Theoretical Modeling of Ionization Processes in Anode Cavity of CAMILA Hall Thruster

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Abstract: The paper is devoted to a development of a simplified theoretical model of ionization processes in an anode cavity of CAMILA Hall thruster. The CAMILA (Co-Axial Magneto-Isolated Anode) is a new concept of the low power Hall thruster. The theoretical analysis is conducted in three fluids hydrodynamic approximation. As a result of the theoretical consideration, the analytical estimations for the main parameters of the anode cavity: the length of the cavity and the magnitude of the longitudinal magnetic field are obtained.

Nomenclature

B = magnetic field induction
E = electric field strength
e = unit positive charge
D_{mc} = middle diameter of the anode cavity
G = quantity, defined by the expression (19)
h = width of the anode cavity
J_d = discharge current
J_e = electron current in the end of the acceleration channel
j_{ey} = Y component of the electron current density
L = length of the anode cavity
M = ion mass
m = electron mass
$\dot{m}$ = mass flow rate
$\dot{m}_{i0}$ = initial mass flow rate of the ions
$\dot{m}_{iL}$ = mass flow rate of the ions as they leave the anode cavity
n = number density of electrons (ions)
n_0 = initial number density of electrons (ions)
n_l = number density of electrons (ions) in the end of the anode cavity
n_a = number density of atoms
p_e = electron pressure
Q = quantity, defined by the expression (20)
Q_{ea} = cross-section of the electron-atom collisions

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S_c = cross-section of the anode cavity
T_e = electron temperature
U = potential drop
V_a = velocity of the Xenon gas
V_i = velocity of the ion "gas"
V_e = velocity of the electron
α = dimensionless initial mass flow rate of the ions
β = coefficient of the ionization
γ = fraction of electron current in the discharge current in the end of the acceleration channel
η_M = mass efficiency of the thruster (without taking into account cathode mass flow rate)
ξ = dimensionless mass flow rate of the ions in the end of the anode cavity
τ_{ea} = time between the collisions of the electron with atoms
ω_e = electron cyclotron frequency

I. Introduction

Considerable attention is presently being focused on development of small power Hall thrusters (50 – 300 W) for applications on micro-spacecraft[1-9]. At building such thrusters, a problem of a compatibility of high efficiency and acceptable lifetime arises. It is due to the fact that reducing mass flow rate causes decreasing the rate of ionization and in order to achieve acceptable mass efficiency in the conventional design of the Hall thruster (HT) a significant reduction of acceleration channel sizes is needed (See, for example, Ref. 10). This, in turn, leads to a fast fall of the thruster lifetime.

In order to solve the problem, a new concept of low power Hall thruster has been proposed in Ref. 11. It has been called CAMILA Hall thruster. (CAMILA – Co-Axial Magneto-Isolated Anode). Experimental investigations of CAMILA-HT-55 (Ref. 12) yielded good results: at power of 149 W, anode efficiency is 41.4 %, at 260 W – efficiency is 54.8 %, at 318 W – efficiency is 59.8 %, at 492 W – efficiency is 65.9 %. The expected lifetime is above 4000 h at power of 200 W, which is due to the relatively low density of the ion current (the outer diameter of an acceleration channel and its width are 55 mm and 12 mm, respectively) and rather large thickness of the walls. With taking into account the large evaluated lifetime of the model, it is possible that the performance of CAMILA HT-55 is the best in the power range of 150 – 500 W in comparison with presently available Hall thrusters.

