The Optimal Control of Thrust Vector of Hall-thrusters

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Introduction

For spatial orientation of a satellite powered by electric propulsion with a fixed direction of a thrust vector, it is necessary to use several of such propulsion units or to employ mechanical joints. This leads to an increase in satellite mass; and accordingly, to a decrease in satellite payload. Besides, using mechanical joints is undesirable, because of lower reliability in space. Thus, it looks very attractive to implement thrust vector control in one electric propulsion.

For any electric propulsion, thrust defined by formula $F = m_a V$ is obviously dependent on only two parameters: a gas flow rate and an average ion velocity which is a function of voltage - $V = f(U_a)$.

In such a way, there are two independent ways to control a thrust value: whether through changing a gas flow rate or through varying the voltage. Obviously, there is only one parameter to control a thrust vector, which is ion velocity, since it is a vector. The ion velocity can be changed by using an additional electrode, but this method is not efficient. The point is that the electric field of a non-emitting electrode does not penetrate into the plasma, and the use of an additional emitting electrode (cathode) will lead to a decrease in the propulsion efficiency, because of the useless particle exchange between the electrode and the plasma.

It is worth noting that it is possible in principle, to control a thrust vector through creation of azimuthal nonuniformity in the gas flow rate, but this method is extremely inefficient. In this case a moment of forces, in the first approximation, will be proportional to the gas flow rate nonuniformity, multiplied by the channel diameter which is significantly smaller than the size of a satellite. Besides, a shaft of the propulsion as a rule, goes through the mass center of a satellite.

In such a way, none of the electric thrusters: such as ion, arcjet, pulsed plasma thruster, etc. is suitable for efficient control of the thrust vector. The only exception in this list is the Hall thruster.

The point is that, in Hall thrusters, the electric field is firmly connected to the magnetic field (equipotentials are on the magnetic lines) which, opposed to the first, can freely penetrate into the plasma. This is why, through changing the shape of the magnetic lines, it is possible to change the direction of the ion velocity, i.e. to control the thrust vector of the propulsion. For this reason, among all electric propulsion types, the Hall thruster is the most promising for building a controlled thrust vector.

This very thruster has become a subject of our investigation.

Nomenclature

z, φ, r – cylindrical coordinates,
e, m, c – charge and mass of electron, speed of light,
Te, ne – temperature and density of electrons,
B, B_{out}, B_{in}, Br,max – magnetic induction, magnetic induction by inner and outer walls, its maximum value in the middle of the channel,
E, Φ, U_a, - electric intensity, potential, potentials of anode, cathode and infinity,
R_{out}, R_{in}, l_a, d – inner and outer radii of the channel, length of the acceleration zone, channel width,
J_H, V_d, υ_e – Hall current, drift velocity and thermal velocity,
F, ma - thrust, gas flow rate through anode,

Analysis of Options to Control a Thrust Vector

According to [1], a thrust as a function of the azimuth in a Hall thruster can be represented in cylindrical coordinates as follows:

$$F(\varphi) = e \int_{R_{in}}^{R_{out}} dr \int_{0}^{la} n_e(z,\varphi,r)E(z,\varphi,r)dz \qquad (1)$$

To make it possible to control a thrust vector it is necessary to make it nonuniformal in the azimuthal direction, i.e. at least to meet an evident condition:

$$\partial F(\varphi)/\partial\varphi = e\int_{R_{in}}^{R_{out}} dr \int_0^{la} \partial\{n_e(z,\varphi,r)E(z,\varphi,r)\}/\partial\varphi \times dz = f(\varphi) \neq \mathrm{const} \quad (2)$$

Thus, a necessary condition for a thrust vector control looks like:

$$\int_0^{la} \partial\{n_e(z,\varphi,r)E(z,\varphi,r)\}/\partial\varphi \times dz \neq 0 \quad (3)$$

Now, writing the expression for Hall current in the acceleration zone

$$J_H = e\int_{R_{in}}^{R_{out}} dr \int_0^{la} n_e(z,\varphi,r)V_d(z,\varphi,r)dz \quad (4)$$

