Characterization of High Frequency plasma oscillations in a Hall Effect Thruster

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Abstract: Plasma fluctuations appear in the ionized discharge of a Hall Effect Thrusters (HET) in a large range of frequencies, from a few kHz up to the plasma frequency (GHz). Low frequencies (10-30kHz) usually named "breathing mode" correspond to an ionization process in the channel of the HET. These fluctuations are experimentally observed in all type of HET and they are described by numerical codes. High frequency bursty fluctuations typically in the range of 5-150 MHz are also frequently observed in HET. They are detected by electrostatic probes located at the exit of the annular channel. The high frequency bursts are correlated to decay periods of breathing oscillation component of discharge current and previous analysis of signals taken from azimuthally separated probes have shown clear evidences of an azimuthal propagation at a velocity of the order of the $E \times B$ drift. A three-probes arrangement with azimuthal and axial separation makes it possible to study both azimuthal and possibly axial propagations of the HF component of the plasma oscillations. Cross-correlating time series simultaneously recorded at each probe gives access to possible propagation velocities. Moreover by making use of the Empirical Mode Decomposition we were able to obtain dispersion relations. Intermittency is also a general property of HF fluctuations. By calculating the wavelet bicoherence we are able to demonstrate strong nonlinear coupling phenomena and energy transfers between fluctuations of low and high azimuthal mode numbers. The physical mechanism of these HF bursts is not yet clarified, but the experimental evidence of the combined axial and azimuthal propagation could bring new ideas on the excitation mechanism and on their role regarding the cross-field transport.

I. Introduction

The magnetized plasma of a Hall thruster displays numerous type of oscillations, which encompass many kind of physical phenomena each of them with its own length and time scales [1]. The oscillation spectrum ranges from ion cyclotron frequency in the kHz band up to the electron plasma frequency in the GHz domain. In between, several well-defined frequency bands can be distinguished. Low frequency oscillations of about 20 kHz carry most of signal energy and are explained as a result of displacement of the ionization front inside the thruster channel. They are often referred to as "breathing oscillations" [2]. Such oscillations are of primary importance for thruster performances and lifetime. Observations of oscillations with frequencies in the range of a few MHz and associated with electron density fluctuations propagating azimuthally have

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also been reported [3,4]. These high frequency bursts are correlated to decay periods of breathing oscillation component of discharge current and previous analysis of signals taken from azimuthally separated probes have shown clear evidences of an azimuthal propagation at a velocity of the order of the $E \times B$ drift [5]. In particular such high frequency oscillations are expected to be responsible for turbulent electron transport across the magnetic barrier [6] and therefore could play a dominant role in regulating the plasma transport.

In this paper we present new insights into propagation properties of the HF fluctuations bursts obtained by processing time series recorded from a three-probes arrangement with azimuthal and axial separation.

II. Experimental set-up

The experimental data were obtained with the PPS-X000 Hall thruster, which is a 5 kW-class laboratory model of the PPS-5000 Snecma's thruster [7], at the French national facility PIVOINE [8]. Typical values of plasma parameters in 1.5 kW Hall thrusters are: an axial electric field $E_z \sim 10^4$ V/m, a radial magnetic field $B_r \sim 20$ mT, an electron temperature $T_e > 10$ eV, a charged particle density (under quasi-neutrality condition) $n_i \approx n_e \sim 10^{17}$ m^{-3}, and a mass flow rate of gas $\dot{m}_a \sim 5$ mg/s.