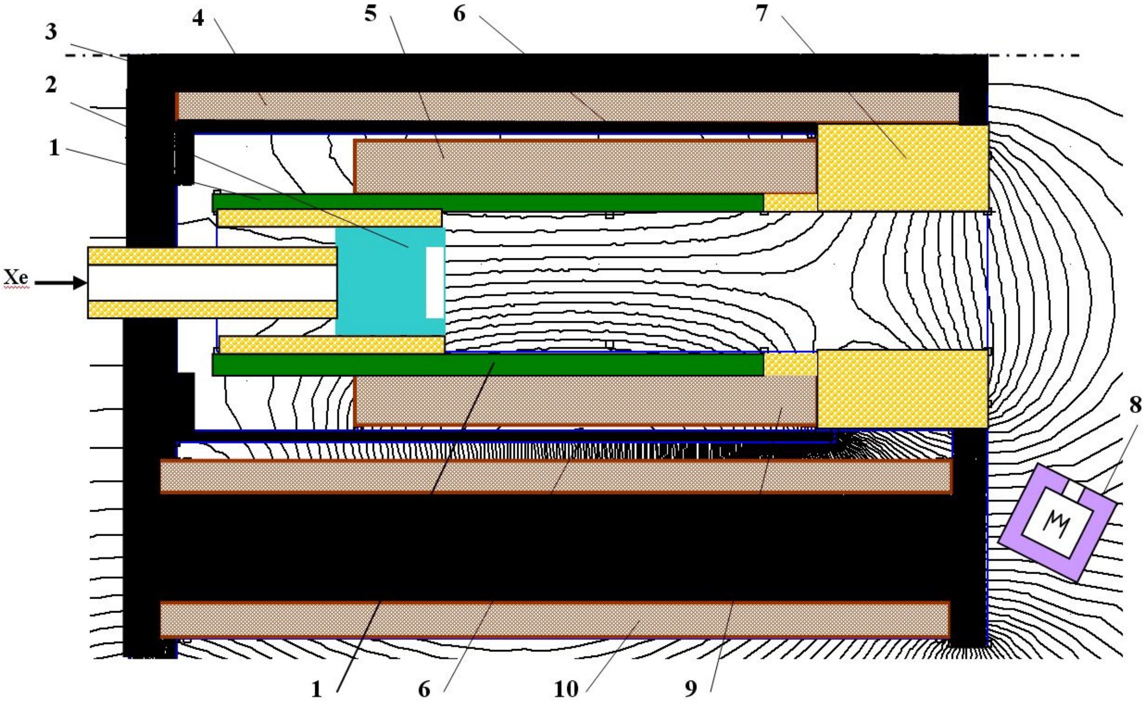
The paper is devoted to a development of a simplified theoretical model of ionization processes in an anode cavity of the CAMILA Hall thruster. The aim is to obtain analytical estimations for the main parameters of the anode cavity: length of the cavity and magnitude of longitudinal magnetic field in it.

II. Basic Principles of CAMILA Hall Thruster Operation

A schematic of the thruster with simulated magnetic field is shown in Fig.1. The CAMILA Hall Thruster consists of the following essentials:

- Magnetic system involving magnetic coils, central magnetic pole, magnetic flange, magnetic screens and magnetic circuit;
- Co-axial acceleration channel;
- Co-axial anode;
- Gas distributor;
- Cathode-neutralizer.

The ionization of propellant is produced mainly in the anode cavity. The anode cavity is formed by two co-axial metallic cylinders, which are kept under anode potential, and an end face of the gas-distributor which is under floating potential. In the anode cavity, the longitudinal magnetic field is applied. It is produced by outer and inner anode magnetic coils with opposite directions of an electric current and the magnetic screens. The propellant, entered the cavity through the gas distributor, is ionized by electrons, oscillated between the end face of the gas-distributor and an exit of the cavity. At strong enough magnetic field in the cavity, it is possible to form a radial electric field, directed to middle surface of the cavity, in spite of a radial electron pressure gradient available. Such radial electric field keeps ions from colliding with the cylindrical walls of the cavity. (This, in turn, allows choosing a length of the cavity from the condition of providing the high degree of the propellant ionization.) After leaving the anode cavity, the ions are accelerated by a longitudinal electric field in the acceleration channel.



1 – Anode, 2 – Gas-distributor, 3 – Magnetic circuit, 4 – Central magnetic coil, 5 – Inner anode coil, 6 – Magnetic screens, 7 – Acceleration channel wall, 8 – Cathode-neutralizer, 9 – Outer anode coil, 10 – Outer magnetic coils

Figure1. Schematic of CAMILA Hall thruster

III. Theoretical Model of Ionization Processes

The length of the cavity and magnitude of the magnetic field induction in it belong to the main parameters of the anode cavity. However, accurate theoretical calculations of these parameters, especially the magnitude of the magnetic induction, are an extremely complicated problem. This is due to

1) two-dimensional nature of the ion motion in the cavity;
2) dispersions of electron and ion velocities that demands, generally speaking, a kinetic description ;
3) the fact that fluctuations in the plasma of the cavity can play important role in an electron conductivity transversely to the magnetic field.

