Three Body Invariant Manifold Transition with Electric Propulsion

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The work presented in this paper was aimed at analyzing the possibility to successfully exploit the characteristics of three-body systems using different manifolds, combined with Electric Propulsion within the Uranus planetary system. Electric Propulsion is used to link the low energy paths between stable and unstable invariant manifolds computed for the different Uranus - moon systems. A tour of the planetary system involving orbiting phases around the five main moons was designed. This mission requires only a minimum of onboard resources and can be carried out with an acceptable transfer duration. The study demonstrated that the combination of three-body dynamics and Electric Propulsion provides a promising, alternative mission approach that can maximize the mission outcome.

Nomenclature

a	Acceleration
C	Jacobi constant
d	Perturbation
E	Energy
g_0	Earth gravitational acceleration
I_{sp}	Specific impulse
L_1 - L_2	First-Second libration point
m	Mass
P	Power
T_{hr}	Thrust
u	Control vector
v	Eigenvector
x	State of the system
α	Thrust angle
θ	Initial moon's angle
η	Thrust efficiency
λ	Eigenvalue
μ	Mass parameter
τ	Thrust scaling factor
Subscript	
0	Initial state
el	Electric
$mani$	Manifold
s	Stable
u	Unstable

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I. Introduction

During the last years various studies have investigated the possibility to develop tours in planetary systems, often performed in the Jovian system[1–3] , based on the exploitation of three-body dynamics where the necessary velocity changes are provided by chemical means. Investigation of an entire planetary system, including temporarily orbiting the planet's moons, significantly increases the scientific outcome of a mission. For example, this could have been adopted by the Cassini spacecraft currently exploring the Saturnian system, however, without the specific consideration of three-body model advantages.

An approach similar to the mentioned Jovian tour is implemented, however, in this study Electric Propulsion is considered for the execution of the required manifold transitions. A similar strategy was investigated by Topputo[4] , developing a low thrust assisted version of the Jovian tour.

The Uranus vicinity provides an interesting dynamical environment because its main moons are sufficiently massive and near enough to form several three body models with the planet acting as the principal body. In this framework a low energy passage from one moon to another is possible, where propellant requirements can be further reduced under consideration of an electric thruster.

The main purpose of the proposed study is to develop a tour within the Uranus system that visits each one of the selected moons, including a temporary capture obtained by a ballistic arrival and departure arc.

The trajectory starts with a ballistic approach arc towards the outer moon, Oberon, and ends with a stable orbit around Uranus with a radius smaller than the inner moon Miranda's distance. During the complete transfer several closed orbits around Oberon, Titania, Umbriel, Ariel and Miranda are executed. This would enable scientific studies of the main moons for a considerably longer duration when compared with fly-by's.

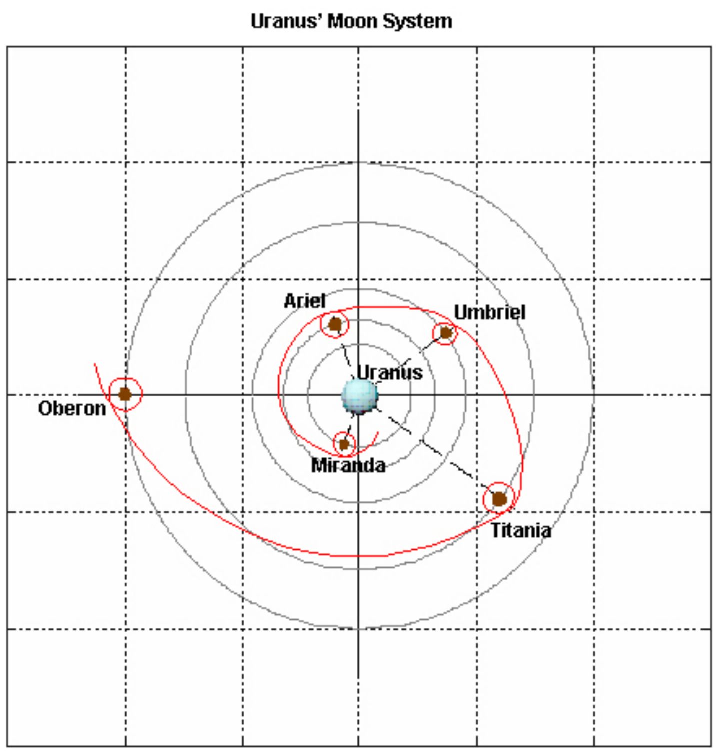

Figure 1. Schematic representation of the Uranus Moons Tour

The developed tour is schematically represented in Fig. 1, where the approaching trajectory to Uranus has been studied in previous works[5,6] .

A. Planetary System

The orbital characteristics of the five main moons[7] present in the Uranus system are presented in Table 1.