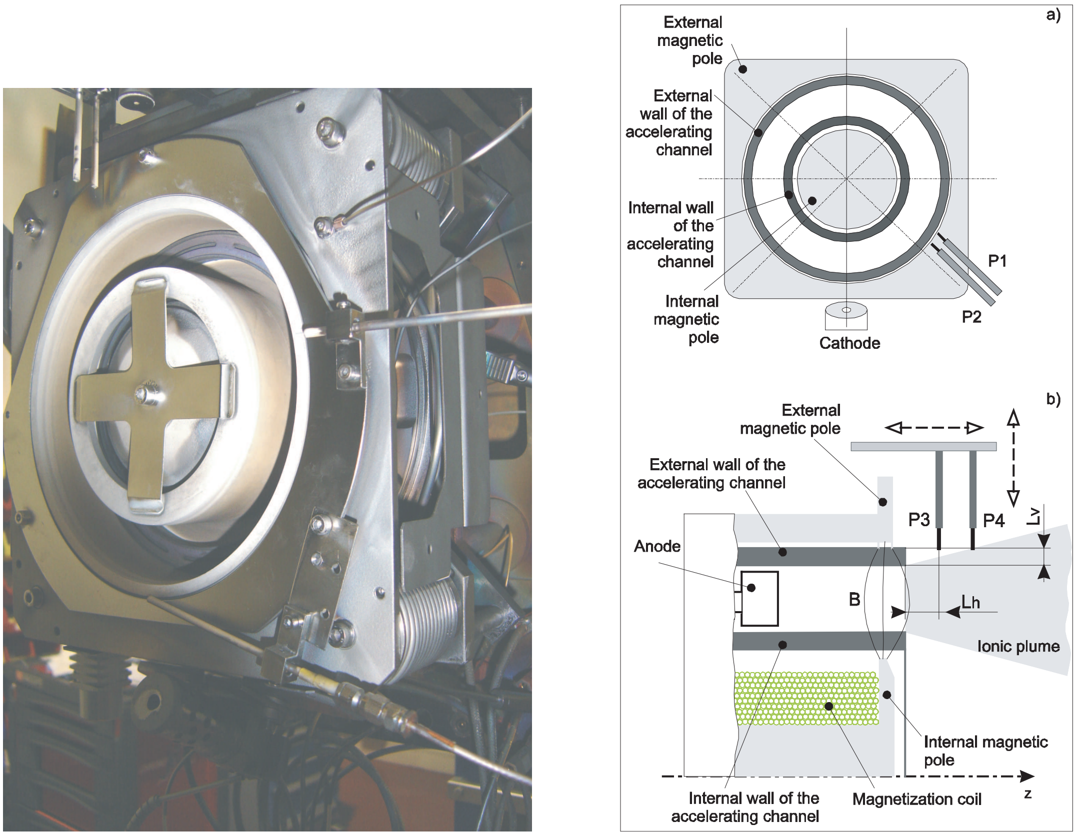

Figure 1. View of the PPS-X000 thruster and probe location. a) front view: for estimating azimuthal correlations, $L_{P1-P2} = 1.1$ cm; b) side view (transversal cut): for estimating axial correlations, $L_{P3-P4} = 1$ cm; the magnetic field geometry is shown schematically.

HF fluctuations were recorded by using shielded Langmuir probes [8] installed around the accelerating channel external wall and outside the ionic plume. Their active part is made of 0.125 Ta wire and has a length of ~ 6 mm. To access the azimuthal propagation properties of the fluctuations two probes (P1 and P2) were aligned in the channel cut-off plane separated by a fixed distance of 1.1 cm between their tips, as sketched in Fig. 1a.

To access the axial propagation properties of the HF fluctuations two probes spaced at 1 cm (P3 and P4) were aligned along the thruster axis (Fig. 1b). In this second configuration the probe pair could be moved in axial and radial directions. The axial position L_h of the P3 tip relatively to the channel cut-off and the radial position L_v relatively to the channel external wall define the probe pair location (see Fig. 1b). The probe signals are transferred through 8 m coaxial lines and are observed and recorded on a digital 4-channel Tektronix 5104B oscilloscope in AC 50 Ω mode. Typical recording length is 80 μs at a sampling rate of 1.25×10^9 Samp/s. Each dispersion relation is reconstructed from two separately recorded data sets. More information about the HF oscillations acquisition system as well as about the collected data can be found in Ref. 9.

III. Results and discussion

A. Azimuthal and axial correlations

By calculating the cross-correlation functions from time series of bursts of floating potential fluctuations taken from a three-probes arrangement with azimuthal and axial separation we get evidence of propagation properties of these fluctuations. Nevertheless because of he strong intermittent and non monochromatic behavior of these HF fluctuations it is not possible to extract from the measured time delays reliable propagation velocities. Such cross-correlations are depicted in Fig. 2.

