STEADY ACCELERATION IN THE HALL THRUSTER

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I. ABSTRACT

A one-dimensional, quasi-neutral, steady-state model is employed to study the dependence of the Hall thruster performance on governing dimensionless parameters, the normalized ionization rate and electron mobility. We focus on the case of a low electron temperature and solve the equations in the supersonic regime. Despite the simplicity of the model, some of the characteristics of the solutions presented here agree with experimental measurements. Such are the existence of more than one steady-state solutions in certain domains of the parameter space and the nonexistence of solutions in other domains, both possibly resulting in oscillations, and also the general dependence of the thruster performance on the ionization rate and the electron mobility.

II. INTRODUCTION

There has been renewed interest in the Hall thruster concept[1–4] in recent years, as new space applications are developed and the operation is being extended to both higher and lower regimes of power and thrust. Hence, in recent years, there has been a continuous effort to explore the dependence of the thruster performance, thrust, specific impulse, and efficiency, on various working parameters of the Hall thruster.[5–20] In guiding detailed numerical simulations, it is useful to have a simple theoretical model. Indeed, in recent years we have developed a one-dimensional steady-state model of the Hall thruster.[14] Employing this model, we identified dimensionless parameters that govern the thruster performance. Despite the simplifying assumptions in the model we often found a good agreement between the theoretical calculations and the experimental measurements.[14,16] We have also shown that the flow in the Hall thruster is sensitive to the plasma parameters in the vicinity of the sonic transition, and have suggested ways of affecting these plasma parameters through active and passive means at the channel walls.[15,17]

In this work we wish to examine how the thruster performance varies when the governing dimensionless parameters vary. For simplicity we assume that the electron temperature is low, so that the sonic transition occurs close to the anode. We then treat the flow as supersonic along the whole channel.

In Sec. II we describe the model. In Sec. III we present some numerical results, and in Sec. IV we present conclusions.

III. THE MODEL

We employ the one-dimensional, quasi-neutral, steady state model of the Hall thruster that was employed in Refs. 14–17. The governing equations for the nondimensional ion current $J = j_i m_i A/e\dot{m}$, ion velocity $V = v_i/v_0$, and electric potential $\psi = \phi/\phi_A$, as functions of the axial normalized location $\xi = x/L$ along the thruster channel, are the ion continuity equation

$$\frac{dJ}{d\xi} = p\frac{J}{V}(1-J), \quad (1)$$

the ion momentum equation

$$(V^2 - C_s^2)\frac{dV}{d\xi} = -p(1-J)(V^2 + C_s^2) + \frac{1}{2\mu}V^2\frac{J_T - J}{J}, \quad (2)$$

and the electron momentum equation (Ohm's law)

$$-\frac{d\psi}{d\xi} = \frac{1}{\mu}V\frac{J_T - J}{J} - 2C_s^2\frac{V}{J}\frac{dN}{d\xi}. \quad (3)$$

The normalized ion (and electron) density is $N = J/V$. In writing these equations, we assumed that the electron temperature is uniform along the thruster channel, and that the ion pressure and the loss terms at the walls are zero. The parameters that appear in the equations are:

$$p \equiv \frac{L\beta \dot{m}}{v_a v_0 m_i A}, \tag{4}$$

$$\mu \equiv \frac{\bar{\mu}\phi_A}{L v_0}, \tag{5}$$

the square of the dimensionless ion acoustic velocity $C_s^2 \equiv T_e/m_i v_0^2$ and the dimensionless total current J_T. In the expression for p the parameter β is the average of σv, where σ is the ionization cross section and v is the electron velocity. In the expression for μ the dimensional electron mobility is $\bar{\mu} \equiv e\nu_{col}/m_e\omega_c^2$, which is valid under the condition that the collision frequency is much lower than the electron cyclotron frequency ($\nu_{col} \ll \omega_c$).

In the above definitions, e is the elementry charge, m_i and m_e are the ion and electron masses, j_i and v_i are the ion current density and flow velocity, T_e is the electron temperature, L and A are length and cross section of the thruster, $\dot{m}$ is the mass flow rate, v_a is the (assumed constant) neutral flow velocity, and x is the coordinate in the direction from the anode to the cathode. Also, $v_0 \equiv (2e\phi_A/m_i)^{1/2}$ and ϕ_A is the applied voltage. In writing the equations, we used current conservation, so the normalized electron current J_e is written as $J_e = J_T - J$. In our dimensionless units the propellant utilization becomes $\eta_p = J$, the current utilization is $\eta_j = J/J_T$, the energy utilization is $\eta_e = V^2$, and the total efficiency, therefore, is $\eta_t = V^2 J^2/J_T$. In the definitions of the efficiencies, J and V are calculated at $\xi = 1$.