Keeping in mind that in stationary case J_H=const along azimuth and T_e is small as compared to U_a, i.e. $T_e \rightarrow 0$, let us rewrite condition (4) in a form as follows:

$$\partial\left\{\int_{R_{in}}^{R_{out}} dr \int_0^{la} n_e(z,\varphi,r)E(z,\varphi,r)/B(z,\varphi,r)dz\right\}/\partial\varphi = 0 \quad (5)$$

or

$$\int_0^{la} \partial\{n_e(z,\varphi,r)E(z,\varphi,r)/B(z,\varphi,r)\}\partial\varphi \times dz = 0 \quad (6)$$

Since in any fixed azimuthal sector the m_a or B can be changed in only one direction (increase or decrease) then the change of the under integral function along the azimuth (or streamlines of motion) will have the same sign. Thus for the under integral function in (6) it must fulfill condition

$$\partial\{n_e(z,\varphi,r)E(z,\varphi,r)/B(z,\varphi,r)\}\partial\varphi = 0 \quad (7)$$

And, if the magnetic field is uniform in the azimuthal direction, expression (7) can be reduced to the following:

$$\partial\{n_e(z,\varphi,r)E(z,\varphi,r)\}/\partial\varphi = 0 \quad (8)$$

When comparing (3) to (7), (8), conclusions can be made as follows:

1. There exists only one parameter that allows control of the trust vector of the Hall thruster. This parameter is magnetic field.
2. It is not possible to control a thrust vector of the Hall thruster through making a gas flow rate azimuthally nonuniform.
3. With insignificant approximations complete, even if it is possible to control a thrust vector through a gas flow rate, its effectiveness will be by one order of magnitude lower than doing this using a magnetic field.
4. The conjoint (B and m_a) control of a thrust vector is possible.

The importance and the paradox of conclusion 2 merit special attention. As a matter of fact, magnetic lines in any Hall thruster are originally inclined towards the thruster's axis. That is why the ion velocities, beside an axial component, have an azimuthal component. At first glance it seems obvious that it is possible to turn the thrust vector by creating azimuthal nonuniformity in the gas flow rate through the anode. Condition (8) proves the opposite. Obviously, it is not possible to change ne, in one of the channels without changing E, since $J_H \approx n_e E l_a = n_e U_a$ = const, i.e. n_e = const (for the sake of simplicity, a one-dimensional case is shown). This effect is conditioned by an azimuthal drift of the electrons.

In such a way, if for other electric propulsion types there is at least the possibility to control the thrust vector through a gas flow rate (see Introduction), than for Hall thruster it is fundamentally not possible. Since magnetic field is the only parameter allowing control of the thrust vector, it is necessary to appropriately describe the relationship between magnetic and electric fields in the acceleration zone of the Hall thruster.

Relationship between Magnetic and Electric Fields

In the acceleration zone of the Hall thruster, the electron velocities along the azimuth and magnetic lines (V_r) are much higher than those in the direction of the anode. This is why the electric potential in these directions levels off fast and becomes constant. As a result, an electro-optical lens is formed in the acceleration zone with its equipotential surfaces placed on two orthogonal vectors V_d and V_r. In the case of a cold plasma, the equation for equipotential surfaces arrives from the equation for electron travel and can be written as [2]

$$B\nabla\Phi = 0 \qquad V_d\nabla\Phi = 0 \quad (9)$$

and for hot plasma it is converted into

$$B[\nabla\Phi + \nabla(T_e n_e)/n_e] = 0 \quad V_d\nabla\Phi = 0 \quad (10)$$

Specific values for the potential and the temperature are found from a solution of the energy equation [3]. The main result of this work for us is z_b a coordinate of the acceleration zone bottom which is defined from relation $B_r(z_b) \cong ½B_{r,max}$.

Thus, it can be considered in the first approximation that the characteristics of the electric field are known.

Now looking at how the azumithally closed arrangement of the thruster affects the electric field. Since the effect is conditioned by the inner processes taking place in plasma, the extent of its interaction will be proportional to $T_e/U_a << 1$, or the so called second approximation.

There exist two of such effects. The first one is the initiation of radial electric field in the channel due to azimuthal drift of electrons. In order to maintain an electron on a stationary orbit, it is necessary to have a centripetal force, i.e. radial electric field. A value of potential U_w between the channel walls can be estimated from the following formula

$$U_w = 2mdV^2_d/e(R_{out}+R_{in}) = 2m(cE)^2/eB^2(R_{out}+R_{in}) \qquad (11)$$

and for modern SPT it equals ≅ 10V.