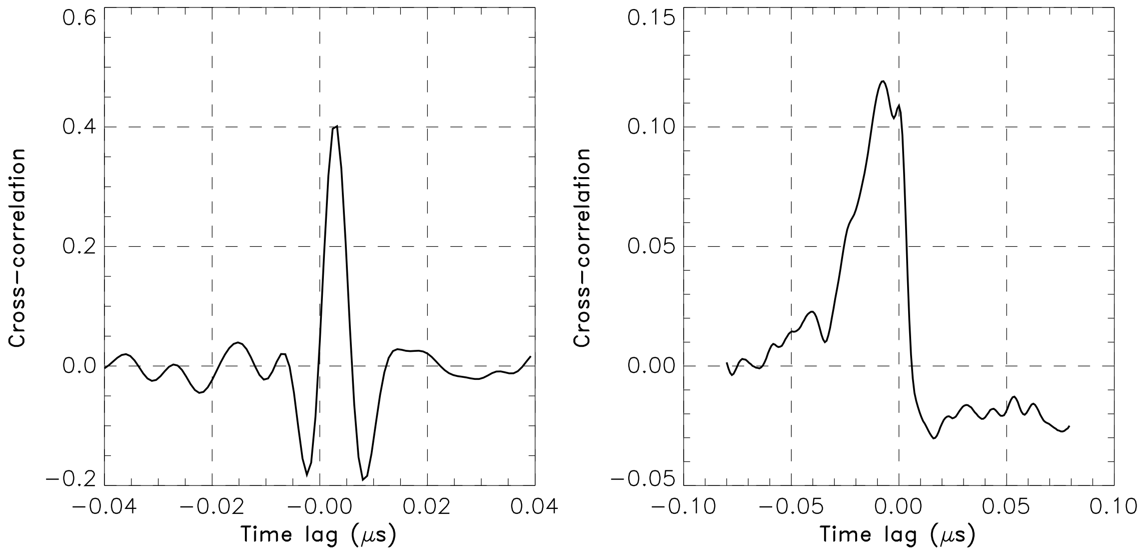

Figure 2. a) Azimuthal correlation. Plot of the cross-correlation between time series from a HF burst recorded at probes P1 and P2 ($L_{P1-P2} = 1.1$ cm, mass flow rate $\dot{m}_a = 6$ mg/s, discharge voltage Ud = 350 V. b) Axial correlation. Plot of the cross-correlation between time series from a HF burst recorded at probes P3 and P4 ($L_h = 5$ mm, $L_v = 10$ mm, mass flow rate $\dot{m}_a = 8.34$ mg/s, discharge voltage Ud = 550 V. .

By making use of the Empirical Mode Decomposition and cross-correlating the obtained Intrinsic Mode Functions (IMFs) [5,11] we were able to measure the time delay in each frequency range and thus obtain dispersion relations.

Fig. 3 shows the set of IMFs obtained from the original time series corresponding to a given burst measured at two azimuthally separated probes. From the time delays measured on each cross-correlation function calculated for each IMF pair we can plot the dispersion curves depicted in Fig. 4.

Fig. 4a shows the result obtained for azimuthally separated probes. The points corresponding to the different IMF pairs fit very well a line crossing the origin, which slope is 3.15×10^6 m/s, of the order of the $E \times B$ drift velocity. This corresponds to a non dispersive behaviour of azimuthally propagating waves with mode number from $m = 1$, $f \sim 7$ MHz up to $m \sim 20$, $f \sim 120$ MHz. This result is in good agreement with previous studies [5].

The situation depicted in Fig. 4b is much more difficult to understand. It corresponds to two axially separated probes. We observe a change in the sign of the time shift in the cross-correlation functions with increasing frequencies. We still do not know if such time shifts correspond to a true axial propagation property of the HF fluctuations or if this only a consequence of some special phase variation along the axial direction, i.e. the inhomogeneity direction.