Table 1. Uranus Moons characteristics

Uranus Moon	Mass [kg]	Semi-major Axis [km]	Eccentricity	Inclination [deg]
Oberon	$3.014 \cdot 10^{21}$	583519	0.0016	0.700
Titania	$3.526 \cdot 10^{21}$	435910	0.0011	0.340
Umbriel	$1.200 \cdot 10^{21}$	266300	0.0039	0.205
Ariel	$1.350 \cdot 10^{21}$	190020	0.0012	0.260
Miranda	$6.590 \cdot 10^{19}$	129390	0.0013	4.232

It is worth noting that the inclinations presented in the Table 1 are referred to the Uranus equatorial plane. Due to the unique inclination of the axis of rotation of the planet, which has an axial tilt of 98 degrees and thus lies approximately in the ecliptic plane, the moons are orbiting almost perpendicular to this plane. All trajectories calculated in this study are with reference to the Uranus equatorial plane, where the moons are considered to move in circular, equatorial orbits. Therefore application of different, coupled Planar Restricted Three Body Problems (PCR3BP) is considered a valid approximation.

II. Mathematical Models

The tour is computed within the planar circular restricted three body model, based on which the stable and unstable manifolds associated with the libration points of that system are computed. The connection of two states on an unstable and stable manifold, respectively, enables the passage from one system to the other[8] . The connection of two states requires an energy change provided by the Electric Propulsion, where the thrust direction and modulus are two parameters of optimization, explained in more detail further on.

If the manifolds intersect in the position space a single ΔV can suffice to establish transition, however, for the system studied in this work this is not possible. Due to the low mass parameters of the different systems [Table 2], no intersections in the position space are present (Fig. 3). Thus, the only approaches feasible to perform the tour are either using a multi-burn strategy or a continuous thrust, which modifies the spacecraft energy during propelled arcs.

A. System Dynamics

The moons, together with the planet Uranus itself, form different three body environments in which the motion of the spacecraft is studied. Here, the planet and its specific moons are the *primaries* of the relative dynamic system. The dynamics of the CR3BP and its governing equations are briefly recalled in the planar version (PCR3BP):[9]

$$\begin{cases} \ddot{x} - 2\,\dot{y} &= \Omega_x \\ \ddot{y} + 2\,\dot{x} &= \Omega_y \end{cases} \tag{1}$$

where the subscripts denote the partial derivates and Ω is the function:

$$\Omega = \frac{1}{2}\left(x^2 + y^2\right) + \frac{1-\mu}{r_1} + \frac{\mu}{r_2} + \frac{1}{2}\,\mu\,(1-\mu) \tag{2}$$

The distances between the two primaries and the third body are given by:

$$\begin{cases} r_1^2 &= (x+\mu)^2 + y^2 \qquad \text{Uranus} - \text{Spacecraft distance} \\ r_2^2 &= (x-1+\mu)^2 + y^2 \qquad \text{moon} - \text{Spacecraft distance} \end{cases} \tag{3}$$

The CR3BP has one first integral of motion, the *Jacobi integral* (C), which represents the energy (E) in the rotating non-dimensional frame. These are given by:

$$
\begin{aligned}
C &= -(\dot{x}^2 + \dot{y}^2) + 2\,\Omega(x, y) \\
E &= -C/2
\end{aligned}
\tag{4}
$$

Both the x and y coordinates and their derivates are computed with respect to a non dimensional, synodic reference frame centered in the center of mass of the planet and the generic moon, moreover rotating with their relative angular velocity. The *mass parameter* (μ) of the system is the only parameter necessary for the characterization of the specific three body system, which in this case depends on the second primary (moon) selected.

This parameter and the non-dimensionalizing quantities are different for each three body systems and are dependent on the specific moon involved in the trajectory arch (Table 2).

Table 2. Uranus-moon identification parameters

Primaries	μ	Distance Unit [km]	Time Unit [sec]	Mass Unit [kg]
Uranus-Oberon	$3.4792 \cdot 10^{-5}$	583519	$1.8539 \cdot 10^{5}$	$8.6628 \cdot 10^{25}$
Uranus-Titania	$4.0703 \cdot 10^{-5}$	435910	$1.1970 \cdot 10^{5}$	$8.6629 \cdot 10^{25}$
Uranus-Umbriel	$1.3853 \cdot 10^{-5}$	266300	$5.7157 \cdot 10^{4}$	$8.6626 \cdot 10^{25}$
Uranus-Ariel	$1.5584 \cdot 10^{-5}$	190020	$3.4452 \cdot 10^{4}$	$8.6626 \cdot 10^{25}$
Uranus-Miranda	$7.6075 \cdot 10^{-7}$	129390	$1.9358 \cdot 10^{4}$	$8.6625 \cdot 10^{25}$

The system (1) has five equilibrium points; three are collinear with the two primaries and unstable (L_1, L_2 and L_3), the other two form an equilateral triangle with the two primaries and are stable (L_4 and L_5).

