Ionization Efficiency in Electric Propulsion Devices

Jerry L. Ross *
Lyon B. King †

While it is widely known that propellant ionization represents an energy loss mechanism that acts to decrease the efficiency of electric propulsion devices, it is not apparent how or where this loss can be measured in thruster data. Furthermore, the manner in which the propellant is ionized holds potential inefficiencies (power losses) above and beyond those arising from the ionization potential. This paper presents a simplified phenomenological model that sheds light on how the ionization dynamics can affect measurable thruster parameters. The results of the model indicate that there is as much as 6% variation in maximum thrust efficiency simply due to the method of ionization.

Nomenclature

$\dot{m}$	propellant flow rate, (kg/s)
η_T	thrust efficiency
η_B	beam divergence
η_c	current efficiency
η_T	thrust efficiency
η_{vdf}	velocity distribution function
η_v	voltage utilization efficiency
ϕ	ionization potential, (eV)
Ψ_{avg}	average potential at which ions are born at, (eV)
e	elementary charge, (c)
E_{ion}	specific ionization energy, (eV/kg)
f_i	ionization mass fraction
g	acceleration due to gravity at the earths surface, (m/s^2)
I_d	discharge current, (A)
I_{sp}	specific impulse, (s)
M	atomic mass, (kg)
P_s	power supply output, (W)
P_{ion}	power required to ionize, (W)
P_{kin}	exhaust kinetic power, (W)
Q	average particle charge number
q	particle charge number
V_d	discharge voltage, (V)
x	rows of ionization

*PhD Candidate, Michigan Technological University, Houghton MI
†Associate Professor, Michigan Technological University, Houghton MI

I. Introduction

Electric propulsion devices, such as ion and Hall thrusters, accelerate plasma to provide thrust. Propellants for these thrusters must be ionized from a gaseous state to become plasma. This process requires an input of energy for each atom equal to the ionization potential.

In 2005 a study was conducted entitled "Energetics of Propellant Options for High-Power Hall Thrusters" where the ionization cross section and ionization potential for different propellants were compared.[1] In that study the fraction of power consumed by ionization processes was defined as

$$\frac{P_{ion}}{P_{kin}} = \frac{2E_{ion}}{g^2 I_{sp}^2}. \tag{1}$$

The term E_{ion} was defined as the specific ionization potential ϕ/M. This equation is valid for a simple energy balance, however, is not sufficient as a measure of efficiency. The manner in which the propellant is ionized holds potential inefficiencies (power losses) above and beyond the ionization potential. This study explores the method of ionization in ion and Hall thrusters and how this is reflected in thrust efficiency, the most common benchmark of operation.[2]

Due to inherent limitations of available onboard power, there has been recent interest in high thrust-to-power Hall thrusters where the voltage is scaled downward and the mass flow rate (current) is scaled upward to maintain a nominal level of power. Experimentally, it has been shown that reducing the voltage of Hall thruster (independent of the current/power) results in reduced thrust efficiency.[3–5] This study will attempt to prove that a portion of that reduction in efficiency is due to the ionization process above what equation 1 predicts.

Herein I will refer to the physics of a Hall thruster, however, all statements are equally valid for any DC discharge plasma accelerator. The energy to ionize the neutral propellant in a Hall thruster comes from energetic electrons in the system. These electrons can originate from one of two sources: 1. the cathode or 2. a propellant neutral that has been ionized. The first category we shall call recycled electrons, and the second plasma electrons. Now in reality, ionization occurs from a combination of each source, however, for the purposes of this study we will explore them individually. The process of ionization by each category of electron is substantially different. Therefore the paper will be divided as follows: First we will present a detailed explanation of how ionization can occur from plasma electrons followed by a detailed explanation of how ionization can occur from recycled electrons. Then, a mathematically rigorous derivation of thrust efficiency will be presented so that the we can see how the ionization process fits in. Next, the loss mechanisms associated with the differing ionization schemes are cast in terms of maximum thrust efficiency. Lastly, we give a brief a look what actual control we can exercise over the ionization processes and how this is exhibited in previous work.

II. Plasma Electrons

When a neutral is ionized a free electron escapes. It is this group of electrons that we refer to as plasma electrons. Once a plasma electron is free from the atom (newly born ion), the electron gains kinetic energy as it accelerates from the local potential towards the anode. This energy can be deposited in two forms: collisions, and joule heating of the anode. Electron collisions occur with neutrals, ions, and chamber walls but only electron-neutral collisions are 'beneficial' means of depositing energy in a Hall thruster. All other energy sinks cause electron-temperature quenching and are considered loss mechanisms.

