

## EARTH'S MAGNETIC FIELD EFFECT UPON PLASMA PLUMES EXPANSION

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### Abstract

The problems about expansion of electric thruster's plasma plumes in space should be solved with consideration of the effect of Earth magnetic field  $B$ . In the paper the authors show that under the geomagnetic field  $B$  effect plasma jets get some 3D shapes elongated along the field  $B$  at some distance  $x_B$  from a source. Expressions for  $x_B$  and for cross-section of a plume  $r_B$  from which the geomagnetic field effect becomes noticeable are represented. Magnetic interaction parameter  $S$  which is equal to *transverse electromagnetic forces scale-to-pressure gradient* ratio, serves as a measure of the field effect. In the paper calculated values of typical distances  $x_B$  and  $r_B$  are depicted for the cases of popular thruster plumes.

### Nomenclature

$a, l$	= transverse and longitudinal spatial scales of plasma jet;
$B$	= Earth's magnetic field induction;
$e$	= electron charge;
$j$	= electric current density;
$k=(a'_m)^{-2}$	= factor of flow divergency;
$M$	= Mach number;
$m$	= ion mass;
$\dot{N}$	= particles flow rate;
$n$	= particles concentration;
$S$	= electromagnetic interaction parameter;
$T$	= temperature;
$x, r, \varphi$	= cylindrical coordinates;
$u, v$	= x- and r- components of flow velocity;
$x, y, z$	= Cartesian coordinates;
$V$	= flow velocity;
$\alpha$	= pitch-angle, the angle between $V$ and $B$ vectors;
$\gamma$	= specific heat ratio;
$\rho, \theta, \varphi$	= polar coordinates;
$\sigma$	= plasma electric conductivity.

### Introduction

Artificial plasma formations (PAF) generated during the work of the near-future electric thrusters (TAL and SPT), are a flow of energetic ions ( $300 \div 3000$  eV) in a background plasma with relatively hot electrons  $T_e = 1 \div 10$  eV. A flow of such kind can be kept efficiently by geomagnetic field from expansion across the lines of force. Thus, a 3-dimensional PAF of relatively high density appears.

As a plume expands into vacuum the pressure in the plume drops and at some distance from a source it equals to magnetic pressure. In a dense and low-temperature plasma the geomagnetic effect can be measured with the value of electromagnetic interaction parameter, that is Ampere parameter  $S$ , equal to *transverse electromagnetic forces scale-to-pressure gradient* ratio<sup>1</sup>. Conditionally, there are three stages of a plume expansion process:  $S < 1$ ,  $S \sim 1$  and  $S > 1$ . Every of the stages has its own phenomenology.

At the initial stage of expansion, while  $S < 1$ , the magnetic field has no effect on a plume behaviour. The nature of the plume expansion is inertial. An analysis of this stage is represented in<sup>2</sup>. Parameter  $S$  grows while a plume expands, and at some point  $x = x_B$  the expansion of plasma across the magnetic field decelerates to the speed of cross diffusion of plasma in magnetic field  $B$ . The speed of expansion along the field  $B$  is equal to the speed of 1-dimensional inertial expansion. As a result the difference of the speeds affects the configuration of a plasma plume.

The  $B$  field influence upon a plasma exhaust depends on electric currents  $j$ , generated in the plasma under the effect of polarisation field  $E = V \times B$  ( $V$  - local velocity of plasma). If plasma concentration in an exhaust plume is not much higher than that of the ionospheric background, the currents  $j$  close themselves mainly through the background plasma and de-polarisation of the plasma occurs along a short distance. Papers<sup>3,4</sup> consider the above case of plasma

expansion. The subject of the present paper is the problem about dynamics of rather powerful and dense jets when the ionosphere plasma has no effect.

**Boundaries of the Zone of a Plume Free from the Magnetic Field Influence**

In figure 1 the hatched area is the zone of a thruster plume where the plasma flow is not distorted by the longitudinal geomagnetic field. In the figure one can see lines of equal electron density in the plume of TAL D-55<sup>2</sup>.