The equilibrium points are also critical points for the effective potential, Ω, and the level surface of the Jacobi constant represent the *energy surface* that is an invariant 3-dimensional manifold in a 4-dimensional phase space:

$$
M(\mu, C) = \{(x, y, \dot{x}, \dot{y}) \mid C(x, y, \dot{x}, \dot{y}) = constant\} \tag{5}
$$

Its projection onto position space is the so called *Hill's region*:

$$
M(\mu, C) = \{(x, y) \mid \Omega(x, y) \geq C/2\} \tag{6}
$$

The presence of the spacecraft is forbidden in this region, as the specific kinetic energy would assume negative values. The boundaries of this surface are the zero velocity curves, which are not possible to be crossed for a given energy value. During the transfer, however, the electric thruster modifies the energy of the spacecraft and during the passage from one manifold to the other the zero velocity curves can be crossed.

For increasing spacecraft energy the forbidden region opens a necks around each equilibrium point, where in Fig. 2 the Hill regions for each Uranus-moon system investigated are shown. These are produced for an energy value that assures an opening being present even for the smallest μ, Miranda, considered.

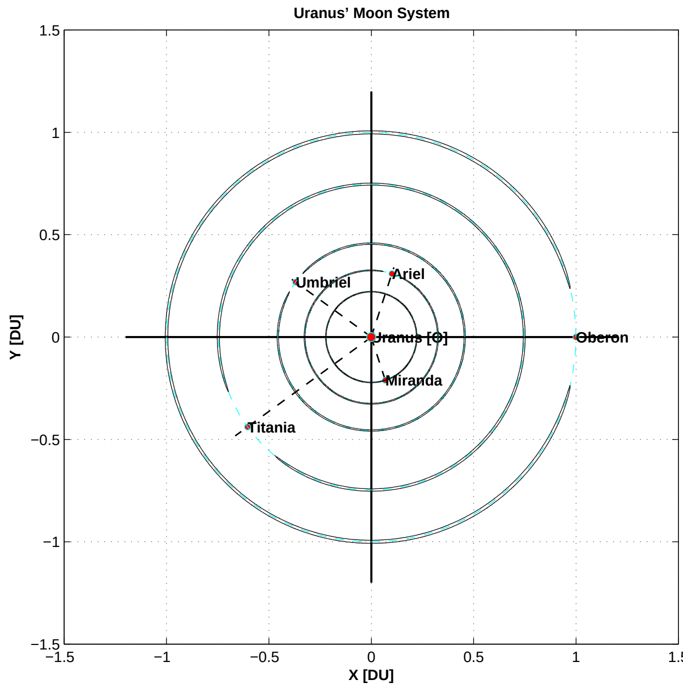

Figure 2. Hill regions of the chosen moons

Using the passages provided by the opened Hill region, transits can be established going from one realm to the other. The lowest energy value that permits a transit from the outer realm, traversing the moon and going to the inner realm is the energy value associated with the second libration point L_2, therefore the manifolds computed in this study are based on this libration point, thus energy level.

These are given by the stable and unstable eigenvalues of the coefficient matrix of the linearized dynamics, and compute two one dimensional manifolds associated with the libration points[10] . The manifold associated with the stable eigenvalues computes to a ballistic trajectory leading towards the libration point, whereas the unstable eigenvalues compute to ballistic trajectories going away from the libration points. The manifolds, in the PCR3BP, represent the saddle part of the dynamics in the vicinity of the unstable libration points (L_1 and L_2 are studied in this work).

The linearized equations of motions can be written as:

$$\dot{\vec{x}} = A \cdot \vec{x} \tag{7}$$

where A is the Jacobian matrix. If λ_s and λ_u represent the stable and the unstable eigenvalues ($\lambda_s < 0$ and $\lambda_u = -\lambda_s$) and v_s and v_u the associated eigenvectors, the computation of the manifolds associated with the point in which the linearization has been made, only requires the propagation of a small perturbation in the direction of v_s and v_u. If x_0 represents the state of the libration point of interest and d the small perturbation the manifolds associated with L_1 and L_2 can be obtained by:

$$\begin{aligned} x_0^s &= x_0 \pm d\, v_s \\ x_0^u &= x_0 \pm d\, v_u \end{aligned} \tag{8}$$

The first of Eqs. 8 must be propagated backward and the second forward, where there are two legs for each manifold. The manifolds associated with L_1 and L_2 for each of the five moons are shown in Fig. 3. It is worth noting that in the figure only the planetary distance is scaled, moreover the manifolds are shown as seen each in their rotating reference frame.