The goal is to identify the minimum penalty, or maximum efficiency of ionization by plasma electrons. To that end our model includes the following idealizations: 1. All of the kinetic energy from the electron is transfered to ionization processes (collisions with neutrals). 2. The only electrons entering the system from the cathode are the catalyst electrons (defined later). 3. The processes continue until the propellant is fully and singly-ionized.

Now the participating members of any electron-neutral ionizing collision include: the impinging electron, the newly born ion, and the newly born free electron. We will assume a simplified impact ionization process in which the newly born free electron has no kinetic energy in the moments immediately following it's escape from the atom. The impinging electron has ϕ less energy than it began with. The presence of the electric field causes both electrons to gain kinetic energy and travel toward the anode. If the free electron is to take part in an ionizing collision of its own, it must gain ϕ eV of kinetic energy, which is to say that it must travel to a location where the potential is ϕ eV greater than the place of its birth. Then, if the electron does ionize a neutral the ion created from this second collision is born at a potential of ϕ eV higher than that of the first ion. This means that the second ion will have a greater available potential than first ion to which it can accelerate through by ϕ. Ignoring post ionization events, the second ion will be accelerated to a larger exit velocity as compared with the first ion.

A perfectly efficient Hall thruster is one where all of the ions are born at anode potential and are accelerated to cathode potential. However, this cascading process of ionization as described eliminates this possibility. Cascading ionization results in ions being ejected with a spread of velocities with a characteristic average. As we will show, the thrust efficiency is not only a function of the average velocity but of the distribution as well.

Now we will expand this simple understanding of individual ionization events by plasma electrons to create a schematic that encompasses all of the ionization events. Thus far, we know that ionization at anode potential is most desired. So let us assume that a maximum of Y atoms can be ionized at anode potential, which is to say that Y electrons arrived with ϕ energy. Therefore, those electrons were came from a location of $\Delta\phi$ lower potential, where we can assume $Y/2$ ions were ionized because that equates to Y electrons. Expanding this we have what appears to by a pyramid as seen in figure 1. There are only a limited number of ϕ eV steps that can be taken from the anode before cathode potential is reached. Therefore, the number of steps or rows can be calculated as:

$$r = \frac{V_d}{\phi}. \tag{2}$$

The average local plasma potential for all ionization processes is

$$\Psi_{avg} = \frac{V_d(2^{r-1}) + (V_d - \phi)(2^{r-2}) + ... + (V_d - (r-1)\phi)(2^{r-r})}{2^r - 1}. \tag{3}$$

Where $2^r - 1$ is the total number of ions. This only accounts for the number of ions in a given pyramid, however, since each pyramid is identical all calculations for one pyramid are representative of the propellant as a whole. In other words, there will be multiple pyramids operating in parallel whose quantity is a function of the mass flow rate.

The ionization by plasma electrons requires the addition of a catalyst electron at the top of the pyramid as seen in figure 1. This electron originates from the cathode and must gain ϕ eV of kinetic energy to ionize the first neutral and start the process. The catalyst electron accounts for $1/(2^r - 1)$ additional current to the anode above and beyond the current from the plasma electrons. Figure 2 depicts the necessary fraction of additional charge collected by the anode from catalyst electrons. Since the voltage is fixed in operation, additional current results in additional power devoted to ionization. The impact is appreciable in the lower voltage regime but when the pyramid grows large (higher discharge voltage) the catalyst electron has negligible effect.

So, for cascading ionization (ionization by plasma electrons) the average accelerating potential (voltage) is necessarily lower than the anode-cathode potential, and there is an additional current to the anode from the catalyst electrons. We will see how these findings are reflected in the thrust efficiency, but first let us look at ionization by recycled electrons.

