A Review of the Hall Thruster Scaling Methodology

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Abstract: Hall thruster scaling methodology is a powerful tool to carry out a preliminary design of a Hall Effect Thruster (HET). This methodology is based on a number of scaling relations between all the relevant system parameters. It consists of several "black-boxes", each one modelling a physical process taking place inside HETs. With the present work some of these models have been re-analyzed and properly refined, in order to better reproduce the behaviour of a real thruster. If the old models were already good to give a taste of what really happens inside the thruster, this new models considerably improved give a more realistic link between all the phenomena and so the parameters involved. Finally, the "black-boxes" so modified have been reintegrated in the scaling methodology modified also adding a new physical scaling fundamental parameter. So the scaling method has been applied to scale a chosen reference thruster (SPT100), and the results obtained have been compared with the available data on existing devices; some comparison have been done also with the results of the old models previously utilized in the scaling methodology.

Latin

b	Channel width
B	Magnetic field
D	Sheath length
E_I	First ionization energy
d	Channel mean diameter
$d_* = D/\lambda_{Debye}$	Non dimensional Sheath length
e	internal energy
E	energy
k	Rate constant
L	Channel depth
m	mass
M_p	propellant atomic mass
n	Number density
N_A	Avogadro Number
T	Temperature
u_i	velocity components
$\underline{V}$	Velocity
$\bar{v}_e$	$\left(\frac{8T_e}{\pi m_e}\right)^{1/2}$
x	x-coordinate
y	y-coordinate
z	z-coordinate

Greek

ε_0	dielectric Vacuum constant
ε	loss fractions
λ_{Debye}	Debye Length
$\lambda_{sheath} = \frac{D}{b}$	Nondimensional sheath length
λ_i	channel fraction of ionization length
λ_{diff}	channel fraction diffusion length
η	Nondimensional potential
Γ	Fluxes to the wall
ρ	Density
σ	Cross section
ν	Collision Frequency
ω	Frequency

Subscripts

a	neutral atoms, anode
ce	electron oscillation
e	electronic
i	ionic, ionization
r	radiative
tot	total

I. Introduction

The complex physical mechanisms involved in a Hall Effect Thruster promote to describe in a more complex way the processes involved such as: ionization, near wall effects affecting the overall thruster efficiency. If on one hand the scaling methodology requires simplicity in the analytical formulation on the other hand is necessary to prove if using more complex formulae, opportunely simplified, the scaling relations, that are the base of the methodology, change in a substantial way.

The present work is the sequel to previous papers[1-2] and deals with a physical deepening and an elaboration of the scaling methodology developed between Pisa Aerospace Engineering Dept. and ALTA in the last few years. In particular the scaling relations used to describe each singe physical process have been is some cases proved with more accurate treatments, in the other cases properly refined in order to account effects that was neglected up to this time. In the next sections first are obtained the scaling relations from more accurate formulas, then the scaling matrix is rewritten according to the changes. Among the fundamental scaling parameter the electronic temperature has been introduced in order to account losses and fundamental lengths in a more accurate way when a similarity performance way of scaling is chosen (even if the electronic temperature is not a verifiable parameter).

Then a simple example of application in similar way of scaling are applied to an high power thruster in order to reproduce correctly performances. Finally, a comparison with the previous methodology is carried on in efficiency prediction.

II. Scaling Model and physical relations

The scaling model that has been widely described in the reference papers[1-2] is here revisited only in the scaling relations that have been verified and in some cases modified according more accurate relations. In particular all the main processes such as ionization, plasma wall interactions are briefly analyzed and the scaling laws are derived and compared with the previous ones.

A. Plasma wall interaction

In the previous paper the relations used to find the scaling laws of the wall losses followed sheath / presheath model with secondary electron emission described by Ahedo[3].

The scaling law was found by considering the power loss to the wall as $P_w = \int_0^L \frac{\Gamma_{i,w}(z)}{1-\delta_s(z)} \cdot \frac{3}{2} kT_e(z) \cdot 2\pi d_m \cdot dz$

$\varepsilon_w \propto \frac{T_e^{3/2} \cdot n \cdot d \cdot L}{P}$.