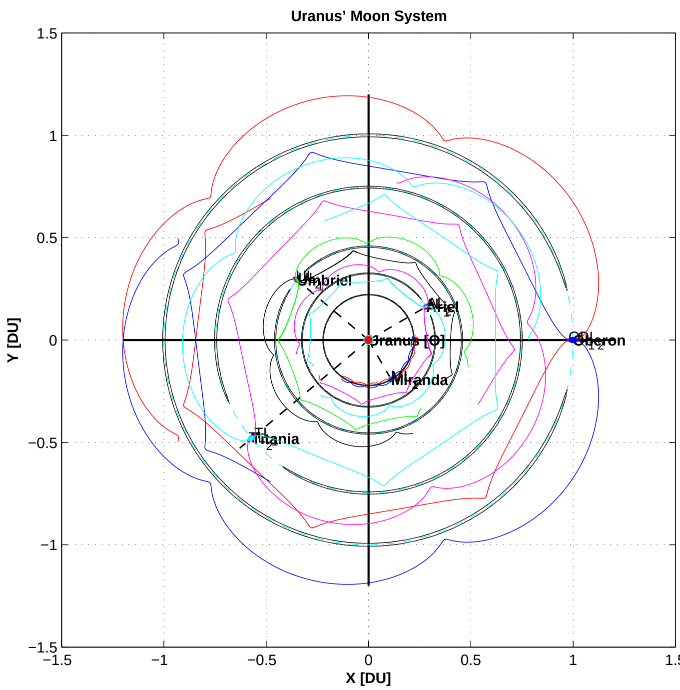

Figure 3. Manifolds of the libration points of the chosen moons

In order to perform the passage among the different three body systems, an in-plane thrust has been included adding an acceleration term to the equations of motion (Eq. 1). To modulate the thrust value a scaling parameter, $\tau \in [0, 1]$, and a thrust angle (α) have been included as shown below. Furthermore an equation for the mass variation has been added.

$$\vec{a} = a\,cos(\alpha)\,\hat{T} + a\,sin(\alpha)\,\hat{N} \tag{9}$$

$$\left\{ \begin{array}{rcl} \ddot{x} - 2\,\dot{y} & = & \Omega_x + \vec{a} \cdot \hat{i} \\ \ddot{y} + 2\,\dot{x} & = & \Omega_y + \vec{a} \cdot \hat{j} \\ \dot{m} & = & -(|\vec{T_{hr}}|\,\tau)/(I_{sp}\,g_0) \end{array} \right. \tag{10}$$

where

$$a = \frac{|\vec{T_{hr}}|\,\tau}{m} \qquad\qquad |\vec{T_{hr}}| = \frac{2\,\eta\,P}{I_{sp}\,g_0} \tag{11}$$

The thrust angle α is measured counterclockwise from the velocity direction, $\hat{i}$ and $\hat{j}$ are the unit vectors of the synodic frame and $\hat{T}$ and $\hat{N}$ are the unit vectors tangent and normal to the trajectory, respectively.

B. Design Approach

The manifolds associated with the libration points of each moon are computed in the relative synodic frame and subsequently translated and scaled to the Uranus - Oberon system, which is chosen as the main system of reference for the tour construction.

The transformation between the two systems takes into account the initial phase of the moons and the associated non-autonomous phase difference during the entire transfer. A manifold of the generic moon, in this system, appears as a trajectory that flows from a radius greater than the radius of the circular orbit of the moon, wraps around the moon's orbit and finally arrives inside the moon's circular orbit. In the main reference frame the manifolds of Oberon are time independent, whereas the manifolds associated with the other moons are time dependent (periodic).

It is worth noting that the manifold used for the construction of the capture arc of each moon is the stable manifold associated with L_2, because this is the ballistic trajectory that leads the spacecraft towards the moon from the outer realm. Its computation requires a propagation of the initial conditions (Eq. 8) for a time span that must begin at the same final time as the powered phase of the previous step. The propagation is performed backward for a time span that identifies the time duration for which the spacecraft lies on the stable manifold. The duration of this time span (t_{mani}) and the initial position of the relative moon (θ) are terms of the control vector. Furthermore, the exit time from the previously considered, unstable manifold of L_1, (t_0), is also considered a term of the control vector.

An appropriate thrust law, based on (α,τ), is required in order to establish the connection between the final conditions of the propulsion phase and the insertion conditions on the manifold of the target moon. The thrust must be considered for a time span to be determined, being (t_{el}), where these parameters are determined by the optimization scheme. The definition of the control vectors elements, for the first passage (Oberon - Titania), are shown in Fig. 4.