Therefore, we will restrict our calculations of given parameters by the approximated evaluations which should be refined in a course of the experimental investigations of the thruster. The evaluations are based on the development and application of one-dimensional, three-liquid model of the ionization processes in the anode cavity. The fields and current in the cavity, used at building the theoretical model, are shown in Fig. 2.

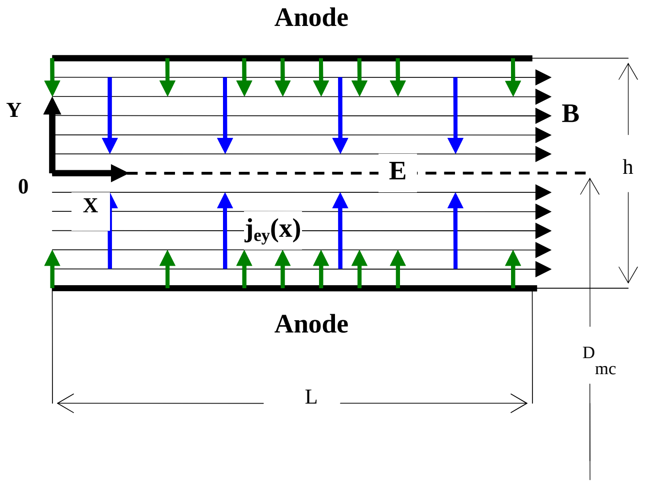


Figure 2. Simplified structure of fields and current in the cavity

For simplicity, we assume that $\frac{h}{D_{mc}} << 1$, where h – the width of the anode cavity, D_{mc} – the middle diameter of the cavity. It means, that we can use the Cartesian frame of reference

At building the theoretical model, we will use steady state equations of continuity and motion for the ions, atoms, and electrons. The ions are considered as non-magnetized. Inertia of the electrons is ignored. The electrons are well magnetized. The latter means that parameter $\omega_e \tau_{ea} >> 1$, where ω_e – the electron cyclotron frequency, τ_{ea} – the time between collisions of the electron with atoms, and Larmour radius of the electrons much less than sizes of the cavity. The plasma is assumed to be quasi-neutral, that is, $n_e \approx n_i \equiv n$, where n is the number density of the electrons (ions).

At the directions of the electric field, shown in Fig. 2, the ions, arisen as the result of the ionization, oscillate between anode surfaces. The average velocity of the ions (the velocity of the ion “gas”) in the Y-direction is equal to zero. The steady state is provided by that the ions leave the area of origin along the magnetic field (i.e. along the X -axis), and the electrons – across the magnetic field (i.e. along the Y –axis). In this case, the equations of continuity for ions and neutral atoms take respectively the following forms:

$$\frac{d(nV_i)}{dx} = \beta n n_a \tag{1}$$

$$\frac{d(n_a V_a)}{dx} = -\beta n n_a \tag{2}$$

where V_i - the velocity of the ion “gas”,

V_a - the velocity of the Xenon gas,
n_a - the number density of the atoms,
β – the coefficient of the ionization.

We assume that no velocities of the ions and atoms depend on *X*. Therefore, the equations of the motion of the ions and atoms may be written just as

$$V_i = constant \tag{3}$$

$$V_a = constant \tag{4}$$

As the equation of the electron motion is convenient to take an expression for *Y* component of the electron current density in generalized Ohm's law:

$$j_{ey} = \frac{e^2 n}{m\omega_e^2 \tau_{ea}}\left(E + \frac{1}{en}\frac{\partial p_e}{\partial y}\right) \tag{5}$$

Where j_{ey} – the *Y* component of the electron current density,

$$\omega_e = \frac{eB}{m} \tag{6}$$

E – the electric field strength in the cavity,
m – the electron mass,
e – the unit positive charge,
B – the magnetic field induction,
$p_e = nT_e$ – the electron pressure,
T_e – the electron temperature.