In this paragraph we follow the energy loss expression proposed by Hobbs and Wesson[4] in space charge saturation condition. The unit length power loss proposed is:

$Q_w = \frac{1}{4} n\bar{v}_e 2T_e F(\sigma)$ where $F(\sigma_0) = 0.33 + 2.2\left(\frac{m_e}{m_i}\right)^{1/2}$. Integrating over the channel length and keeping constant the function shapes along the channel the resulting scaling law is the same find with the previous model $\varepsilon_w \propto \frac{T_e^{3/2} \cdot n \cdot d \cdot L}{P}$. So, also the anode power loss, lifetime and heat flux to the wall have the same scaling laws.

Sheath length evaluation

The previous scaling relation used to obtain sheath length scaling laws refers to the model developed by Ahedo in ref[3] and conducted to the following relation[1-6]

$$\lambda_{sheath} = \frac{2 \cdot 10 \cdot \lambda_{Debye}}{b} \propto \frac{T_e^{1/2} / n^{1/2}}{b} \tag{2.3}$$

In this section the result of the previous model are compared with another model from literature.

If we consider the ***collisionless*** limit we recover the Child's law[5]. When the potential drop across the sheath is large $\eta_{wall} = -e\phi_w / k_b T_e << 1$, we make two approximations: the electron term of Poisson equation is neglected and in absence of collision energy is conserved.

With these simplification the solution of the governing equations is Child's law $\eta = 3^{4/3} 2^{-5/3} u_{Bohm}{}^{2/3} \xi^{4/3}$. For the evaluation of sheath thickness the previous equation can be solved in $\xi = d$ near the wall, so the solution is $d_* = 2^{5/4} 3^{-1} u_{Bohm}{}^{-1/2} \eta_w{}^{3/4}$. Using for wall potential $\eta_w = \frac{-e\phi_W}{k_B T_e}$ and $\phi_W = -\frac{T_e}{2e}\sqrt{\frac{m_i}{2\pi m_e}}$

the scaling law so obtained is $\lambda_{sheath} \propto \frac{u^{-1/2}{}_{Bohm} \lambda_{Debye}}{b} \approx \frac{T_e^{1/2} / n^{1/2}}{b}$.

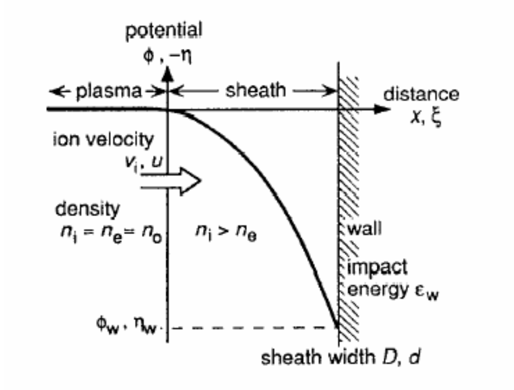
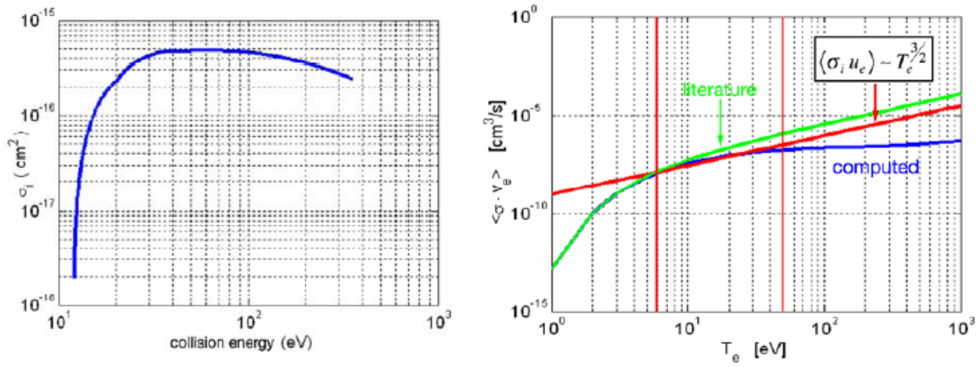


Figure 1. Sheath scheme[5]

These model are in good agreement with exact numerical solutions if the collision parameter in the sheath $\alpha = \frac{\lambda_{Debye}}{\lambda_{mfp}} = \lambda_{Debye} n_n \sigma_s > 0.3$[5]. In this way it has been proved that the previous scaling relation used in the scaling model is coherent with the other used here.