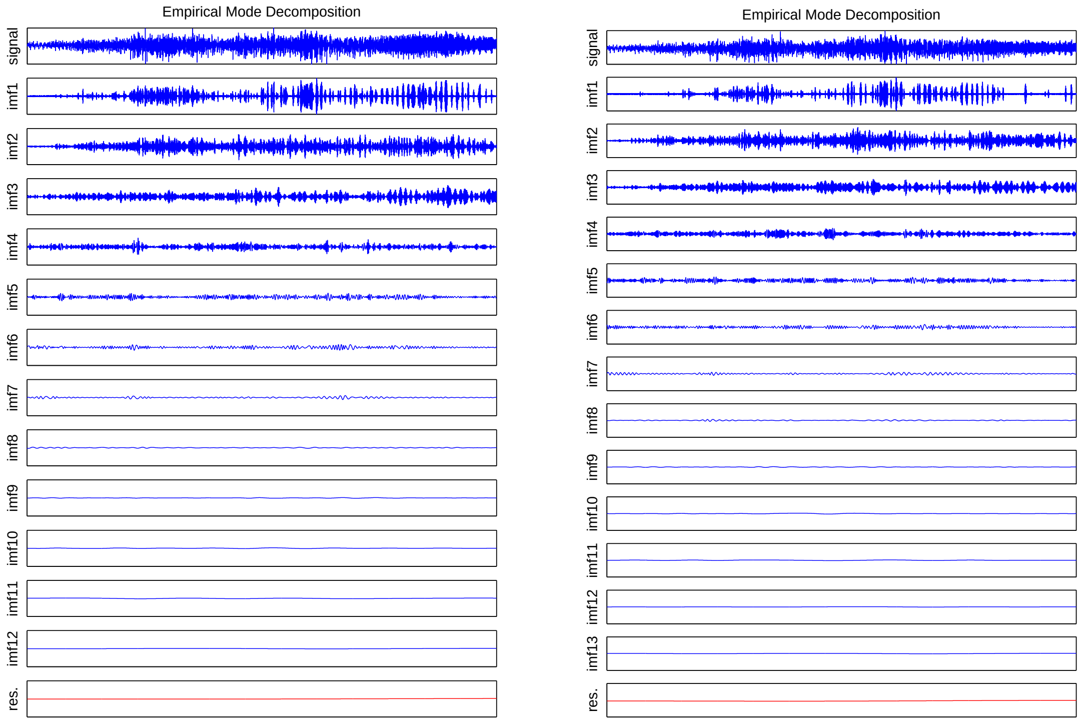

Figure 3. Empirical Mode Decomposition of a HF burst (same as case a) above recorded simultaneously at probes P1 (left) and P2 (right). Mass flow rate $\dot{m}_a = 6$ mg/s, discharge voltage Ud = 350 V.

B. Wavelet bicoherence analysis

Intermittency is also a general property of HF fluctuations. This feature is already obvious from a direct inspection of the time series. For a more quantitative approach, the time evolution of the frequency is investigated by means of a wavelet analysis.

The wavelet transform of a signal $s(t)$ is given by:

$$W_s(a,\tau) = \int s(t)\Psi_a(t-\tau)dt$$

where Ψ_a is the wavelet function, which is a function of both time t and scale a. Many different wavelets exist, and in this present work we chose the commonly used Morlet wavelet, which is a sinusoidal oscillation convoluted with a Gaussian:

$$\Psi_a(t) = (2a\pi)^{-1/2}\exp[i2\pi t/a - (t/a)^2/2]$$

The frequency corresponding to the scale a is given by $\omega = 2/a$. The frequency and time resolution corresponding to the wavelet Ψ_a are, respectively, $\Delta\omega = \omega/4$ and $\Delta t = 2a$.

The result of the wavelet analysis of the time series depicted in Fig. 5 shows that all mode numbers are not all the time present and that probably strong nonlinear coupling processes occur from which new modes may appear or disappear.