In order to remain in the frame of one dimensional problem, we replace in Eq. (5) the real gradients of electric potential and electron density by their simplest approximations. In doing so, we assume that *E* and T_e are constant over all bulk of the anode cavity. The sign of the term with T_e is chosen from the assumption that maximum of the electron (ion) density is placed on the central surface of the cavity. As a result, we obtain:

$$E + \frac{1}{en}\frac{\partial p_e}{\partial y} \approx \frac{U}{h/2} + \frac{T_e}{e\, h/2} \tag{7}$$

where *U* is the drop of the potential between the anode and central surface of the cavity.
In the subsequent treatment of the problem, as well as in Eq. (1) and Eq. (2), the value of n is assumed to be averaged over the width of the anode cavity.

The time between the collisions of the electron with atoms can be represented as

$$\tau_{ea} = \frac{1}{< Q_{ea} V_e > n_a} \tag{8}$$

Taking into account Eq. (6), Eq. (7), and Eq. (8), Eq.(5) may be written as

$$j_{ey} = \frac{2m < Q_{ea} V_e > n n_a \left(U + T_e/e\right)}{hB^2} \tag{9}$$

where Q_{ea} – the cross-section of the electron-atom collisions,

V_e – the electron velocity.

Instead of the equation of continuity for the electrons, we will use the equation of an integral balance of the electrons in the anode cavity (all electrons go to both anode surfaces):

$$2\pi D_{mc} \cdot \int_0^L j_{ey}(x)dx = J_d \qquad (10)$$

Where L - the length of the cavity,
J_d – the discharge current.

Taking into consideration Eq. (3) and Eq. (4), from Eq. (1) and Eq (2), we obtain:

$$V_i \frac{dn}{dx} = \beta n n_a \qquad (11)$$

$$V_a \frac{dn_a}{dx} = -\beta n n_a \qquad (12)$$

Joining Eq. (11) and Eq (12), one can obtain:

$$V_i \frac{dn}{dx} + V_a \frac{dn_a}{dx} = 0 \qquad (13)$$

The integral of Eq. (13) has the form:

$$nV_i + n_a V_a = const = \frac{\dot{m}}{S_c M} \qquad (14)$$

where S_c is the cross-section of the anode cavity,

$$S_c = \pi D_{mc} h \qquad (15)$$

$\dot{m}$ - the mass flow rate,
M – the ion mass.

Substituting n_a from Eq. (14) into Eq. (11), we have:

$$V_i \frac{dn}{dx} = \frac{\beta \dot{m} n}{V_a S_c M} - \frac{\beta V_i n^2}{V_a} \qquad (16)$$

Equation (16) is the ordinary differential equation of the first order. Its solution, with taking into accounts that at x = 0, $n = n_0$ has the following form:

$$\ln\left(\frac{n(Qn_0 - G)}{(Qn - G)n_0}\right) = Gx \qquad (17)$$

where
$$G = \frac{\beta \dot{m}}{V_a S_c M V_i} \qquad (18)$$

$$Q = \frac{\beta}{V_a} \tag{19}$$

Let n_L be the ion number density in the end of the anode cavity. In this case from Eq. (17), it follows:

$$L = \frac{1}{G}\ln\left(\frac{n_L(Qn_0 - G)}{(Qn_L - G)n_0}\right) \tag{20}$$

Substituting in Eq. (20) the values of *G* and *Q* from Eq. (18) and Eq. (19), respectively and introducing new quantities:

$$\alpha = \frac{\dot{m}_{i0}}{\dot{m}} \tag{21}$$

$$\xi = \frac{\dot{m}_{iL}}{\dot{m}} \tag{22}$$

where $\dot{m}_{i0} = Mn_0V_iS_c$ - the initial mass flow rate of the ions,

$\dot{m}_{iL} = Mn_LV_iS_c$ -the mass flow rate of the ions in the end of the anode cavity,

we obtain the expression for the length of the anode cavity, corresponding to an acceptable level of the ionization of the flux in the cavity ξ:

$$L = \frac{1}{G}\ln\left(\frac{1-\frac{1}{\alpha}}{1-\frac{1}{\xi}}\right) \tag{23}$$

In order to define the demanded magnitude of the longitudinal magnetic field in the anode cavity, we will transform Eq. (10). For this purpose, from Eq. (17), with taking into account Eq. (.21),
1)we define *n* as a function of *x*:

$$n(x) = \frac{n_0 e^{Gx}}{1-\alpha+\alpha e^{Gx}} \tag{24}$$

2) we calculate the integral $\int_0^L n(x)n_a(x)dx$, substituting into it instead of n and n_a their expressions from Eq. (24) and Eq. (14), respectively, and taking into consideration Eq. (18) and Eq. (21). As a result, we have:

$$\int_0^L n(x)n_a(x)dx = \frac{V_i n_0}{\beta}\left[\frac{e^{GL}(1-\alpha)-1+\alpha}{1-\alpha+\alpha e^{GL}}\right] \tag{25}$$

