

# Indirect Optimization Method for Low-Thrust Interplanetary Trajectories

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**Abstract:** The indirect optimization method, which has been used at the Politecnico di Torino to compute the trajectories for the 1st and 2nd editions of the Global Trajectory Optimisation Competition, is presented. In particular, the general features of the optimization method, the numerical technique for the solution of the boundary value problem, and the procedure for finding suitable tentative solutions are described; details of the solutions presented at the competitions are also given.

## I. Introduction

THE optimization of electric propulsion (EP) trajectories for interplanetary missions is a quite difficult task, because of the low thrust level and the existence of a large number of local optima. Moreover, the addition of planetary gravity assists increases the number of parameters and the sensitivity to the initial conditions, making the problem more complex. Indirect optimization methods, which are based on the optimal control theory (OCT), provide accurate solutions but the convergence is difficult and can be obtained only when the initial estimation of the unknown values, which must be provided by the user, is sufficiently close to the actual solution. Direct methods convert the trajectory optimization problem into a nonlinear programming problem and require a large number of parameters to describe the trajectory with sufficient accuracy; they also require an initial guess, and are sensitive to the tentative solution even though it is common opinion that they are characterized by greater robustness than indirect methods. For both direct and indirect methods, an inappropriate initial guess may cause convergence to suboptimal solutions (e.g., a local optimum instead of the global optimum). Evolutionary algorithms have also been proposed for trajectory optimization; they should converge toward the global optimum without requiring a preliminary guess at the solution; however, they are more suitable for the optimization of impulsive trajectories, whereas they suffer when EP trajectories are dealt with, as a large number of parameters are necessary to describe the trajectory and the control history with sufficient accuracy, making the convergence difficult.

An indirect optimization method has been developed by the aerospace propulsion research group of the Politecnico di Torino.<sup>1,2</sup> The method has been applied to a large number of problems concerning different topics of aerospace propulsion, and, in particular, has been widely used to find optimal trajectories for interplanetary missions that employ EP.<sup>2-5</sup> The trajectory is split into arcs at relevant points where constraints on the state variables are applied and/or the control variables experience jumps. The OCT provides the boundary conditions that must be satisfied at the extremities of each arc and a procedure based on Newton's method solves the multipoint boundary value problem, starting from a tentative solution which must be provided by the user.

In the recent past, the concept of global optimization has drawn much interest from the scientific community; many local optima usually exist when complex problems concerning trajectory optimization are dealt with, and the capability of finding the absolute optimum is desired. Indirect optimization methods are however the typical example of local optimizers, as OCT provides conditions for the local optimality of the

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solution; only when the initial guess is close to the correct solution, convergence to the global optimum is obtained; otherwise, only a local optimum is found (if any convergence is at all obtained).

The search for the global optimum becomes particularly difficult when multiple planetary gravity assists are exploited, or a large number of multiple targets must be reached. In these cases, the indirect optimization procedure requires the specification of the sequence of encounters, and this preliminary choice of the mission structure requires user's insight and good knowledge of astrodynamics; an uncorrect choice of the sequence may lead to suboptimal, and often poor-performance, solutions. Once the preliminary structure has been assessed, the whole trajectory can be split into elementary legs, which are separately optimized with suitable boundary conditions at the leg junctions. The solution for each leg is usually found without great efforts, and these basic solutions are then joined and constitute the tentative solution for the optimization code, that eventually converges to a local optimum for the complete mission.

The aerospace propulsion research group of the Dipartimento di Energetica at the Politecnico di Torino took part in the first and second editions of the Global Trajectory Optimisation Competition<sup>a</sup> (GTOC). The first edition<sup>6</sup> was organized by the Advanced Concepts Team (ACT) of the European Space Agency (ESA). The low-thrust trajectory from Earth to asteroid 2001 TW229 that maximizes the change in the asteroid orbital energy after the spacecraft impact was sought. Planetary flybys were allowed. The competition has been won by a team from the Jet Propulsion Laboratory (JPL); the Torino group was fifth.

The second competition was organized by JPL, as the winner of the previous edition. A multiple asteroid rendezvous mission had to be designed for a low-thrust spacecraft which launches from Earth and subsequently performs a rendezvous with one asteroid from each of four defined groups of asteroids. A minimum 90-day stay time at each asteroid was prescribed. Maximization of the ratio of final spacecraft mass to flight time was sought, and no gravity assist was allowed. The team of the Politecnico di Torino won the competition, thus proving that indirect methods can be profitably applied to very complex global optimization problems, once procedures to find suitable tentative solutions have been developed. In this paper, the generalities of the optimization method, the numerical technique for the solution of the boundary value problem, and the procedure for finding suitable tentative solutions are described; some details of the solutions proposed for both competitions are also given.