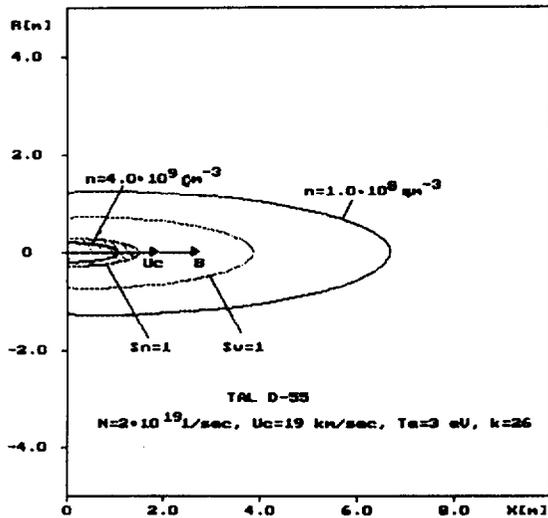


Fig.1

At first an external magnetic field affects the longitudinal distribution of plasma density in a plume. One can write down the condition of this effect as follows:

$$S_n = \frac{(jB)}{(\nabla nT)} = \frac{\sigma_c u_c^2 B^2 \pi a^3 a'}{NT} \geq 1$$

Then, a little farther along a plume the magnetic field distorts also lines of the ion flow. So, a plume is a  $\theta$ -pinch which is inhomogeneous along the magnetic field, and it is as if drawn to the force lines of the magnetic field. The condition of the  $B$  field influence on the distribution of ion streams in a plume can be written down as

$$S_v = \frac{[jB]}{m \frac{dv}{dx}} = \frac{\sigma_c u_c^2 B^2 \pi a^2}{m\dot{N}} \geq 1$$

Based on the above, the zone of a plume undisturbed by the geomagnetic field can be described with two dimensions:

$$x < x_B \cong \left( \frac{\dot{N} T k^2}{\sigma_c u_c^2 B^2 \pi} \right)^{1/3}$$

$$r < a_B \cong \left( \frac{\dot{N} T \sqrt{k}}{\sigma_c u_c^2 B^2 \pi} \right)^{1/3}$$

For example, in the plume of TAL D-55, generated at the altitude of 1300 km ( $B=0.3$  Gauss), the values are :  $x_B = 1.2m$  and  $r_B = 0.24m$ . In case of the plume generated by the source of EPICURE<sup>5</sup>, operating at 800 km ( $B=0.37$ Gauss) one can get  $x_B = 3.3m$  and  $r_B = 1.8m$ . Out of this zone the external magnetic field changes the nature of parameter distributions in a plume.

**The Magnetic Field Effect upon the Configuration of a Plume.**

**A Plume in a Longitudinal Magnetic Field**  
Configuration of PAF generated by operating plasma thrusters in the vicinity of a spacecraft, depends on  $\alpha$  pitch-angle. If the axis of a plume is directed along the field  $B$  ( $\alpha=0^\circ$ ), isoconcentrals (the lines of equal density) have a 'knitting needle' shape, i.e. their longitudinal sizes are much more than their cross ones. At  $x > x_B$  and  $r > r_B$  radial ion streams are much less than in case of a conical expansion. Their velocity decelerates to the speed of cross diffusion of plasma in the magnetic field. The speed along the  $B$  field is equal to the speed of 1-dimensional inertial expansion. As a result, the difference of these speeds affects the configuration of plasma plumes.

