3D Transient Modeling of Hall Thrusters: a Fully Fluid Approach

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Abstract: **The implementation of the physical laws governing Hall thruster phenomena may be fairly different as the numerical approach to the problem is varied (fluid, hybrid fluid-PIC, fully kinetic, etc.). However, fundamental characteristics and some results (thrust, specific impulse, breathing-mode frequency-range, etc.) are shared by all these models. Due to large computational time of hybrid codes (particularly for multidimensional codes), our current focus is to make a 3D transient model able to obtain some local and global performances of Hall thrusters with a low computation cost in terms of CPU. Our efforts were also focused to an optimization approach, in order to include the design of magnetic circuit in the simulation cycle. The code assumes the quasi-neutrality of the plasma in the discharge channel, so sheaths and high-frequency electron phenomena are not directly included in our investigations. However, the plasma sheaths are included in simulations by using the classical models that permit to express the sheath properties in terms of simulation parameters. The magnetic induction is assumed to be fixed and it is assumed to be described in terms of a stream function which permits to fulfill the solenoidality condition. The neutral atoms are considered entering to the channel with a uniform velocity along the axial direction. The neutral property we considered be varying is only the numerical density. The ion dynamics is described using the continuity and the momentum equations. The momentum equation for the ions considers only the friction force due to ionization and neglects the elastic momentum collisions with neutrals. Finally, the electrons are modeled using continuity, momentum, and energy equations, by eliminating the electric field from the energy equation with the electron momentum equation. The near-field zone was not modeled but we extended the simulations to a channel length corresponding to the position of a virtual cathode with a fixed electron temperature.**

Nomenclature

Roman Symbols

A_c	=	area of the channel cross section
b	=	channel depth
d	=	channel mean diameter

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B_0	=	maximum magnetic induction intensity
B	=	magnetic induction field
E	=	electric field
E_i	=	ionization potential: for Xenon =12.13 eV
I_d	=	discharge current
k_i	=	ionization coefficient
$\dot{m}$	=	mass flow rate
n_n	=	neutral density
n	=	plasma density
r	=	radial coordinate
t	=	time
T_e	=	electron temperature
u_B	=	Bohm velocity $=(T_e/m_i)^{1/2}$
u$_e$	=	electron velocity
u_e	=	electron velocity intensity
u$_i$	=	ion velocity
U_n	=	neutral velocity (constant, and uniform along x)
V_d	=	discharge potential
x	=	axial coordinate

Greek Symbols

c_i	=	ionization cost = 3
ε_{ew}	=	electron-wall energy
v_e	=	total electron collision frequency
v_{ei}	=	electron-ion momentum collision frequency
v_{en}	=	electron-neutral momentum collision frequency
v_{ew}	=	electron-wall collision frequency
v_i	=	ionization collision frequency
v_{iw}	=	ion-wall collision frequency
σ$_e$	=	electron mobility tensor
σ	=	SEE coefficient
θ	=	angular coordinate

Physical Constants

e	=	electron charge = $1.602176487 \times 10^{-19}$ C
m_e	=	electron mass = $9.10938215 \times 10^{-31}$ kg
m_i	=	Xe$^+$ ion mass = 2.1802×10^{-25} kg

Acronyms

FVM	=	Finite Volume Method
HET(s)	=	Hall Effect thruster(s)
SCS	=	Space Charge Saturation
SEE	=	Space Electron Emission

I. Introduction

THIS paper is mainly focused upon the modeling and the analysis of a Hall thruster using a fully-fluid approach (also called, in the literature, as a "hydrodynamic" approach) in order to extend some results shared by the literature and make a more complete picture of HETs operation in all the four space-time dimensions.

A literature survey shows that 3-dimensional models use kinetic or hybrid approach to describe HET dynamics[1,2], whereas hybrid or fully fluid are 2-dimensional in space[3,4,5].

The need to take into account a 3-dimensional geometry is due to the necessity to make a numerical model to be sensitive to 3-dimensional configurations of the magnetic induction field (these conditions typically happen by making a non axysimmetric design of the magnetic circuit or also they may be the result of self-induced magnetic induction fields that arise during the ordinary thruster's operation).

Another important need to develop a fully fluid model is that the theory of the oscillations and waves occurring in a HET channel is written using a fluid framework[6,7]. By extending the linear approach followed in frequency-domain analysis to a non-linear viewpoint (using the "optimal balance" of equation terms used in the different approaches), we can speculate about results derived using linear fluid theory and compare numerical simulations with experiments. As well known, linear oscillations/waves can disappear in a non-linear evolution of the thrusters due to intrinsic nature of the non-linear equation system.

Finally, we remark that fluid codes have intrinsically a computational cost lower than other kind of simulation approaches (fluid/PIC, kinetic, etc.). The motivation because we consider the simulation time in terms of CPU to be as a crucial aspect of our model, is that the code must be embedded within an optimization toolbox that itself increase the computational time.

