Study on Plasma Lens of Hall Thrusters in Different Operating Modes

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Abstract: In this paper, the characteristic of plasma lens and the rules for ion motion control are investigated in a 1kW P70 Hall thruster in different operating modes. Based on the theory of thermalize potential, magnetic field line curvature is introduced to characterize the focusing effect of plasma lens. The optimized plasma lenses were obtained and analyzed when P70 operated at different propellants or discharge voltages. Two dimensional ion motions were simulated in optimized plasma lens through employing Monte Carlo method. Accordingly, wall loss and ion emission angle were calculated and single ion trajectories were also traced. Both experimental and numerical results show that the curvature in the discharge channel is decreased when the thruster operates at high voltage or uses the propellants with smaller atom mass. It suggests that the focusing ability of plasma lens should be changed for different operating conditons and the optimum plasma lens can be realized. The statistic results indicate that, in the optimized plasma lenses, the ratio of ion loss is less than 2% and ion emission angle is generally less than 25°. In addition, it is found that the distribution of magnetic field curvature is unsymmetrical, which is related with the typical annular effect of Hall thrusters. These results have implications on the magnetic field optimization of Hall thrusters.

Nomenclature

α	=	magnetic field line curvature
β	=	angle of rotation
γ	=	magnetic field line
κ	=	coefficient
ϕ	=	thermalized potential
ϕ_p	=	plasma potential
Ψ	=	magnetic flux
$\vec{B}$	=	Magnetic flux density
B_r	=	radial magnetic field
B_z	=	axial magnetic field
$\vec{e}$	=	unit vector
$\vec{E}$	=	electric field
I_d	=	discharge current
I_1, I_2, I_3	=	coil current

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i	=	direction of magnetic field
j	=	direction of perpendicular to magnetic field
k, θ	=	azimuthal direction for different coordinate
K	=	Boltzmann's constant
m_i	=	ion mass
n_0	=	reference number density
n_e	=	electron number density
N	=	ion number
T_e	=	electron temperature
T_{i0}	=	ion initial temperature
r	=	axial direction
R_c	=	curvature radius
S	=	area of volume surface
U_d	=	discharge voltage
$\vec{v}_i$	=	ion velocity
v_{i0}	=	initial ion velocity
v_r	=	radial velocity of ion flow.
v_z	=	axial velocity of ion flow
z	=	radial direction

I. Introduction

HALL thrusters are a type of electric propulsion devices that use crossed electric and magnetic fields in discharge channel to ionize atoms and accelerate ions, and then generate thrust. In the two dimensional $\vec{z}\times\vec{r}$ plane, the thrust mainly depends on the axial velocity of ion flow, v_z, whereas energy loss always attributes to the radial velocity, v_r. Many investigations have demonstrated that ion motion in radial direction can result in wall erosion, plume divergence, lower efficiency and thrust of Hall thrusters. At present, the magnetic field topography in discharge channel is always designed or optimized to control ion motion. It is the so-called plasma lens, which forms the convergent electric field based on the theory of thermalized potential.[1] Ions are focused toward the discharge channel centerline by the plasma lens, and accordingly, wall erosion and plume divergence are decreased.[2]

The Plasma lens has been applied to many kinds of Hall thrusters. For example, ATON-type Hall thrusters, which were developed by Russia and French in 1996,[3, 4] realized the plasma lens through mending and optimizing magnetic circuit. The experimental results showed that ATON had higher performance, and efficiency and plume divergence half-angle were about 70% and 10°, respectively. In addition, University of Michigan studied NASA-173M series Hall thrusters, which also realized plasma lens through the use of an auxiliary coil. The results indicated that NASA-173M operated better in a wide voltage range. Above 400 V, efficiencies were maintained at >50% and above 900 V, specific impulses >3000 s. The largest gains in performance were observed at 1000 V, where the thrust, specific impulse, and efficiency improved by 10 mN, 200 s, and 5.5%, respectively, to 165 mN, 3360 s, and 51.5%.[5- 8] It can be seen that plasma lens is important for improving the performance of Hall thrusters. During the investigations about the effects of plasma lens on performance in different operating modes, however, we also find that the plasma lens is always adjusted or designed for better control of the ion motion.[8,9] Therefore, study on the characteristic of plasma lens and the rules that control ion motion in different operating modes is significant for designing and optimizing Hall thrusters.