Assuming the angular velocity of electrons to be constant along the radius, we can obtain an intensity of the radial electric field from the following function: $E_r \sim r$.

The second effect is initiation of a radial electric field, due to the impossibility of the electrons to penetrate into the area of powerful magnetic fields, so cold «magnetic mirror». Because of the cylindrical form of the structure, the magnetic field by the inner wall is always higher than that by the outer wall. The potential of this field can be estimated by the following formula:

$$U_w = 2T_e(B_{in} - B_{out})/e(B_{in} + B_{out}) \qquad (12)$$

and for modern SPT it equals ≅ 20V.

The occurrence of the radial electric field in the channel causes deflection of equipotentials from the magnetic lines (Fig.1). This effect was observed in the experiment [4] with no explanation found. It is worth noting that the characteristics of the radial electric field can be found directly, although in a more complicated way, from (10), if looking at all of the parameters as functions of coordinate r. Due to the effect of the field, ions tend to move towards the outer channel wall, provoking erosion of the latter. In order to avoid this adverse phenomena, magnetic lines can be made slightly tilted towards the thruster axis (this gave rise to a desire to use gas flow rate for the thrust vector control, see the above), although this causes a decrease in thrust and efficiency. The inclination angle of the magnetic lines equals $U_w/U_a \cong$ 5-7 degrees in a first approximation. Note, that the inclination angle grows with a decrease in the thruster size, and accordingly, the output parameters of the thruster get worse.

Figure 2 shows a preferred configuration of the main magnetic field and a corresponding position of the acceleration zone.

In such a manner, knowing the relationship between the magnetic and electric fields in the Hall thruster, and a preferred configuration of the main magnetic field, it is possible to formulate requirements to be imposed on the deflecting magnetic field.

Requirements Imposed on Deflecting Magnetic Field

It is obvious that in order to turn a thrust vector, a deflecting magnetic system must change the axial component of the main magnetic field in one of the azimuthal sectors of a channel. At the same time, it is not desirable to change the radial component of the main magnetic field along the azimuth. The latter requirement follows directly from (10), since the radial component of the magnetic field is orthogonal to the electric field in the channel, which has here only an axial component. By preserving B_r, we can preserve azimuthal uniformity for all processes in the channel which allows avoiding a number of adverse phenomena, such as growth of current oscillations, occurrence of axial drift of electrons etc. The latter will occur in the $[\nabla B_r \times B_r]$ field and will cause tilt of equipotentials in the orthogonal direction with respect to the deflecting magnetic field.

Consequently, the magnetic field of the deflecting system should meet requirements: $B_z >> B_r$ and $B_r \rightarrow 0$, in the acceleration zone, at least.

In order to depress erosion of the channel walls, which is likely to increase with deflection, it can be done as follows. To push the acceleration zone out of the channel as far as possible, that is, to shift $B_{r,max}$ towards the cathode. To produce an effect on the main magnetic field with the help of the deflection system in the second part of the acceleration zone, that is outside the channel. The required configuration of the deflecting magnetic field should be as shown in Fig. 2

Review of Existing Deflecting Magnetic Systems.

At present, there are two deflecting magnetic systems patented. [5,6]. Both of them have an outer part of the main magnetic systems divided into four independent parts, including an outer pole. But they also have certain differences. One of them has four independent solenoids to initiate both the main and deflecting magnetic fields [5]. In the other [6], the main magnetic field is initiated by one outer solenoid, and the deflection – by four additional fields. Since in both cases the deflecting magnetic field in the channel is initiated by two outer poles (in [5] these are the main pole and the end of the outer magnetic screen, and in [6] these are two ends of outer screens) arranged in a similar way, their fields practically do not differ from each other (Fig.6).

Analysis shows that both deflecting magnetic fields do not meet the requirements of the above section: $B_z >> B_r$, at least in the acceleration zone, and $B_r \to 0$. In both cases $B_r \cong B_z$, $B_r \neq 0$. Besides, when turning the deflecting field on, $B_{r,max}$ shifts towards the direction of the anode. The relative amplitude of the change in B_r of the entire magnetic field along azimuth on the coordinate where B_r is placed, makes up more than 22%.

Note that the main magnetic field [6] has an advantage before [5] because of the greater extension of the field maximum from the channel (Fig.3).