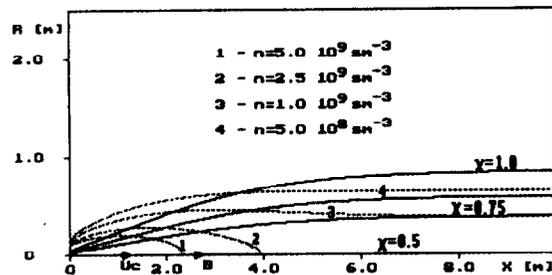


Fig.2

In figure 2 the calculation results of a TAL D-55 plume expanding in the longitudinal field

of  $B=0.3\text{ Gauss}$ . The dotted lines show three levels of isoconcentrals:  $10^9\text{ cm}^{-3}$ ,  $5 \cdot 10^8\text{ cm}^{-3}$ ,  $10^8\text{ cm}^{-3}$ . The solid lines are the lines of currents covering the ion flow which is  $\chi$ -part of the total flow.

The following three figures illustrate how the magnetic field can influence the distribution of parameters in a plume expanding along magnetic lines. In fig. 3 behaviour of plasma density of a flow core along the axis of a plume is depicted for three values of external magnetic field induction: 0.5 Gauss, 0.3 Gauss и 0.05 Gauss. One can see that the greater values of the magnetic field correspond to the higher concentrations along the axis. In other words, more quantity of substance is kept around the core.

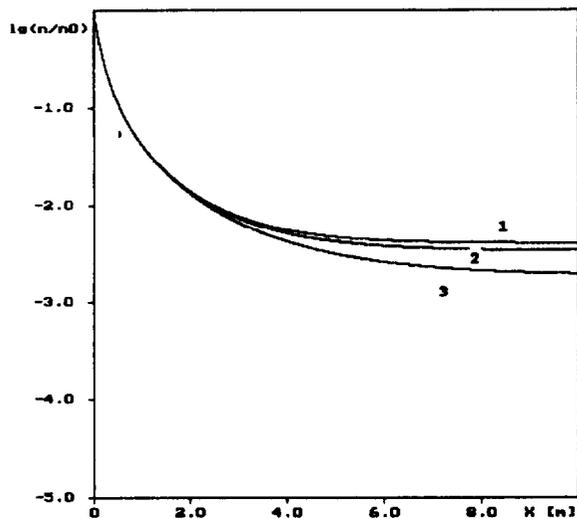


Fig.3

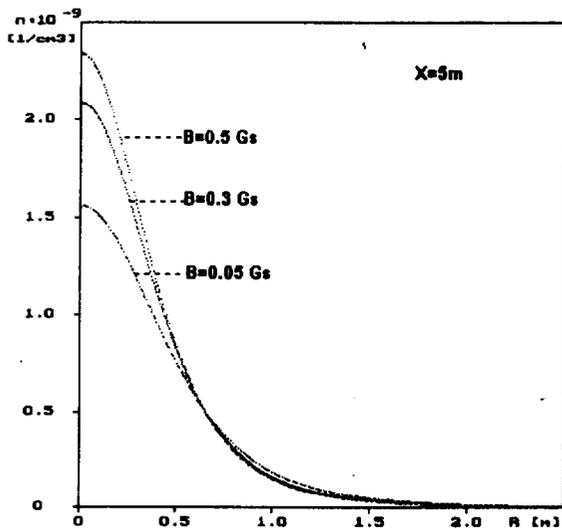


Fig.4

Figure 4 illustrates it. There, concentration distribution in the cross-section  $x=5m$  is plotted for a  $D-55$  plume. The greatest  $n$  corresponds to the greatest magnetic induction  $B$ . In this case the peripheral flow density appears to be slightly less than in case of expansion in a weaker magnetic field. This variation of density influences the flow configuration.

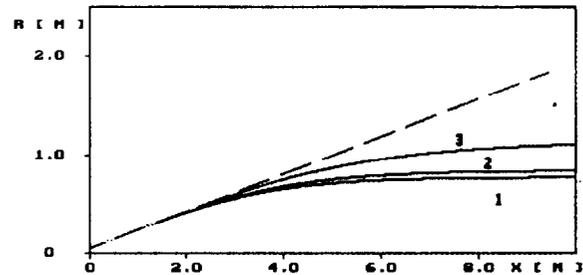


Fig.5

Figure 5 illustrates how the geomagnetic field influences the cross-section size of a  $D-55$  plume expanding along the force lines. Curve 1 is the case of geomagnetic field induction  $B=0.5\text{ Gauss}$ . Curves 2 and 3 are the cases of 0.3 and 0.05 Gauss respectively. The dotted straight line is the case of inertial expansion with no magnetic field. One can see that in case 1 a plume expansion is under the magnetic field effect even at  $x > x_B = 0.8\text{ m}$ . In case 3 of a weak magnetic field which can be produced on the orbit of 7000 km altitude, the field  $B$  does not disturb the nature of a plume expansion until  $x \leq 5\text{ m}$ .