This paper is organized as follows. In Sec. II the parameters describing the plasma lens and corresponding computational method are introduced. In Sec. III variation law of the parameters is investigated experimentally when 1kW P70 Hall thruster worked at different voltages or propellants. In Sec. IV the numerical simulation of ion motion is conducted by Monte Carlo method. Wall loss and distribution of ion emission angle are calculated statistically to estimate the focusing effect of plasma lens; ion trajectories in different operating modes are also traced.

II. Physical Background and Description of Plasma Lens

The magnetic field in the discharge channel of Hall thrusters makes electron magnetized and ion unmagnetized. Electric potential distribution between anode and cathode is decided by the magnetized electron motion, which controls ion motion. The relation between magnetic field line γ and electric potential is given by Eq.(1), which was described first by A. I. Morozov:[3]

$$\phi(\gamma) = \phi_p - \frac{KT_e(\gamma)}{e}\ln\left(\frac{n_e}{n_0}\right) \tag{1}$$

Where $\phi(\gamma)$ is the so-called "thermalized potential" and $\frac{KT_e(\gamma)}{e}\ln\left(\frac{n_e}{n_0}\right)$ is the thermalized item.

Since the thermalized potential is constant along a given field line, the plasma potential along a magnetic field line only varies with the logarithm of the density variation (the density varies roughly by a factor of five). If the electron temperature is low, the potential difference along the magnetic field line is low too and the magnetic field line coincides with the electric field equipotential line. This result has been proved to be an invaluable tool in Hall thrusters design.[5] Thus, we study the characteristics of plasma lens in the corresponding two-dimensional static magnetic field. As shown in Fig. 1, magnetic field topography is characterized by magnetic field line curvature which actually reflects the control ability of plasma lens to ions.

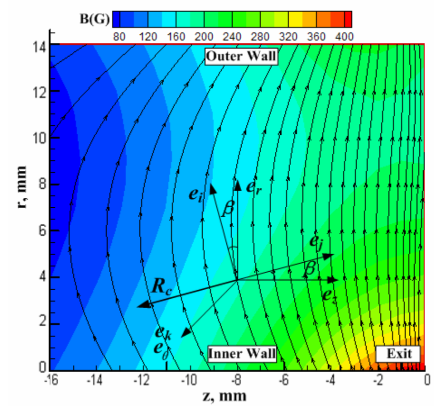


Figure 1. The distribution of two-dimensional static magnetic field.

In order to give the expression of the curvature, we change the coordinate system to be relative to the magnetic field lines. This transformation can be seen in Fig. 1 and described by Eqs. (2) and (3)

$$\begin{pmatrix} \vec{e}_i \\ \vec{e}_j \\ \vec{e}_k \end{pmatrix} = \begin{bmatrix} -\sin\beta & \cos\beta & 0 \\ \cos\beta & \sin\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} \vec{e}_z \\ \vec{e}_r \\ \vec{e}_\theta \end{pmatrix} \tag{2}$$

$$\tan\beta = \frac{B_r}{B_z} \tag{3}$$

Where $\vec{e}_z, \vec{e}_r, \vec{e}_\theta$ are unit vectors in axial, radial, and azimuthal direction, respectively, and $\vec{e}_i, \vec{e}_j, \vec{e}_k$ are also unit vectors, but they are parallel to magnetic field, perpendicular to the magnetic field in the $\vec{z} \times \vec{r}$ plane, and along the azimuthal direction, respectively.[10] β is the angle of rotation. R_c is curvature radius.

Through the transformation, we know that the directions of magnetic field gradient $\nabla\vec{B}$ and curl $\nabla\times\vec{B}$ are *j*-direction and *k*-direction, respectively. In the static magnetic field $\nabla\times\vec{B}$ =0 exists and can be unfolded in new coordinate system, which is expressed by Eq. (4).

$$\begin{aligned} \nabla\times\vec{B} &= \left(\frac{\partial B_j}{\partial k} - \frac{\partial B_k}{\partial j}\right)\vec{e}_i + \left(\frac{1}{j}\frac{\partial B_k}{\partial i} - \frac{\partial B_i}{\partial k}\right)\vec{e}_j + \left(\frac{1}{j}\frac{\partial}{\partial j}(jB_i) - \frac{1}{j}\frac{\partial B_j}{\partial i}\right)\vec{e}_k \\ &= \frac{1}{j}\frac{\partial}{\partial j}(jB_i) = 0 \end{aligned} \tag{4}$$

