Near Wall Conductivity in Hall Thrusters. Cylindrical geometry effect

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Abstract: Electron transport transverse to external magnetic field in Hall Effect Thrusters is analyzed. A crucial role of electron scattering on the isolating walls is demonstrated for the case of "slow" electrons not passing the sheath. The detailed shape of sheath equi-potentials becomes responsible for electron-wall collision contribution to electron transverse mobility in Hall Thrusters.

Nomenclature

n_a ,n_i, n_e	=	atom, ion and electron densities
V_a, V_i, V_e	=	atom, ion and electron velocities
T_e	=	electron temperature
β	=	ionization rate
σ_i	=	ion loss rate on the walls
v_{ea}, v_{ew}	=	electron-atom and electron-wall collision frequencies
v_e	=	$v_{ea} + v_{ew}$ "total" electron collision frequency
t	=	time
z	=	axial coordinate
m	=	electron mass

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M	=	ion (atom) mass
E	=	electric field
B	=	magnetic field
R_1, R_2	=	inner and outer radii of discharge channel
L	=	length of the discharge channel
U_d	=	discharge voltage
τ	=	electron secondary emission coefficient
V_s	=	sheath potential
$\dot{M}$	=	mass flow rate

I. Introduction

Modeling Hall Thruster plasmas we are faced with problem of proper description of electron transport transverse to applied magnetic field. It has been well known from the time of first study of such thrusters, that in general both classical and anomalous processes should govern electron transport transverse to magnetic field as it was pointed out in Ref. 1. There, as a classical process, it is meant electron collision with other plasma particles (atoms, ions) as well as electron scattering on the dielectric walls.

A. I. Morozov in Ref. 2 first introduced near-wall conductivity transverse to magnetic field. In this paper it appears as a result diffusive electron reflection on the dielectric walls and, what's more, the final result was computed for electron current in an infinite half-space. In further development of near-wall conductivity theory and in its experimental verification (see e.g. Ref. 3) authors take into account the role of sheath distinguishing "slow" and "fast" incident electron which cross (or do not cross respectively) the sheath. Only fast electrons are diffusively scattered on the wall. In case of "slow" electrons the no vanishing contribution to near-wall conductivity will appear if, during the scattering on a sheath, electrons lose their drift velocity. This can happen if equi-potentials within a sheath follow the microstructure of wall material. Taking into account that in Hall thrusters we have plasma bounded between two cylindrical walls it was shown that in such a case near-wall conductivity will produce electron current varying periodically along the discharge channel cross section and therefore preferring electrons having specific values of velocity along magnetic field lines. Introducing velocity averaging with thermal electron velocity distribution it was shown[4], that in the range of parameters corresponding typical working conditions of Hall thruster near-wall conductivity is strongly reduced when compared with a value derived using theory presented in Ref. 1.

These contributions to electron-wall scattering effect in electron transport were summarized in review paper of Zhurin, Kaufman and Robinson – Ref. 5. In recent review[6] devoted to electron transport transverse to magnetic field in selected plasma device authors introduce additional factor determining effective electron-wall collision frequency i.e. computing electron trajectory when crossing the sheath as an effect of combined electric fields: - radial field of a sheath and axial field sustaining discharge.

The hypothesis that only "fast" electron (crossing the sheath and reemitted by the wall) contribute electron transport were confronted with experimental results for different wall materials[7]. This was done by means of fluid 1-D model of Hall thruster plasma where additionally electron temperature anisotropy has been allowed. These model predictions were in good agreement with experimental results especially when comparing character of discharge parameter variation caused by varied wall material parameters. At the same time predicted values of discharge current were systematically lower than those in experiment. This allows us to conclude, that in such a model electron wall collision frequency responsible for electron momentum transfer was systematically underestimated. The same conclusion is to be stated when looking for the results of fitting 2-D hybrid model predictions to static and dynamic Hall thruster discharge characteristics[8]. This model contains three adjustable parameters; one of them represents electron-wall collision frequency in electron momentum transfer equation. The best fitting was achieved when electron-wall collision frequency has to be taken as: $\nu_{ew} = 10^7 s^{-1}$. This value is much greater than electron-wall collision frequency as computed for electron temperature and secondary emission coefficient in the experiment conditions.

The aim of this paper is to examine the evaluation of electron-wall frequency. The main point will be determination of the "slow" electrons contribution. Since the expected complicated small scale geometry of equi-potentials and the cylindrical shape of the walls make the fluid and simple kinetic description ineffective we apply molecular dynamic methods. In section II we present how the electron-wall collision enter the fluid model of electron transport and the way of electron-wall collision frequency evaluation if the electron distribution function has been known. Section III is devoted presentation of molecular dynamic approach in description of electrons in crossed electric and magnetic fields colliding with isolating walls. In section IV we present primary computation results and finally (Section V) we present conclusion.

