

A Two-dimensional (azimuthal-axial) Particle-In-Cell of a Hall Thruster

IEPC-2013-252

*Presented at the 33rd International Electric Propulsion Conference,
The George Washington University • Washington, D.C. • USA
October 6 – 10, 2013*

L. Garrigues^{1,2†} and P. Coche^{1‡}

¹LAPLACE (Laboratoire Plasma et Conversion d'Energie),
Université de Toulouse, UPS, INPT Toulouse
118, route de Narbonne, F-31062 Toulouse cedex 9, France
²CNRS; LAPLACE; F-31062 Toulouse, France

We have developed a two-dimensional fully kinetic model of the Hall thruster. Simulations are able to reproduce the “breathing mode” already observed, with a periodic depletion of neutral atoms. Results show that during the increase of the discharge current, an azimuthal instability (frequency in the range of MHz and wavelength on the order of mm) is formed close to the channel exhaust. During the current decrease, an axial electric wave is formed (whose frequency is on the order of 400 kHz) that can accelerate a fraction of the ion population to energy larger than the potential drop applied between the anode and the cathode.

Nomenclature

roman characters

B	=	magnetic field strength (T)
d	=	channel width (m)
e	=	electron charge constant (1.6×10^{-19} C)
E_x	=	axial electric field
k_B	=	Boltzmann constant (1.38×10^{-23} JK ⁻¹)
m_e	=	electron mass (kg)
n_e	=	plasma number density (m ⁻³)
T_e	=	electron temperature (K)
V	=	electric potential (V)
v_x	=	axial electron mean velocity (ms ⁻¹)
v_z	=	radial electron velocity (ms ⁻¹)
x	=	axial direction (m)
y	=	azimuthal direction (m)
z	=	radial direction (m)

greek characters

[†] Research Scientist at CNRS, laurent.garrigues@laplace.univ-tlse.fr

[‡] Ph.D., philippe.coche@laplace.univ-tlse.fr

α	=	scaling factor
Δt	=	time step (s)
Δx	=	axial grid spacing (m)
Δy	=	azimuthal grid spacing (m)
ϵ_0	=	free permittivity (8.85×10^{-12} F.m ⁻¹)
λ_d	=	Debye length (m)
ν_{ew}	=	electron-wall collision frequency (s ⁻¹)
ν_m	=	electron momentum collision frequency (s ⁻¹)
$\mu_{\perp,c}$	=	electron collisional mobility (m ² V ⁻¹ s ⁻¹)
$\mu_{\perp,fluid}$	=	fluid electron mobility (m ² V ⁻¹ s ⁻¹)
ω_c	=	electron cyclotron frequency (s ⁻¹)
ω_p	=	electron plasma frequency (s ⁻¹)

I. Introduction

All thrusters (HT) are devices mounted on board of satellites used for trajectory corrections, or as a main thrust system on probes designed for deep space exploration. They measure a few centimeters and have the particularity to use an external magnetic field to constrain the electric potential applied between an anode and an outside cathode to penetrate inside a quasi-neutral plasma. This electric field accelerates the ions to provide the thrust, without the use of accelerating grids. They are of great interest among the community for their high specific impulse compared to chemical thrusters, which is a measure of the thrust produced by the engine with respect to the propellant consumption per unit of time. The difference of specific impulse exceeds easily a factor of 10 leading to an important mass saving of the total system [1], [2].

Despite decades of research and many success of the engine operating in outer space, there remains some unanswered questions about the physic of the discharge, bringing some complexity in the way to optimize the engine. In particular, electron transport across the B-field cannot be described by collisions in volume, as stated by the classical theory because of a discrepancy of a factor of 10 between the mobility measurements and the theoretical values. Two possible causes have been deeply explored in order to solve the discrepancy which reveals the so-called anomalous transport: electron-wall collisions and turbulence.

Numerous works have tried to clarify some aspects of the anomalous transport. Using fluid modeling, Barral *et al.* [3] were able to recover the experimentally observed behavior of the wall material influence on the current-voltage characteristic, at high applied voltage. It was found that in this regime, secondary electron emission under high energetic electrons impacting on the discharge walls plays a role on the electron transport across the magnetic field lines. Kaganovitch *et al.* [4] performed a similar study with a one-dimensional Particle-In-Cell (PIC) model and conclude that the fraction of secondary electrons was much lower, reducing the influence of the walls on the axial current. The Electron Energy Distribution Function (EEDF) described self-consistently by Kaganovitch *et al.* appears to depart from a Maxwellian EEDF assumed in the study of Barral *et al.*, explaining the two opposite conclusions.

Adam *et al.* [5] numerically evidenced the role of the turbulence using (axial, azimuthal) geometry and a kinetic approach. They showed the development of a high frequency azimuthal wave of electric field and were able to obtain self-consistently an electron mobility perpendicular to the magnetic field of the same order of magnitude than the experiments. Tsikata *et al.* [6] have confirmed the existence of this wave using collective scattering measurements. The transport of magnetized electrons coupled with unmagnetized collisionless ions has been theoretically studied by Gary and Sanderson [7], [8]. They show that an instability develops in the ExB drift direction due to the coupling of the Bernstein mode of electrons and ion acoustic waves. Finally, McDonald [9] investigated experimentally a lower frequency azimuthal wave, the so-called rotating spoke, and demonstrates its ability to induce a significant axial transport of electrons.