From Eq. (4), it can be obtained that $B_i \propto \frac{1}{j}$ which means $|B| \propto \frac{1}{R_c}$. Therefore, magnetic field line curvature α can be represented by the following equation:[11]

$$\alpha = \left|\frac{\nabla|B|}{|B|}\right| = \left|-\frac{\vec{R}_c}{R_c^2}\right| = \frac{1}{R_c} \tag{5}$$

III. Experimental Study on the Plasma Lens in Different Operating Modes

A. Experimental Apparatus

Plasma lens of the Hall thruster has been investigated experimentally by HIT Plasma Propulsion Laboratory (HPPL). The test facility includes a stainless-steel vacuum chamber which is 1.5m in diameter and 4m in length as shown in Fig.2. Chamber pressure during thruster operation was maintained at about 3×10^{-2} Pa by a cryogenic pumping system. High-purity research grade xenon and krypton, of which purity level is 99.9995% and 99.999% respectively, were used as propellants for the following measurements. The propellants were supplied through mass flow controllers. The anode and cathode mass flow rate were 3mg/s and 0.2mg/s during all the following experiment.

Figure 3. The vacuum chamber in HPPL.

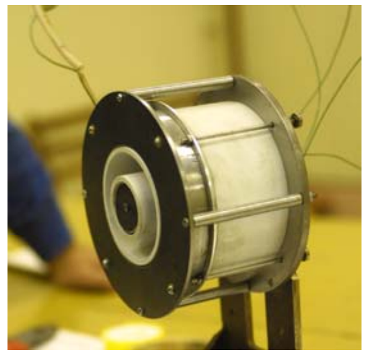


Figure 2. The picture of 1kW P70 Hall thruster.

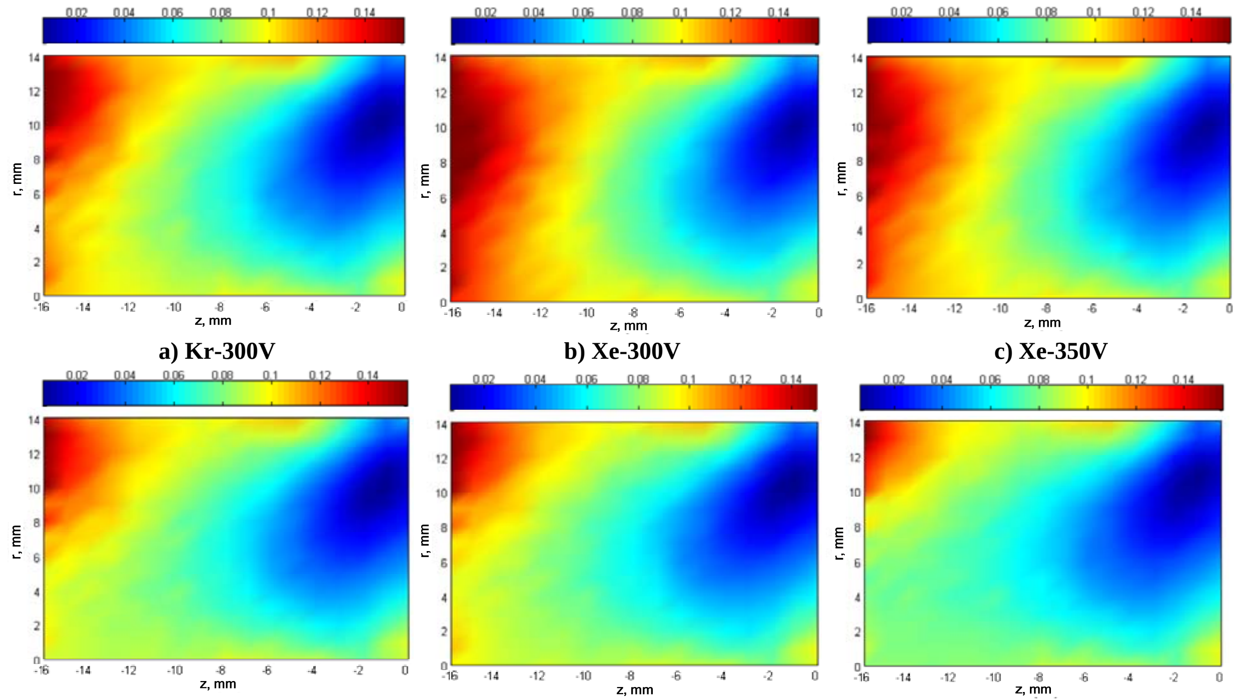
A 1kW P70 Hall thruster as shown in Fig.3, which was developed jointly by MIREA [3] and HPPL, was used in the thruster system. The thruster magnetic system consists of magnetic circuit and three coaxially placed magnetic coils – two primary and one additional. Additional coil has been widely used in laboratory Hall thruster models to increase the opportunities to control the magnetic field topography.[7] The Finite Elements Method Magnetics (FEMM),[12] a user-friendly magnetic field solver available on the internet, was used to calculate the two dimensional distribution of static magnetic field in the $\vec{z}\times\vec{r}$ plane. Thus the corresponding curvature distribution is calculated by Eq.(5) when the parameters of magnetic circuit are known