By calculating the wavelet bicoherence we are able to study these nonlinear coupling phenomena and energy transfers between fluctuations of low and high azimuthal mode numbers. To investigate this hypothesis, a wavelet bicoherence analysis is used. The squared wavelet bicoherence of the signal is defined as:

$$[b^W(a_1,a_2)]^2 = \frac{|B^W(a_1,a_2)|^2}{[\int |W_s(a_1,\tau)W_s(a_2,\tau)|^2 d\tau\ ]\ [\int |W_s(a,\tau)|^2 d\tau\ ]}$$

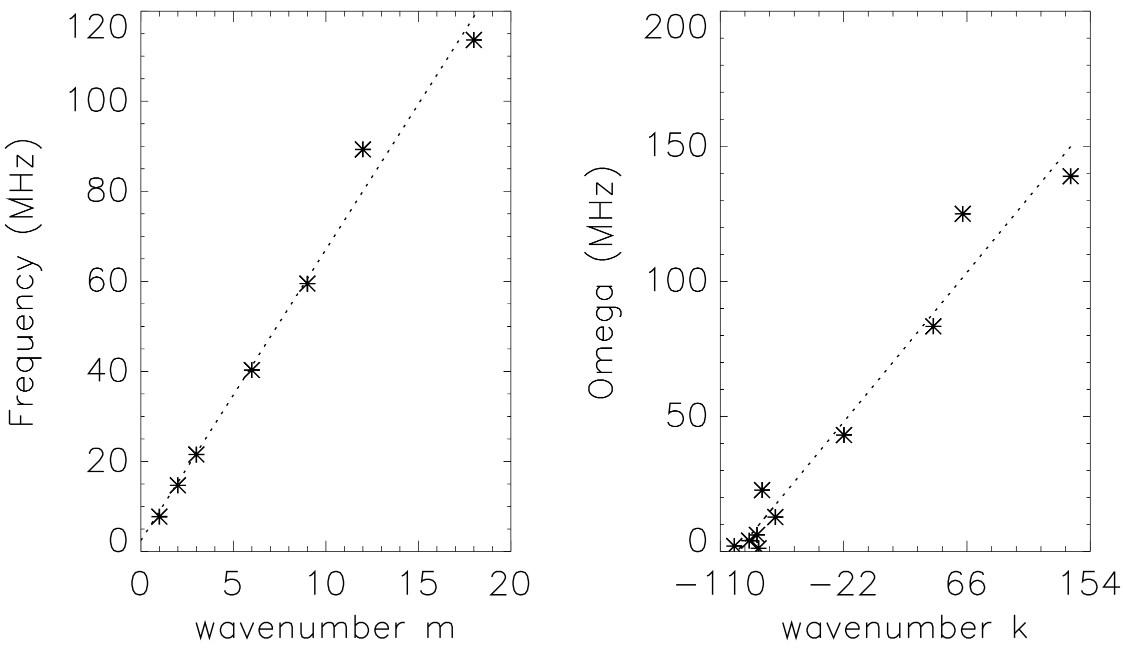

Figure 4. a)Azimuthal dispersion relation (left. Mass flow rate $\dot{m}_a = 6$ mg/s, discharge voltage Ud = 350 V.) and b) axial dispersion relation. Mass flow rate $\dot{m}_a = 8.34$ mg/s, discharge voltage Ud = 550 V.

which is the normalized wavelet bispectrum defined as:

$$B^W(a_1, a_2) = \int W_s^*(a, \tau) W_s(a_2, \tau) d\tau$$

where $1/a = 1/a1 + 1/a2$. The wavelet bispectrum is a measure of the amount of phase coupling between components of scale lengths a_1, a_2, and a of the signal $s(t)$. More details on the calculations as well as on the theory of wavelet bicoherence can be found in Ref. 11.

By inspecting the plot (Fig. 7) of the wavelet bicoherence calculated for a burst sequence we can observe evidences of direct (generation of high mode numbers from the lowest ones) and inverse cascades (birth of mode numbers).

IV. Conclusion

By processing time series of floating potential fluctuations taken from a three-probes arrangement with azimuthal and axial separation we obtained new insights into propagation properties of the high frequency bursts observed in Hall Effect Thrusters.

The strong intermittent and non monochromatic behavior of these HF fluctuations makes it impossible to fully determine their propagation properties from the cross-correlation functions of two long times series taken at two azimuthally or axially separated probes.

By making use of the Empirical Mode Decomposition and cross-correlating the obtained Intrinsic Mode Functions we were able to obtain dispersion relations.