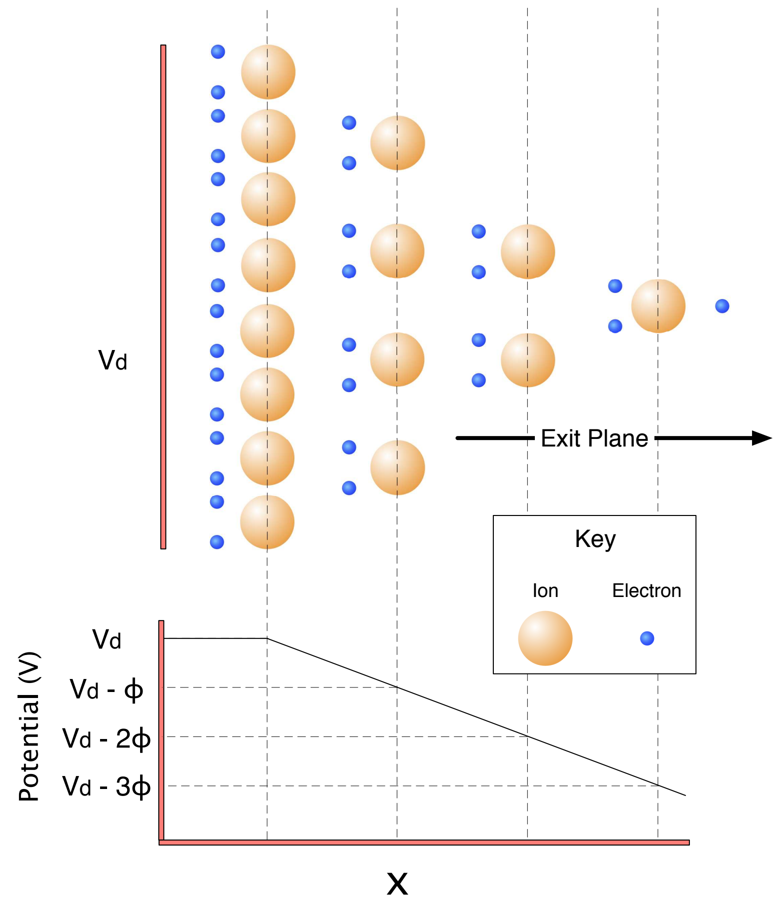

Figure 1. This graphic displays the end result of cascading ionization. The process begins on the right with the catalyst electron and progress until the atoms nearest the anode are ionized. The bottom graph shows how the local potential is increasing by ϕ with each row of ionization.

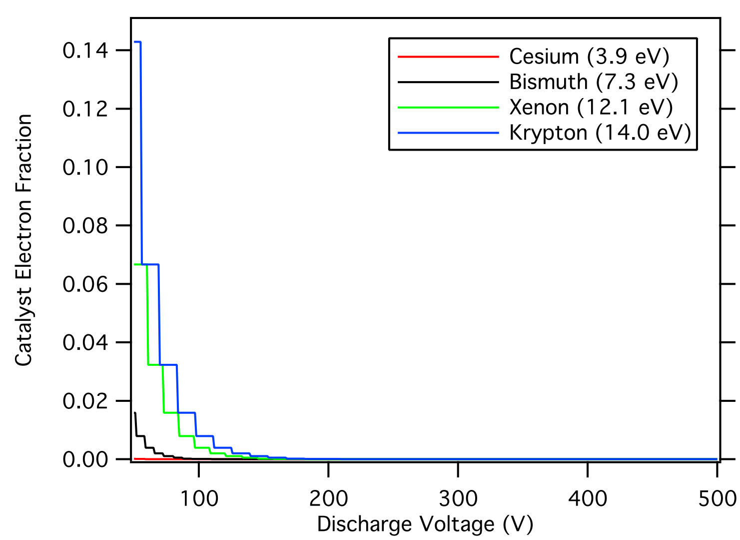

Figure 2. The necessary fraction of current from catalyst electrons collected at the anode for ionization occurring solely through plasma electrons. The discretization is explained in detail in the appendix.

III. Recycled Electrons

The idealizations for ionization by recycled electrons are as follows: 1. All propellant atoms are at anode potential. 2. All cathode electrons are born at cathode potential and are accelerated to anode potential. 3. The processes continue until the propellant is fully and singly-ionized. 4. The recycled electrons dump all of their energy into ionization (no joule heating of the anode). The benefit of this is that all ionization will occur at anode potential. Furthermore, each cathode electron gains the full potential and is capable of ionizing many neutruals. The penalty is that a considerable amount of extra current is now collected at the anode.

From the energy acquired by acceleration through the anode-cathode potential, each cathode electron can ionize V_d/ϕ neutrals. Each ion is then accelerated out across the full accelerating potential baring any downstream events. The tradeoff is that an additional

$$\frac{\phi e\dot{m}}{MV_d} \tag{4}$$

of current is collected at the anode above and beyond the inherent current from free electrons in the plasma. An increase in current is a decrease in current efficiency. Current efficiency is outlined in the efficiency analysis. Figure 3 shows the fraction of additional current for a variety of different ionization potentials as a function of anode voltage. This fraction increases rapidly with decreasing discharge voltage for all ionization potentials. At a discharge voltage of 50 V, 28% of the current collected is recycled electrons in the case of Krypton. Unlike cascading ionization, the increased current is the only penalty incurred here.