Rebuilding of a plume occurs within a small area in the vicinity of  $x = x_B$ , where the magnetic interaction parameter  $S_n \sim 1$ . Here, the velocity of a jet decelerates a little and the temperature of plasma rises. The integration results of a set of equations describing a flow transition are represented in fig.6.

The values of velocity  $V$  and temperature  $T$  are represented as non-dimensional. Their values in the initial cross-section are taken as the scale parameters.

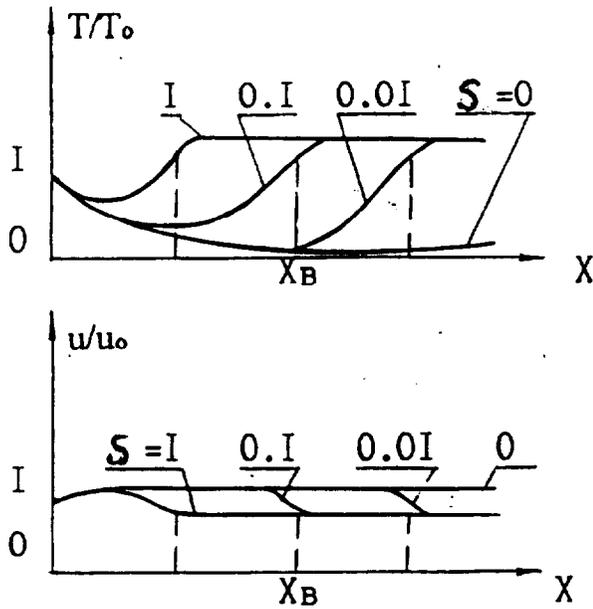


Fig.6

**A Plume in a Transverse Magnetic Field**

If a jet expands across the field  $B$  ( $\alpha = 90^\circ$ ), the isoconcentrales transform themselves into a petal, the plane of which is parallel to vectors  $V$  and  $B$ . The system of currents, arising in a jet, compresses the jet in the direction of  $\pm Y \parallel [V \times B]$  and decelerates it along the axis  $X \parallel V$ . The ion streams expanding across the petal plane at  $x > x_B$  and  $r > r_B$  are much less than at a conical expansion, but along the  $B$ , i.e. parallel to the petal plane, they are a little greater. Such PAF are characterised with  $\nabla_{\perp} n \gg \nabla_{\parallel} n$ , variation of  $n$  is weak along a jet ( $n \sim x^{-1/2} \div x^{-10/9}$ ). So, by their radiophysics properties these PAF are similar to the discontinuities of planar-laminar type. In figures 7a,b,c calculations of a 3D configuration of a TAL D-55 plume surface  $n=const$  are represented. Lines of equal density are shown for three levels: 1)  $n=5 \cdot 10^8 \text{ cm}^{-3}$ ; 2)  $n=2.5 \cdot 10^8 \text{ cm}^{-3}$ ; 3)  $n=1 \cdot 10^8 \text{ cm}^{-3}$ ; the pitch-angle  $\alpha=90^\circ$ , the geomagnetic field induction  $B = 0.3 \text{ Gauss}$ . One can see that sizes of PAF across the symmetry plane are essentially less than longitudinal ones. Such a 3D configuration of the plasma petal causes high anisotropy of radiophysics properties of space in the vicinity of a spacecraft during on-board operation of a plasma thruster. In addition, when solving the problems about plasma jets interaction with a spacecraft some singularities of plasma jet expansion in a

magnetic field should be taken into consideration.

