathode should lead to a lower plasma density when compared to an NSTAR cathode at the 13.1 A operating point. The calculated plasma density of $3.6 \times 10^{19} \text{ m}^{-3}$ is between one and two orders of magnitude less than the experimentally measured plasma density of the NSTAR cathode, which is close to the expected effects of orifice area and current differences.

The second step was to use the plasma properties calculated in the first step to determine the temperature of the C12A7 electrde surface. For the data presented in this paper, it was assumed that there was both radiative and conductive heat transfer between the electrde sliver and the graphite tube. The temperature was calculated over a range of work functions because both the emitting area and the work function of the electrde are unknown at this point in time. An additional assumption made was that the plasma properties found in the first step remained constant over the range of work functions assumed for the electrde.

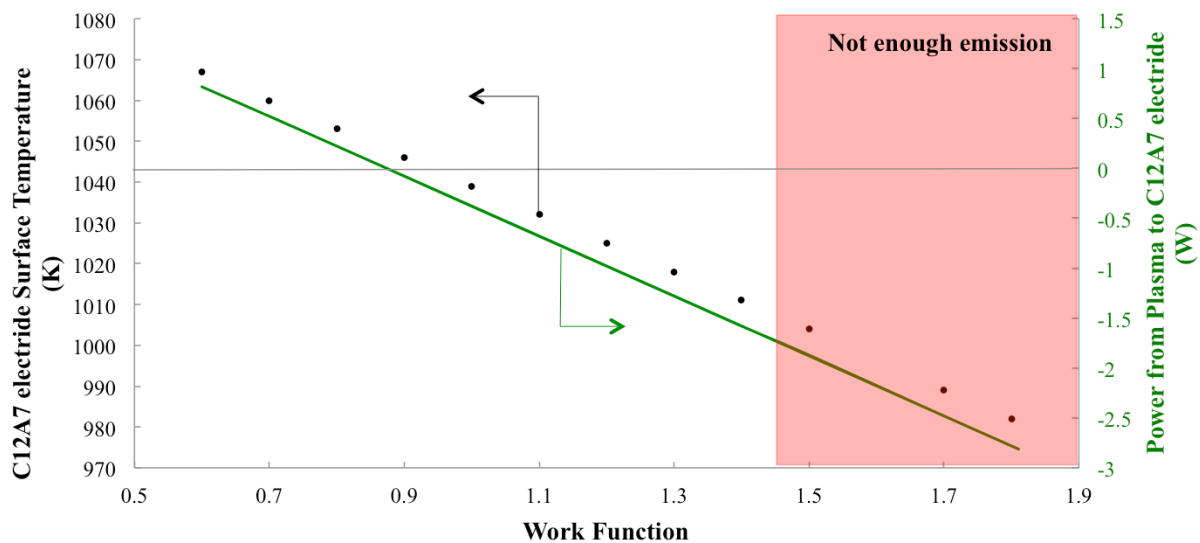


Figure 8. A plot of the C12A7 electrde surface temperature (left axis) and the heat transfer from the C12A7 electrde to the interior plasma (right axis) as a function of the C12A7 electrde work function. The model was based on 3 A of emitted current, 4 sccm of xenon flow, a 1.42 mm diameter orifice, and an experimentally measured barrel temperature of 775°C. At work functions greater than 1.4 eV, the C12A7 electrde can no longer emit the discharge current.

Figure 8 shows that the direction of heat transfer between the C12A7 electrde sliver and the graphite tube is dependent on the C12A7 electrde work function. At a work function of approximately 0.9 eV, the C12A7 electrde transitions from being hotter than its surroundings to cooler. Additionally, given the assumptions made, it is physically impossible for the C12A7 electrde to emit the desired current if it has a work function greater than approximately 1.4 eV (as calculated by the Schottky-enhanced Richardson-Dushman equation^{4,25}). Thus, according to the model results, the electrde inside the C12A7 electrde hollow cathode has a work function less than or equal to 1.4 eV. Additionally, it is possible that the C12A7 electrde emitting surface is cooler than the operating temperature of the cathode as it is calculated to be at a work function between 0.9 eV and 1.4 eV, implying that the cathode operating temperature is not solely a reflection of the insert work function.

VI. Conclusion

A hollow cathode utilizing C12A7 electride as an insert has been developed at Colorado State University. The cathode has no heater, and is capable of starting near-instantaneously from room temperature. Several cathodes have been fabricated and operated under different conditions, and a single insert was operated for 50 hours with xenon with no sign of degradation. There appears to be no incompatibility between C12A7 electride and iodine after 20 hours of operation at discharge currents up to 15 A. The application of a 0-dimensional indicates that the interior plasma of a C12A7 electride hollow cathode is similar to that of a traditional hollow cathode. Although the operating temperature of a hollow cathode with a 0.76 mm orifice is higher than expected, the model suggests this may be due to mechanisms not strongly linked to the low work function of the insert. Additionally, it appears possible for the C12A7 electride insert to be cooler than the operating temperature of the cathode, depending on its work function.

