Effects of non-Maxwellian electron velocity distribution function on two-stream instability in low-pressure discharges

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Abstract: Recent analytical studies and particle-in-cell simulations suggested that the electron velocity distribution function in a Hall thruster plasma is non-Maxwellian and anisotropic. The electron average kinetic energy in the direction parallel to walls is several times larger than the electron average kinetic energy in direction normal to the walls. Electrons are stratified into several groups depending on their origin (e.g., plasma discharge volume or thruster channel walls) and confinement (e.g., lost on the walls or trapped in the plasma). Electron emission from discharge chamber walls is important for plasma maintenance in many low-pressure discharges. The electrons emitted from the walls are accelerated by the sheath electric field and are injected into the plasma as an electron beam. Penetration of this beam through the plasma is subject to the two-stream instability, which tends to slow down the beam electrons and heat the plasma electrons in the direction of its propagation. In the present paper, a one-dimensional particle-in-cell code is used to simulate these effects both in a collisionless plasma slab with immobile ions and in a cross-field discharge of a Hall thruster. The two-stream instability occurs if the total electron velocity distribution function of the plasma-beam system is a non-monotonic function of electron speed. Low-pressure plasmas can be depleted of electrons with energy above the plasma potential. This study reveals that under such conditions the two-stream instability depends crucially on the velocity distribution function of electron emission. It is shown that propagation of the secondary electron beams in Hall thrusters may be free of the two-stream instability if the velocity distribution of secondary electron emission is a monotonically

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decaying function of speed. In this case, the beams propagate between the walls with minimal loss of the beam current and the secondary electron emission does not affect the temperature of plasma electrons in Hall thrusters.

Nomenclature

x = coordinate normal to the walls and for the applied magnetic field
z = coordinate parallel to the walls and for the applied electric field
t = time
$v_{x,y,z}$ = velocity components of an electron
w = kinetic energy of an electron
$w_{x,y,z}$ = kinetic energy of electron motion in x,y,z direction respectively
H = width of the plasma slab
λ_c = electron mean free path between two collisions

I. Introduction

There is a reliable experimental evidence of the wall material effect on operation of a Hall thruster.[1,2] The existing fluid theories explain this effect invoking a strong secondary electron emission (SEE) from the channel walls. The SEE is predicted to weaken insulating properties of the near-wall sheaths and, thereby, (i) to cause cooling of plasma electrons and (ii) to enhance the electron conductivity across the magnetic field. From a practical standpoint, a strong SEE from the channel walls is expected to cause additional inefficiencies due to enhanced power losses in the thruster discharge, and intense heating of the channel walls by almost thermal electron fluxes from the plasma[3]. Moreover, because the SEE may lead to lower values of the sheath potential drop, ion-induced erosion of the channel walls can be also affected. Although these predictions can be certainly applied for plasmas with a Maxwellian electron velocity distribution function (EVDF), there is no consensus between the existing fluid [2,4,5,6,7] and kinetic models[8,9] on how strong the SEE effects on the thruster plasma are. According to kinetic simulations[10,11,12,13] the EVDF in a collisionless plasma is depleted at high-energies due to electron-wall losses. Under such conditions, the electron losses to the walls can be tens of times smaller than the losses predicted by the fluid theories. A similar depletion of EVDF at high energies was also reported for other kinds of low-pressure gas discharges.[14,15,16] Note that the deviation of the EVDF from a Maxwellian does not necessarily mean that the SEE cannot play a significant role in the thruster discharge. In experiments with a Hall thruster operating at high discharge voltages, the maximum electron temperature and the electron cross-field current were strongly affected by the SEE properties of the channel wall materials.[17,18]

Recent particle-in-cell (PIC) simulations[10,11,12] and the kinetic analytical study[14] revealed that the SEE effect on power losses in a thruster discharge is quite different from what was predicted by previous fluid and kinetic studies. In simulations, the EVDF was found to be strongly anisotropic, depleted at high energies, and, in some cases, even non-monotonic. The average kinetic energy of electron motion in the direction parallel to the walls is several times larger than the average kinetic energy of electron motion in the direction normal to the walls. Secondary electrons form two beams propagating between the walls of a thruster channel in opposite radial directions[10,11] (also predicted in Ref. 19 in the modified fluid approximation). In Refs. 10-14 it is shown that for a typical high-performance Hall thruster, the electron fluxes to the walls are limited by the source of electrons overcoming the wall potential and leaving the plasma. The flux of these electrons is determined mainly by the frequencies of elastic electron collisions with atoms and ions. The sheath insulating properties depend on the electron fluxes to the walls and, therefore, on the rate of elastic scattering of plasma electrons.