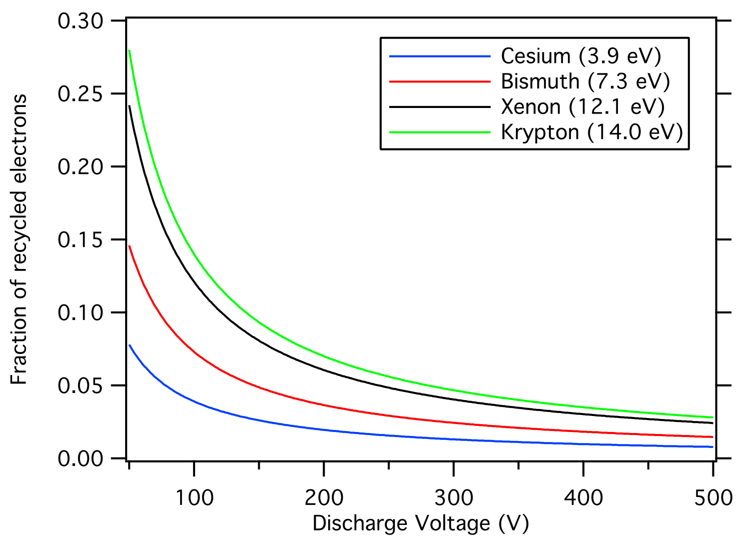

Figure 3. The necessary fraction of current from recycled electrons collected at the anode for ionization occurring solely through recycled electrons

IV. Efficiency Analysis

The energy required to ionize the propellant of a Hall thruster is 'cost' of operation, in that it is a un-retrievable sink of available on-board power. The magnitude of this 'cost' depends on how the propellant is ionized. Ionization by recycled electrons significantly increases the current to the anode. Cascading ionization fractionally increases the current to the anode but it also reduces the average accelerating potential below the anode potential. In order to quantify these effects the penalties must be cast in terms of thrust efficiency.

Thrust efficiency is a real-time gauge of operation using thrust stand data, mass-flow rates, and power supply values

$$\eta_T = \frac{T^2}{2\dot{m}P_s}. \tag{5}$$

This efficiency can be broken down into the product of four loss mechanisms of operation:[6] velocity distribution function,

$$\eta_{vdf} = \frac{< |V| >^2}{< |V|^2 >} \tag{6}$$

beam divergence efficiency,

$$\eta_B =< cos\beta >^2 \tag{7}$$

voltage utilization efficiency,

$$\eta_v = \frac{\frac{1}{2}m < \mathbf{V}^2 >}{eV_d}\frac{1}{f_iQ} \tag{8}$$

and current efficiency

$$\eta_c = \frac{\dot{m}e}{mI_d}(f_iQ) = \frac{I_i}{I_d}. \tag{9}$$

The variable f_i is the ionization mass fraction of the propellant

$$f_i = f_1 + f_2 + f_3...$$

$$f_0 + f_i = 1$$

where f_0, f_1, f_2, f_3 are the exit mass fractions of Xe, Xe^+, Xe^{2+}, Xe^{3+} and where Q equals the average charge state of the propellant species

$$Q = \frac{1}{f_i}\Big(f_1 + 2f_2 + 3f_3\Big).$$

The velocity distribution function is a loss mechanism arising from non-uniformity it the velocity distribution of ions from multiple ionic species and acceleration inefficiencies. The beam divergence is a measure of the collimation of the ion beam also known as cosine losses. Perpendicular components of the beam exert a force radially inward compressing the thruster as opposed to propelling it forward and therefore are energy losses. The voltage utilization efficiency is another measure of the acceleration inefficiencies that prevent ions from being accelerated across the entire anode-cathode potential. Lastly, current efficiency is a measure of the actual current collected by the anode as opposed to the necessary current collected if the propellant was fully and singly ionized.

Mathematically, the thrust efficiency is reconstructed as:

$$\eta_T = \frac{< |V| >^2}{< |V|^2 >} < cos\beta >^2 \frac{\frac{1}{2}m < \mathbf{V}^2 >}{eV_d}\frac{1}{f_iQ}\frac{\dot{m}e}{mI_d}(f_iQ) \tag{10}$$

$$\eta_T = \frac{< \mathbf{V} >^2}{< \mathbf{V}^2 >}\frac{\frac{1}{2}\dot{m} < \mathbf{V}^2 >}{I_dV_d} \tag{11}$$

$$\eta_T = \frac{\frac{1}{2}\dot{m} < \mathbf{V} >^2}{I_dV_d} \tag{12}$$

$$\eta_T = \frac{T^2}{2\dot{m}P_s}. \tag{13}$$

V. Efficiency Penalties

As previously mentioned, the cascading ionization modifies the effective accelerating potential of the ions and thus the exit velocity is altered. Hence, both the velocity distribution function and the voltage utilization function will be affected. Furthermore, ionization by plasma electrons requires the addition of catalyst electrons that increases the current collected at the anode. Thus, the current efficiency will be penalized. Figure 4 shows the maximum velocity distribution function as achievable with ionization by plasma electrons. Likewise, Figure 5 and Figure 6 depict the maximum voltage utilization efficiency and current efficiency respectively. Note that the discrete jumps occur on multiples of the ionization potential. These are real effects that are discussed in the appendix.