## II. Description of the Method

The patched-conic approximation is usually sufficient for a preliminary analysis of interplanetary trajectories and is adopted. The planetary flybys can be modeled as discontinuities in the state variables and only the heliocentric motion can be considered. The spacecraft status is described by seven state variables, i.e., the position vector  $\mathbf{r}$ , the velocity vector  $\mathbf{v}$ , and the spacecraft mass  $m$ . To improve the numerical accuracy, the variables are made nondimensional by using the Sun-Earth mean distance ( $1.4959787066 \cdot 10^8$  km), the corresponding circular velocity (29.78469049 km/s, computed assuming  $1.32712428 \cdot 10^{11}$  km<sup>3</sup>/s<sup>2</sup> for the solar gravitational constant), and the spacecraft initial mass as the reference values. The spacecraft leaves the departure planet, and reaches the target object after a succession of planetary encounters (i.e., flybys) or asteroid rendezvous, which is assumed a priori. The state equations are

$$\frac{d\mathbf{r}}{dt} = \mathbf{v} \quad (1)$$

$$\frac{d\mathbf{v}}{dt} = \mathbf{g} + \frac{\mathbf{T}}{m} \quad (2)$$

$$\frac{dm}{dt} = -\frac{T}{c} \quad (3)$$

where an inverse-square gravity field ( $\mathbf{g} = -\mathbf{r}/r^3$ ), and a constant effective exhaust velocity are assumed. The thrust  $\mathbf{T}$  (namely, its magnitude and direction) is the problem control variable. Nuclear electric propulsion with constant specific impulse and available thrust was assumed by both competitions. A given performance index  $J$  is maximized (any index is however permitted, and different indexes are actually considered for the optimization of the basic legs).

An adjoint variable is associated to each state equation to apply OCT. The Hamiltonian is defined as

$$H = \boldsymbol{\lambda}_r^T \mathbf{v} + \boldsymbol{\lambda}_v^T \left( \mathbf{g} + \frac{\mathbf{T}}{m} \right) - \lambda_m \frac{T}{c} \quad (4)$$

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<sup>a</sup><http://www.esa.int/gsp/ACT/mad/op/GTOC/index.htm> [cited 31 August 2007]

and the Euler-Lagrange equations that rule the evolution of the adjoint variables are derived

$$\frac{d\lambda_r^T}{dt} = -\frac{\partial H}{\partial \mathbf{r}} = -\lambda_v^T \left[ \frac{\partial \mathbf{g}}{\partial \mathbf{r}} \right] \quad (5)$$

$$\frac{d\lambda_v^T}{dt} = -\frac{\partial H}{\partial \mathbf{v}} = -\lambda_r^T \quad (6)$$

$$\frac{d\lambda_m}{dt} = -\frac{\partial H}{\partial m} = \frac{\lambda_v}{m^2} T \quad (7)$$

According to Pontryagin's Maximum Principle (PMP), the optimal controls maximize the Hamiltonian. The thrust must be parallel to the primer vector<sup>7</sup>  $\lambda_v$  and the Hamiltonian is rewritten as

$$H = \lambda_r^T \mathbf{v} + \lambda_v^T \mathbf{g} + TS_F \quad (8)$$

where the switching function

$$S_F = \frac{\lambda_v}{m} - \frac{\lambda_m}{c} \quad (9)$$

has been introduced. The thrust magnitude must assume its maximum assigned value  $T_{\max}$  when the switching function  $S_F$  is positive (thrust arcs), whereas it must be set to zero when  $S_F$  is negative (coast arcs), in accordance with PMP.

The optimization procedure requires that the trajectory is split into arcs; state and/or control variables can experience jumps at the junction of two arcs. Thrust arcs (T) and coast arcs (C) are here considered; the sequence of arcs and the type of discontinuity at the arc extremities, either flyby (F) or thrust magnitude discontinuity, that is, the trajectory switching structure, must be specified a priori. The differential problem, which has been outlined, is completed by the boundary conditions at the arc junctions. The constraining equations define the characteristics of the mission that is subject to analysis and only concern the state variables in particular points of the trajectory (e.g., departure, arrival and flybys); they are written in the form  $\chi = 0$ ; a vector of adjoint constants  $\mu$  is associated to  $\chi$ .