In fluid models, authors use to describe the anomalous transport through fixed mobility terms (scaling as B^{-1} or B^{-2} according to the Bohm anomalous transport assumption) as it is difficult to derive a self-consistent mobility term [10-13]. Consequently, even if the progress toward a better description of the thruster working is remarkable, these

models still miss an accurate description of time dependent phenomena. Indeed, Garrigues *et al.* [14] used averaged quantities extracted from measurement obtained by mean of Laser Induced Fluorescence setup (LIF) to empirically fit a more accurate mobility term instead of imposing an analytical function. Unfortunately, calculated ion velocity distribution functions exhibit a finer dispersion in velocity compared to measurements, due to the fix in time deduced mobility. In light of this, we have developed a 2D (axial, azimuthal) model to study the transport phenomena which successively develops during the time evolution of the thruster discharge.

We describe the model based on the PIC approach we have developed in Section II. In section III, results are shown. The phenomena occurring during the time evolution of discharge are chronologically presented and their role for the electron transport is also studied. We finally conclude in section IV.

II. Description of the model

The model is 2-dimensional and based on a Particle-In-Cell (PIC) description with an explicit discretization scheme (leap-frog) [15], [16]. The domain extends along the axis and azimuthal directions. Only a sector is retained in the azimuthal direction because of computational time considerations. The curvature is neglected so that a Cartesian coordinate system is employed. 2 cm are described in the azimuthal direction, 4 cm in the axial direction. The domain is visible in Fig. 1. In the rest of the paper, the axial direction will be refereed as the x-direction and the azimuthal direction as the y-direction. Periodic boundary conditions are imposed along the latter direction, both for the particles and the electrical potential. Particles leaving the domain at the anode ($x = 0$) and the cathode ($x = 4$ cm) are suppressed. Dirichlet conditions are imposed: 300 V at the anode and 0 V at the cathode. An exhaust plane lies at $x = 2.5$ cm and splits the domain into a wall-included area (the inside of the thruster) and a wall-free area (the outside). Its implementation and its effect on the transport will be discussed further.

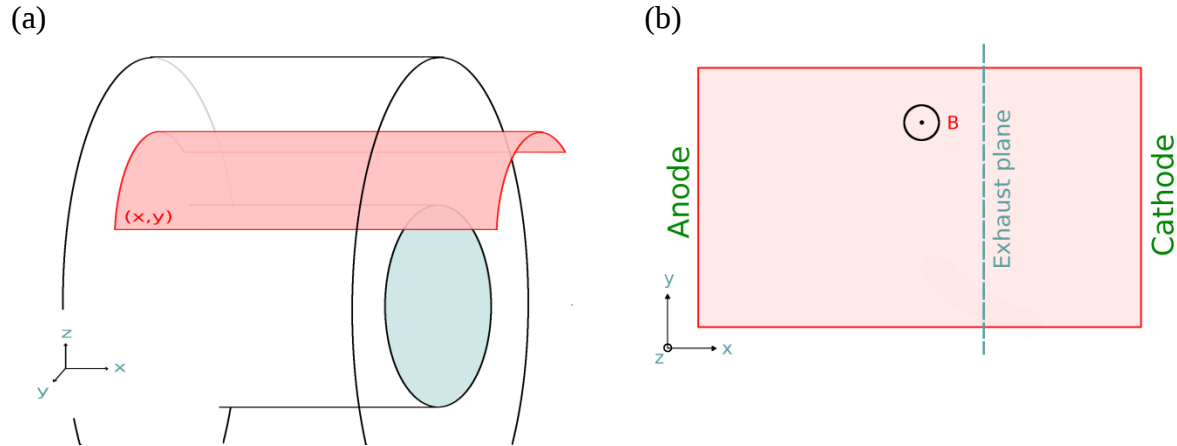


Figure 1: (a) computational domain, (b) computational domain in the (x,y) plane, the magnetic field is along the z direction; the position on the exhaust plane at $x=2.5$ cm is also presented.

Particle injection is calculated self-consistently. Each time step, charge balance between the particles leaving the anode is calculated and the amount of excessive electrons is injected at the cathode. Their distribution is chosen from a Maxwellian distribution with a temperature of 1 eV. The magnetic field profile is plotted in Fig. 2. Its maximum corresponds to the exhaust plane. Typically, a value of 170 G is imposed. The neutral gas is xenon. Only electrons and single charged ions of xenon are considered. Neutrals are described using fluid equations (as in Ref. [5]).

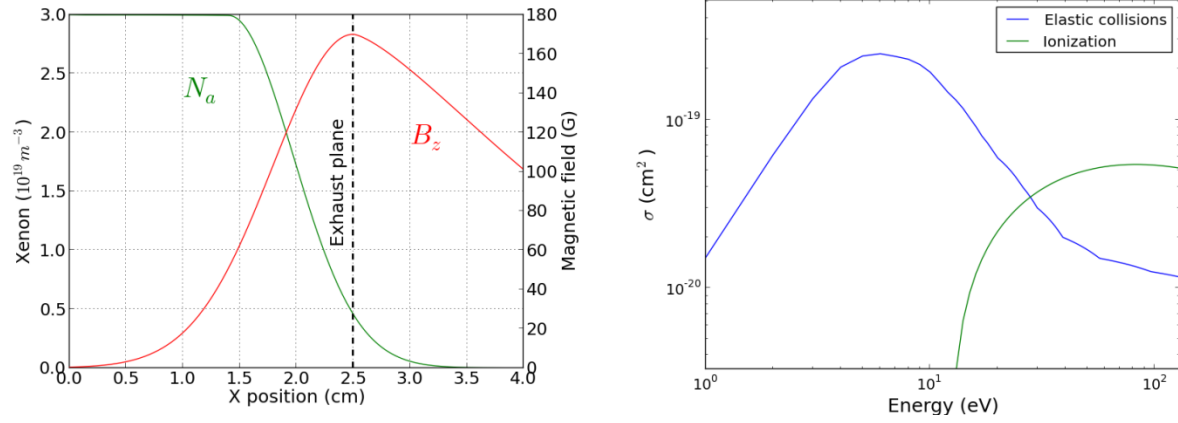


Figure 2: (a) axial profiles of initial neutral density and magnetic field strength, (b) electron-neutral collision cross sections.