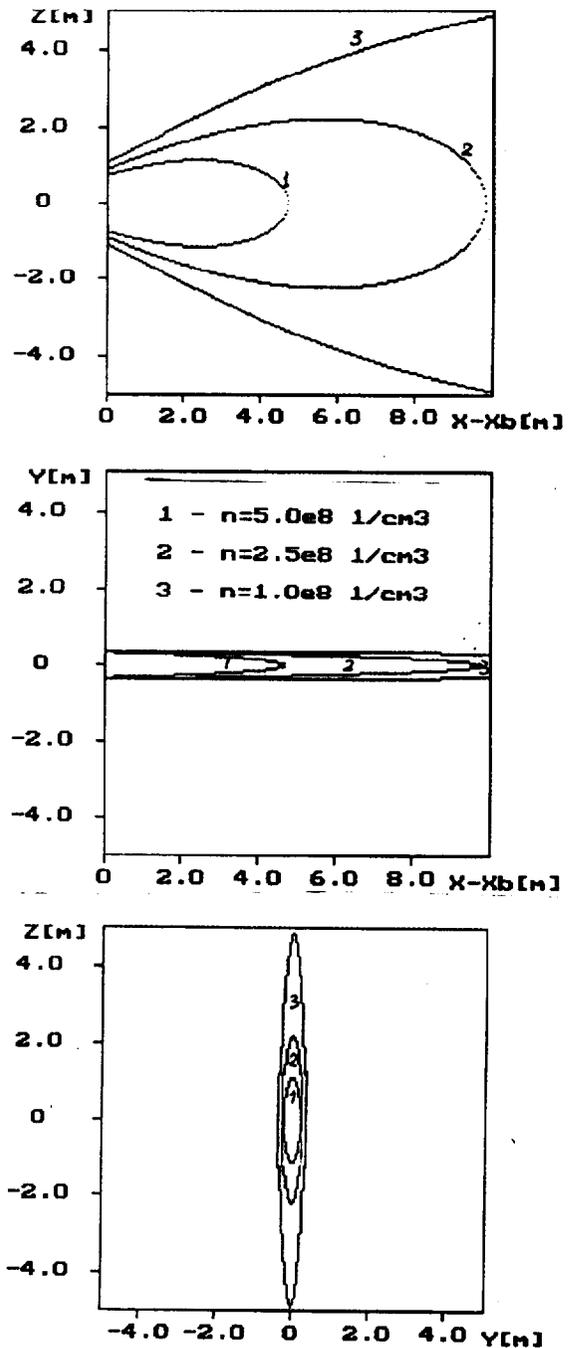


Fig.7

**The Magnetic Field Influence Boundaries Dependence on the Orbit Altitude**

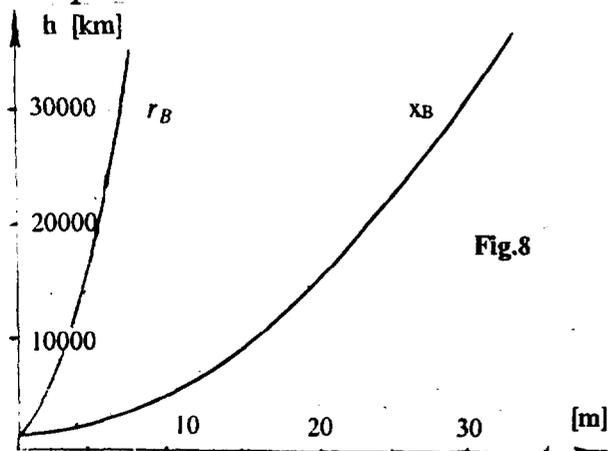
In Table 7 data on the most popular thrusters are arranged and evaluated distances  $x_B$  along the jet axis starting from which the

geomagnetic field effect should be taken into account, are represented. Computations of  $x_B$