The boundary conditions for optimality are provided by OCT and depend on the function  $J$  that is maximized and on the constraining equations; they involve the values of the adjoint variables. In a generic point  $j$ , which is the boundary between two arcs of different nature, one has:<sup>2,8</sup>

$$H_{j-} + \frac{\partial J}{\partial t_{j-}} + \mu^T \frac{\partial \chi}{\partial t_{j-}} = 0 \quad j = 1, \dots, f \quad (10)$$

$$\lambda_{j-}^T - \frac{\partial J}{\partial \mathbf{x}_{j-}} - \mu^T \frac{\partial \chi}{\partial \mathbf{x}_{j-}} = 0 \quad j = 1, \dots, f \quad (11)$$

$$H_{j+} - \frac{\partial J}{\partial t_{j+}} - \mu^T \frac{\partial \chi}{\partial t_{j+}} = 0 \quad j = 0, \dots, f - 1 \quad (12)$$

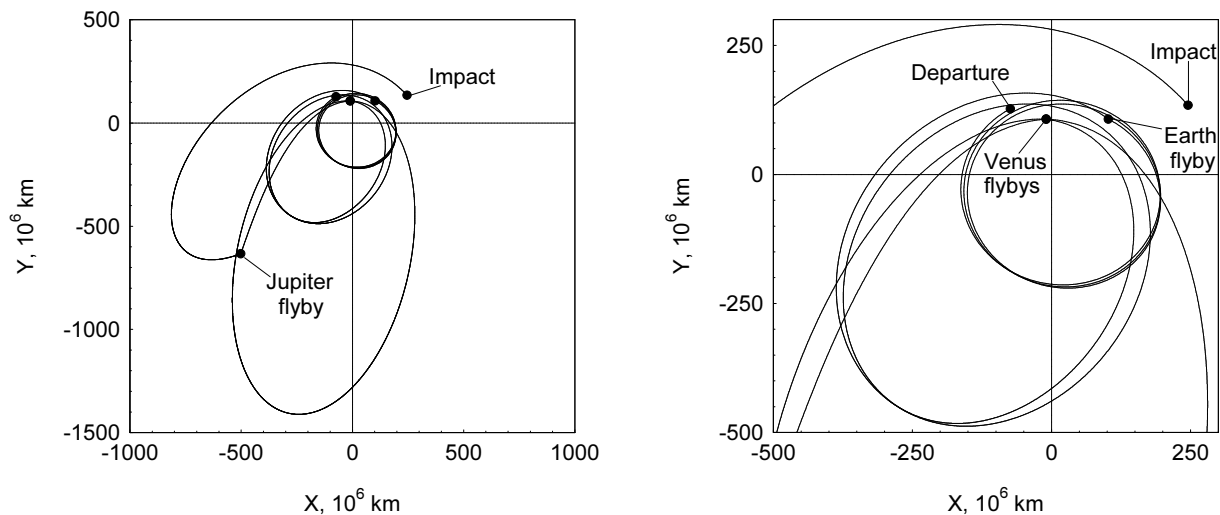
$$\lambda_{j+}^T + \frac{\partial J}{\partial \mathbf{x}_{j+}} + \mu^T \frac{\partial \chi}{\partial \mathbf{x}_{j+}} = 0 \quad j = 0, \dots, f - 1 \quad (13)$$

where subscripts  $j-$  and  $j+$  distinguish the values just before and after the  $j$ -th point; 0 and  $f$  denote the initial and final point of the whole trajectory, respectively.

A systematic application of Eqs. (10)-(13) is sufficient to complete the boundary conditions needed for the numerical optimization of a specific mission. The adjoint constants  $\mu$  are eliminated from Eqs. (10)-(13); the remaining equations and the constraining equations are then collected and written in the form  $\chi^* = 0$ , which contains all the problem boundary conditions. The details of the boundary conditions are not given here for the sake of conciseness and can be found in previous works.<sup>1,2,9</sup>

Equations (1)-(3) and (5)-(7), and the set of boundary conditions  $\chi^* = 0$  define a multipoint boundary value problem (BVP). Constant parameters (e.g., the arc time-lengths) and the initial values of some variables are unknown, and are determined by means of a procedure<sup>10</sup> based on Newton's method. Tentative values  $\mathbf{p}$  are assumed for the unknowns and progressively modified to fulfill the boundary conditions. At each iteration the errors on the boundary conditions  $\chi^*$  and the error-gradient matrix  $\partial \chi^* / \partial \mathbf{p}$  are numerically evaluated; the unknowns are corrected aiming at nullifying the errors under the assumption of linear behavior

$$\Delta \mathbf{p} = -K [\partial \chi^* / \partial \mathbf{p}]^{-1} \chi^* \quad (14)$$



a) Trajectory to impact 2001 TW 229.

b) Enlargement.

Figure 1. Projection on the XY plane of the solution presented at the 1<sup>st</sup> GTOC.