II. Electron-wall interaction in fluid model of Hall thruster plasmas

Fluid model of Hall Thruster plasma is in general reduced to lower than 3-D dimensionality exploring this way the assumed axial symmetry and relatively short distance between discharge channel isolating walls (i.e. corresponding electron transit time between the walls much less than any other characteristic time scale). Usual three-fluid (electrons, ions, neutrals) continuity, momentum and energy equations are further reduced by:

- Assuming cold fluid approximation for neutrals and ions
- Neglecting electron inertia terms in electron momentum equations
- Assuming electric neutrality for bulk plasma and
- Averaging all equations over the channel cross section

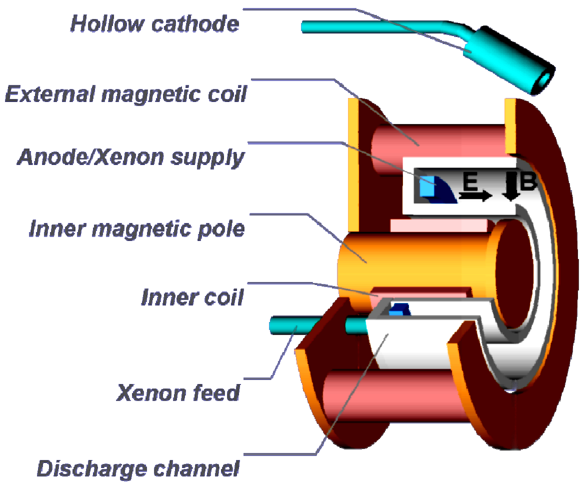

Fig. 1 Schematic view of discharge channel.

Derivation of above fluid model equations was discussed in details e.g. in Ref. 7,9. Here we present the final version, where all above approximations were performed.

Ions:
$$\frac{\partial n_i}{\partial t}+\frac{\partial (n_i V_i)}{\partial z}=\beta n_a n_e-\sigma_i n_i \quad (1)$$

$$\frac{\partial (n_i V_i)}{\partial t}+\frac{\partial (n_i V_i^2)}{\partial z}=\frac{eEn_i}{M}+\beta n_a n_e V_a-\sigma_i n_i V_i \quad (2)$$

Neutrals
$$\frac{\partial n_a}{\partial t}+\frac{\partial (n_a V_a)}{\partial z}=\beta n_a n_e+\sigma_i n_i \quad (3)$$

$$\frac{\partial (n_a V_a)}{\partial t}+\frac{\partial (n_a V_a^2)}{\partial z}=(-\beta n_a n_e+\sigma_i n_i)V_a \quad (4)$$

Electron momentum:
$$\frac{\partial (n_e V_{ez})}{\partial t}+\frac{\partial (n_e V_{ez}^2)}{\partial z}\approx 0=-\frac{eEn_e}{m}+n_e\omega_b V_{e\theta}-n_e(\nu_{ea}+\nu_{ei}+\nu_{ew})V_{ez} \quad (5)$$

$$\frac{\partial(n_e V_{e\theta})}{\partial t}+\frac{\partial(n_e V_{ez} V_{e\theta})}{\partial z}\approx 0=-n_e\omega_b V_{ez}-n_e(\nu_{ea}+\nu_{ew})V_{e\theta} \tag{6}$$

Electron energy
$$\frac{3}{2}\frac{\partial T_e}{\partial t}+\frac{3}{2}V_{ez}\frac{dT_e}{dz}+T_{ez}\frac{dV_{ez}}{dz}=-\beta n_a\gamma E_{ion}-(\beta n_a-\sigma_i)\left(3T_e/2+E_{kd}\right)- \nu_{ew}(2T_e+E_{kd}+(1-\tau)eV_s)+\nu_e(2E_{kd})\equiv Q_{02} \tag{7}$$

These equations are subject to boundary conditions:

$$\begin{aligned} &n_i(0)=n_0;\quad V_i(0)=V_a(0)=V_0;\quad n_iV_i(0)+n_aV_a(0)=\dot{M};\\ &T_e(L)=T_0;\quad \Phi(0)=0;\quad \Phi(L)=\int_0^L Edz=U_d; \end{aligned} \tag{8}$$

Electron-wall collision frequency ν_{ew} and ion loss rate σ_i appear in above equation as result of averaging over the channel cross-section, i.e.