In this paper we investigate the case of a low electron temperature. We assume that the electron temperature is high enough for ionizing the gas, but is low enough so that the effect of the electron pressure in Ohm's law (3) is negligible. In the limit that C_s^2 is zero Eqs. (2) and (3) become

$$\frac{dV}{d\xi} = -p(1-J) + \frac{1}{2\mu}\frac{J_T - J}{J}, \tag{6}$$

and

$$-\mu\frac{d\psi}{d\xi} = V\,\frac{J_T - J}{J}. \tag{7}$$

The governing equations are therefore Eqs. (1), (6) and (7), supplemented by the boundary conditions $J(0) = 0$, $V(0) = 0$ and $\psi(0) - \psi(1) = 1$. However, examination of the equations shows that they have no regular solutions for these boundary conditions of zero ion velocity and current at the anode. Solutions for the equations in the case of cold plasma may still exist if the requirement of quasi-neutrality is removed. Indeed, it is possible that in the limit of a cold plasma a non-neutral region is formed near the anode that cannot be described by our quasi-neutral model. However, since we are interested in quasi-neutral solutions, we choose to assume that even though C_s^2 is small, it is still non-zero. In that case the full equations (1), (2) and (3) are valid near the anode and admit solutions even for the boundary conditions $J(0) = 0$ and $V(0) = 0$. The ion flow acquires finite J and V in the quasi-neutral plasma in a small subsonic regime near the anode. For simplicity, we solve the equations in the supersonic regime only, which, in the low temperature case occupies most of the thruster channel. We derive the boundary conditions at the anode from the requirement of a smooth sonic transition, as was done previously.[14–17,7,18]

The condition for a smooth sonic transition is that the right hand side (RHS) of Eq. (2) is zero at the sonic transition plane. We approximate this condition by requiring that the RHS of Eq. (6) vanishes at the sonic transition, that, in turn, we assume to occur at the anode. This boundary condition dictates a relation between J and μ at the anode and p and J_T. The velocity V at the anode V_0 is taken as the asssumed low ion acoustic velocity. We assume that there is a small voltage drop in the subsonic regime. Therefore, the boundary condition for the electric potential is $\psi(0) - \psi(1) = 1 - V_0^2$. As in our previous papers, we obtain an eigenvalue problem, in which the discharge current is determined by the requirement on the value of the applied voltage. In some cases more than one solution can be found for one set of operating parameters. In such a case we choose the solution of the lowest total current.

In addition to the requirement that the RHS of Eq. (6) vanishes at $\xi = 0$, we also require that the ion acceleration be positive throughout the thruster. In the region in which J is small this implies that

$$\frac{d}{d\xi}\left(\frac{1}{\mu J}\right) > 0, \tag{8}$$

which must be satisfied even though $dJ/d\xi > 0$. This can be satisfied if the mobility decreases rapidly enough along ξ, as indeed happens if the magnetic field intensity is lower in the vicinity of the anode. For the calculation we assume a mobility dependence on ξ of the form:

$$\mu = \mu_0\, e^{-a\xi}. \tag{9}$$

In order to satisfy inequality (8) a is chosen to satisfy $a > p/V_0$.

IV. SOLUTION OF THE EQUATIONS

The governing equtions (1), (6) and (7) were solved for a large number of values of the dimensionless parameters p and μ_0. For a specified mobility profile (9) (where $a = 3.5$), and a specified ion velocity at the anode (V_0=0.2), an eigenvalue problem for J_T was solved, as described in the previous section. Figures 1–5 show contours of constant discharge current, propellant utilization, current utilization, energy utilization and total efficiency in the $p\mu_0$ plane. The areas of the maps where contour levels are absent show the parameter regimes at which steady state solutions do not exist.

As said above, in some cases more than one solution can be found for one set of operating parameters. It is possible that the existence of several steady state solutions for the same working parameters, as well as the nonexistence of such solutions for other sets of working parameters, result in oscillations in the thruster.