Taking into account that

$$e^{GL} = \frac{1-\frac{1}{\alpha}}{1-\frac{1}{\xi}} \qquad (26)$$

one can reduce the integral (26) to the following:

$$\int_0^L n(x)n_a(x)dx \approx \frac{V_i n_0}{\beta}\frac{(\xi-\alpha)}{\alpha} \qquad (27)$$

3) The discharge current J_d may be represented as

$$J_d = \frac{e\eta_M \dot{m}}{M(1-\gamma)} \qquad (28)$$

where η_M - the mass efficiency of the thruster (without taking into account cathode mass flow rate),

$\gamma = \frac{J_e}{J_d}$ - the fraction of the electron current in the discharge current in the end of the acceleration channel,

J_e – the electron current in the end of the acceleration channel

If we substitute Eq (28) and Eq (9) into the equation of the balance (10) and suppose that all quantities besides n and n_a do not depend on x, and take into account Eq. (27), then, after arrangement, we will obtain the requirements to the magnetic field in the anode cavity:

$$B \geq \sqrt{\frac{4 < Q_{ea}V_e > m(\xi-\alpha)(1-\gamma)\left(U+\frac{T_e}{e}\right)}{h^2 e\eta_M \beta}} \qquad (29)$$

The approach, used at obtaining inequality (29), resembles A. V. Zharinov's estimation of the length of an ionization and acceleration layer in Hall accelerators (See, for example, Ref. 13). However, in our case, it is essentially complicated by the fact that collecting the electron current is produced by the anodes transversely to the ion flow. Besides, the finite temperature had to be taken into consideration.

The magnitude of needed potential drop U, involved in inequality (29), is chosen from two requirements: 1) to keep the temperature of the electrons in the cavity at level, which provides the effective ionization of Xenon in the cavity (T_e ~ 15-20 eV); 2) to prevent the collisions of the ions, arisen as a consequence of ionization, with the anode walls. The value of U of 40 V is expected to be sufficient to meet both demands. The values of other parameters, directly and indirectly involved in inequality (29) and Eq.(23) may be taken as follows:
T_e =16 eV, $\beta(T_e = 16 \text{ eV}) = 6.6\cdot10^{-14}$ m^3/s (The value of β has been calculated with the use of convenient approximation given in Ref. 14), $<Q_{ea}V_e> = 4\cdot10^{-13}$ m^3/s, h = 0.012 m, α =0.05, ξ=0.9 (It is supposed that 90 % of the propellant is ionized in the anode cavity, the remaining fraction - in the acceleration channel), γ = 0.15, η_M =1. In this case, the induction of the longitudinal magnetic field in the anode cavity should be not less than B = 0.0062 T.

Using Eq. (.23), we find that at $\dot{m} = 8.25\cdot10^{-7}$ kg/s (It corresponds to a HT power of 200 W and the discharge voltage of 300 V), D_{mc} = 0,043 m (S_c = 0.00162 m^2), $M = 2.2\cdot10^{-25}$ kg, $V_i = 3.8\cdot10^3$ m/s, V_a = 150 m/s, the length of the anode cavity is L = 0.019m.

IV. Conclusion

1. The simplified theoretical model of ionization processes in the anode cavity of the CAMILA Hall thruster has been developed.
2. In the frame of the collision mechanism of the conductivity transverse to the magnetic field, the requirements to the length of the anode cavity and magnitude of the longitudinal magnetic field have been found.
3. The presented theoretical model does not take into consideration real two-dimensional motion of the ions. In the future theoretical models, it should be done.
4. The most important and challenging problem is a quantitative estimation of a role of possible plasma instabilities in the transferring the electrons across the magnetic field in the anode cavity. However, taking into account state of the art in prediction of the enhanced transfer of the electrons in the similar conditions, it is highly impossible to do this without conducting before the relevant experiments.