The relaxation parameter  $K$  can be set to one when the tentative guess is close to the solution, and the convergence is quite easy; a reduced value (typically  $K = 0.01$ ) must be adopted when the convergence is more difficult. The use of an Adams-type integrator with variable step-size and order of accuracy<sup>11</sup> guarantees a very high precision.

### III. Mission Structure and Tentative Solutions

A global optimization problem is, in general, not apt to indirect methods, that instead efficiently explore to region surrounding a local optimum. Aiming at the optimal solution (hopefully, the global optimum) two steps are necessary to apply the indirect procedure described in the present paper: first, a suitable switching structure must be assumed for the trajectory; then, a tentative initial guess of the unknowns, close enough to the actual solution, must be provided to the optimization code to allow for convergence. The number of unknown parameters may be quite large: departure and arrival dates, flyby dates, initial velocity, position, and adjoint variables, velocity and adjoint variables soon after flybys, and the times when the engine is switched on and off. Both competitions were extremely challenging for different reasons and the procedures adopted to deal with the proposed problems are described in the following.

#### A. 1st Global Trajectory Optimisation Competition<sup>b</sup>

In the first edition multiple gravity assists could be exploited; the global optimum was found by choosing the correct sequence of planetary flybys in order to make the orbit retrograde and intercept the target asteroid with a large relative velocity. Since the available time was long (30 years), a large number of planetary encounters could be performed, thus greatly reducing the propulsive effort; solutions with up to 12 flybys have been presented by other teams. The best solution has 7 gravity assist from Venus, Earth (three times), Jupiter (twice), and Saturn, uses thrust only during the first leg from Earth to Venus, and a ballistic trajectory is flown from Venus encounter to the impact with asteroid 2001 TW229. This problem seems to be more suitable to direct methods or, better, to evolutionary algorithms, at least in order to define the flyby sequence; since such a means was not available for the Torino team (a procedure based on evolutionary algorithms for the optimization of impulsive trajectories with multiple flyby has been developed<sup>12</sup> only after the competition), the search was limited to a local solution with a reduced number (four) of flybys, that can be managed by the described indirect procedure (more recently missions with up to six gravity assists<sup>13</sup> have been optimized). The flyby sequence (Earth, Venus, Venus, Jupiter) was suggested by simple considerations concerning astrodynamics,<sup>14</sup> and the presented solution is shown in Fig. 1.

<sup>b</sup>URL: <http://www.esa.int/gsp/ACT/mad/pp/GTOC1/gtoc1.htm> [cited 31 August 2007]

The initial guess at the solution was found by means of a preliminary optimization of simple legs that were then joined together. The boundary conditions at the leg extremities are chosen to assure the mission feasibility (e.g., mass continuity across the leg junctions). A 3:2  $\Delta V$ -EGA trajectory is first flown to obtain a suitable hyperbolic excess velocity at the Earth encounter in order to send the spacecraft toward Venus. An Earth-Venus leg, which performs two revolutions around the sun and reaches Venus with a large velocity to move the spacecraft toward Jupiter, constitutes the second elementary leg. A 1:1 resonant orbit with Venus is the third leg, as two Venus flybys are necessary to obtain the required velocity turn. Finally a Venus-Jupiter-2001 TW229 trajectory completes the mission. First, the relative planet positions are left free and optimized in order to find optimal-phasing legs. The desired positions are compared to the planets' actual positions to find mission opportunities. The joining of the legs does not usually require much effort, at least if the boundary conditions at the leg extremities are properly chosen, and the complete solution is eventually obtained.

## B. 2nd Global Trajectory Optimisation Competition<sup>c</sup>

The second edition problem required the choice of the most favorable asteroids among the hundreds of asteroids of each group; an exhaustive search was however not possible due to the extremely large number of combinations (41 billions); in fact, only a limited number of targets could be considered; again, basic astrodynamics suggests the most favorable asteroids. The desire of a monotone increase of the spacecraft energy suggests the group sequence; an asteroid from the low-energy group 4 first, then from the medium-energy groups 3 and 2 (or vice versa) and finally from the large-energy group 1. Four asteroids with the lowest energy and inclination were selected from group 1 as the more favorable final targets; 18 asteroids from group 4 were selected according to semiaxis (which should be large), eccentricity (close to 0), and inclination (less than or close to the value that can be reached with the departure  $V_\infty$ , i.e., about 7 degrees). The group 2 and 3 asteroids must act as “elevators” for the spacecraft, and the aphelion of the second and third asteroids should be close to the perihelion of the third and fourth ones, respectively. The second, third, and fourth asteroids should preferably be intercepted slightly after their perihelion passage. Three windows for the arrival at the final asteroid (2024/2025, 2032/2033, and 2040/2041) were identified in the prescribed time-frame, and a mission length ranging from 9 to 10 years was estimated. The projection on the ecliptic plane of the asteroid orbits for the best solution is shown in Fig. 2; note that the characteristics of the orbital parameters closely match the desired features.