In a typical experiment the optimal working point is found by increasing the magnetic field intensity until the total current reaches a minimum. Beyond that point, the discharge suffers severe oscillations.[9,10] In the contour maps of Figs. 1–5, this procedure corresponds to moving down along vertical lines through a decrease of μ_0 until steady-state solutions do not exist anymore. The efficiency is maximal at that minimal value of the mobility. From Fig. 1 it can be seen that the decrease of the mobility is followed by a decrease in J_T as seen in the experiment only when $p > 0.7$, while for smaller values of p an opposite phenomenon occurs. The total current then increases with the decrease of μ_0. A possible explanation for this current increase may be that, at low values of p, reducing the electron mobility increases the density and thus the ionization rate (for a constant p), and therefore also the total current increases. At high p values, densities are high already and reducing the mobility simply reduces the electron current.

As may be expected, the propellant utilization is primarily affected by p, while the current utilization is primarily affected by the electron mobility. Again, the main deviation from these dependencies occurs at low values of p where the electron mobility affects the ionization rate. The total efficiency exhibits a combined dependence on the propellant utilization and on the current utilization, as the energy utilization is always relatively high.

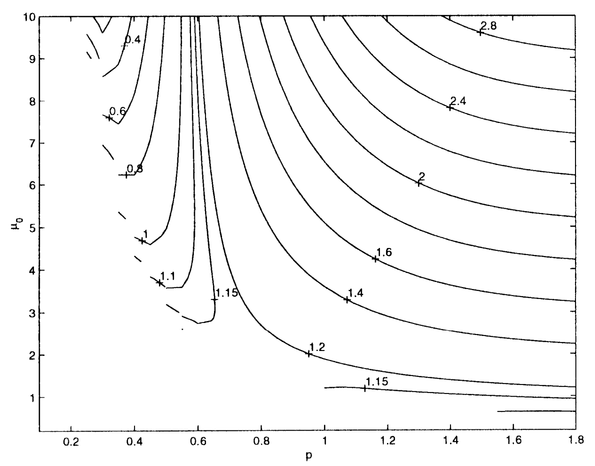

FIG. 1. Total current (J_T) contour levels in the (p, μ_0) plane, $a = 3.5$, $V_0 = 0.2$.

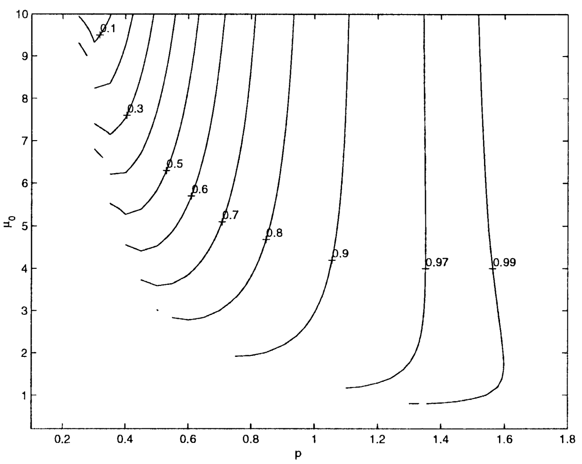

FIG. 2. Propellant utilization (η_p) contour levels in the (p, μ_0) plane, $a = 3.5$, $V_0 = 0.2$.

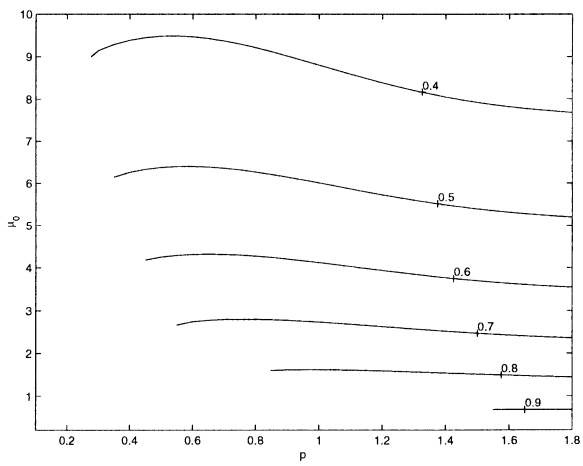

FIG. 3. Current utilization (η_j) contour levels in the (p, μ_0) plane, $a = 3.5$, $V_0 = 0.2$.

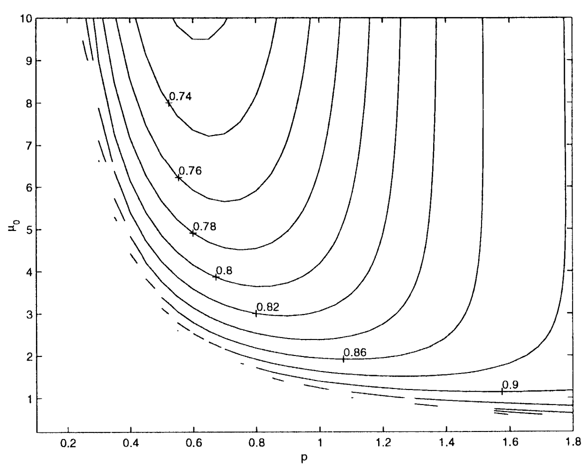

FIG. 4. Energy utilization (η_e) contour levels in the (p, μ_0) plane, $a = 3.5$, $V_0 = 0.2$.