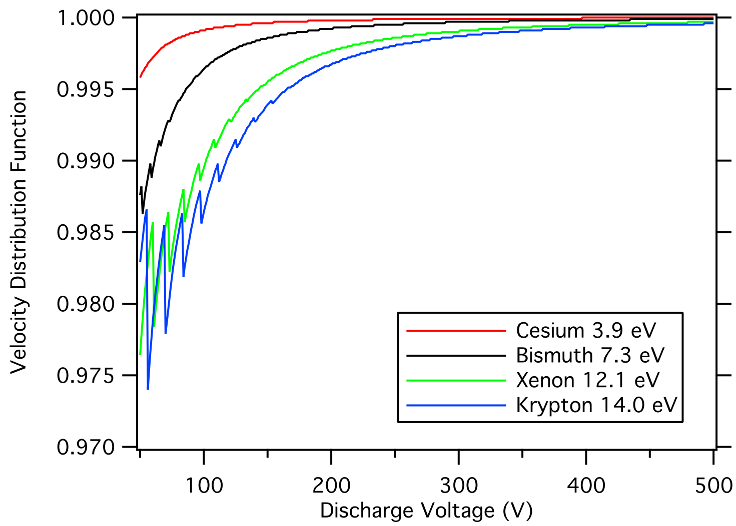

Figure 4. cascading ionization

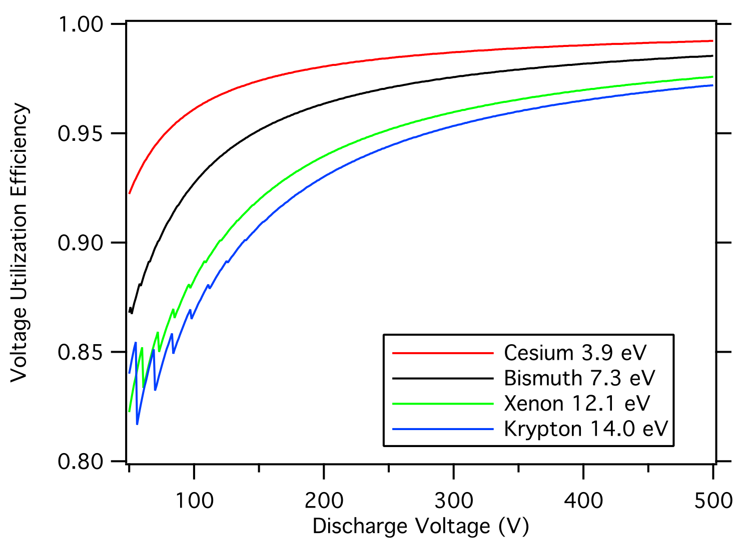

Figure 5. cascading ionization

Ionization by recycled electrons results in increased current to the anode, and thus only affects

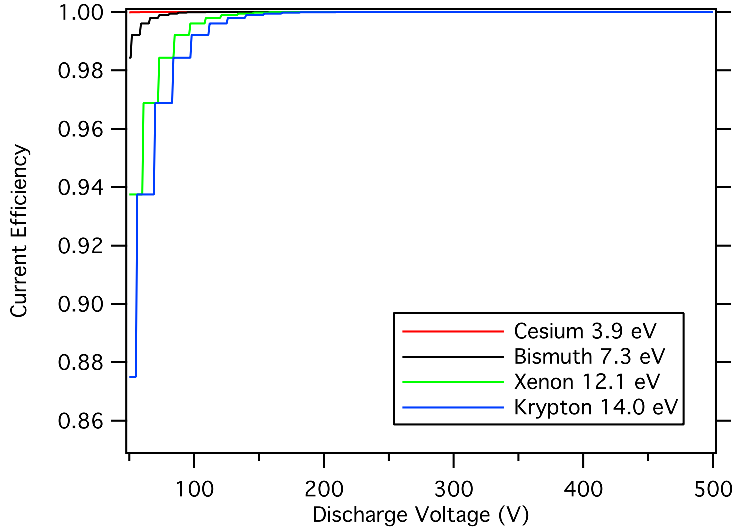

Figure 6. cascading ionization

the current efficiency. Figure 7 displays the penalty to the current efficiency from ionization in this manner. Although, ionization by recycled electrons affects the current efficiency in a much greater magnitude than ionization by plasma electrons does, the latter process also affects two other efficiencies. The net effect is the product of all efficiencies (thrust efficiency) as seen in Figures 8-9. Thrust efficiency for ionization by recycled electrons is identical to the current efficiency because the other three efficiencies are unaffected (unity).