$$\sigma_i=\frac{2V_{Bohm}}{R_1-R_2}\left(\frac{R_1 n_i|_{r=R_1}+R_2 n_i|_{r=R_2}}{(R_1+R_2)n_i}\right);\quad V_{Bohm}\equiv\sqrt[2]{\frac{k_B T_e}{M}} \tag{9}$$

$$\int d^3v\, v_r v_{z,(\theta)}\left(f_e^{entering}(\vec{r},\vec{v},t)+f_e^{outgoing}(\vec{r},\vec{v},t)\right)\Big|_{r=R_1,R_2}\equiv\nu_{ew}V_{ez,(\theta)} \tag{10}$$

Here the Bohm criterion for sheath existence[10] was applied. It is evident in eq. (10), that for mirror reflection and magnetic field exactly perpendicular to the wall surface, electron momentum loss on the wall disappears. However, electrons entering sheath see the wall as a sequence of equi-potential surfaces with local electric field defining local normal to surface outward direction. This is the case when axial electric field (sustaining discharge) remains comparable with radial electric field at the sheath edge (radial electric field allowing smooth transition sheath-presheath). Moreover from the measurement and theory of high frequency plasma oscillations[11-13] we can expect quickly oscillating electric field having no vanishing component along azimuth. Since for most of electrons transit time across the sheath is much less than the period of oscillations we can expect electrons entering the sheath with the angle of incidence (with respect to local electric field) stochastically varying.

Having in mind that in absence of above factors the electron-wall frequency contributed by "fast" electrons only can be expressed as:

$$\nu_{ew}=\left(\frac{2\sqrt{T_e/M}}{R_2-R_1}\right)\frac{1}{1-\tau}$$

(which even in case of maximal τ value (≈ 0.98) and for the typical condition like in SPT100 thrusters does not approach $10^7 s^{-1}$) we are searching for method of determining ν_{ew} directly simulating individual electron trajectories.

III. Reconstruction of electron trajectory

Let us determine the ensemble of electron trajectories starting from one wall and approaching another (or the same, which can happen for electron starting from outer cylinder) i.e. between the moments of leaving wall up to the next approaching wall moment. Between the walls electrons are moving under the action of axial electric field and radial magnetic field. The ensemble is created taking the discrete set of initial velocities. Hence taking any sequence of ensemble members by joining the final position of previous member of ensemble with the starting point of the next member we reconstruct the electron trajectory with randomly scattered velocity at every collision with the wall.

$$X_n = \left\{ \vec{r}_n\left(t; \vec{R}_{0n}, \vec{V}_{0n}\right), \vec{v}_n\left(t; \vec{R}_{0n}, \vec{V}_{0n}\right), t_n \right\}$$

where: $\vec{R}_{0n}, \vec{V}_{0n}, t_n$ are the initial conditions and the time of next approaching the wall. In reconstruction of trajectory we can use recursive procedure:

$$X_{n+1} = \left\{ \vec{r}_{n+1}\left(t; \vec{r}_n(t_n), \vec{V}_{0n+1}\right), \vec{v}_n(t;), \vec{r}_n(t_n), \vec{V}_{0n+1}, t_{n+1} \right\}$$

and each V_0 is randomly chosen independently of previous choice. Hence as far as force field does not depend on *z* and *θ* and all the initial velocities are equally probable, the final z-coordinate will not depend on the order of ensemble member entering the sequence. Hence we have:

$$\text{"}mean\ axial\ displacement\text{"} \equiv \Delta z = \sum_n z\left(\vec{R}_0, \vec{V}_{0n}, t_n\right)$$

where: $\vec{R}_0 = \vec{0}$ and displacement has been achieved at the moment:

$$\Delta t = \sum_n t_n \qquad \text{Hence:} \qquad V_{ez} \equiv \Delta z / \Delta t$$

IV. Results and Conclusion

Computation like described above were performed for the thruster parameters like in SPT100, i.e.

$$R_1 = 0.03m;\ R_2 = 0.05m; \quad \vec{E} = \begin{pmatrix} 0 & 0 & E \end{pmatrix} \quad \vec{B} = \begin{pmatrix} B_0\left(2r/(R_1+R_2)\right) & 0 & 0 \end{pmatrix};$$
$$E = (2500 \div 4000)V/m; \quad B_0 = 0.02T; \quad T_e = 20eV \Rightarrow v_{Te} = 2.7\times10^6 m/s;$$

initial velocities were taken as: $|\vec{v}| = (1.3;\ 1.8;\ 2.3,\ 2.8)\times 10^6\ m/s$;
and velocity directions were from the hemisphere division for equal are fragments like in Fig.2.