Favorable orbital parameters do not guarantee a good solution, as the correct phasing is mandatory. An approach, based on an impulsive transfer with departure at either interception of Earth's orbit with the asteroid orbital plane and arrival at the asteroid perihelion, was used to obtain a preliminary estimation of the first leg performance; good asteroids are found if the departure hyperbolic excess velocity is lower (or slightly larger) than the available  $V_\infty$  (3.5 km/s); the required phase angle was also estimated and optimal-phasing minimum-time low-thrust transfers from the Earth to the selected asteroids were computed; the actual asteroid position on the possible arrival dates was compared to the arrival position, and actual rendezvous trajectories were computed when the difference was low. A set of about 100 feasible first legs was found (6-18 months long), but only departures about 10 years before the envisaged dates for rendezvous with the last asteroid had to be considered, thus reducing the relevant legs to 14 (toward 8 group 4 asteroids) with departure around 2015/2016, 11 (toward 7 asteroids) in 2022/2033 and 13 (toward 7 asteroids) in 2030/2031.

An approach based on Edelbaum's approximation<sup>15</sup> was developed to estimate if two asteroids are cor-

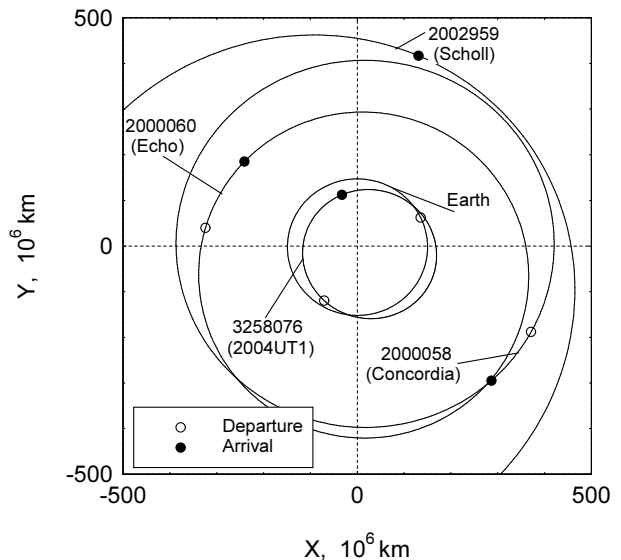


Figure 2. Projection on the XY plane of the asteroid orbits of the 2<sup>nd</sup> GTOC winning trajectory.

<sup>c</sup>URL: <http://www.nasa.gov/vision/universe/solarsystem/asteroidf-20070404.html> [cited 31 August 2007]

rectly phased for the final legs; the procedure requires the estimation of the propulsive  $\Delta V$  which must be provided to move the spacecraft from the departure to the arrival asteroid and considers separately the in-plane and out-of-plane maneuvers. Edelbaum considers constant thrust acceleration and assumes quasi-circular trajectories for the transfer between circular orbits. In the planar case, the optimal thrust direction is parallel to the velocity, resulting in a propulsive  $\Delta V$  which is equal to the absolute magnitude of the change in spacecraft velocity

$$\Delta V = |V_f - V_i| \quad (15)$$

When the spacecraft radius is increased, as in the cases considered here,  $V_f < V_i$  and the propulsive  $\Delta V$  and the actual velocity change  $V_f - V_i$  have the same magnitude and opposite sign. The actual velocity change is the difference between the propulsive  $\Delta V$  and velocity losses, which are only related to gravity, according to Edelbaum's model. Therefore, the gravitational losses, which are related to the change in radius, can be expressed as twice the change in circular velocity between departure and arrival orbit. A plane change maneuver, which introduces misalignment losses, can also be dealt with by Edelbaum approach, but is here treated separately.

A global search among groups 2 and 3 provided suitable third asteroids for a given fourth asteroid and intercept date, and a global search among the other group provided suitable second asteroids for a given third asteroid and intercept date; the approach was not used for the first leg (quite large eccentricity changes are not suitable for Edelbaum's approach) and second leg (long arcs with multiple revolutions allow for phase corrections). After departure and target asteroids have been selected, the procedure assumes the leg time-length ( $\Delta t$ ) and arrival date ( $t_f$ ), which also define the departure date ( $t_0 = t_f - \Delta t$ ), and consequently determines the spacecraft position and velocity at departure and arrival. The mission angular length  $\Delta\vartheta = \vartheta_f - \vartheta_0$  and mean angular velocity  $\dot{\vartheta} = (\dot{\vartheta}_f + \dot{\vartheta}_0)/2$  are then computed, while the mission  $\Delta V$  is estimated as the sum of the in-plane velocity change, i.e.,  $V_f - V_0$  plus gravitational losses  $2(\sqrt{1/r_0} - \sqrt{1/r_f})$  in agreement with Edelbaum's formulation, and the out-of-plane velocity change, approximated with an impulse at the mean velocity between departure and arrival

$$\Delta V = \sqrt{\left[ V_f - V_0 + 2 \left( \sqrt{1/r_0} - \sqrt{1/r_f} \right) \right]^2 + [(V_0 + V_f) \sin(\Delta i/2)]^2} \quad (16)$$