In the PIC method, collisions calculation is separated from the particle motions. A null collision Monte-Carlo method is employed [17]. Only elastic collisions and ionization (energy threshold of 12.3 eV) are taken into account. The cross sections used are plotted in Fig. 2b. Due to the geometry employed, inner and outer walls of the coaxial cylinders are not included in the computational domain. However, a simple model of electron-wall collision is included in order to take into account any wall effects on the transport of electrons. This model applies only in the area defined between $x = 0$ and $x = 2.5$ cm. Electrons whose energy directed along the radial direction is larger than a fixed value of sheath potential (infinitely thin) defined as an input parameter equals to 20 V, are likely to undergo wall collisions. For each of these electrons, we calculate a collision frequency that can be roughly estimated by $\nu_{ew} = v_z/d$, where v_z is the electron velocity in the radial direction and d is the channel width (as in Ref. [18]). If the collision occurs, the electron is isotropically reflected and does not suffer from any energy losses. Secondary electron emission is not taken into account.

The simulation is started with a uniform plasma density of 10^{18} m^{-3} . Initial particle velocities are chosen from a Maxwellian distribution with a temperature of 1 eV. The initial neutral gas profile is plotted in Fig. 2. As an explicit scheme is employed, strong numerical constraints arise. The Debye length λ_d must be resolved by a fine grid spacing $\Delta x, y$ and the time step Δt must be smaller than $0.2/\omega_p$, where ω_p the electron plasma frequency. For typical conditions of the HT, that is an electron density of $n_e = 10^{18} \text{ m}^{-3}$ and an electronic temperature T_e of 50 eV, the Debye length is equal to 50 μm and the plasma frequency is $5.6 \times 10^{10} \text{ s}^{-1}$. For $\Delta x = \lambda_d$, 3×10^5 mesh grid points would be needed for a domain of 4 cm long and 2 cm large, along with a time step Δt equals to $3 \times 10^{-12} \text{ s}$. In order to alleviate the numerical constraints on the relevant physical quantities, a scaling on the permittivity is used. It is simply increased by a coefficient α . This scaling technique has been already successfully used in the context of HT modelling or low temperature plasmas with magnetized electrons [19-21]. The plasma frequency and Debye length thus become (starred quantities):

$$\lambda_d^* = \sqrt{\frac{\alpha \epsilon_0 k_B T_e}{e^2 n_e}} = \lambda_d \alpha^{1/2} \quad (1)$$

$$\omega_p^* = \sqrt{\frac{e^2 n_e}{\alpha \epsilon_0 m_e}} = \omega_p \alpha^{-1/2} \quad (2),$$

where e is the electron charge, k_B is the Boltzmann constant, ϵ_0 is the free permittivity, n_e is the electron density, and m_e is the electron mass. Modified $\Delta x, y$ and Δt now read:

$$\Delta t^* = \Delta t \alpha^{1/2} \quad (3)$$

$$\Delta x, y^* = \Delta x, y \alpha^{1/2} \quad (4).$$

For the simulation results presented in this paper, $\alpha = 80$. The number of grid points has been reduced by one order of magnitude and the time step is increased by the same factor.

III. Results

For all the results presented here, we use the same magnetic field profile plotted in Fig. 2 with a maximum strength of 170 G, an applied voltage of 300 V, a xenon flux of 5 mg.s^{-1} , a sheath potential of 20 V and a scaling parameter α of 80. It is well known that the Hall thruster behavior is strongly time-dependent and a rich variety of oscillations, ranging from a few kHz to MHz develops [22]. For a thorough investigation of the electron transport mechanisms, we must define a time scale which comprises all of the relevant oscillating phenomena. The most noticeable of such oscillations is the so-called breathing mode [18], [23] which strongly impacts the discharge and ion currents. Its shape is plotted on Fig. 3a as a function of time. This mode is due to the periodic depletion of the neutrals, as can be seen on Fig. 3b, as a function of the x-coordinate and time. The xenon density has been averaged on the y- coordinate. As the front enters the region of high ionization rate (close to the maximum of magnetic field), the flux of incident electrons increases and due to the strong ionization rate, the front move backwards, toward the anode. The ionization rate decreases as the energy of incident electron gets lower, the flux of the incident electrons decreases and the front can move forwards. For the discharge parameters considered, the period of the oscillation is around $50 \mu\text{s}$, that is a frequency of 20 kHz.