Consequently, both of the existing deflecting magnetic systems are too far from being perfect.

New Deflecting Magnetic System

In order to meet the requirements for the deflecting magnetic field, it is necessary to introduce at least one additional pole divided into several parts (≥3) along the azimuth, and each part should have an independent magnetic source. One of the SPT versions with such a deflecting magnetic system is shown in Fig. 4 [7]. The magnetic fields initiated by the main and deflecting systems are shown in Fig.5, 6.

$B_{r,max}$ of the main magnetic field is carried out of the channel for 3 mm along the axis from the outer pole, and for 10 mm from the inner pole. This allows us to significantly depress erosion of the channel walls, especially for the outer wall that is more exposed to ion attacks. Besides, the main magnetic field is in better compliance with the requirements (Fig.2) than the fields in [5,6].

Unlike the existing deflection systems [5,6], the magnetic field of the new deflecting system meets all the requirements (Fig.2): $B_z >> B_r$, $B_r \to 0$ (at least $\int B_r dz = 0$), $B_{r,max}$ is placed in the second half of the acceleration zone (outside the channel). Relative amplitude of change in the entire magnetic field along the azimuth on the coordinate where $B_{r,max}$, is placed, makes up 2.5%.

Since the additional pole is placed far from the plasma jet, its exposure to erosion is slight, and the changes in the pole geometry will make only a slight effect on the magnetic field in the acceleration zone.

In addition, let us note that for a better thruster operation, slots in the outer screen can be cut to be oriented in the magnetic flow direction, and also it is desirable to place the cathode on the magnetic line connecting the poles of the main magnetic system.

Conclusions

Of all types of electric propulsion, the Hall thruster is the most suitable candidate for building a thruster with a controllable thrust vector. The efficient control of the thrust vector can only be achieved with the help of the magnetic field. For more efficient control of the thrust vector the deflecting magnetic system should have an additional pole displaced towards the cathode with respect to the main poles.

References

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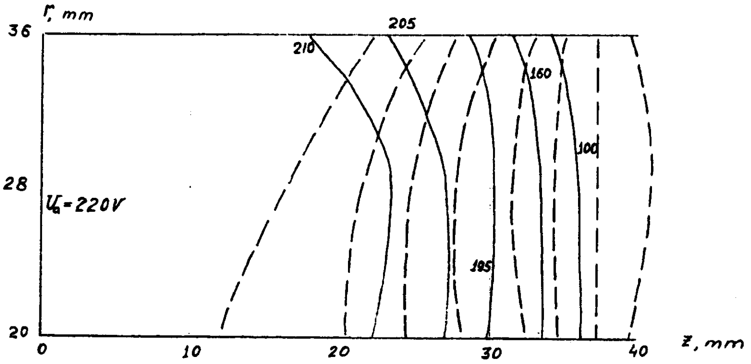

Fig.1. Distributions of equal-potentials (solid lines) and magnetic lines (broken lines) in the acceleration zone of the channel of SPT, *Ua* = 220V [4].

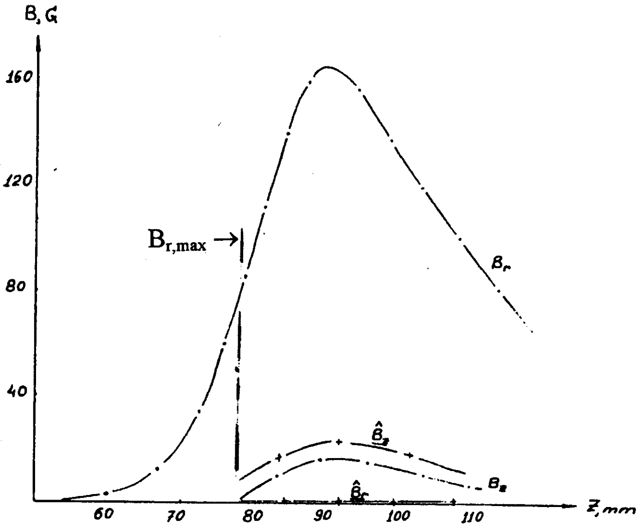

Fig.2. Preferable distributions of *Br, Bz* (the main magnetic field) and $\underline{Br}$, $\underline{Bz}$ (the steering magnetic field) along the middle of a channel for T-140: the anode` s coordinates ≅ 40mm, the vertical broken line is the beginning of the acceleration zone.