B. Ionization

In extended channel (SPT type) HETs ionization takes place in the portion of the channel upstream of the region of high potential gradient due to the radial magnetic field. To see how ionization modalities obtained in existing thrusters can be preserved in scaled devices, a simple model of the ionization process is considered.
Based on results published in the literature for singly ionized atoms $\sigma_i(T_e)$ has been shown to grow from a value near to zero at the first ionization energy (12.1 eV) to a maximum around 50 eV. This in the previous work was approximated[1] by a polynomial. Based on this approximation, the ionization rate factor can be computed, as shown in Fig 2.



a b

Figure 2 Polynomial cross section (for single ionization xenon atoms) approximation (a) and ionizationrate factor possible approximation[1] (b)

This procedure gives the following scaling law for the ionization rate:

$$R_{ion} \propto n_a \cdot n_e \langle \sigma_i v_e \rangle \approx n_a \cdot n_e \cdot T_e^{3/2}$$

In order to account second ionization, a different procedure to find the rate of ionization has been followed in this work.

The ionization cross section in the case of multiple ionization can be written as follows[7]

$$\sigma_{ion}(T_e) = \sigma_0 \left[1 + \frac{\overline{T}_e}{\left(1+\overline{T}_e\right)^2} \right] \exp\left(-\frac{1}{\overline{T}_e} \right) \tag{2.4}$$

where $\overline{T}_e = \dfrac{T_e}{E_{ion}}$ and $\sigma_0 = 5 \cdot 10^{-20}\,\mathrm{m}^2$. The expression of the rate of ionization is now $R_{ion}(T_e) = n_a n_e \overline{v_e}(T_e)\sigma_{ion}(T_e)$ where $\overline{v_e}(T_e) = \left(\dfrac{8T_e}{\pi m_e} \right)^{1/2}$ and the cross section is represented by 2.4, this expanded in Taylor series (in the interval of *Te* typical of the Hall Thrusters) as reported in the *Appendix A* of the work. The resulting law is oddly the same of the one found following the previous methodology $R_{ion} \propto n_a \cdot n_e \cdot T_e^{3/2}$.

So all the parameters depending only from the ionization rate as λ_i (channel fraction of ionization length) and λ_{diff} channel fraction diffusion length) do not change the respective scaling law.

For what concern the ionization losses the model used was fairly simple. Considering the energy for ionization $E_{ion} = 3 \cdot E_I$ the power loss for ionization $P_{ion} = \dfrac{\dot{m}_p E_{ion}}{\mathrm{M_p}}$ and so the loss factor is $\varepsilon_i = \dfrac{P_{ion}}{P_D} \propto \dfrac{\dot{m}}{P_D}$.

Following the work of Dugan[7] the constant factor that multiplies the Ionization energy can be written in function of the T_e:

$$c_{ion} = 2 + \frac{1}{4}\exp\left(\frac{2}{3\overline{T}_e} \right)$$

So the resulting scaling lows is different respect to the previous model: $\varepsilon_i = \dfrac{P_{ion}}{P_D} \propto \dfrac{\dot{m}}{T_e P_D}$.

III. The scaling matrix

A. Fundamental Parameters

To build our scaling model we have to chose a group of fundamental parameters which can be independently tailored in the preliminary design phase. A new scaling parameter, the Electronic Temperature have been added to the set of parameter in order to better reproduce performances parameters even if this is not verifiable parameter from the external. All the scaling relations that has been used depends on these fundamental parameters[2]. The chosen parameters are the following ones:

1) d channel mean diameter
2) b channel height
3) L channel length
4) V discharge voltage
5) n gas particle density in the injection plane
6) T_e electronic temperature

B. New Scaling matrix

The scaling matrix is derived from the scaling relation presented here and in the companion paper[2]:

PARAMETERS	*Channel mean diameter (L)*	*Channel height (R)*	*Channel length (A)*	*Applied voltage (V)*	*Gas inlet density (N)*	*Reference Te (T)*
d/d_{ref}	ζ_d	1	1	1	1	1
b/b_{ref}	1	ζ_b	1	1	1	1
L/L_{ref}	1	1	ζ_L	1	1	1
V/V_{ref}	1	1	1	ζ_V	1	1
n/n_{ref}	1	1	1	1	ζ_n	1
$\overline{T}_e/\overline{T}_{e_{ref}}$	1	1	1	1	1	ζ_t
J_D/J_{D_ref}	ζ_d	ζ_b	1	1	ζ_n	1
P/P_{ref}	ζ_d	ζ_b	1	ζ_V	ζ_n	1
$B_{\max}/B_{\max\_ref}$	1	1	$(\zeta_L)^{-1}$	ζ_V	1	1
$\lambda_L/\lambda_{L\_ref}$	1	1	1	$(\zeta_V)^{-1}$	1	$(\zeta_t)^{1/2}$
$\lambda_i/\lambda_{i\_ref}$	1	1	$(\zeta_L)^{-1}$	1	$(\zeta_n)^{-1}$	$(\zeta_t)^{-3/2}$
$\lambda_{sheath}/\lambda_{sheath\_ref}$	1	$(\zeta_b)^{-1}$	1	1	$(\zeta_n)^{-1/2}$	$(\zeta_t)^{1/2}$
$\lambda_{diff}/\lambda_{diff\_ref}$	1	1	$(\zeta_L)^{-1/2}$	$(\zeta_V)^{-1/2}$	$(\zeta_n)^{-1/2}$	$(\zeta_t)^{3/4}$
$\varepsilon_w/\varepsilon_{w\_ref}$	1	$(\zeta_b)^{-1}$	ζ_L	$(\zeta_V)^{-1}$	1	$(\zeta_t)^{3/2}$
$\varepsilon_a/\varepsilon_{a\_ref}$	1	1	1	$(\zeta_V)^{-1}$	1	$(\zeta_t)^{3/2}$
$\varepsilon_i/\varepsilon_{i\_ref}$	1	1	1	$(\zeta_V)^{-1}$	1	$(\zeta_t)^{-1}$ / 1
Θ_a/Θ_{a_ref} , Θ_W/Θ_{W_ref}	1	1	1	1	ζ_L	$(\zeta_t)^{3/2}$
t_{life}/t_{life_ref}	1	ζ_b	1	1	ζ_L	$(\zeta_t)^{-1/2}$

Tab. 1.Scaling Matrix (in grey are highlighted the old models)

IV. Example of application and comparison between the models

The thruster selected for the comparisons of the scaling results with the experimental data is the P5 Hall Thruster. The University of Michigan and the United States Air Force Research Laboratory designed and built the P5 5 kW Hall thruster for research purposes. The P5 had a discharge chamber outer diameter of 173 mm and was designed to emulate the characteristics of commercial Hall thrusters. Measurements showed that this thruster had performance characteristics comparable to commercial models. This thruster was used for long term research projects at the University of Michigan.

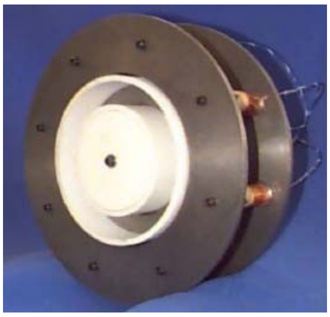


Figure 3 P5 Hall Thruster

The operating conditions selected are reported in the following table:

d_med [mm]	b[mm]	L[mm]	P [W]	Te_max	Voltage[V]	Isp[s]	Thrust[mN]	Current[A]	Tot.Efficiency	Peak B [G]
160.5	25	38	5020	44	500	2259.06	238.9	10.4	0.53	190

Tab. 2 P5 operating point selected[8-9]

In order to obtain the P5 performance using the scaling model the new scaling matrix permit to fix 6 parameters; the parameter selected are : the geometry parameters d, b, L, and physical parameters: Voltage, Current and Electronic Temperature (for the reference thruster SPT-100parameter used ref.to[2]).
The system of equations is

$$\underbrace{\begin{bmatrix} \ln\left(d/d_{ref}\right) \\ \ln\left(b/b_{ref}\right) \\ \ln\left(L/L_{ref}\right) \\ \ln\left(V/V_{ref}\right) \\ \ln\left(J_d/J_{d\,ref}\right) \\ \ln\left(T_e/T_{eref}\right) \end{bmatrix}}_{\vec{B}} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{\underline{\underline{A}}} \underbrace{\begin{bmatrix} \ln\zeta_d \\ \ln\zeta_b \\ \ln\zeta_L \\ \ln\zeta_V \\ \ln\zeta_n \\ \ln\zeta_T \end{bmatrix}}_{\vec{X}} \Rightarrow \vec{X} = \underline{\underline{A}}^{-1}\vec{B}$$

The results in terms of scaling factors are

ζ_d =1.8882, ζ_b =1.6667, ζ_L =1.7273, ζ_V =1.6667, ζ_n = 0.70895, ζ_T =1.3333.