References

1. Hruby, Vlad, (Busek Co. Inc), "Review of Electric Propulsion Activities in the US Industry," *The 39th AIAA/ASME/SAE/ASEE Joint Propulsion Conference and Exhibit*, Huntsville, Alabama, 20 -23 July 2003, AIAA 2003-4441.
2. Kulagin, S. N., Statsenko, I. N., Protsan, Yu. V., Ozimin, V. O., Dubovik, L. G., Gontarev, Yu. K., "Electric Rocket Engines of Small Power for Micro-Satellites," *Universities' Projects of Microsatellites: Trends, Technologies, Realization*, Suzir'ya, Dniepropetrovsk, Ukraine (in Russian), 2002, pp. 67 – 72.
3. Bugrova, A. I., Desiatskov, A. D., Kaufman, H. R., Kharchevnikov, V. K., Morozov, A. I., Zhurin, V. V., "An Improved Small Closed Drift Thruster with Both Conducting and Dielectric Channels," *The 28th International Electric Propulsion Conference*, Toulouse, France, March 17 –21, 2003, IEPC-2003-263.
4. Bugrova, A. I., Lipatov, A. S., Baranov, S. V., Morozov, A. I., and Daren, Yu., "SPT Operation in Mashine-Gun Mode," *The 29th International Electric Propulsion Conference*, Princeton University, NJ, October 31 – November 4, 2005, IEPC-2005-161.
5. Ashkenazy, J., Shitrit, S., and Appelbaum, G., "Hall Thruster Modification for Reduced Power Operation," *The 29th International Electric Propulsion Conference*, Princeton University, NJ, October 31 – November 4, 2005, IEPC-2005-080.
6. Antropov, N. N., Kim, V., Kozlov, V. I., Popov, G. A., and Skrylnikov, A. I., "Small Electric Propulsion Developed by RIAME MAI," *Europian Conference for Aerospace Science (EUCASS 2005)*, Moscow, Russia, July 4 – 7, 2005.
7. Loyan, A. V., Maksymenko, T. A., "Performance Investigation of SPT-20M Low Power Hall Effect Thruster," *The 30th InternationalElectric Propulsion Conference,* Florence, Italy, September 17 – 20, 2007, IEPC-2007-100.
8. Belikov, Mikhail B., Gorshkov, Oleg A., Dyshlyuk, Evgeny N., Lovtzov, Aleksandr S., and Shagayda , Andrey A., "Development of Low-Power Hall Thruster with Lifetime up to 3000 Hours," *The 30th InternationalElectric Propulsion Conference,* Florence, Italy, September 17 – 20, 2007, IEPC-2007-129.
9. Raitses, Yevgeny, Smirnov, Artem, Granstedt, Erik, and Fisch, Nathaniel J., "Overrun Discharge Current Operation of Low Power Cylindrical Hall Thrusters," *The 30th International Electric Propulsion Conference,* Florence, Italy, September 17 – 20, 2007, IEPC-2007-222.
10. Ahedo, E., and Gallardo, J. M., "Scaling Down Hall Thrusters," *The 28th International Electric Propulsion Conference*, Toulouse, France, March 17 –21, 2003, IEPC-2003-104.
11. Kapulkin, Alexander, Guelman, Mauricio Moshe, Balabanov, Vladimir, and Rubin, Binyamin, "Low-Power Hall Thruster," EU, US Application No./Patent No. 07827357.0 – 1226 PCT/IL2007001384, filed 11 Nov. 2007, Priority US/09.11.06/USP 865033.

12. Guelman, M., Kapulkin, A., Balabanov, V., Rabinovich, L., Appelbaum, G., “A New Low-Power Hall Thruster Concept,” *The 59th International Astronautical Congress*, Glasgow, Scotland, 29 Sep – 3 Oct 2008, IAC-08-C4.4.6.
13. Grishin, S. D., Erofeev, V. S., Zharinov, A. V., “Accelerators with closed Hall current,” *Plasma accelerators* (in Russian), edited by L. A. Artsimovich, Mashinostroenie, Moscow, 1973, pp. 54 – 61.
14. Morozov, A. I., and Savelyev, V. V., “Fundamentals of Stationary Plasma Thruster Theory,”*Reviews of Plasma Theory*, Vol. 21,edited by B.B Kadomtsev and V. D. Shafranov, Kluwer, New York, 2000, pp. 203 -391.