II.   HET Fluid Model

A. Equation System

The simulation model we developed is a 3-dimensional transient solver that assumes a HET's plasma as a quasi-neutral fluid having three components: neutrals, single-ionized ions, and electrons.

The fluid model exposed in the following is written using the continuity equation for neutrals, the continuity and momentum equations for ions, and continuity, momentum, and energy equations for electrons (we assume the quasi-neutrality approximation, so we omit the index for the ion and the electron density). The continuity equation for electrons will be embedded in the current conservation whereas the electron momentum equation reduces to the generalized Ohm's law.

The source terms in the neutral, ion, and electron continuity equations include only the ionization effect because we assume that the recombination in bulk of the plasma is much lower than the ionization production.

The neutrals are treated using only the continuity equation in a one-dimensional approximation whereas ions are considered in the fully 3D cylindrical geometry.

The electron energy equation is derived assuming to be verified the hypothesis on a thermalization of electrons along the magnetic field lines. So, we write the electron energy balance in an integral form using the divergence theorem across magnetic field surfaces[3].

Neutral Continuity

$$\frac{\partial n_n}{\partial t} + U_n \frac{\partial n_n}{\partial x} = -v_i n \tag{1}$$

Ion Continuity

$$\frac{\partial n}{\partial t} + \nabla \bullet \left(n\mathbf{u}_i \right) = v_i n \tag{2}$$

Ion Momentum

$$\frac{\partial \left(n\mathbf{u}_i \right)}{\partial t} + \nabla \bullet \left(n\mathbf{u}_i \otimes \mathbf{u}_i \right) = \frac{e}{m_i} n\mathbf{E} + v_i n\mathbf{u}_n \tag{3}$$

Electron Energy

$$\frac{\partial \left(\frac{1}{2} m_e nu_e^2 u_{ex} + \frac{5}{2} nT_e u_{ex} \right)}{\partial x} = -enE_x u_{ex} - v_i nc_i E_i - v_{ew} n\varepsilon_{ew} \tag{4}$$

In order to reduce the problem complexity, we assume that the electron current density along the x direction, at different radial positions, with x fixed, may be considered as a constant. The axial electron velocity can be connected to the axial electric field using the definition of the axial electron mobility. So, the electron axial velocity and the electron mobility along the x direction are given by

$$u_{ex} = \frac{1}{n}\left(\frac{I_d}{A_c} - e\int_{A_c} nu_{ix}dS \right) \qquad \mu_{ex} = \frac{e}{m_e}\frac{1}{v_e\left(1+\dfrac{\omega_{ce}}{v_e}\right)^2} \tag{5}$$

The electron collision frequency is given by the sum of the elastic collisions between electrons and neutrals, the Coulombian collisions between electrons and ions, and the electron-wall collisions (our choice was that to explain the electron mobility only in terms of classical mobility and wall mobility). So, we can write

$$v_e = v_{en} + v_{ei} + v_{ew} \tag{6}$$

The electron-neutral elastic collision frequency is assumed to be given by the relation $3.1\cdot10^{-13}n_n$ (as a linear function in terms of the neutral density)[8], the electron-ion collision frequency can be calculated using results from the plasma physics theory[9], and finally, the electron-wall collision frequency is given by[8]

$$v_{ew} = \frac{4u_B}{3b(1-\sigma)} \tag{7}$$

The constitutive relations for the ionization coefficient, for the SEE coefficient, and for the energy loss to the wall are assumed to be expressed by the models

Ionization Coefficient[8]

$$k_i = k_i^0 \max_{T_e}\left(0, \frac{T_e - E_1}{T_e + E_2}\right) \tag{8}$$

$$k_i^0 = 2.5\cdot10^{-13}\ \frac{\text{m}^3}{\text{s}} \qquad E_1 = 5\ \text{eV} \qquad E_2 = 40\ \text{eV}$$

SEE Coefficient[10]

$$\sigma \triangleq \begin{cases} \sigma_0 + \dfrac{2T_e}{E_c}(1-\sigma_0) & T_e < T_e^* = \dfrac{E_c(\sigma^*-\sigma_0)}{2(1-\sigma_0)} \\ \sigma^* & T_e \geq T_e^* \end{cases} \tag{9}$$

$$\sigma_0 = 0.5 \qquad E_c = 53\ \text{eV} \qquad \sigma^* = 0.983$$

Wall Energy[10]

$$\varepsilon_{ew} = \begin{cases} 2+(1-\sigma)\log\left[(1-\sigma)\left(\dfrac{m_i}{2\pi m_e}\right)^{1/2}\right] & \text{No SCS} \\ 2+1.02(1-\sigma^*) & \text{SCS Regime} \end{cases} \tag{10}$$

The electron mobility is obtained by writing the momentum equation for the electrons in a derivative-free form and considering the electron pressure term effects.