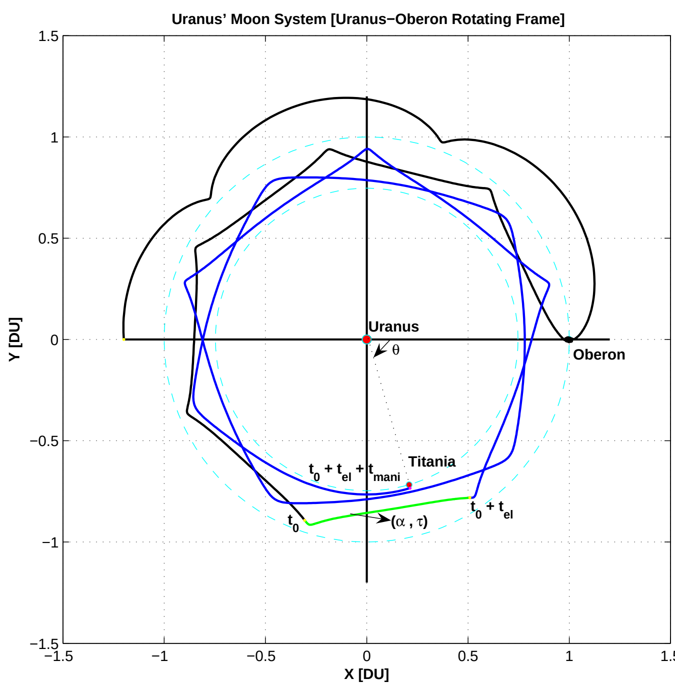

Figure 4. Representation of the control vector parameters for the Oberon - Titania transfer

The complete control vector (u) used for each passage of the tour is:

$$u = \{t_0, t_{el}, t_{mani}, \alpha, \tau, \theta\} \tag{12}$$

The control vector elements are determined by an optimization process that must compute the passage using minimum propellant mass subjected to the constraint that the final state of the propulsion phase must match with the initial state of the L_2 stable manifold of the target moon. It must be noted that the stable manifold associated with the second libration point of the specific moon, when propagated for a ballistic time greater than t_{mani}, performs various closed orbits around that moon after which it passes onto the unstable manifold of L_1 of the same moon. This transition is called a *heteroclinic connection*[11] and is used to obtain the starting conditions for the next passage. In fact, t_0 is the exit time from the unstable manifold associated with the first libration point of the previous moon, considered in the tour.

1. *Non Linear Programming*

The problem is stated as a constrained minimization approach with equality constraints on the final state function of the control vector and with inequality constrains on the elements of the control vector, which has an upper and a lower bound (u_b, l_b), these identify the feasibility envelope (U) for u.

$$\begin{aligned} &\min_U f(u) \;\; subjected\ to: \\ &c_{eq}(u) = 0 \\ &u_{lb} \le u \le u_{ub} \end{aligned} \tag{13}$$

This is a nonlinear programming problem with only active constraints[12] . The functional to be minimized, $f(u)$, is the propellant mass required during the propulsion phase, this is a nonlinear objective function with multiple nonlinear constraints.

A sequential quadratic programming technique has been implemented to find the optimal solution. This technique converts the objective function to quadratic form and linearizes the constraints. Moreover, at each iteration an approximation of the Hessian of the Lagrangian is made using a Quasi-Newton updating method.

This type of optimization process is quite dependent on the initial guess chosen and is possible a high computational load if the initial guess is very poor or if it lies on the boundary of the feasible region.

In this method the thrust law (α,τ) is included in the control vector by a time discretization of the propulsion phase. It has been divided into N-mesh points and at each point the thrust modulus and angle have been considered as elements of the control vector. So the total dimension of u equals: $2 \cdot N + 4$. The thrust law between two consecutive mesh points has been interpolated in a linear way.

Due to the extreme sensibility of the three body system to the initial conditions this kind of approach is not sufficient to assure the passage from one manifold to the other. In fact, the chaotic dynamics of the model lead to completely different solutions even for very similar initial conditions.

In order to improve the precision of the conjunction points a further optimization process has been implemented starting from the solution of the nonlinear programming problem. In this second step the function to be minimized is only the distance in the phase space between the end point of the propelled phase and the initial condition required for the insertion onto the stable L_2 manifold. A simplex algorithm has been used taking the output of the previous step as initial guess. This approach assures a local solution that requires approximately the same propellant mass as given by the minimization process.

It is worth noting that the small perturbation introduced in Eq. 8 has not been considered an optimization parameter. It was chosen an arbitrary small value that assures the manifolds existence for all values of μ considered in the tour. However, from some numerical experiments it seems to affect only the number of closed orbits around the moons, which increase with the diminution of d, and the minimum altitude above them. For some values of the perturbation even impact trajectories are possible, moreover it also marginally affects the time spent on the manifold.

III. Results

The spacecraft's input characteristics, taken from previous studies[5,6] , are summarized in the Table 3:

Table 3. Fixed Parameters

Initial Mass [kg]	500
Specific Impulse [sec]	3200
Thruster Power [W]	1000
Thrust Efficiency [-]	0.5

The initial guess for the thrust angle has been chosen for each powered phase and it is always near 180 because the tour is computed from the outer to the inner moon and an anti-tangential thrust (in a synodic frame) assures a diminution of the spacecraft energy. Without loosing generality, the starting point of the entire tour has been arbitrarily fixed on the intersection of the x-axis of the stable L_2 manifolds of Oberon.