Appendix A

At a basic level of understanding, the plasma inside the hollow cathode can be modeled using a 0-dimensional energy balance model. A number of groups have done this type of analysis^{1,21,24} and they follow a similar strategy by equating the power lost by a section of the cathode with the power deposited there. For the data presented herein, the electron temperature inside the hollow cathode was iteratively solved for using the cylindrical diffusion equation discussed in Goebel & Katz¹.

$$\left(\frac{R}{\lambda_{01}}\right)^2 n_o \sigma_i(T_e) \sqrt{\frac{8kT_e}{\pi m}} - D = 0 \quad (1)$$

In equation (1), R is the radius of the internal cathode volume, n_o is the neutral density, $\sigma_i(T_e)$ is the xenon ionization cross section averaged over an (assumed) Maxwellian electron temperature distribution, D is the diffusion coefficient found from the ion and electron mobilities, and the electron thermal velocity is

$$v_{th} = \sqrt{\frac{8kT_e}{\pi m}}$$

The barrel wall thickness is 0.508 mm, the inner radius of the tube is 2.67 mm, the orifice diameter is taken to be 1.42 mm and the orifice plate thickness is 0.76 mm. The graphite tube inner radius is 1.27 mm, its outer radius is 2.54 mm, and its length is 12.7 mm. The neutral gas temperature is assumed to be three times the wall temperature. In calculating the diffusion coefficient, the xenon charge exchange cross section is assumed to be $1 \times 10^{-18} \text{ m}^2$.

Once the electron temperature is known, the plasma density and sheath potential can be found by combining the conservation of energy applied to a control volume around the internal cathode plasma and the conservation of energy applied to a control volume around the insert surface. The energy balance applied to the internal cathode plasma can be written as presented in Goebel & Katz.¹

$$I_i \phi_s + RI_e^2 = I_i U^+ + \frac{5}{2} T_{ev} I_e + (2T_{ev} + \phi_s) I_r e^{-\phi_s/T_{ev}} \quad (2)$$

Equation (2) is structured so that that the control volume around the plasma is bounded prior to the sheath at the insert surface. The left side of (2) is the energy flowing into the plasma control volume, and the right side is the energy flowing out. $I_i \phi_s$ is the energy brought into the plasma by thermionically-emitted electrons that flow from the internal cathode surface that are accelerated through the sheath potential and RI_e^2 is the joule heating of the plasma as the discharge current is pulled through the control volume to the orifice. The average conduction length used to calculate R is taken to be the hypotenuse of the right triangle with legs formed by the graphite tube length and radius. $I_i U^+$ is the ionic potential energy removed from the plasma in the form of ionization potential and $(5/2)T_{ev} I_e$ is the enthalpy of the electrons leaving the control volume that flow to the orifice. The term

$$I_r e^{\frac{-\phi_s}{T_{ev}}} (2T_{ev})$$

is the kinetic energy of the electrons lost from the plasma to the internal cathode surface and

$$I_r e^{\frac{-\phi_s}{T_{ev}}} (\phi_s)$$

accounts for the potential energy carried by these electrons. Both terms include an attenuation factor to account for the electrons that make it partially through the sheath but have inadequate amounts of energy and are returned to the plasma before reaching the cathode surface. The ratio used to calculate the average plasma density from the plasma density at the sheath edge is assumed to be 0.145.

An energy balance is also applied to the control volume placed at the internal cathode surface as presented in Goebel & Katz.¹

$$H(T) + I_i \phi_{wf} = I_i \left(U^+ + \phi_s + \frac{T_{eV}}{2} - \phi_{wf} \right) + (2T_{eV} + \phi_{wf}) I_r e^{\frac{-\phi_s}{T_{eV}}} \quad (3)$$

The terms on the right side of the equation account for the energy leaving the cathode surface, where $H(T)$ is the total heat transfer from the internal cathode surface to the cathode structure and $I_i \phi_{wf}$ is the thermionic emission cooling term describing the energy removed by the emitted electrons when they are boiled off the surface. In calculating the heat transfer term, the cathode base is assumed to be at 423°C and the vacuum chamber surrounding the cathode is taken to be at room temperature. The first grouping of terms on the right side of the equation accounts for the power deposited into the cathode surface by the ion bombardment current. $I_i U^+$ is the ionization energy supplied by the ions when they hit the cathode surface, $I_i \phi_s$ is the energy gained by the ions when they fall through the plasma sheath en route to colliding with the cathode surface,

$$I_i \frac{T_{eV}}{2}$$

is the energy gained by the ions prior to arriving at the sheath boundary edge, also known as the pre-sheath or Bohm energy, and $-I_i \phi_{wf}$ is the energy lost by the cathode surface when electrons in the surface conduction band jump to the surface to neutralize ions on the surface.

The second grouping of terms on the right side of the equation is the power deposited into the cathode surface by the returning electron current. The rate of electrons arriving at the surface is equal to the random thermal electron current multiplied by an attenuation term

$$e^{\frac{-\phi_s}{T_{eV}}}$$

The kinetic energy deposited by the electrons that make it to the cathode surface (assuming a Maxwellian velocity distribution) is

$$I_r e^{\frac{-\phi_s}{T_{eV}}} (2T_{eV})$$

and the energy given up by the electrons to the surface when they transition from the vacuum level to the conduction band is

$$I_r e^{\frac{-\phi_s}{T_{eV}}} (\phi_{wf})$$

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