B. Boundary Conditions and Numerical Solution Strategy

The neutrals are considered injected at the anode throughout with a uniform velocity along the x direction, and by continuity with the external mass flow rate we can calculate the neutrals density to the anode as

$$n_n(0) = \frac{\dot{m}}{m_i U_n A_c}$$

(11)

In the relation above we neglected the ion flux to the anode because we assumed that it was a low effect with respect the neutral flux.

The ion boundary condition at the anode is assumed due to a sonic reversal-flow condition, as in several one-dimensional fluid simulations, whereas they are considered entering to the side wall sheaths having a radial velocity equal to the Bohm velocity.

The discharge current at each time step is obtained from the current continuity along the x direction, and expressed in term of the external discharge potential

$$V_d = \phi(0) - \phi(L)$$

(12)

The boundary condition for the electrons at the anode is obtained from the total current conservation whereas the boundary conditions at the side sheaths are obtained considering floating conditions.

Finally, the electron temperature is implicitly fixed at the exit of the computational domain (we neglect the presence of a near-field and we consider a channel longer than the real channel length in order to impose a fixed electron temperature for the virtual cathode).

Although we tested finite different schemes during the code development, we used the FVM because this method is inherently conservative.

By writing the equation system for heavy particles in a conservation-law form, we obtain a vector of the conserved variables **W**, a flux function vector **F**, and a source term **G**. So, by taking volume integrals within each computational cell, of volume V_c, we can write a differential equation having the form

$$\frac{d}{dt}\left(\frac{1}{V_c} \int_{V_c} \mathbf{W}dV \right) + \frac{1}{V_c} \int_{S_c} \mathbf{F} \bullet d\mathbf{S} = \frac{1}{V_c} \int_{V_c} \mathbf{G}dV$$

(13)

In order to solve this equation, although we implemented different discretization strategies, we used a FVM scheme in the Roe formulation adapted to the cylindrical geometry. The numerical algorithm uses the Godunov splitting method, so the multi-dimensional problem is treated assuming two problems: an axial problem and an azimuthal-radial problem[11].

The implementation of the model is made in a dimension-less form in order to split up the dependence of the universal physics of a HET from the "problem scale" (we talk about scale in order to consider all physical parameters involved in the equation system).

III.   Results

The numerical simulations we performed using Fortran 90/95 on a computational grid of 30 (radial) x 100 (azimuthal) x 50 (axial) equally spaced grid points. Typical simulation times are about 5-10 minutes, depending on the input parameters to the simulation and also depending on the initial profiles we used to initialize the flow.

The fundamentals parameters of the simulation in the following are assumed to be

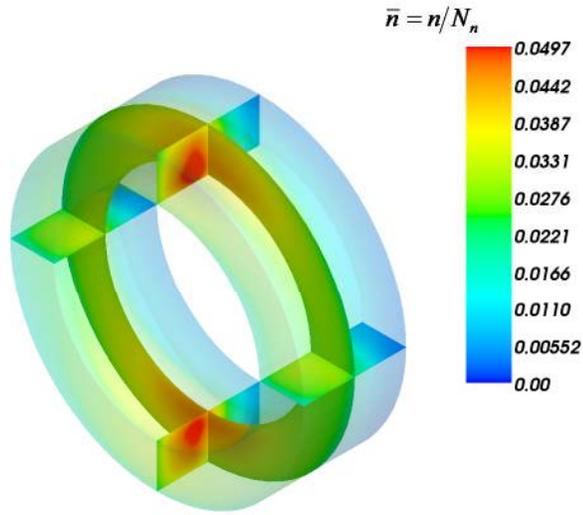| V_d	$\dot{m}$	L	b	D	B_0
220 V	3.8 mg/s	35 mm	15 mm	85 mm	18 mT

igure 1. Dimensionless Plasma Density (fixed time). *The reference numerical density is equal to* $N_n = 1.4505 \cdot 10^{19} \, \text{m}^{-3}$. *The figure above highlights the presence of an asymmetric solution in terms of plasma density distribution along the azimuthal direction.*

IV.   Conclusion

The numerical procedure exposed here is the first step of a 3D transient modeling of a HET using fluid equations. The code development was focused to an optimization approach, so may appear that we neglect some HET's phenomena. However, classical properties of simulations presented in the literature are recovered, despite the model may be improved in terms of physical behavior (the presence of asymmetric solutions along the azimuthal direction must be accurately investigated in the future).

Because of we used a dimension-less formulation for the internal implementation of the code, the result of our technique was a universal-like equation system, dependent only on a finite set of dimension/less parameters. The dimension-less approach we follow is very effective also because permits to make elaborations about the scaling of a HET. Keeping constant the dimension-less parameters, of course, the physics of the thruster is kept unchanged. We remark first that it is very difficult to keep constant all dimension-less parameters by varying thruster physical parameters, and second that the dimension-less parameters are intrinsically embedded in a non-linear formulation given by the plasma equation system, the constitutive relations, and the boundary conditions.

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