At this position it is taken $t_0 = 0$. The number of mesh points has been arbitrary fixed at 10 and only doubled for long propulsion phases in order to limit the computational time. Sequential quadratic programming has been applied until the convergence of the solution and the relative error on the equality constraints taken. In order to limit the computational time a tolerance of 5% has been imposed on the phase-distance of the conjunction states for the simplex algorithm.

The main system used for the computation of the different transfers is the Uranus - Oberon reference frame, therefore, the complete transfer is shown in this frame, as seen in Fig. 5.

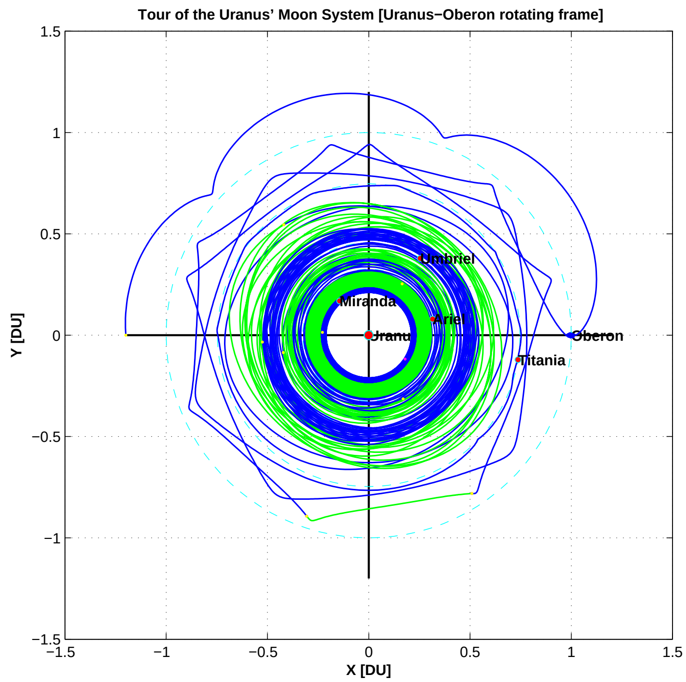

Figure 5. Uranus Moons Tour [Uranus-Oberon rotating frame]

The propulsion phases are represented by green lines and the ballistic arcs by blue lines. With the same convention the trend of position, velocity and mass are also shown in Fig. 6. It is must noted that the velocities are again in the Uranus - Oberon frame in which the velocity of the moon is zero. The passages near each moon are clearly visible from the rapid oscillations in the velocity and position plots.

The passage between the Oberon and the Titania systems requires a very short propulsion phase (approximately 11.5 days) due to the proximity of the two moons and their similar physical conditions. On the other hand the passage to Umbriel and Miranda require very long propulsion arcs (approximately 150 and 151 days respectively), as they are physically very different from the previous moon and with relatively large radial difference. It is worth noting that the PCR3BP approximation for Miranda does not hold as well due to its higher inclination (Table 1).

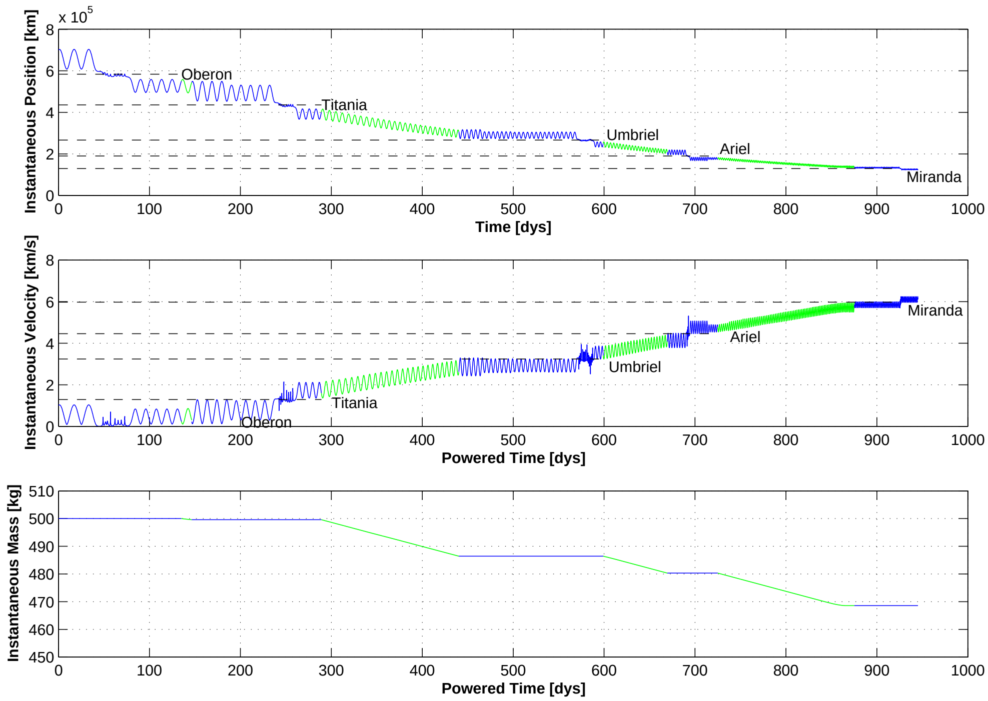

Figure 6. Position, Velocity and Mass Evolution [Uranus-Oberon rotating frame]

The same tour seen in an inertial reference frame centered on the planet is shown in Fig. 7.