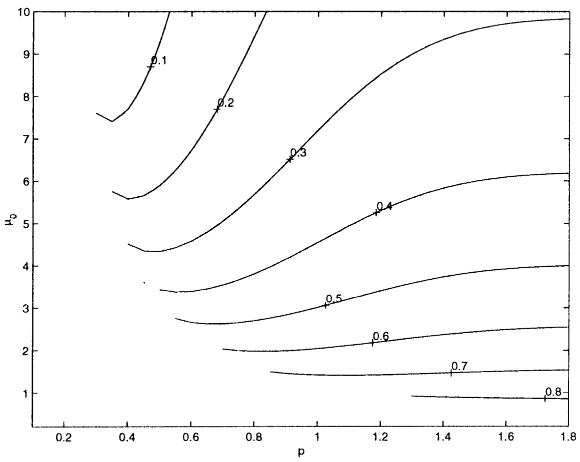

FIG. 5. Total efficiency (η_t) contour levels in the (p, μ_0) plane, $a = 3.5$, $V_0 = 0.2$.

In the low ion current case, when $J \ll J_T$ and $J \ll 1$, we can write analytical expressions for some quantities. The low ion current case can be attributed to very low magnetic fields or low ionization rates. From Fig. 1 it can be seen that the two conditions are related.

In this case the vanishing of the RHS of Eq. (6) is reduced to a linear relation between J_T and J_0 of the form: $J_0 = J_T/(2\mu_0 p)$. The ion current and velocity can thus be expressed as:

$$\frac{1}{J} = \frac{1}{J_0}\left[1 + \frac{1}{b}\ln\left(\frac{1 + b/\zeta}{1 + b}\right)\right], \tag{10}$$

$$V = V_0\left(\frac{\zeta + b}{1 + b}\right)\left[1 + \frac{1}{b}\ln\left(\frac{1 + b/\zeta}{1 + b}\right)\right], \tag{11}$$

where

$$\zeta = e^{a\xi}, \tag{12}$$

and

$$b = \frac{V_0 a}{p} - 1. \tag{13}$$

The electric field becomes:

$$\frac{d\psi}{d\zeta} = -2\frac{p^2}{a^2}(\zeta + b)\left[1 + \frac{1}{b}\ln\left(\frac{1 + b/\zeta}{1 + b}\right)\right]^2. \tag{14}$$

Note that the electric field depends neither on J_T nor on μ_0. Hence, a quasi neutral solution seems to exist for a particular combination of the values of the parameters a, V_0 and p. For $a = 3.5$ and $V_0 = 0.2$ used here p is calculated to be 0.54 in order to satisfy the boundary condition for the applied voltage.

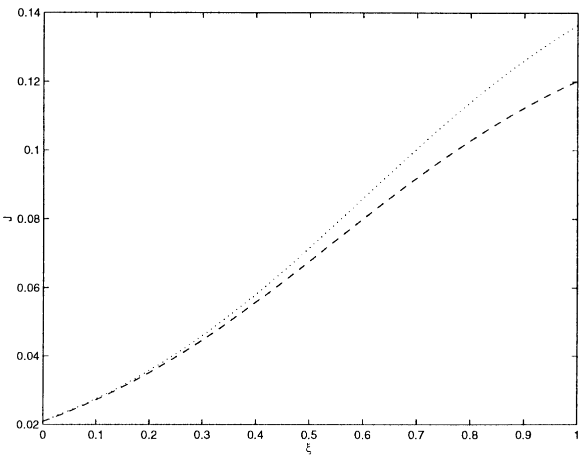

FIG. 6. The ion current along the thruster, accurate (dashed) versus approximate (dotted) solutions, $p = 0.54$, $\mu_0 = 40$, $a = 3.5$, $V_0 = 0.2$.