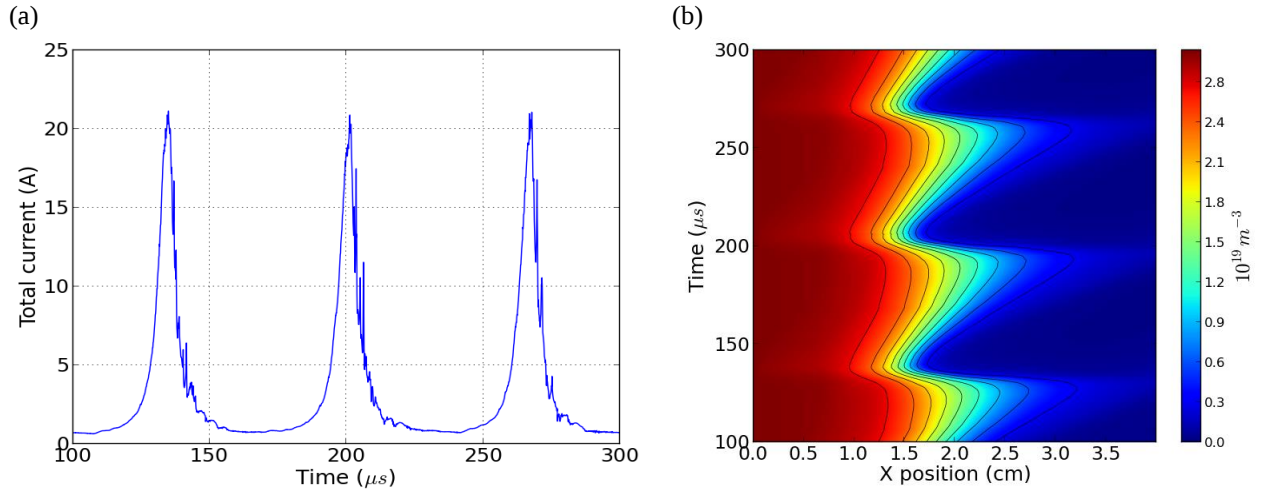


Figure 3: (a) total current as a function of time, (b) contour plot of the xenon density as a function of space (averaged along the y-direction) and time.

The main characteristic of this mode, the periodic depletion of the neutrals, plays a key role in the transport of electrons. Indeed, as the neutral density varies with time, the collision frequency consequently varies with time, providing a time-dependence of the collisional transport of electrons. As we will see, the magnitude of the transport ensured by turbulence and wall-collisions is closely related to the magnitude of the collisional transport in volume. Thus, both transport phenomena, turbulent and wall-collisional originate from a particular configuration of the system at a given time, whose consequences on the discharge are visible through the breathing mode and the discharge current. In order to articulate the study, we are going to isolate one oscillation of the discharge current and observe along with time, the evolution of the physical phenomena impacting the discharge. In Fig. 4, we have distinguished the increase and the decrease of the discharge current because of the very different nature of the phenomena developing for each phase, as we will see next. It should be noted that the time scale has been centered at the top of the oscillation. Negative times refer to the current increase, positive times to the current decrease.

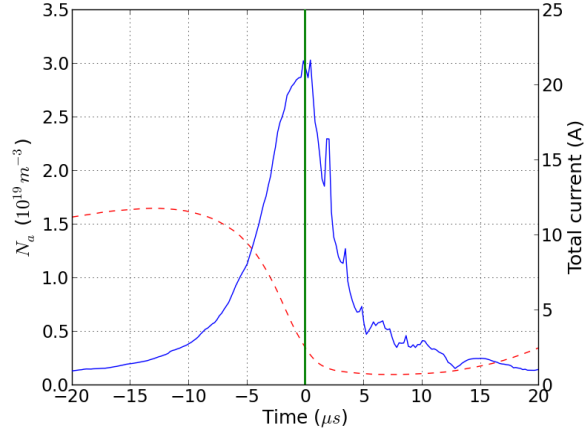


Figure 4: total current as a function of time and xenon density as a function of time (at $x = 2.5$ cm and averaged along the y-direction).

We have plotted in Fig. 5 the collision frequencies of the three processes taken into account in our model: elastic, ionization, and electron-wall collisions. They have been plotted as a function of space and time for a whole oscillation period. These profiles have been averaged along the azimuthal direction.

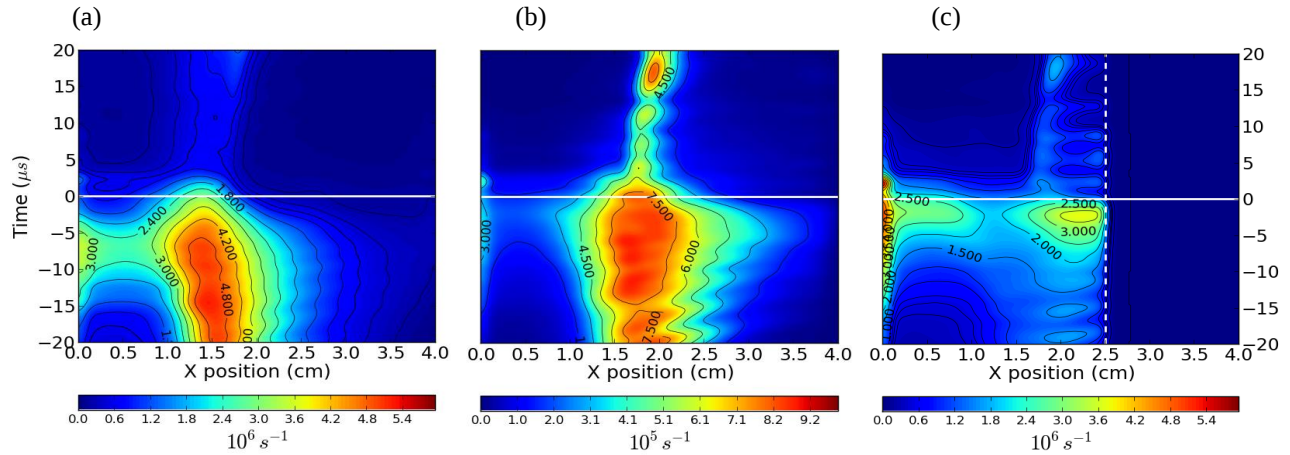


Figure 5: (a) elastic, (b) ionization, (c) electron-wall collision frequency profiles as a function of space and time (averaged along the y-direction).

A. Current increase

We first focus on the current increase that corresponds to the negative times (see Fig. 4). When the current starts increasing ($t = -20$ μs), the front of the xenon density lies in the area of strong ionization near the maximum of the magnetic field. The xenon density is high and the consequence is a high collision rate. This is easily seen in Fig. 5 (a), from -20 μs to 0 μs. The collision frequency is the highest (over 5×10^6 s $^{-1}$) and the maximum strengths are localized in the region between 1 and 2.5 cm. In the beginning of the oscillation, the ionization frequency, plotted on Fig. 5b is concentrated in the same area. Then, it expands on the side, especially at the cathode side until the time $t = -5$ μs, where the ionization frequency and the electron-neutral collision frequency shrink. In Fig. 4, along with the time evolution of the discharge current is plotted a profile of the time evolution of the xenon density, taken at $x = 2$ cm. It is clear that the decrease of the xenon density becomes stronger around $t = -5$ μs. The elastic collision frequency which depends on the xenon density follows the same trend and consequently decreases.