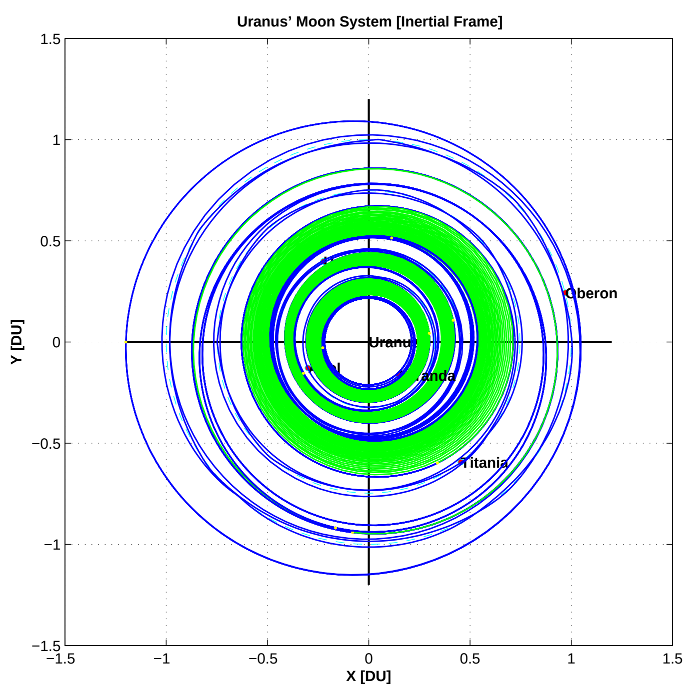

Figure 7. Uranus Moons Tour [Uranus centered inertial frame]

The ballistic arcs that correspond to heteroclinic connections between the stable manifold of L_2 and the unstable one of L_1 of the same moon are shown in Fig. 8. These are propagated for the effective time for which the spacecraft lies on them and are shown in the relative non-dimensional synodic frame with the Hill region relative to the L_2 energy value associated with the specific moon. The ballistic capture performed around Oberon is already shown in Fig. 5

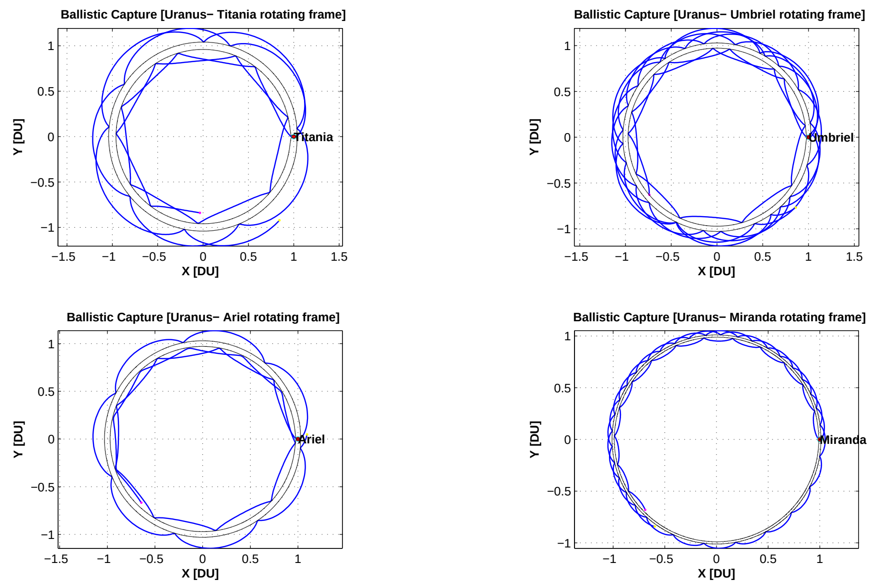

Figure 8. Ballistic Captures and Escapes

The number of closed orbits around each moon and the duration of the ballistic capture are strongly dependent on the value of the perturbation used to compute the relevant manifold; however, for the tour studied the time spent around each moon ranges from several days to almost a month.

A close-up of the realms of the moons is presented in Fig. 9, where it is possible to identify the closed orbits performed around each moon and the ballistic incoming and outgoing arcs. The trajectories are going from the right to the left. During the passage between the realms the trajectory passes near the libration points; in particular near L_2 during the approach phase and near L_1 during the departure phase, always respecting the forbidden regions.