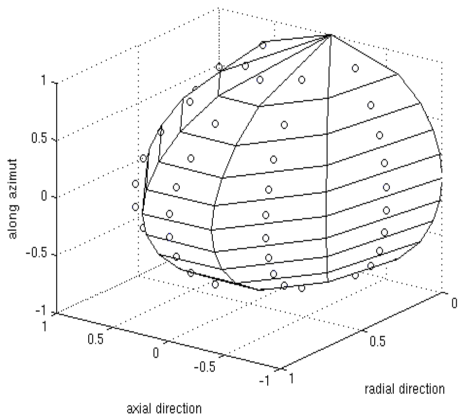

Fig.2 Initial velocity directions on hemisphere

Below we present some examples of individual trajectories taken from the ensemble with trajectory computations performed for: $E = 2500\ V/m$.

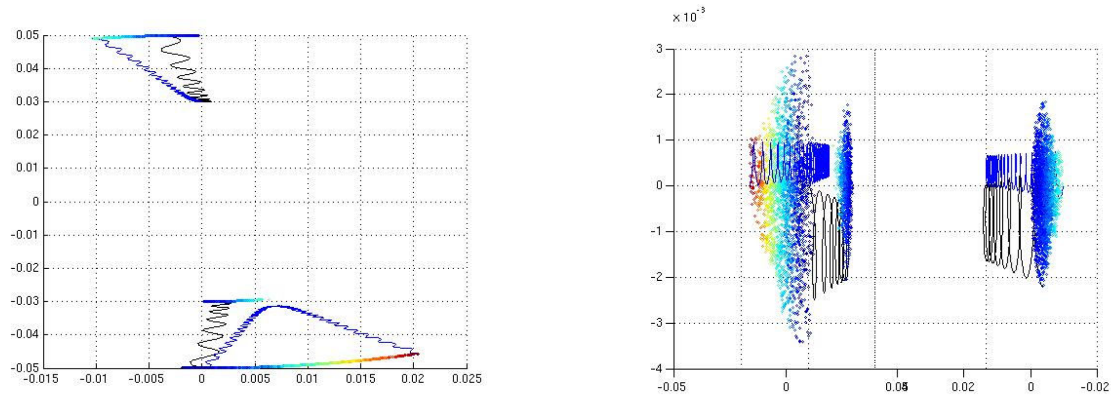

Fig.3 Examples of electron trajectories: for the cross section *z = const* (left) and for the cross section *θ = const* (right).

Electron axial velocity as governed by electron-wall collisions only was computed by means of above procedure for 16 cases (4 electric field values × 4 electron velocity modulus values). The computed electron velocities are presented in the Table 1.

Table 1

	E = 2500 V/m	E = 3000 V/m	E = 3500 V/m	E = 4000 V/m
$V = 1.3\times10^6$m/s	-0.3171e+04 m/s	-0.3451e+04 m/s	-0.5136e+04 m/s	-0.5781e+04 m/s
$V = 1.8\times10^6$m/s	-0.5406e+04 m/s	-0.5154e+04 m/s	-0.5772e+04 m/s	-0.5196e+04 m/s
$V = 2.3\times10^6$m/s	-0.3419e+04 m/s	-0.6377e+04 m/s	-0.8602e+04 m/s	-0.8195e+04 m/s
$V = 2.8\times10^6$m/s	-1.2666e+04 m/s	-1.2193e+04 m/s	-1.4530e+04 m/s	-1.5451e+04 m/s

Assuming electron temperature equal to 20eV we estimate averaged V_{ez} for each electric field and corresponding electron-wall frequency. These estimations are summarised in Table 2

Table 2

Electric Field [V/m]	V_{ez} averaged [m/s]	Equivalent electron-wall frequency [1/s]
2500	- 0.8e+04	1.8e+08
3000	- 0.9e+04	1.7e+08
3500	- 1.1e+04	1.7e+08
4000	-1.2 e+04	1.7e+08

Summarising we can state that reconstructing electron trajectory we are able to determine the equivalent electron-wall collision frequency greater than that which result from measurements in thrusters in the same condition. At the current stage the method is oversimplified since:

- Completely stochastic scattering on the wall is over estimated; it is necessary to introduce at least accommodation coefficient like in classical gas dynamics,
- Applying reconstructed trajectory with large number of collisions with the wall it will be necessary to include the effect of electric and magnetic field variation along *z;* collision with other plasma particles has to included in such a case, but
- Beside above the method at this stage is very exact in solving the classical equations of motion between collisions with the wall.

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