At the same time, the electron-wall collision frequency increases around its maximum at $t = -5 \mu\text{s}$. There is a close correlation between the sudden shrink of the collision frequencies in the bulk of the plasma and the increase of the electron-wall collision frequency. We remind that the electrons are likely to make wall collisions if their energy, directed along the z-direction exceeds an imposed potential of 20 V. In our model, no radial electric field is described; so that the only way for the electrons to gain energy in the z-direction is by mean of electron-atom collisions which reorganize their velocity components (see Refs. [24], [25] for a similar process). Thus, two conditions for electron-wall collisions are necessary: electron-atom collisions and a high enough radial kinetic energy. As can be noted on Fig. 9 (b) which is a profile at $x = 2 \text{ cm}$ of the kinetic energy as a function of time and averaged on the azimuthal direction, at $t = -5 \mu\text{s}$, there is an increase of the total kinetic energy. This is due to the fall of collisions in volume and especially of the ionization which resulted in a limitation of the energy level. However, even fewer, there remains enough electron-neutral collision to distribute this energy increase in the z-direction until the time $t = 0 \mu\text{s}$ where collisions in volume reach their lowest values around $x = 2 \text{ cm}$.

During the discharge current increase, collisions are mainly responsible for the transport of electrons. We have shown that when collisions in volume become too low, the contribution of wall-collisions on transport becomes stronger. However, beyond the exhaust plane, no wall collisions can exist and compensate the decrease of collisions in volume. $5 \mu\text{s}$ before the top of the current oscillation, we show now that an azimuthal electric wave develops and provides the required conductivity. The azimuthal electric field is plotted in Fig. 6a and the electron density in Fig. 6b as a function of space, at time $t = -3 \mu\text{s}$. We will see further that this wave can no longer exist after the top of the oscillation.

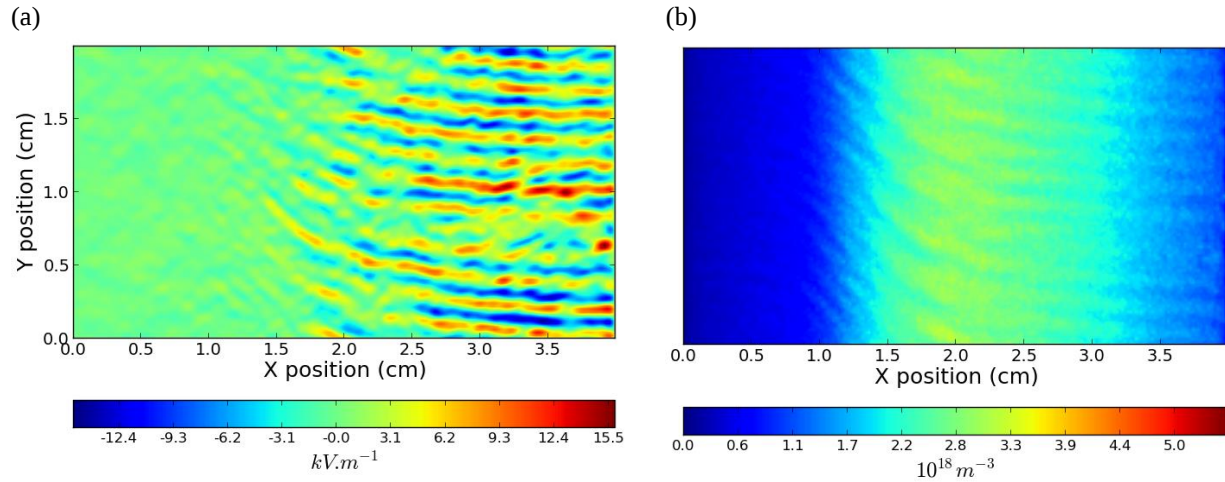


Figure 6: (a) azimuthal electric field and (b) electron density profiles as a function axial position (averaged along the y direction) and time.

This wave is similar to the wave observed by Adam *et al.* [5]. However some differences arise. Firstly, this wave has a dominant mode number (number of pattern) equals to 13 and its amplitude is stronger after the exhaust plane, where collision frequency becomes too low. This is also observed in Adam's model but the dominant mode number is equal to 20. The maximum amplitude observed is twice larger and is equal to 30 kV.m^{-1} instead of 15 kV.m^{-1} in our model. Finally, the most striking characteristic is the appearance time of the wave: it exists at any time of the discharge current oscillation in the implicit PIC model whereas it appears only during $5 \mu\text{s}$, at the end of the current increase in the explicit PIC model.