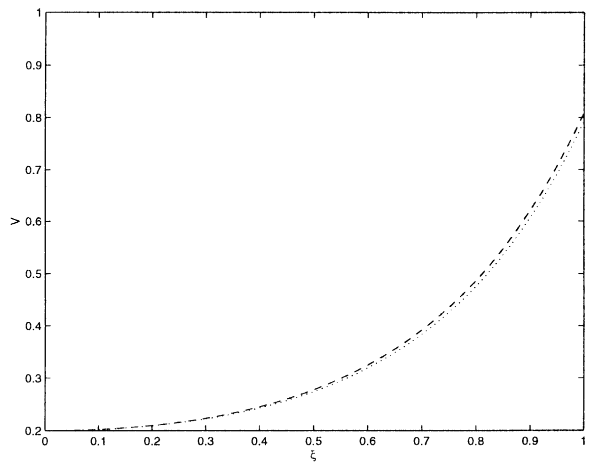

FIG. 7. The ion velocity along the thruster, accurate (dashed) versus approximate (dotted) solutions, $p = 0.54$, $\mu_0 = 40$, $a = 3.5$, $V_0 = 0.2$.

The profiles of the ion current, velocity and density, and of the electric potential along the thruster for a case of a low ion current are shown in Figs. 6–9. In each figure are shown both the more accurate profile, as found by solving Eqs. (1), (6) and (7), and the approximate profile, as found by employing Eqs. (10), (11) and (14). In this example we used $\mu_0 = 40$.

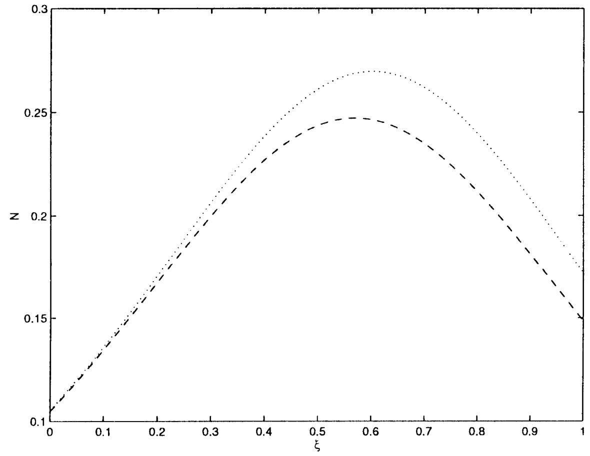

FIG. 8. The plasma density profile, accurate (dashed) versus approximate (dotted) solutions, $p = 0.54$, $\mu_0 = 40$, $a = 3.5$, $V_0 = 0.2$.

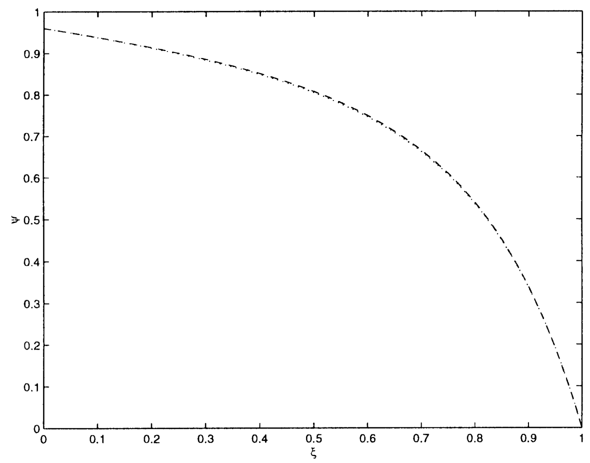

FIG. 9. The potential distribution, accurate (dashed) versus approximate (dotted) solutions, $p = 0.54$, $\mu_0 = 40$, $a = 3.5$, $V_0 = 0.2$.

V. CONCLUSIONS

In this paper we employed a one-dimensional, quasineutral, steady-state model to study the dependence of the Hall thruster performance on governing dimensionless parameters, the normalized ionization rate and electron mobility. We focused on the supersonic regime of a low temperature plasma, while making some simplifying assumptions about the plasma parameters at the sonic transition plane that was assumed to be located at the anode. We have also identified regimes of parameters where several steady state solutions exist.

Despite the simplicity of the model, some of the characteristics of the solutions presented here seem to agree with the experimental results. Such are the existence of more than one steady-state solutions in certain domains of the parameter space, the nonexistence of steady-state solutions in other domains, both possibly cause oscillations, and the general dependence of the thruster performance on the ionization rate and the electron mobility.

The present parametric study will be extended in the future to include both subsonic and supersonic regime. We also intend to investigate the implications of the coexistence of several steady-state solutions in the framework of a time-dependent model.

VI. ACKNOWLEDGEMENTS

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