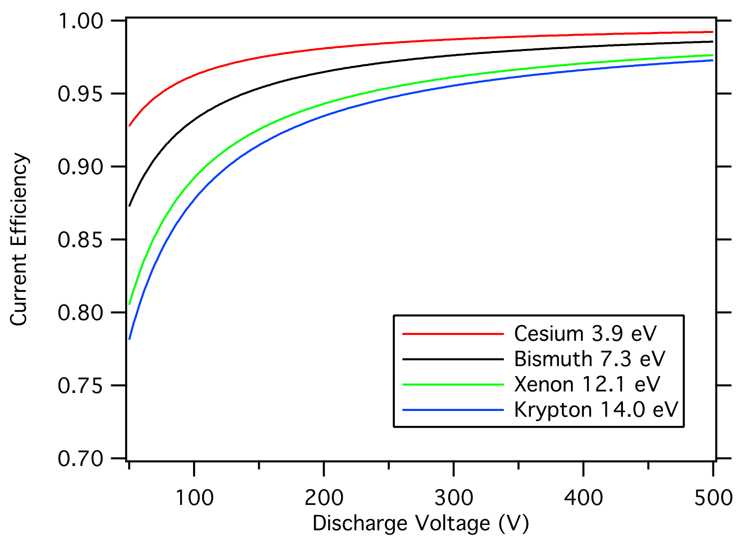

Figure 7. This is graph represents the penalty associated with the additional current to the anode for ionization by recycled electrons.

VI. Discussion

The schematic setup for cascading ionization used single step ϕ eV in potential space. It is entirely possible that each row could be separated by any multiple of ϕ and maintain that the

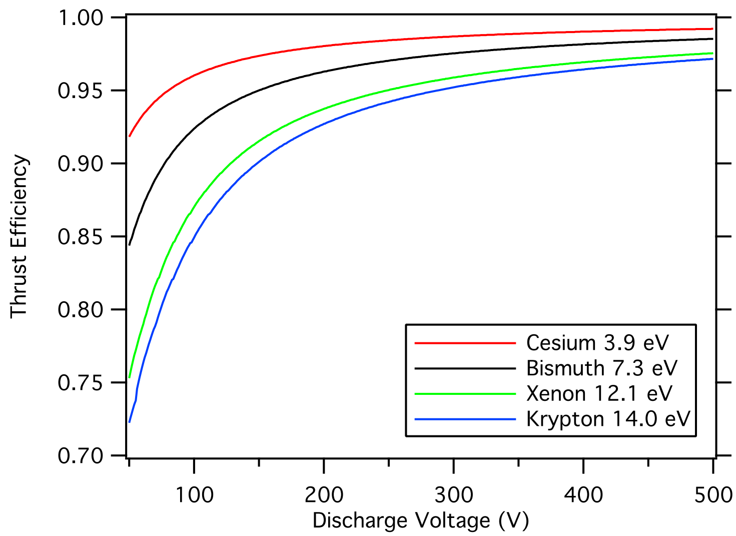

Figure 8. The product of the loss mechanisms combine to represent total thrust efficiency for ionization by plasma electrons (cascading ionization).

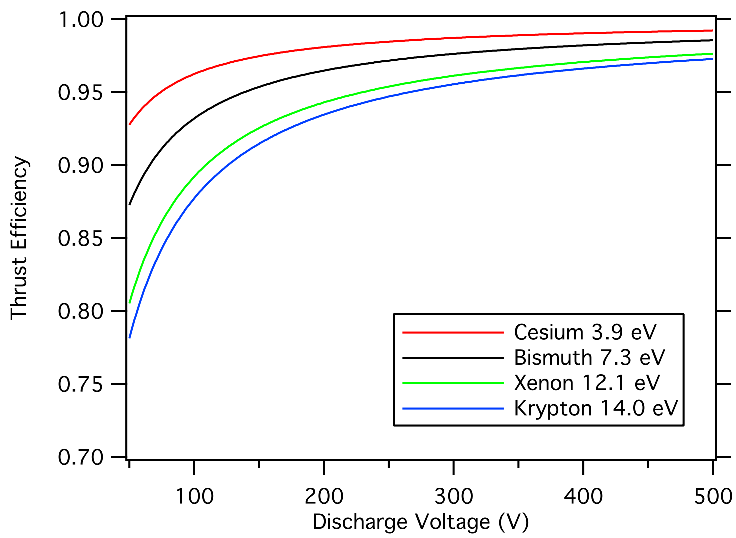

Figure 9. The product of the loss mechanisms combine to represent total thrust efficiency for ionization by cathode electrons (which in this case is only the current efficiency).

electrons deposit all of their energy into ionizing collisions. Figure ?? shows the pyramid for a separation of 2ϕ between rows. Figure 10 however, depicts the total thrust efficiency for cascading ionization with various step sizes. Within the accuracy of the model, cascading ionization efficiency is independent of step size in potential space.

