Mathematical Model and Calculation Method for Hollow Cathodes Lifetime Forecast

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Abstract: The complex mathematical model and the calculation method are presented, which permit to calculate main local and operation descriptions of high emission gas-discharge hollow cathodes, including the forecast of life-time in steady state operation mode. The mathematical model uses the kinetics description of electrons and electric gas dynamics description of atoms and ions inside the cathode. The state of activator film and surface temperature field are also calculated. Semi-empiric erosion model is based on the thermally stimulated cathode sputtering theory, which is developed by authors.

Nomenclature

e	=	electron charge
m_e, m_i	=	electron and ion masses
P	=	sputter coefficient
T_e, T_i	=	electrons and ions temperature
f	=	electrons velocity distribution function
v	=	velocity
V_e, V_i	=	electrons and ions mass flow velocity
β	=	reply function
Γ	=	electrons flow density in velocity space
Γ_p	=	sputtered atoms flow density
Γ_i	=	bombardment ions flow density
ε_i	=	bombardment ions energy
ε_m	=	energy of surface atoms desorbtion
θ, ϑ	=	polar angle in spherical coordinates
φ	=	electric potential, longitude angle in spherical coordinates
φ_i	=	ionization potential
ψ	=	target angle

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I. Introduction

The building of mathematical models of base degradation parameters (BDP) change is one of the most important tasks, which are solved while the developing of fast test methods. The creation of quantitative mathematical models of hollow cathodes, which permit to realize the optimal work routines and to forecast their life-time, is complex and actual task. The complete and precise mathematical model is necessary for the development of life-time design methods, which describe the temporal evolution of main work routines, which are responsible for reliability and life-time.

II. General Guidelines

The complex mathematical model of high emission gas discharge hollow cathodes (HEGHC) is developed firstly in Science and Technology Center of Space Power and Engines in KhAI [1-6], which differs by completeness and precision of description of work routines determining HEGHC emission ability, erosion strength. This model permits with enough for design practice precision to make the calculations and forecast of main local and integral HEGHC descriptions, including life-time. This model includes the space-kinetic diffusion model of processes, which considers the plasma components dynamics inside the hollow, electron spectrum features, activator dynamics, Langmuir layer descriptions, energy transfer between the surface and the volume, processes in external discharge column; the model of film thermal emitters work; erosion model, which is built on the heat stimulated cathode sputtering (HSCS) theory (Fig. 1). The complex non-linear mode of surface and volume processes mutual influence was taken into consideration. The expressions for ion surface recombination, electron back current, emission were used like the boundary conditions for plasma dynamics equations. Plasma descriptions on Langmuir border were in turn included into boundary conditions.

The critical expression was used in boundary conditions on Langmuir border:

$$m_e V_e^2 + m_i V_i^2 = k\left(T_e + T_i\right). \tag{1}$$

This condition can be realized either in the most thin flow section (exit orifice) or because of quasi neutrality break-up (inside Langmuir layer).

The emission current depends on the state of activator film. The equations describing the activator dynamics in the volume and on the surface were also considered like mutual boundary conditions. Also activator dynamics and emission currents depend on temperature surface distribution, which in turn is determined on heat flow from the volume. The rate of ionization and excitation as well as electron back current density depends on the peculiarities of electron energy distribution in high energy area.

According to these ideas the complex model includes the following sub-models:

- the processes inside hollow main volume;
- the processes near the orifice;
- Langmuir cathode layer;
- electrons energy distribution;
- heat stimulated cathode sputtering;
- film thermal emitter work and activator dynamics;
- cathode surface temperature distribution;
- external column.

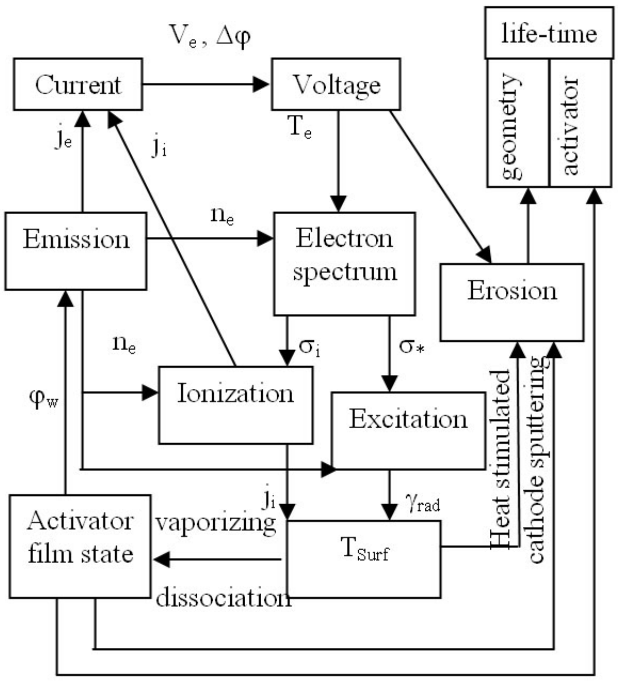

Figure 1. Complex model structure

The complex model unites the continuum methods (in the tasks of electric field and plasma components space behavior) and kinetics approach (in the task of electron energy distribution [4]).

It is shown that the main collision factors, which must be considered in plasma components space dynamics, are ion mass flow velocity decrease and velocity dispersion (temperature) increase due to appearance of new ions inside already accelerated ones because of ionizations. We have named these factors like “ionization deceleration” and “ionization heating”.