Previously, Meezan and Cappelli[8] developed a kinetic model based on the so-called nonlocal approach. The non-local approach (described for example in Ref. 20) was developed for large gas discharges where the distance between the walls is of the order of tens of centimeters and the neutral gas pressure is above 10mTorr, so that the electron mean free path λ_c is much smaller than the discharge gap width H, $\lambda_c << H$. Because of the smallness of the electron mean free path in the large gas discharges, the EVDF is isotropic even for electrons with energy high enough to overcome the wall potential. In Hall thrusters, however, the characteristic distance between the walls (the thruster channel width) H is typically small compared to the electron mean free path, $\lambda_c >> H$. Therefore, the traditional nonlocal approach is not applicable to Hall thrusters. It has been shown that the EVDF in Hall thrusters is anisotropic.[11] The anisotropy of the EVDF strongly affects the electron flux to the wall. In Ref. 14, practical analytical formulas are derived for wall fluxes, secondary electron fluxes, plasma parameters and contribution to the electron current due to SEE. The calculations based on the analytical formulas agree well with the results of numerical simulations.

An important implication of the present work is that future theoretical and experimental studies need to determine the influence of these kinetic effects on the thruster performance, heating and erosion of the channel walls. For instance, the reduction of the gas density in the thruster channel may significantly reduce the electron fluxes to the walls because in xenon plasmas of Hall thrusters the electron collisions with neutral atoms are the major scattering process while the Coulomb scattering off the ions gives a small contribution.

Under the condition of a large electron mean free path, the major mechanism affecting propagation of electrons emitted by the walls through the plasma is the two-stream instability. This study reveals that the two-stream instability depends crucially on the velocity distribution function of electron emission.[13] It is shown that propagation of the secondary electron beams in Hall thrusters may be free of the two-stream instability if the velocity distribution of secondary electron emission is a monotonically decaying function of speed. In this case, the beams propagate between the walls with minimal loss of the beam current and the secondary electron emission does not affect insulation properties of the sheath and the temperature of plasma electrons.

II. Two-Stream Instability in a Hall thruster

The secondary electrons emitted from the wall and accelerated by the sheath form a beam which propagates through the plasma. During propagation through the plasma, the secondary electron beam may be a subject of two-stream instability, which may lead to effective energy loss of the SEE beam and its scattering.[21] If the beam does not lose its energy, the beam electrons can escape to the opposite wall in case of a symmetric plasma or they can return to the same wall if reflected from an electrostatic barrier in case of an asymmetric plasma. Small energy loss during fast collisionless processes, such as the two-stream instability, can significantly modify the wall fluxes and, consequently, the sheath and plasma properties [19]. Therefore, a detailed numerical study of the two-stream instability in Hall thrusters has been performed.[13]