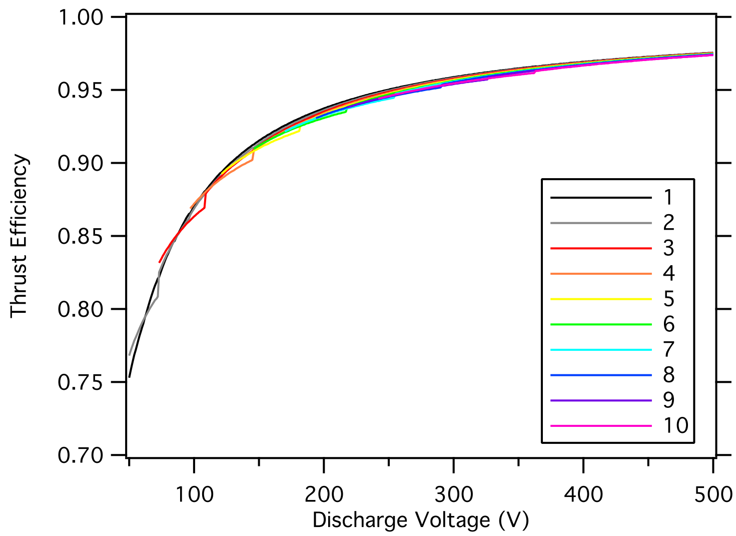

Figure 10. Thrust efficiency for cascading ionization for a range of ionization potential steps.

Figure 12 represents the difference in maximum thrust efficiencies for the two processes. In the higher discharge voltage regime, the difference between the two processes is negligible. At low voltages, however, the difference reaches 6% for Krypton. Hence, if ionization could be forced to occur by recycled electrons, a maximum thrust efficiency gain of 6% could be reaped over cascading ionization.

For high voltage thrusters (>300 V) maximum thrust efficiency corresponds to minimum current to the anode or maximum current efficiency.[2] Figure 13 is from a study conducted earlier this year on the efficiencies of a low voltage Hall thruster. The dashed line represents maximum thrust efficiency, which occurs at a lower magnet current value than the maximum current efficiency. Also appearing on the graph is the voltage utilization efficiency increasing with decreasing magnet current. The magnets are responsible for, among other things, trapping the cathode electrons in the Hall current. Our model plus the empirical data from the previous experiment suggests that by turning down the magnet current, cathode electrons are allowed to penetrate deeper into the discharge chamber and become a larger participant in the ionization of the propellant. This would account for an increase in total thrust efficiency.

VII. Conclusions

Propellant ionization has an inherent energy 'cost' based on the ionization potential of the propellant in use. The manner in which the propellant is ionized, however, introduces additional inefficiencies. In a Hall thruster, or similar device, there are two methods or rather two sources of electrons that can contribute to the ionization process. For missions requiring a Hall thruster operating at discharge voltages > 200 V the penalties associated with the ionization process is negligible. At lower discharge voltages, however, ionization by recycled electrons can benefit thrust efficiency by as much as 6% over cascading ionization. Although complete control over the ionization mechanism cannot be achieved, some degree of control can be exercised by reducing the magnetic

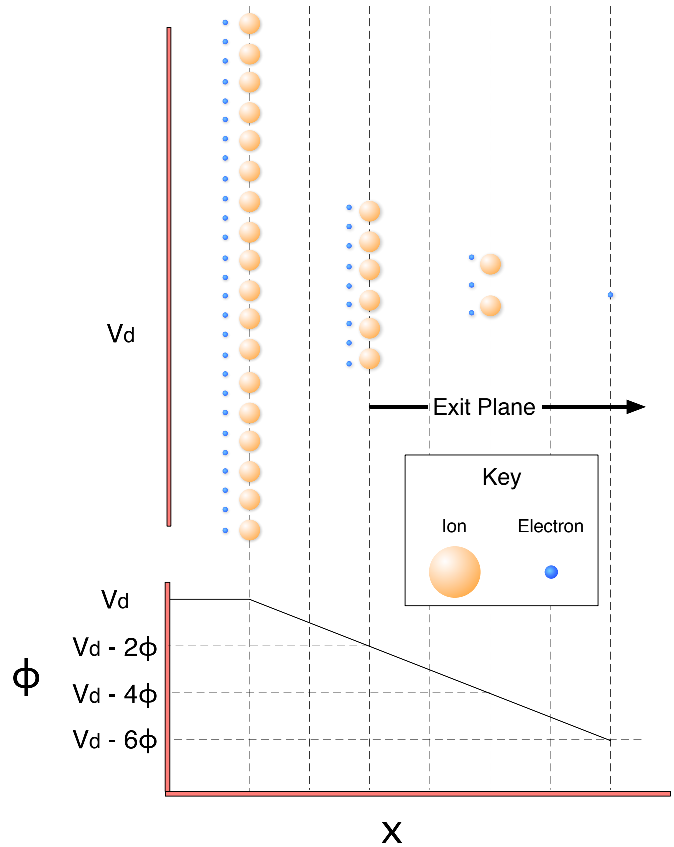

Figure 11. Cascading ionization with a 2ϕ step size in ionization rows.