B. Results and Analysis

The three coil currents of P70 thruster were adjusted for the 300V *Xe* and *Kr* operation points and 350V to 500V *Xe* operation points. The best plasma lens was believed to be realized when the discharge current had a minimum.[3] and the experimental parameters are recorded and listed in Table 1. Accordingly, the distribution of magnetic field curvature α in the $\vec{z}\times\vec{r}$ plane is calculated based on Eq. (5), as shown in Fig.4, where Hall thruster's exit is regard as the origin of axial coordinate. The experimental results indicate that the area of $\alpha > 0.1$ inside the discharge channel increases obviously with the increase of atom mass or the decrease of discharge voltage. It is suggests that the control ability of plasma lens to ion beams should be weakened under the condition of small atom mass or high discharge voltage. We also find that the distribution of α is unsymmetrical and the maximum of α is near the outer wall. We think it relates with the distribution of magnetic field strength. Because of the typical annular effect of Hall thrusters,[13] the magnetic field intensity near outer wall is lower than that near inner wall. Accordingly, electric field intensity is lower near outer wall due to high electron mobility. Therefore, α near outer wall should be increased to enhance the focusing effect of plasma lens. In addition, we speculate that the area with obvious changes in magnetic field curvature may be ionization area.

Table 1. The experimental parameters for the best plasma lens in different operating modes

No.	*Propellants*	U_d(V)	I_d(A)	I_1(A)	I_2(A)	I_3(A)
(*a*)	*Kr*	300	4.46	1.88	2.04	3.39
(*b*)	*Xe*	300	2.93	2.46	2.14	0.88
(*c*)	*Xe*	350	2.88	2.20	2.23	7.80
(*d*)	*Xe*	400	2.97	2.10	1.89	6.05
(*e*)	*Xe*	450	3.02	1.39	1.74	3.96
(*f*)	*Xe*	500	3.4	1.34	1.64	6.20



a) Kr-300V b) Xe-300V c) Xe-350V

d) Xe-400V e) Xe-450V f) Xe-500V

Figure 4. The distribution of the magnetic field line curvature.

IV. Numerical simulation of ion motion in the Plasma Lens

In order to find out the reason why plasma lenses change in different operation modes, we employed the numerical method to investigate the ion motion in different plasma lenses.

A. Basic Equation of the Model

The assumptions on numerical simulation model of ion motion in the plasma lens are as follows:

(i) Ion is unmagnetized because of its large mass.
(ii) Ion is considered as single charged because the portion of multi-charged ion is small.
(iii) No ion collides with particles, and if ion collides with channel wall, it is excluded from ion flow and regarded as ion loss.

Under the above assumptions, the ion momentum can be represented by the following equation：

$$m_i \frac{d\vec{v}_i}{dt} = e\vec{E} \tag{6}$$

Where m_i, $\vec{v}_i$ and $\vec{E}_i$ are ion mass, ion velocity and electric field, respectively.

If the thermalized item in Eq. (1) is not considered, the relationship between the potential ϕ_p and the magnetic flux Ψ in the $\vec{z}\times\vec{r}$ plane can be further expressed by:[3]

$$\phi_p\left(z,r\right) = \kappa\Psi\left(z,r\right) \tag{7}$$

Where κ is a coefficient.

Based on $\phi_p = \int \vec{E}\cdot d\vec{z}$ and $\Psi = \int \vec{B}\cdot d\vec{S}$, Eq.(7) is equivalent to the following scalar equations in axial and radial direction:

$$E_z\left(z,r\right) = \kappa\cdot 2\pi r B_r\left(z,r\right) \tag{8}$$

$$E_r\left(z,r\right) = -\kappa\cdot 2\pi r B_z\left(z,r\right) \tag{9}$$

Where $B_r\left(z,r\right)$ and $B_z\left(z,r\right)$ can be obtained from FEMM.