To quantify the ability of the wave to enhance the mobility of the electrons, we establish two expression of the mobility. The first one is the so-called collisional mobility $\mu_{\perp,c}$ which expresses the collisions as the only detrapping processes across a magnetic field. It writes,

$$\mu_{\perp,c} = \frac{e}{m_e v_m} \frac{1}{1 + (\omega_c / v_m)^2} \quad (5),$$

where ω_c is the electron cyclotron frequency, related to the magnetic field strength and v_m is the momentum exchange frequency (that includes electron-atom and electron-wall collisions). The other expression of the mobility (noticed $\mu_{\perp,fluid}$) is a fluid-based mobility and has been derived from the drift-diffusion approximation. It writes:

$$\mu_{\perp,fluid} = -\frac{v_x}{E_x + \nabla(n_e T_e)/en_e} \quad (6),$$

where v_x is the mean axial velocity of the electrons, E_x is the axial electric field, n_e is the electron density. Unlike the other mobility, it is not required to know the exact processes (such as collisions through v_m) responsible for the observed mobility. If both mobilities coincide, it means that no others processes than collisions induce transport. In Fig. 7, collisional mobility (dotted line) and fluid mobility (continuous line) have been integrated from $-5 \mu s$ to $0 \mu s$ and plotted as a function of the x-direction (averaged along the y-direction). The abrupt step at 2.5 cm is due to the electron-wall collision frequency which is equal to zero after the exhaust plane. The trend followed by the fluid mobility is different. Due to the increase of the azimuthal electric field and the development of the organized wave, the fluid mobility increases whereas the electron-neutral collisions frequency decreases. The role of the azimuthal wave on the electron transport is then confirmed.

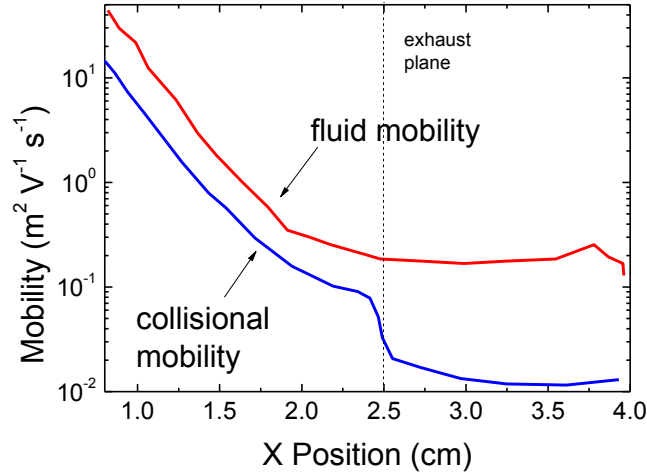


Figure 7: Mobility profile integrated between $t = -5 \mu s$ and $0 \mu s$ as a function of x direction (averaged along the y-direction).

B. Current decrease

The current decrease is affected by the development of an axial wave of electric field and the simultaneous disappearance of the azimuthal wave. The axial wave is the so-called transit time instability already observed and reported in the literature (e.g. [26-29]). The calculated frequency is 400 kHz. During the current decrease, several waves of axial electric field are excited at regular time intervals, from a position located at $x = 2$ cm (5 mm before the maximum of magnetic field), and propagate toward the cathode. On the direction of the cathode, the disturbance interacts with the ions and impacts their energy gain through the potential, as we see in Fig. 8. This is a succession of plots, at successive times, of the distribution function of the ion kinetic energy as a function of the x-axis. The dashed horizontal line locates the kinetic energy level that the singly charged ions are supposed to gain (300 eV for an applied potential of 300 V). As you can see, during the propagation, the beam is disturbed: some ions are heated up and some others are cooled down, depending on the phase of the wave. A significant part of the ions has energy exceeding the maximum energy gained of 300 eV. This kinetic effect is known as ion Landau damping [30] and results from a "wave riding" effect of the ions [28] that is resonant interaction between the axial ion velocity and the phase velocity of the wave. The consequences on the ion beam are also noticeable on the total current (see Fig. 4), where several small peaks appear in the current decrease and are due to abrupt energy gains on a short time.