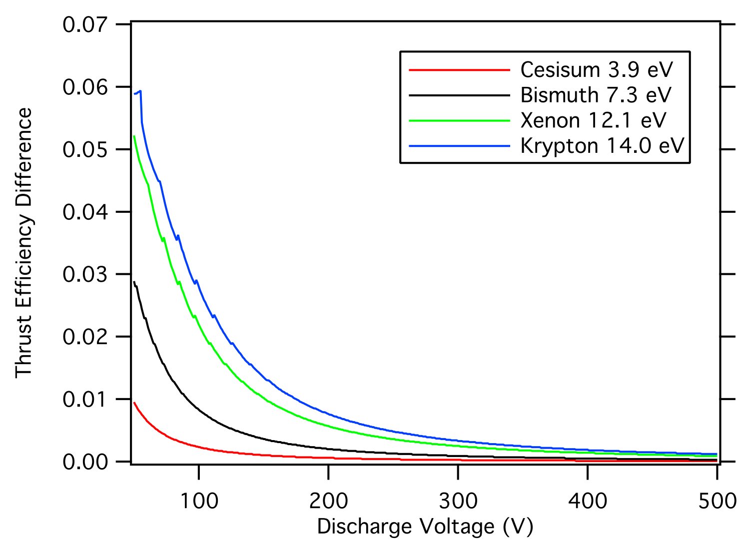

Figure 12. Thrust efficiency for recycled electron ionization minus thrust efficiency for cascading ionization.

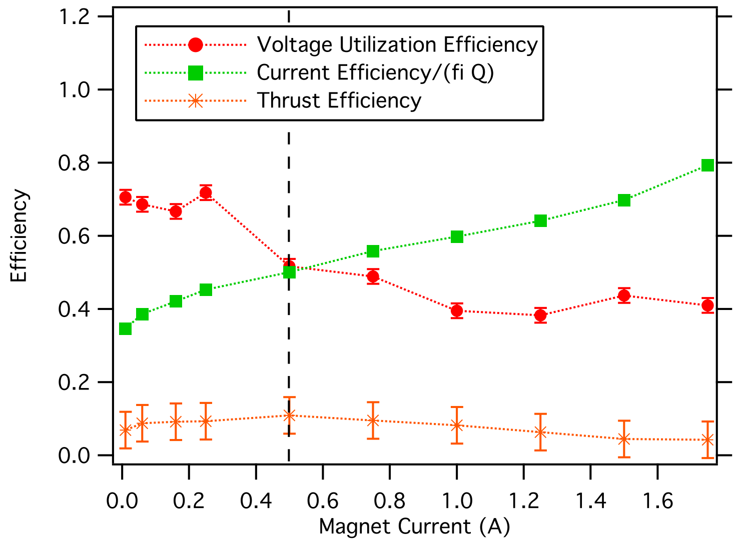

Figure 13.

field strength allowing the recycled electrons to contribute to a larger percentage of the ionization of the propellant. This is empirically evident by results that show peak thrust efficiency occurs at lower than peak values of current efficiency for low voltage thrusters as compared to high voltage thrusters.

A. Appendix

The number of rows, r, available for ionization was defined as $\frac{V_d}{\phi}$. In this model the number is rounded down to the nearest integer since traveling the distance in potential space of a partial row will not contribute enough energy to allow the electron to further ionize any neutrals. The effect of this is different for each individual loss mechanism.

Current efficiency for cascading ionization is the measure of the fraction that the one catalyst electron is as compared to the total number of plasma electrons in the pyramid. The number of electrons in the pyramid only changes when the number of ionization rows, r, changes. Therefore, for a given number of rows the current efficiency will not change and thus it is a step function.

The velocity distribution function and voltage utilization efficiency also exhibit discrete jumps corresponding to a change in the number of rows in the pyramid. These two efficiencies are a function of average ion velocity and the spread of ion velocities. A decrease in the number of rows signifies a decrease in the average ion velocity as well as the spread of ion velocities. Decreasing the spread is actually an efficiency increase. Although these effects are divergent, the losses due to the diminishing average ion velocity is dominant on scale of multiple row changes (multiple changes in the discharge voltage of ϕ or greater). However, at the instance of a row change (decreasing) the effects of the reduced spread in ion velocities causes a discrete increase in efficiency. It is important to note that once all three of the loss mechanisms are multiplied together these effects are negligible in the thrust efficiency

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