The time and angular length required to obtain the  $\Delta V$  are then computed as  $t_{\Delta V} = m\Delta V/T$  (the spacecraft mass  $m$  during each leg was easily estimated after some preliminary calculation) and  $\Delta\vartheta_{\Delta V} = \dot{\vartheta} t_{\Delta V}$ . A good phasing occurs when the estimated flight time and angular length are close to the assumed values, that is, when  $\Delta t \approx t_{\Delta V}$  and  $\Delta\vartheta \approx \Delta\vartheta_{\Delta V}$ . Ten candidates as the third asteroid were usually found for each of the four target asteroid at each perihelion passage, and the corresponding optimal minimum-time legs were computed by the indirect method: the typical length was 20 months; candidates as the second asteroid were sought for the most promising third asteroids: again, usually ten asteroids were found and the minimum-time legs were computed (with arrival date at least 90 days earlier than the start of the next leg); again, the typical length was about 20 months.

Roughly 400 second-third-fourth asteroid sequences were found. For the most promising sequences, suitable first legs were selected (departure 10 years before the final asteroid was reached) and a minimum-time leg, which joined the first and second asteroids was computed for prescribed arrival date (90 days before the start of the next leg). Feasible solutions occurred when the departure date was at least 90 days later than the arrival at the first asteroid. These all-propulsive minimum-time legs were used to build the tentative solution for the optimization of the whole mission. Due to the particular performance index prescribed by the competition, the all-propulsive minimum-time legs are usually an accurate approximation of the optimal legs; short coast arcs were introduced during the optimization of the complete trajectory according to the suggestions of PMP to compensate for non-optimal phasing (first and second legs) and/or increase the final mass  $m_f$ , when convenient (third and fourth legs). In particular, 650 asteroids pairs have been considered (18 Earth - group 4, 500 group 4 - group 2/3 and 112 group 2/3 - group 1). Roughly 3000 minimum-time legs have been computed and 50 complete missions have been tried; 32 feasible solutions have been obtained and are shown in Table 1, according to the chronological order of obtainment; note that only three final targets appear, as no feasible and interesting solution has been found with the fourth asteroid (2003134) originally selected inside the group 1. The solutions that have been obtained during the first days of the competition are shown even though their performance index is low, as the criteria to found suitable asteroids were not yet fully understood (it is worthwhile to remind that four weeks were available for the competition after the

Table 1. Solutions obtained for the 2<sup>nd</sup> GTOC.