The initial ion velocity is set from an isotropic Maxwellian distribution by Eq. (10).

$$v_{i0} = \sqrt{\frac{eT_{i0}}{m_i}} \cdot rand \qquad (10)$$

Where T_{i0} is ion initial temperature, and we used $T_{i0} = 4$ eV in our calculation. *rand* is a white noise randomly distributed in the range [0, 1].

Monte Carlo method is employed to calculate large numbers of ion trajectories and corresponding velocities. In this paper, simulated ion number is N=5000 which is adequate for considering both statistic results and operation time.

B. Results and Analysis

The percentage of ion loss in the discharge channel was first calculated and compared for six operating points (see Table 1), and the results are listed in Table 2, which shows that ion loss is very low and the loss ratio is less than 2%. It suggests that the focusing effect of the plasma lens is so good that ions are far from walls and focused on the centerline of discharge channel. That is to say, the majority ions are accelerated by the electric field in the discharge channel, and then emitted from the channel exit. The status of Hall thruster plume relates with the emissive ions, especially with the ion emission angle which will affect the plume divergence.

Table 2. The percentage of ion loss in optimized plasma lens in different operating modes

Operation points	*Kr*-300V	*Xe*-300V	*Xe*-350V	*Xe*-400V	Xe-450V	*Xe*-500V
Ion loss (%)	0.68	1.50	1.34	0.58	0.32	0.34

The statistic of ion emission angle for six operation points above are shown in Fig.5. The sign of emission angle relates with the coordinate directions. Here the positive angles suggest that ions should be diffuse outward, whereas the negative angles suggest that ions should be focused toward the symmetry axis of the Hall thruster. In Fig.5, all results show that the number of ions with negative angle is more than that with positive angle. The negative angle is generally less than 25° and the positive angle is less than 18°. It means ions are mainly focused on the symmetry axis of the thruster in the optimized plasma lens. Thus plume divergence should be reduced.

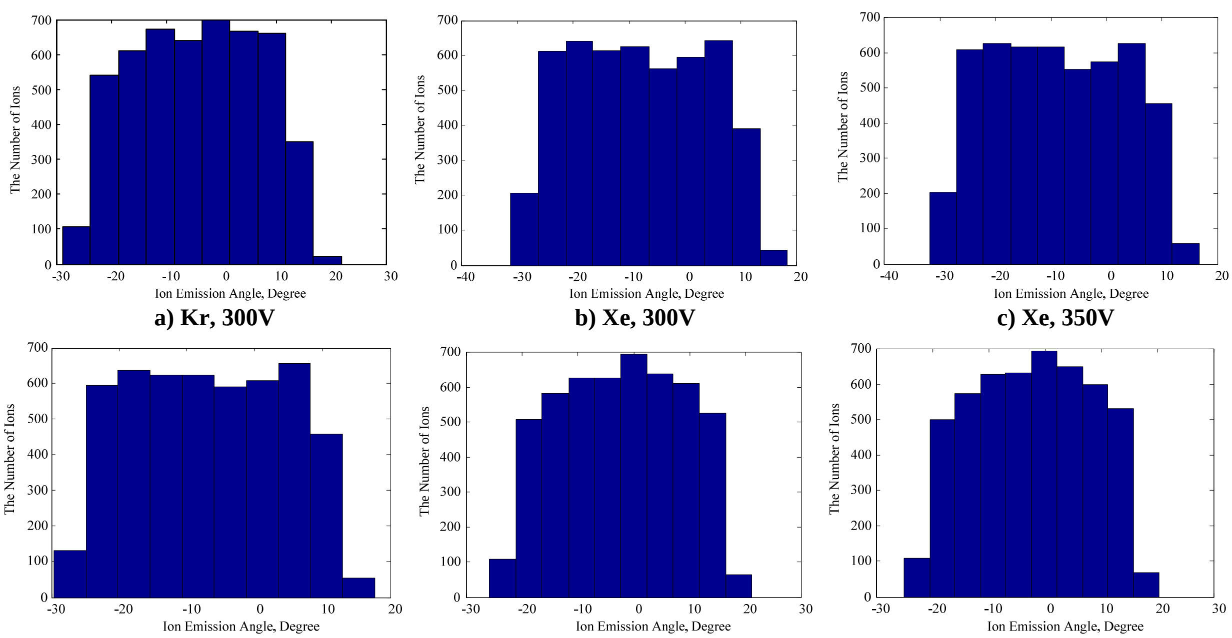


Figure 5. The statistic of ion emission angle at the thruster exit plane.