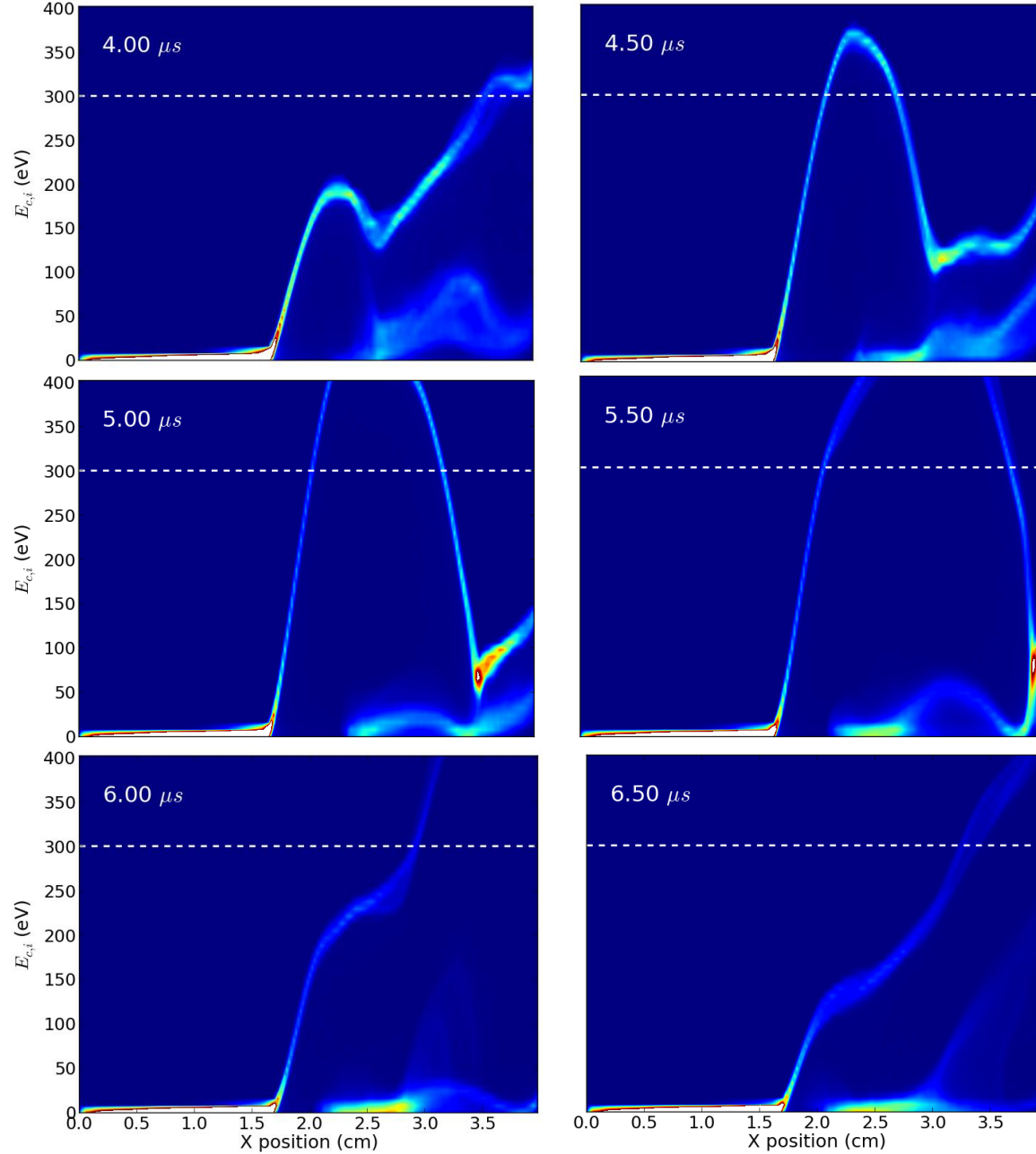


Figure 8: Phase space ($x, E_{c,i}$) of ions at six successive times. The horizontal dashed line locates the maximum corresponding energy that singly charged ions can gain for an applied potential of 300 V.

Each peak corresponds to the propagation of one wave and each wave originates from a brutal increase of the axial electric field. In Fig. 9a, the axial electric field has been plotted as a function of space (x -direction) and time. It has been averaged along the azimuthal direction. We note that during the current decrease and after a first rise, the amplitude of the axial electric field oscillates with time. We remind that in a HT, the axial electric field is created because of a local drop of conductivity forced by the trapping effect of a magnetic field. At the end of the current increase, we noted that the collisional mobility drops and that the electron transport is mainly ensured by the appearance azimuthal wave of electric field. However, as the collisional transport keeps decreasing, the azimuthal wave can no longer provide enough conductivity and the axial electric field consequently rises.

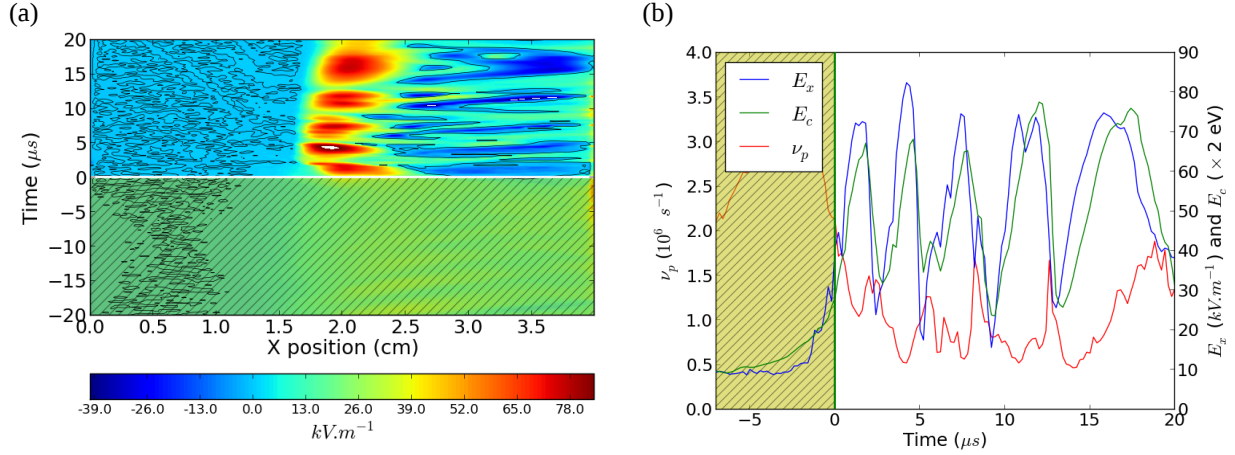


Figure 9: (a) axial electric field profile as a function of time and axial position (averaged along the y-direction), (b) time evolution of the electron-wall collision frequency, axial electric field and electron mean energy at $x = 2$ cm.

The rise of the axial electric field triggers the transit time instability, and the wave associated destroys on its way the azimuthal wave. Actually, the phenomenon is so brutal that the axial electric field changes sign when the disturbance propagates. The azimuthal wave originates from the $E \times B$ drift whose velocity is equal to E/B . As E_x reaches 0, the organized pattern of the azimuthal electric field is broken. The rise of the electric field leads also to an increase of the conductivity through electron-wall collisions. In Fig. 9b, a profile of the electron-wall collision frequency at $x = 2$ cm is superimposed to the profile of the axial electric field at the same location. We can see the correlation between the decrease of the amplitude of the electric field and the increase of the electron-wall collision frequency. Due to the high kinetic energy reached, the collision frequency in volume is more important and enhances further the electron-wall collision frequency (see also Fig. 5c). As the conductivity is now ensured by the electron-wall collisions, the trapping effect is less important and the axial electric field drops (as well as the total kinetic energy), followed by the electron-wall collisions. The electric field will rise again for the same reason stated previously, leading to a short increase of the conductivity, followed by a decrease. This process will happen several time again, until the time $t = 20 \mu s$. At that moment, the xenon front is moving back to the ionization area, leading to an increase of the collisional transport. The current oscillation finally ends. The whole process will start again after the xenon move back in the ionization region.