Asteroid sequence (SPKID)	Rank	Departure	Arrival	$m_f$ , kg	$t_f$ , days	$J$ , kg/yr
3258076 2000020 2000147 2002483	32	Oct. 2023	Feb. 2035	649	4132	57.4
3250293 2000429 2000962 2002483	30	Jan. 2031	Oct. 2042	702	3892	65.9
3250293 2000569 2000962 2002959	31	Dec. 2030	Aug. 2045	913	3861	62.3
3250293 2000569 2000962 2002483	27	Dec. 2030	Apr. 2042	903	4141	79.6
3250293 2000232 2000962 2002483	28	Nov. 2031	Apr. 2042	825	3821	78.9
3070801 2000067 2000569 2002483	29	Mar. 2022	Jul. 2033	811	4148	71.4
3070801 2000182 2000058 2002959	8	Jun. 2022	Nov. 2031	849	3458	89.7
3070801 2001621 2000090 2011542	6	Feb. 2032	Jul. 2041	858	3445	91.0
3250293 2001621 2000090 2011542	11	Nov. 2031	Jul. 2041	861	3538	88.8
3070801 2001621 2000090 2002483	22	Feb. 2032	Apr. 2042	849	3727	83.2
3250293 2000037 2000090 2011542	21	Dec. 2030	Apr. 2041	866	3771	83.9
3070801 2000296 2000058 2002959	2	May 2022	Dec. 2031	915	3485	95.9
3070801 2000060 2000058 2002959	23	May 2022	Jan. 2032	796	3505	83.0
3070801 2000005 2000058 2002959	7	Jun. 2022	Nov. 2031	850	3457	89.8
3070801 2000364 2000058 2002959	9	Mar. 2022	Dec. 2031	871	3555	89.5
3070801 2000951 2000342 2002483	16	Apr. 2023	Jul. 2033	889	3743	86.8
3261681 2000189 2000821 2002483	20	Sep. 2015	Sep. 2025	839	3644	84.1
3261681 2000556 2000010 2002483	26	Sep. 2015	Oct. 2025	817	3667	81.3
3070801 2001314 2002407 2002959	15	Apr. 2023	Feb. 2033	859	3596	87.3
3070801 2000270 2002407 2002959	14	Apr. 2023	Mar. 2033	868	3628	87.4
3261681 2000851 2001445 2002483	18	Nov. 2016	Aug. 2026	834	3542	86.0
3250293 2000851 2001445 2002483	24	Mar. 2017	Feb. 2027	818	3626	82.4
3250293 2000851 2001445 2002483	25	Sep. 2017	Dec. 2027	837	3745	81.7
3250293 2000851 2001445 2011542	4	Sep. 2017	Mar. 2027	868	3459	91.7
3329255 2000060 2000058 2002959	3	Jul. 2022	Nov. 2031	883	3429	94.0
3170221 2000007 2000074 2002483	12	Dec. 2016	Nov. 2026	879	3623	88.6
3170221 2000189 2000821 2002483	19	Dec. 2015	Sep. 2025	832	3553	85.5
3258076 2000060 2000058 2002959	1	Oct. 2022	Nov. 2031	898	3326	98.6
3170221 2000556 2000010 2011542	13	Jan. 2016	Jul. 2025	838	3484	87.8
3329255 2000364 2000058 2002959	5	Jul. 2022	Nov. 2031	860	3436	91.4
3261681 2000851 2001445 2011542	10	Nov. 2016	Nov. 2026	890	3656	88.9
3329255 2001314 2002407 2002959	17	Jul. 2023	Mar. 2033	831	3530	86.0

problem disclosure). In some of the earlier solutions the asteroid phasing is not favorable; the mission is lengthened and the performance index reduced, even though the longer available time allows for a better use of the propellant and a quite large final mass. Note that the best trajectory presented at the competition by other teams would rank 13<sup>th</sup> among the solutions found by the Turin team.

The best trajectory, which has been obtained at the end of the third week, presents the shortest time of flight  $t_f$  among the trajectories which maximize  $J$ , but shortest missions have been obtained when the trip-time was minimized; missions with larger final masses  $m_f$  appear, but they are penalized by longer times of flight. The winning trajectory is presented in Figs. 3 and 4; the four asteroids roughly lie on the same plane and the inclination change is almost entirely provided (free) by the departure  $V_\infty$ ; only limited adjustments are required during the flight. The final asteroid pair (2000058 2002959) is the key for the good performance of the best trajectories; it presents a minimum-time trip time of only 357 days.

The strengths of the present method are the extreme accuracy for equation integration and boundary condition enforcement (less than 1 km error on position) which allows for a rigorous local optimization of