Intermittency is also a general property of HF fluctuations. By calculating the wavelet bicoherence we are able to demonstrate strong nonlinear coupling phenomena and energy transfers between fluctuations of low and high azimuthal mode numbers. The physical mechanism of these HF bursts is not yet clarified, but the experimental evidence of the combined axial and azimuthal propagation could bring new ideas on the excitation mechanism and on their role regarding the cross-field transport

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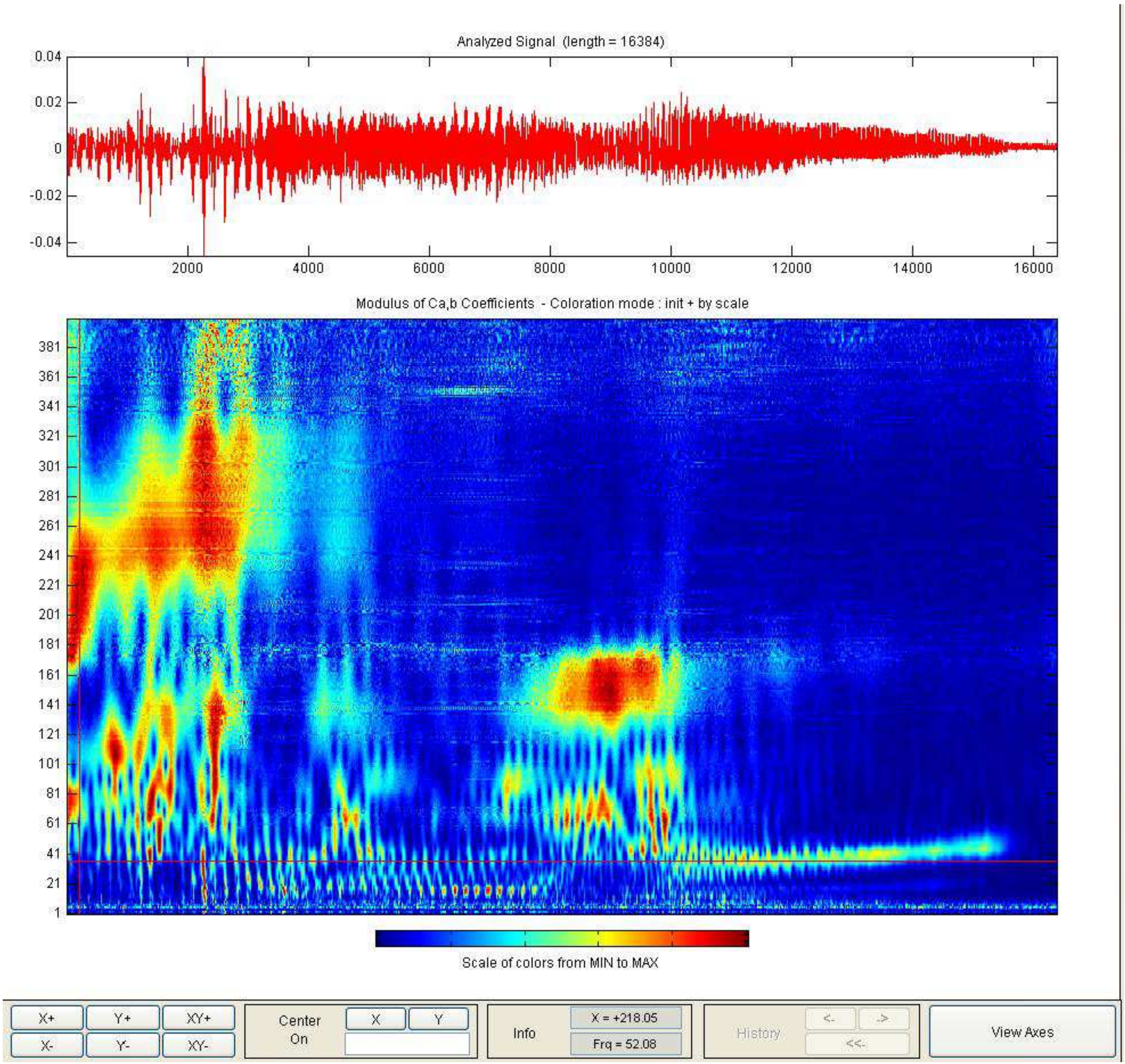

Figure 5. Wavelet time-frequency spectrum of one HF burst (Mass flow rate $\dot{m}_a = 6$ mg/s, discharge voltage Ud = 350 V.). Horizontal axis: time. Vertical axis: scaling factor N, inverse of the frequency $f = 1.5 f_e / N = 1875/N$ in MHz.