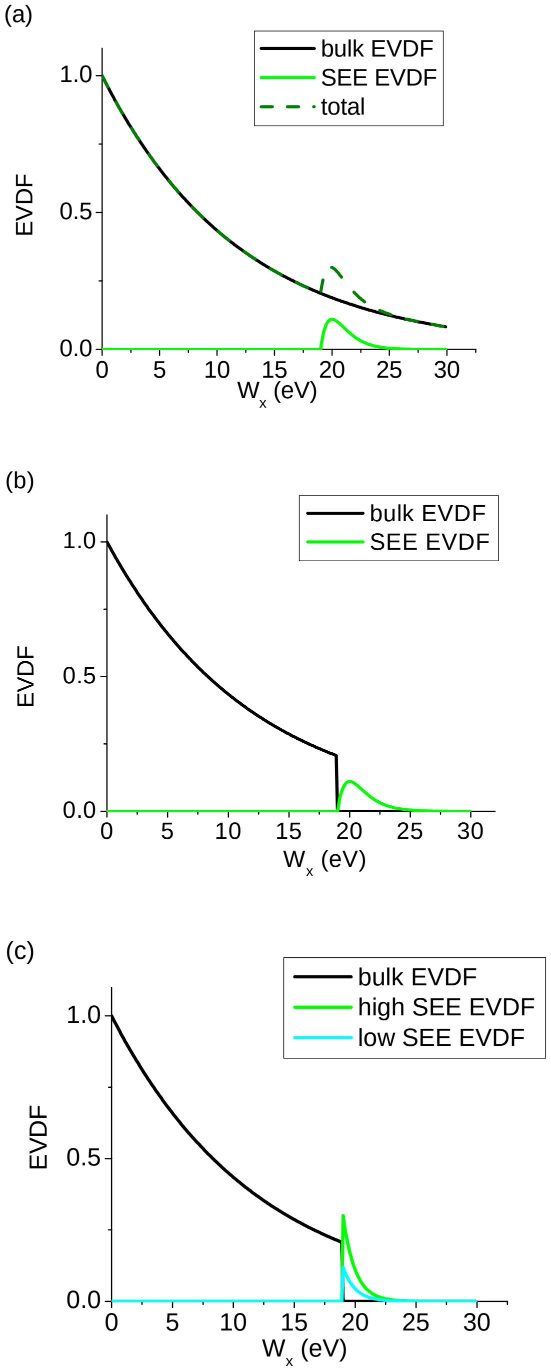

Figure 1. **Schematic of the electron velocity distribution function of bulk plasma electrons and the secondary electron beam. (a) A Maxwellian bulk plasma EVDF (b) realistic EVDF with weak SEE, (c) realistic EVDF with intense SEE.**

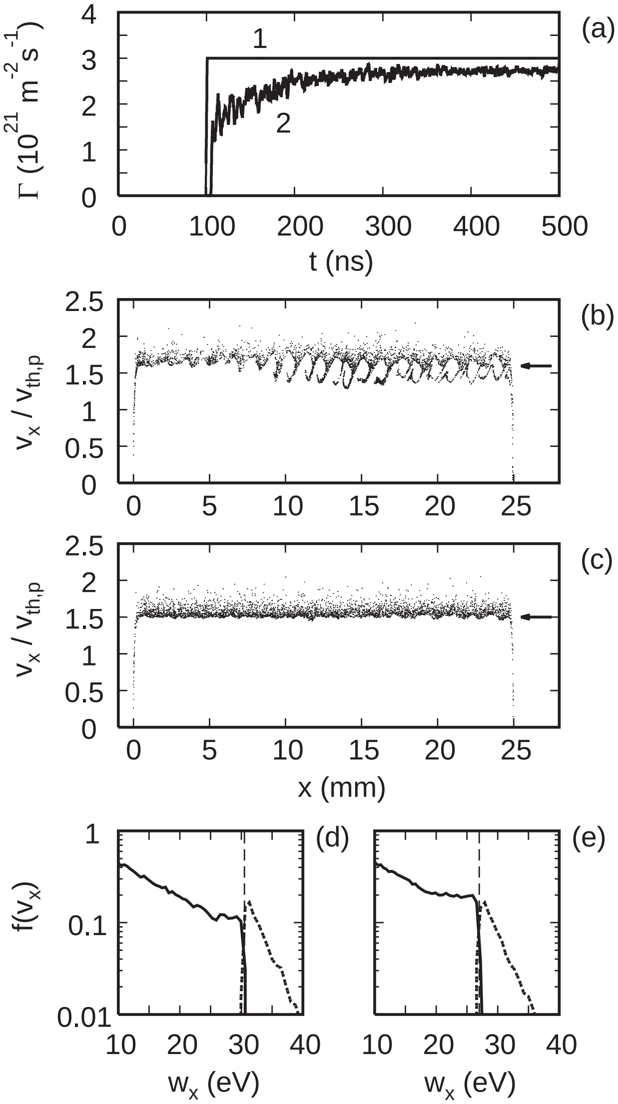

Fig.2 Results of particle-in-cell simulations. (a) The penetrated electron beam flux (curve 2) gradually approaches to the emitted flux value (curve 1). (b) Initially phase plane (t = 199 ns) shows the intense permanently existing two-stream instability. (c) At the end of simulation (t = 499 ns) phase plane shows the unperturbed beam. (d) Initially (t=119 ns), the total EVDF (solid line – plasma, dashed line - beam) near the emitting wall (x=0) is non-monotonic. (e) At the end of simulation (t = 499 ns), the plateau forms on the plasma EVDF (solid curve), so that the total EVDF becomes a monotonically decaying function of speed. As a result, the instability vanishes.

Experimental and theoretical studies[21] showed that electron beam relaxation occurs on the spatial scale of several Debye lengths, which can be small compared with thruster channel width. Typical instability develops due to the fact that the EVDF of bulk plasma electrons and the secondary electron beam is not a monotonic function, as shown schematically in Fig.1. If the bulk plasma EVDF is a Maxwellian, the addition of the secondary electron beam corresponds to the bump-on-tail case and is unstable, as shown in Fig.1 (a). However, in a finite size plasma of a Hall thruster, the tail of the EVDF corresponding to the electron total energy in the direction normal to the walls above the plasma potential is strongly depleted and is practically empty, therefore addition of the secondary electron beam may not produce a pronounced bump on a tail, as shown in Fig.1(b). In case of strong SEE, the density of the beam of secondary electrons is high enough to ensure that the resulting EVDF is non-motonic [see Fig.1(c)].