Electron energy distribution plays the most important role in this model. The kinetics equation in diffusion approximation (Landau mode for collision integral) is formed:

$$\bar{v}\cdot\nabla f(\bar{v})+\frac{e}{m_e}\nabla\varphi\cdot\nabla_v f(\bar{v})=\frac{\delta f(\bar{v})}{\delta t}, \tag{2}$$

with Landau collision integral:

$$\frac{\delta f(\bar{v})}{\delta t}=-\frac{1}{v^2}\frac{\partial}{\partial v}\left(v^2\Gamma(v)\right)\cdot \tag{3}$$

where initial form of electron flow density in velocity space is:

$$\Gamma(v)=\int_0^\infty\int_0^\pi\int_0^{2\pi}\int_0^\infty\int_{-\pi/2}^{\pi/2}\int_0^{\pi}\int_0^{2\pi}\int_{v-\delta v}^{v}\left[f(v'')f(v')-f(v''+\delta v)f(v'-\delta v)\right]\left|v''-v'\right|\times \\ \times v'^2 dv'\sin\theta' d\theta' d\varphi' b\, db\, d\psi\,\sin\theta\, d\theta\, d\varphi\, v''^2 dv'' \tag{4}$$

which can be reduced, taking into consideration the descriptions of processes, into Landau form:

$$\Gamma(v)=\frac{32}{3}\pi^2\sigma_e\left(\frac{e\varphi_i}{m_e}\right)^2\int_0^\infty\left[\frac{f(v)}{v'}\frac{df(v')}{dv'}-\frac{f(v')}{v}\frac{df(v)}{dv}\right]\mathrm{Min}^3(v',v)v'dv'\cdot \tag{5}$$

Other main part of complex model is erosion sub-model based on HSCS theory [3, 5, 6]. Surface atoms flow in this theory is considered as a result of both ion bombardment and surface atoms thermal oscillations with sputter coefficient, which is the function of ion energy and surface temperature:

$$\Gamma_p=P(\varepsilon_i,T)\Gamma_i\cdot \tag{6}$$

The sputter coefficient is calculated using the atoms oscillation faze distribution, velocity direction and value distributions:

$$P(\varepsilon_i,T)=\int_{-\pi/2}^{\pi/2}\int_0^{2\pi}\int_{v_{min}}^{v_{max}} f(\vartheta)f(\varphi)f(v)\sin\vartheta\, d\vartheta\, d\varphi\, dv, \tag{7}$$

where

$$v_{min}=\sqrt{\frac{2\varepsilon_m}{m}}-\sqrt{\frac{2\beta\varepsilon_i}{m}}\cos\vartheta\cos\varphi, \qquad v_{max}=\sqrt{\frac{2\varepsilon_m}{m}} \tag{8}$$

Reply function β here must be taken from experimental data for short-term cathode tests and then be used for long-term life-time forecast.

The calculation results for voltage and orifice radius evolution are shown in Fig. 2 and Fig. 3.

Test and forecast results (Fig. 4) for orifice radius evolution had demonstrated good coincidence.

III. Conclusion

Thus, the models and software developed permit:

- to make life-time design of HEGHC via optimization by the necessary life-time criterion;
- to make the calculation of evolution of cathode main work descriptions;
- to create calculation-experiment methods of HEGHC life-time tests.

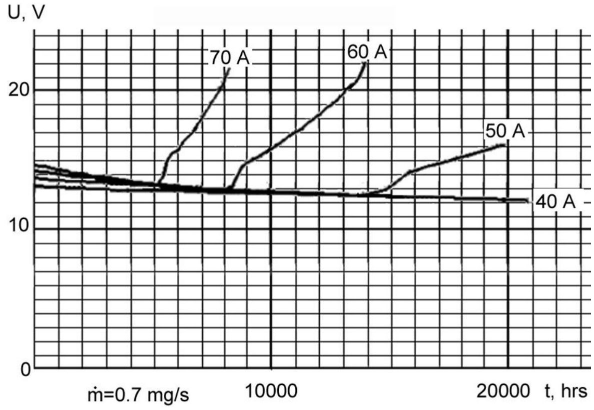

Figure 2. Voltage evolution forecast

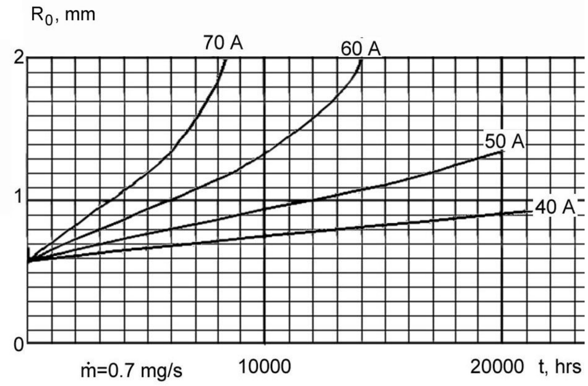

Figure 3. Orifice radius evolution forecast

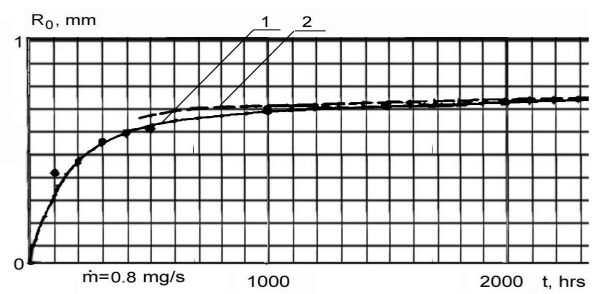

Figure 4. Test (1) and forecast (2) results

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