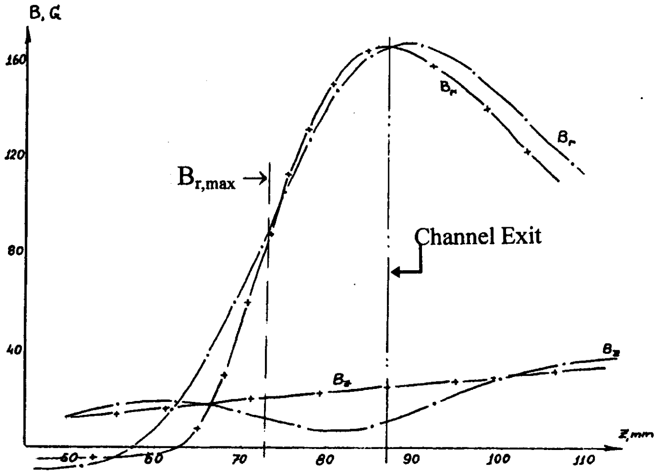

Fig.3. Distributions of *Br, Bz* of the main magnetic fields along the middle of channels for T-160 (—+—+—+—) made according to [5] and for T-160E (—•—•—) made according to [6]: the vertical broken and dashed lines are the beginning of the acceleration zone and boundary of poles.

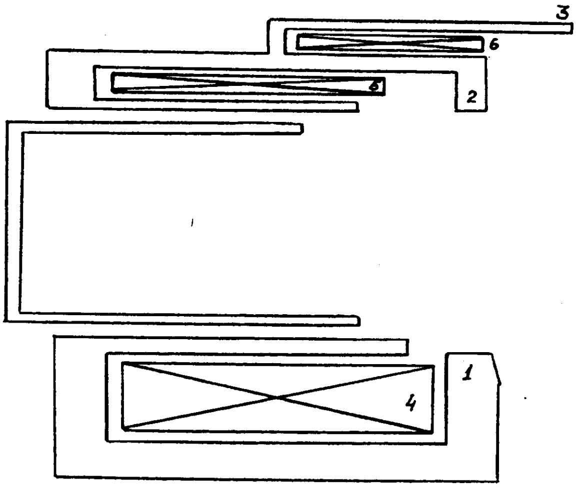

Fig.4. The new magnetic system: inner (1), outer (2), additional (3) poles; inner (4), outer (5), additional (6) coils.

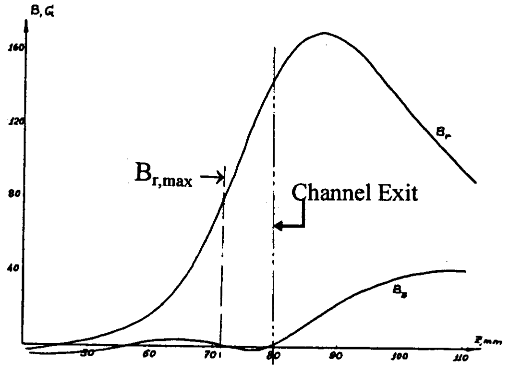

Fig.5. Distributions of *Br*, *Bz* of the main magnetic field along the middle of the channel for T-140 made in SPI: the vertical broken and dashed lines are the beginning of the acceleration zone and boundary of poles.

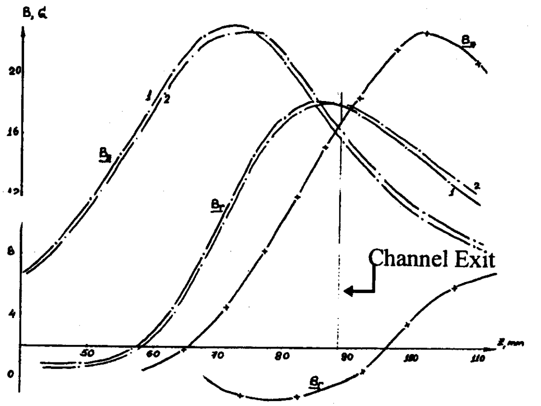

Fig.6. Distributions of Br, Bz of the steering magnetic fields along the middle of channels in T-160 (1), T-160E (2) (—•—•—) and in T-140 (—+—+—); the vertical broken line is the middle of the acceleration zone.