Since no optimization on the value of d has been performed there is no control on the altitude of the capture orbit around the generic moon, therefore also impact trajectories can be present. The study of the kind of orbit around each moon, however, is beyond the scope of this work and the particular closed orbit can be obtained with two very small ΔVs, in the order of some $[m/s]$, which can deform the trajectory into a predefined orbit and afterwards enable continuation on the prescribed trajectory.

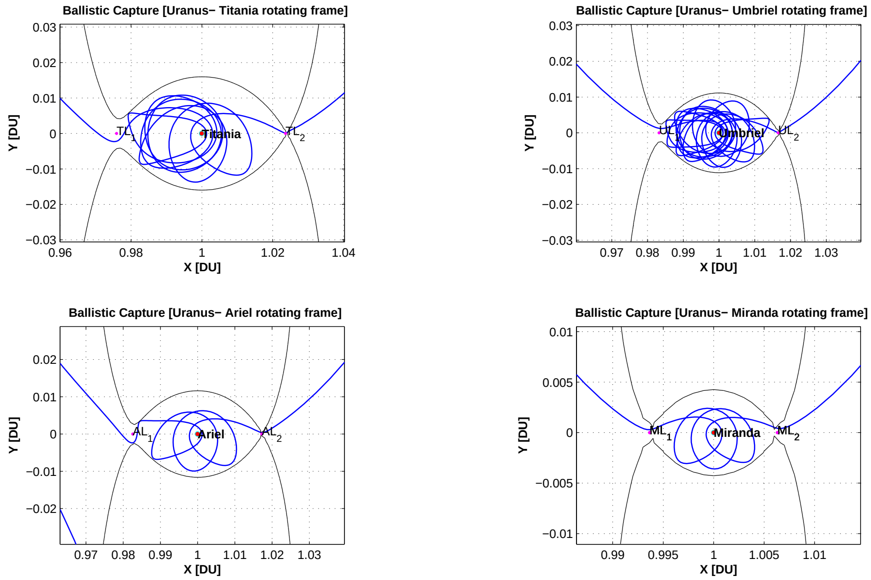

Figure 9. Ballistic Captures and Escapes

Finally also the evolution of the energy and the Jacobi constant during the entire tour are presented in Fig. 10.

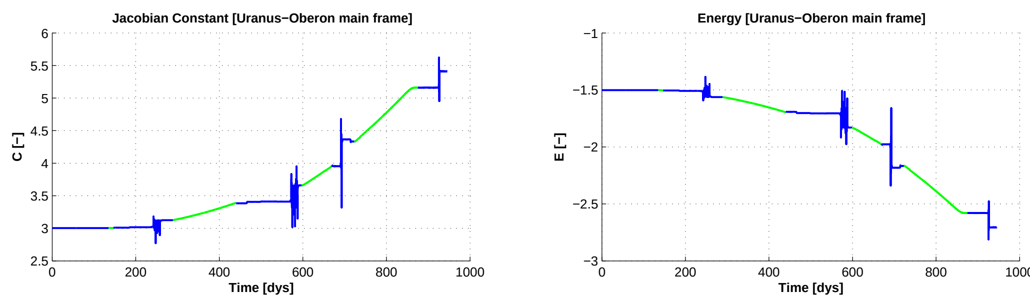

Figure 10. Energy evolution [Uranus-Oberon centered inertial frame]

The energy is shown as seen in the Uranus - Oberon synodic frame. The energy on the manifolds, both stable and unstable, of Oberon is constant also during the passage near the moon. When the energy for the other moon passages is converted in this frame, the transit near the body modifies the energy of the spacecraft. Moreover, also the energy on the other manifolds appears only approximately constant.

IV. Conclusion

The tour proposed uses both the advantages of dynamical system theory within the three body model and of Electric Propulsion. This combination opens a wide range of possibilities and the passages among the moons of Uranus are just a benchmark for different missions. The tour constructed realizes the passages among the five main moons with closed orbits around each one. This enables a wide spectrum of scientific studies for each moon and finally the spacecrafts ends its tour in a closed orbit around Uranus. The propellant mass fraction required for the entire tour is approximately 0.062879 that corresponds to a ΔV of 2.0387 $[km/s]$. This value is very small and can easily be included in the launch mass budget of the spacecraft. The total time required is approximately 930 dys to lead the spacecraft beyond the inner moon. This time must be added to the transfer time towards Uranus that remains the most stringent constraint to the feasibility of this kind of missions.

Further development of the present work could deal with the use of the manifolds associated with the periodic orbits around the libration points. This provides more freedom in the intersection of the different manifolds and also the possibility to control the size and the number of the closed orbits around the moons. Moreover a different optimization approach can be applied in order to also perform a global search with respect to the optimization parameters. Finally, a three dimensional approach could better represent the true gravitational environment of the Uranus moons and the trajectory studied using a three body approximation can be used as first guess for a complete propagation in a multi-body model, such as superposition of bi-circular models.

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