If we substitute the Voltage with the Power the matrix can be rewritten as follows

$$\underbrace{\begin{bmatrix} \ln\left(d/d_{ref}\right) \\ \ln\left(b/b_{ref}\right) \\ \ln\left(L/L_{ref}\right) \\ \ln\left(P/P_{ref}\right) \\ \ln\left(J_d/J_{d\,ref}\right) \\ \ln\left(T_e/T_{eref}\right) \end{bmatrix}}_{\vec{B}} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{\underline{\underline{A}}} \underbrace{\begin{bmatrix} \ln\zeta_d \\ \ln\zeta_b \\ \ln\zeta_L \\ \ln\zeta_V \\ \ln\zeta_n \\ \ln\zeta_T \end{bmatrix}}_{\vec{X}} \Rightarrow \vec{X} = \underline{\underline{A}}^{-1}\vec{B}$$

The resulting scaling factor are the same as before, and this demonstrate the consistency of the model.
The scaling results are listed above:

d [mm]	b [mm]	L [mm]	V [Volt]	lambda_acc [%L]	B [G]	P [W]	m_dot [mg/s]	Efficiency	Thrust_real[mN]	Isp_real[s]	Te[eV]	
160.5	25	38	500	0.366029234	192.98	5020	11.6017778	0.522447	246.6891751	2169.478	44	NEW MODEL
160.5	25	38	500	0.419018509	192.98	5020	11.6017778	0.564765	256.485637	2255.631	33	OLD MODEL
160.5	25	38	500	nd	190	5020	10.34	0.53	238.9	2238.157	44	EXPERIMENTAL

Tab. 3 Scaling results and thruster geometry

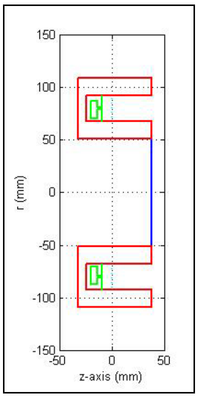


Figure 4 Thruster Geometry from the scaling code.

	Old Model	New Model
ε_w	0.1152	0.1773
ε_a	0.0240	0.0370
ε_i	0.0604	0.0453
λ_{diff}	0.5385	0.4340
λ_i	0.0955	0.1470

Tab. 4 Comparison in loss factors

The results in terms of overall performance shows a good agreement between experimental data and scaling results. It is remarkable that scaling with the new matrix gives a better prediction of the overall efficiency. So the introduction of the T_e among the scaling parameters has an interesting impact on losses evaluation.

Differences can be noticed between the two models in the evaluation of the channel zones length; but even if in the new scaling matrix the physical modeling has probably been refined with the introduction of the Electronic Temperature the lack of experimental data do not permit to assure a real improvement.

V. Conclusions and future work

The results reported in this paper for what concerns the scaling laws shows that the scaling relations except for few cases well predict the parameters behavior in the range of operation of the Hall thruster parameters in study.

For what concern the new scaling matrix with the addition of the T column, it can be said that since the performances depends from the Electronic Temperature, if this value is known, the prediction of the thruster performance and mainly the efficiency evaluation is more accurate (unfortunately few data about electronic temperature of commercial devices are present in literature).

The following steps of this work is to understand how the *T* column can be used in the design of a new thruster even if the T_e is not a parameter like the other five that can be adjusted directly, moreover some experiments could be designed to measure, losses fraction and channel significant lengths in order to validate the model exposed.

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APPENDIX A

The function expressed in Taylor series is:

$f\left(x\right)=\sigma_0\left[1+\frac{x}{\left(1+x\right)^2}\right]\exp\left(-\frac{1}{x}\right)\rightarrow\frac{5}{4e}+\frac{5(x-1)}{4e}+o(x)$ neglecting second order terms the function is $f\left(x\right)\propto x$.