The numerical results above prove that the plasma lens given by experimental results is better. It is necessary to adjust the plasma lens for obtaining the best performance in different propellants or discharge voltage. However, it is still difficult to know directly why the magnetic field curvature increases obviously with the increase of atom mass or the decrease of discharge voltage from the above statistic. So the single ion trajectories are described.

In order to show the difference of ion trajectories clearly, the initial velocity and position of each ion are given with same value and the distribution of α is also same. As shown in Figures .6 and 7, the results suggest that the focusing effect of the same plasma lens is different in the different operating modes. The control ability of the plasma lens decreases with the increase of the atom mass or the decrease of discharge voltage. Because the bigger the ion mass, the bigger the inertia in the same voltage is, it is more difficult for *Xe* ion to change the state of the former motion. That's to say, compare with *Xe* ion, *Ar* ion motion can be controlled easily by the plasma lens. In Fig.6, the direction of *Ar*, *Kr*, *Xe* ion trajectory are changed in sequence. In Fig.7 the same plasma lens is employed to study the *Xe* ion trajectories in the different discharge voltage. The electric field intensity between adjacent equipotential lines will be increased when the discharge voltage is increased. Accordingly, the focusing effect of the plasma lens will be enhanced. In conclusion, the ion trajectories suggest that the curvature should be decreased when the thruster operates at high voltage or uses the propellants with smaller atom mass. These results are accordance to the experimental phenomenon in section III.

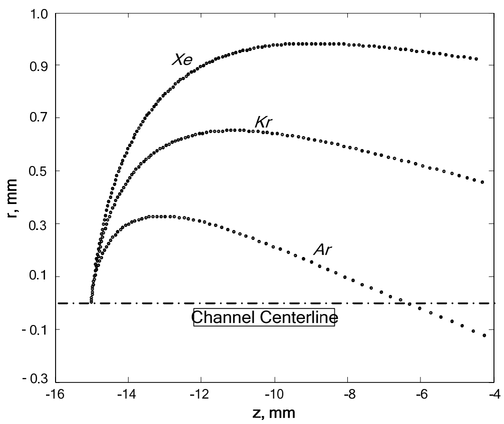


Figure 6. Ion trajectories of the different propellants for the 300V discharge voltage.

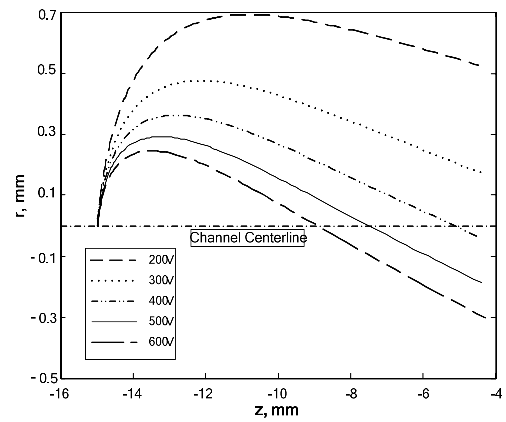


Figure 7. *Xe* ion trajectories for the 200V to 600V discharge voltage.

V. Conclusion

The focusing effect of plasma lens in a P70 Hall thruster has been successfully investigated in different operating modes. The investigations indicate that magnetic field line curvature α can characterize the control ability of plasma lens to ions. The experimental results show that the optimum plasma lenses are different in different operating modes. When the thruster operates at the same discharge voltage, the area of $\alpha > 0.1$ in Hall thrusters discharge channel increases obviously with the increase of atom mass, and when the thruster operates at the same propellant, the area of $\alpha > 0.1$ increases with the decrease of discharge voltage.

The Monte Carlo simulation of ion motion is used to evaluate the wall loss and plume divergence of the optimized plasma lenses. The results show that wall loss is less than 2% and the ion emission angle is less than 25°. The majority of ions are focused toward the center of the discharge channel. The emissive ions finally move to the axis of thruster.

The single ion trajectories are traced in different operating modes with the same distribution of α. The numerical results indicate that the control ability of the same plasma lens to ions is different for different operating points. Ion is more sensitive for plasma lens in high discharge voltage operation and the trajectories are changed easily for small mass. So the curvature should be decreased when the thruster operates at high voltage or uses the propellants with smaller atom mass, which is accordance to the experimental results.

In addition, the unsymmetrical distribution of α in discharge channel is found and considered to be related with typical annular effect of Hall thrusters.

Acknowledgments

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