IV. Conclusions

We have developed a two-dimensional PIC model that describes the azimuthal and axial directions of a HT. To reduce the computational time, a scaling method has been used. A hybrid parallel technique has also been used to permit calculations on the order of a few days for typical conditions.

The model is able to reproduce the so-called breathing mode with a periodic depletion of neutral atoms. We have seen that during the current rise when the neutral density reaches his minimum, a fluctuating azimuthal electric field appears and an instability develops. This instability strongly enhances the transport of electrons toward the anode. On the decrease phase of the discharge current profile, we have noticed that several peaks appear. These peaks are the signature of the transit time instability. The axial electric field is strongly perturbed by the appearance of a single maximum wave propagating toward the cathode.

Due to the computational geometry, the role of the electron-wall interactions cannot be accurately studied. Even if the electron-wall interactions cannot be responsible for the electron transport outside of the channel, the formulation we have chosen is too simplistic to conclude about their effects inside of the channel. The next step of the study is to look in more details the consequences of electron-wall interactions on electron transport.

Acknowledgments

This work has been performed in the frame of the joint-research program CNRS-CNES-Sncema- Universités n° 3161 “Propulsion à Plasma dans l’Espace”. It was granted access to the HPC resources of CALMIP under the allocation 2013-P1125. P. Coche has benefited from a Cnes/Snecma PhD fellowship. The authors want to thank J. P. Boeuf and G. J. M. Hagelaar for fruitful discussions.

References

- [1] V. V. Zhurin, H. R. Kaufman, and R.S. Robinson, *Plasma Sources Science and Technology* **8**, R1-R20, (1999).
- [2] R. G. Jahn and E. Y. Choueiri, *Electric Propulsion in Encyclopedia of Physical Science and Technology*, Third Edition, Volume 5 (The Academic Press, San Diego, 2001).
- [3] S. Barral, K. Makowski, Z. Peradzynski, N. Gascon, and M. Dudeck, *Physics of Plasmas* **10**, 4137 (2003).
- [4] I. D. Kaganovich, Y. Raitses, D. Sydorenko, and A. Smolyakov, *Physics of Plasmas* **14**, 057104 (2007).
- [5] J. C. Adam, A. Héron, and G. Laval, *Physics of Plasmas* **11**, 295 (2004).
- [6] S. Tsikata, N. Lemoine, V. Pisarev, and D. M. Grésillon, *Physics of Plasmas* **16**, 033506 (2009).
- [7] S. P. Gary and J. J. Sanderson, *Journal of Plasma Physics* **4**, 739 (1970).
- [8] S. P. Gary, *Journal of Plasma Physics* **4**, 753 (1970).
- [9] M. S. McDonal, “Electron Transport in Hall Thrusters”, Ph.D. thesis, University of Michigan (2012).
- [10] G. J. M. Hagelaar, J. Bareilles, L. Garrigues, and J. P. Boeuf, *Journal of Applied Physics* **91**, 5592 (2002).
- [11] J. W. Koo and I. D. Boyd, *Physics of Plasmas* **13**, 033501 (2006).
- [12] M. K. Scharfe, N. Gascon, and M. A. Cappelli, *Physics of Plasmas* **13**, 083505 (2006).
- [13] F. I. Parra, E. Ahedo, J. M. Fife, and M. Martinez-Sanchez, *Journal of Applied Physics* **100**, 023304 (2006).
- [14] L. Garrigues, S. Mazouffre, and G. Bourgeois, *Journal of Applied Physics* **111**, 113301 (2012).
- [15] C. Birdsall and A. Langdon, *Plasma Physics Via Computer Simulation* (IOP, 1998).
- [16] R. Hockney and J. Eastwood, *Computer Simulation Using Particles* (Mac Graw-Hill, 1981).
- [17] V. Vahedi and M. Surendra, *Computer Physics Communications* **87** 179 (1995).
- [18] J. P. Boeuf and L. Garrigues, *Journal of Applied Physics* **84**, 3541 (1998).
- [19] H. Liu, B. Wu, D. Yu, Y. Cao, and P. Duan, *Journal of Physics D: Applied Physics* **43**, 165202 (2010).
- [20] G. Fubiani, G. J. M. Hagelaar, J. P. Boeuf, and S. Kolev, *Physics of Plasmas* **19**, 043506 (2012).
- [21] J. P. Boeuf, B. Chaudhury, and L. Garrigues, *Physics of Plasmas* **19**, 113509 (2012).
- [22] E. Y. Choueiri, *Physics of Plasmas*, **8** 1411 (2001).
- [23] L. Garrigues, A. Héron, J. C. Adam, and J. P. Boeuf, *Plasma Sources Science and Technology* **9** 219 (2000).
- [24] D. Sydorenko, A. Smolyakov, I. Kaganovich, and Y. Raitses, *Physics of Plasmas* **13** 014501 (2006).
- [25] I.D. Kaganovich, Y. Raitses, D. Sydorenko, and A. Smolyakov, *Physics of Plasmas* **14**, 057104 (2007).
- [26] Y. B. Esipchuck, A. I. Morozov, G. N. Tilinin, and A. V. Trofimov, *Soviet Physics Technical Physics* **18**, 928 (1973).
- [27] J. M. Fife, “Hybrid-PIC Modeling and Electrostatic Probe Survey of Hall Thrusters”, Ph.D. Thesis, Massachusetts Institute of Technology (1998).
- [28] G. J. M. Hagelaar, J. Bareilles, L. Garrigues, and J. P. Boeuf, *Journal of Applied Physics* **93**, 67 (2003).
- [29] S. Barral, K. Makowski, Z. Peradzynski, and M. Dudeck, *Physics of Plasmas* **12**, 073504 (2005).
- [30] F. F. Chen, *Introduction to Plasma Physics and Controlled Fusion* (Springer, 1984).