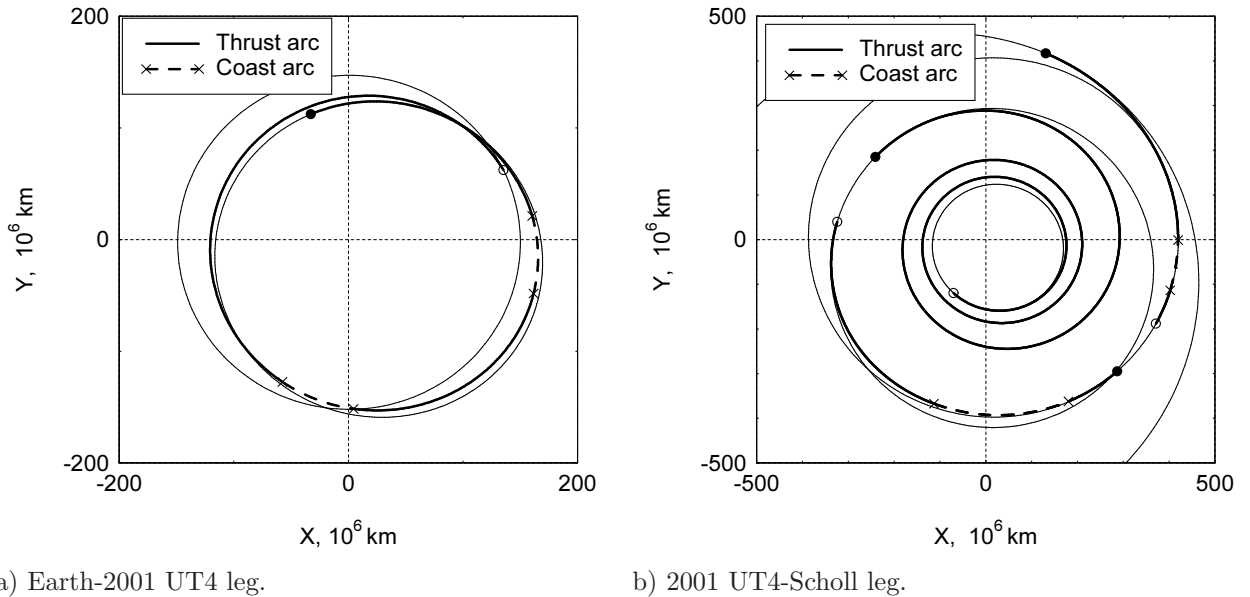


Figure 3. Projection on the XY plane of the 2<sup>nd</sup> GTOC winning trajectory.

basic legs and complete trajectory. This, in turn, is mandatory to obtain a good solution. The competition results show that an accurate optimization of a selected group of sequences provided better results than the approaches based on global approximate optimization tried by other teams, as the exact optimization of the single legs seems fundamental to assess the best sequences, and even a 1-week change in a leg time-length may greatly affect  $J$  or may prevent the completion of the mission.

Indirect methods are very well fit for this problem, because they provide an accurate optimization of the thrust program; moreover, the asteroids had similar parameters and the solution for a given asteroid pair could be used as tentative guess to analyze different options. In fact, the method proved very good but improvements are still possible; the convergence is influenced by the tentative guess, and elementary legs better than those used to build the tentative solutions may exist for the same asteroid pairs, as shown by some solutions of other teams. When an optimal phasing has been assumed, solutions with an improvement up to 30% have been found, suggesting that a sequence of asteroids with favorable phasing could have a large index even though they do not have favorable orbital parameters and have been discharged to reduce the number of options; in particular, only four asteroids have been considered as the final target, but many teams found good solutions aimed at different asteroids from group 1.

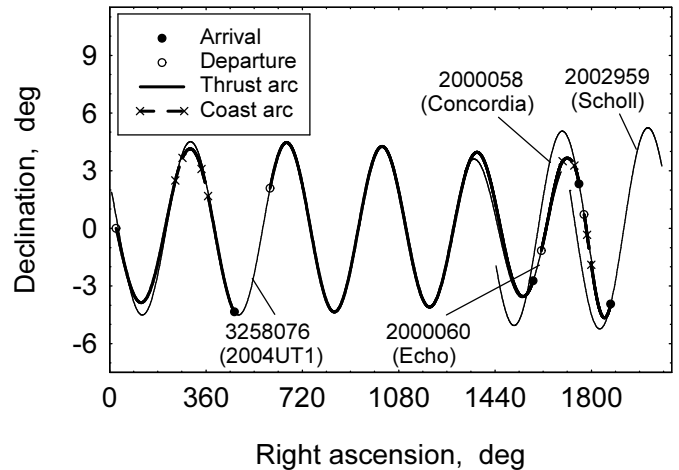


Figure 4. Side view of the 2<sup>nd</sup> GTOC winning trajectory

#### IV. Conclusions

The past editions of the Global Trajectory Optimisation Competition represented a challenging and viable benchmark for the optimization procedure that are currently used worldwide for the preliminary analysis of interplanetary missions. The Politecnico di Torino took part in both competitions using the



indirect procedure which had been developed in the previous years for the search of optimal trajectories using electric propulsion and gravity assists. Even though this optimization procedure was not apt to the first edition problem (which was essentially a ballistic problem with a great number of flybys and marginal use of thrust) the research group was able to find an accurate, locally optimal, (quite) simple solution with good performance. The second edition problem, which required the consecutive rendezvous with four asteroids using EP, was instead very well suited for the indirect procedure of the Torino team that was able to propose a mission with a surprisingly high performance index. Among the method capabilities it is worthwhile to note the extreme accuracy of the numerical solution, corresponding to rendezvous errors less than 1 km in position and 1 mm/s in velocity.

The lessons learned from both competitions show that automatic procedures for global optimization can presently be successful in ballistic problems. Traditional indirect methods are superior in the solution of complex low-thrust problems. For these problems the accuracy of the numerical solver and a good knowledge of the fundamentals of astrodynamics seems to be necessary in order to guide the search for a viable solution. When indirect methods are used, the knowledge of a single solution represents a "gold mine," since it can be used as a first guess to search for other trajectories that may improve the performance index. During the second GTOC the Torino team was therefore able to find a large number of good solutions, and the winning trajectory was among them.

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