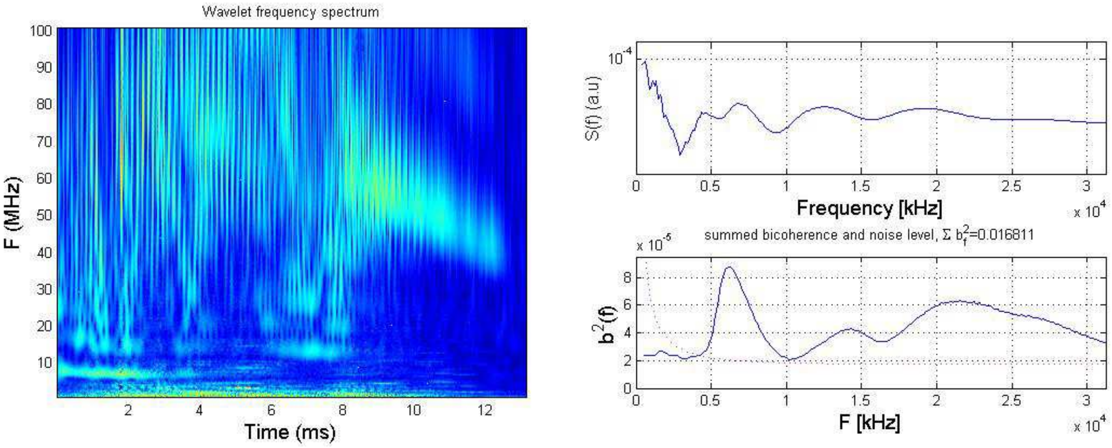

Figure 6. a) Wavelet spectrum. b) Summed bicoherence. Mass flow rate $\dot{m}_a = 6$ mg/s, discharge voltage Ud = 350 V.

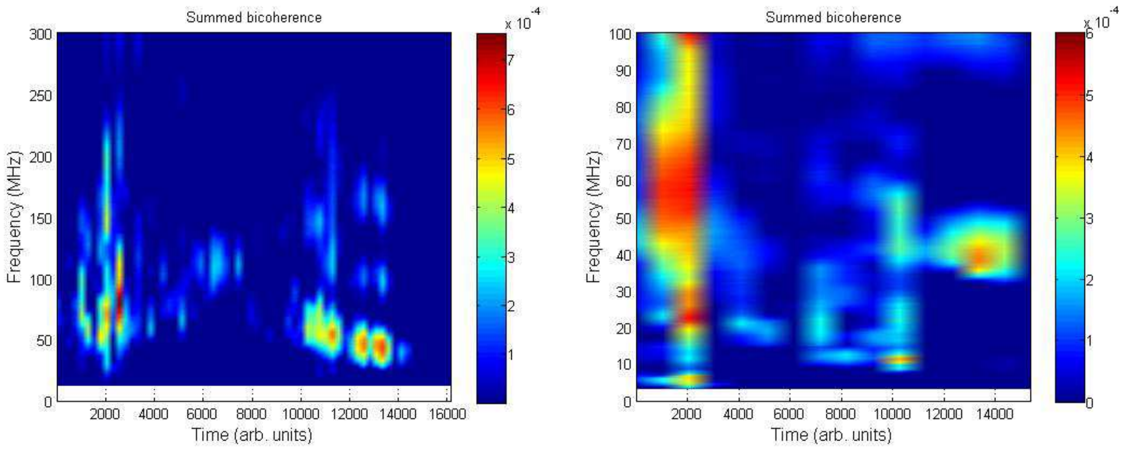

Figure 7. Squared wavelet bicoherence. On the right is displayed a zoomed version to better showing the low frequency range. Mass flow rate $\dot{m}_a = 6$ mg/s, discharge voltage Ud = 350 V.