Thruster type, basic characteristics	Orbite altitude, magnetic field B	$X_B$ , (m)
TAL D-55, SPT-100, SPT T-100 $\dot{N}=2 \cdot 10^{19} \text{s}^{-1}$ , $U=1.7 \cdot 10^4 \text{m/s}$ , $T_e=2.5 \text{eV}$ [AIAA 95-2927]	h=300 km; B=0.5 Gs	0.84
	h=1300 km; B=0.3Gs	1.2
	h=7000km; B=0.05Gs	3.9
End-Hall Thruster EPICURE $\dot{N}=2 \cdot 10^{20} \text{s}^{-1}$ , $U=2 \cdot 10^3 \text{m/s}$ , $T_e=0.3 \text{eV}$ [5]	h=300 km; B=0.5 Gs	2.7
	h=1300 km; B=0.3Gs	3.8
	h=7000km; B=0.05Gs	12.7
Ion thruster (Hg) $\dot{N}=1.25 \cdot 10^{18} \text{s}^{-1}$ , $U=3.2 \cdot 10^4 \text{m/s}$ , $T_e=1 \text{eV}$ [IEPC 95-162]	h=300 km; B=0.5 Gs	0.25
	h=1300 km; B=0.3Gs	0.35
	h=7000km; B=0.05Gs	1.15
Ion thruster NSTAR (Xe) $\dot{N}=3.75 \cdot 10^{18} \text{s}^{-1}$ , $U=3.2 \cdot 10^4 \text{m/s}$ , $T_e=5 \text{eV}$ [IEPC 95-162]	h=300 km; B=0.5 Gs	0.27
	h=1300 km; B=0.3Gs	0.39
	h=7000km; B=0.05Gs	1.27
Arcjet $\dot{N}=2 \cdot 10^{21} \text{s}^{-1}$ , $U=4.5 \cdot 10^3 \text{m/s}$ , $T_e=0.3 \text{eV}$ [NASA TM 103638]	h=300 km; B=0.5 Gs	2.2
	h=1300 km; B=0.3Gs	3.2
	h=7000km; B=0.05Gs	10.6

are given for three cases of the field induction  $B$  corresponding to different orbits.

The limit of the geomagnetic field influence upon a plasma expansion depending on orbital altitude is more clear from fig. 8. The curves are plotted for a plume generated by SPT plasma source or TAL of 1.35 kW. In fig.8 the longitudinal size  $x_B$  and transverse size  $r_B$  dependence on the orbital altitude (or, that is, on the external magnetic field value) is depicted.



On the altitudes of 300...3000 km the undisturbed zone of a plume has the

dimensions  $x_B \sim 2 \div 3 \text{m}$  and  $r_B \sim 0.5 \div 1 \text{m}$ . On GSO it has  $x_B \sim 40 \text{m}$  and  $r_B \sim 6 \text{m}$ .

#### Conclusions

1. Expansion of electric thruster plumes is under the geomagnetic field influence. On the low and medium Earth orbits the effect appears at the distances  $x_B \sim 2 \div 3 \text{m}$  and  $r_B \sim 0.5 \div 1 \text{m}$ , on the high orbits the geomagnetic field effect should be taken into consideration for radial streams of ions starting from  $r_B \sim 5 \text{m}$  only.
2. In comparison with inertial expansion, transversal ion streams are reduced by the geomagnetic field in the  $\perp B$  direction, but they have some increase along  $B$ .
3. The geomagnetic field influence causes specific distribution of electron concentrations in the PAF (generated in the vicinity of a spacecraft during operation of plasma thrusters). The concentration varies weakly along the plume axis and it has an abrupt change in the cross-section of a plume. The PAF of such kind can diffract radio waves effectively like planar-laminar media do.
4. Distribution of electric potentials and currents in a plume depends on their possibility to close their path out of the plume. This possibility is conditioned, for example, by the density of background plasma, by existence of metal walls close to the plume, etc. Correspondingly, the degree of a weak external magnetic field influence upon parameter distributions in a plasma exhaust is also a function of those conditions.

#### References

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