We performed particle-in-cell simulation for this case and studied the development of the two-stream instability shown in Fig.2; a detailed description of the study is given in Ref.13. The two stream instability leads to the formation of a plateau on the EVDF, then the instability disappears.

III. Conclusions

In low-pressure discharges, electron emission from the walls can result in the formation of intense electron fluxes in the plasma. Examples of such emission are secondary electron emission, thermionic emission from heated metal surfaces such as emissive probes, and field emission (e.g., emission from dust particles). The emitted electrons are accelerated into the plasma by the voltage drop across the sheath. The presence of such electron streams in the plasma can lead to the two-stream instability if the total electron velocity distribution function EVDF of the electron stream and plasma has a region with positive derivative with respect to the electron speed. If the plasma electrons are described by a Maxwellian EVDF, the combination of plasma and emitted electrons results in a non-monotonic total EVDF leading to the two-stream instability.

However, in low pressure discharges, the EVDF is not Maxwellian, it is depleted at energies above the plasma potential relative to the wall. Therefore, the development of the two-stream instability in low-pressure discharges is different compared to Maxwellian plasmas. We performed systematic studies of the two-stream instability and found that the pattern of its development depends crucially on the shape of the velocity distribution function of electron emission VDFEE. One type of VDFEE considered in the present paper is a monotonically decaying function of the electron energy, which is maximal if the energy of emission is zero. The total EVDF consisting of the plasma EVDF and the VDFEE accelerated by the plasma potential is a monotonically decaying function of speed if the emission current is below some threshold. In this case, the two-stream instability does not occur. If the emission current is above this threshold so that the total EVDF is a non-monotonic function of speed, then the two-stream instability does occur but quickly vanishes. This happens because the two-stream instability forms a plateau on the velocity distribution function of electrons confined by the plasma potential (i.e., the plasma EVDF). Then the total EVDF becomes a monotonic function of speed and the beam propagates through the plasma without perturbations. Alternatively, the VDFEE may be equal to zero at zero energy of emitted electrons and grow as a function of energy for a few electronvolts. Such a non-monotonic VDFEE is a feature of secondary electron emission from metals.[22] At low pressures, the total EVDF of the plasma-beam system near the emitting wall has a gap of a few electron volts at the energy corresponding to the wall potential. This gap is responsible for the development of the two-stream instability, which is confirmed by simulations with a non-monotonic VDFEE. In these simulations, the two-stream instability reaches the nonlinear saturation stage and exists for as long as the emission lasts. As a result, the plasma electrons accelerate while the emitted electrons decelerate, which leads to the partial trapping of emitted electrons in the plasma. In our simulations with immobile ions and constant emission current, about 50% of emitted electrons become trapped in the plasma during their first flight between the walls. However, the two-stream turbulence accelerates these electrons back to the energy above the confinement threshold (plasma potential relative to the walls expressed in energy units) so that they leave the plasma after several bounces between the walls. In fact, during a steady state, the sum of wall fluxes of emitted electrons that reach the wall after multiple bounces and those that cross the plasma directly is close to the emitted electron flux.[14] For some applications, e.g,, Hall thrusters, it is therefore expedient to assume that the two-stream instability does not affect the beam propagation and that the effective penetration coefficient is close to unity.[14] The plasmas considered in the present paper are confined by a symmetrical potential well between floating or electrically connected walls, as in Hall thrusters or hollow cathode discharges. However, even in these plasma devices, the potential profile between the walls can be non-symmetric due to geometrical effects or applied voltage. Nevertheless, our conclusions on the effects of the VDFEE and the non-Maxwellian plasma EVDF on the two-stream instability can be generalized for plasmas with non-symmetric potential profiles. It is necessary to point out that the effects considered here are essentially one-dimensional and may be modified in cases in which the three-dimensional effects, such as the finite beam width, geometrical expansion in cylindrical or spherical systems, or nonuniform magnetic field effects, become important (see, e.g., Ref. 23). Electron motion along the magnetic field line is affected not only by the electrostatic force, but also by drifts in nonuniform magnetic field.[24] In addition to the electrostatic instability, where the wave number vector is parallel to the external magnetic field, two or three-dimensional systems permit electromagnetic instabilities, where the wave number vector is non-parallel to the magnetic field.[25] In order to investigate these effects, three dimensional kinetic simulations are necessary.

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