

Absence of a steady-state space charge limited regime for a sheath in a weakly collisional plasma bounded by walls with high secondary electron emission

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Recent analytical studies and particle-in-cell simulations suggested that the electron velocity distribution function in a Hall thruster plasma is non-Maxwellian and anisotropic. The electron average kinetic energy in the direction parallel to walls is several times larger than the electron average kinetic energy in the direction normal to the walls. Electrons are stratified into several groups depending on their origin (e.g., plasma discharge volume or thruster channel walls) and confinement (e.g., lost on the walls or trapped in the plasma). This strong modification of the velocity distribution leads to absence of a steady-state space charge limited regime for a sheath even in presence of a high secondary electron emission. The sheath never reaches a steady space charge limited state even though the secondary electron emission produced by the plasma bulk electrons is high, with the corresponding partial emission coefficient exceeding unity. Instead, the plasma-sheath system performs relaxation oscillations by switching quasi-periodically between the SCL and the non-space charge limited states.

Nomenclature

E_z	= accelerating electric field in the 1D model of a Hall thruster (HT)
B_x	= magnetic field in the HT acceleration region
H	= HT channel width (the direction normal to the walls is \mathbf{x})
ν_t	= frequency of electron scattering on micro-turbulence in $\mathbf{y-z}$ plane
T_{RSO}	= period of resonant sheath oscillations
V_d	= electron drift velocity (E_z/B_x); ions are not magnetized
γ	= coefficient of secondary electron emission

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I. Introduction

IN this paper we present recent simulation results for a kinetic model of plasma in crossed electric and magnetic fields, bounded by dielectric walls producing secondary electron emission (SEE). The model had been developed to approximate, in one spatial dimension, the acceleration region in the channel of a Hall thruster (HT).

For a plasma with a Maxwellian electron velocity distribution function (EVDF) bounded on one side by an emitting wall, it has been known in theory that for SEE intensity above a certain critical value, the near-wall sheath undergoes a transition to space charge limited (SCL) regime. In SCL regime, sufficient negative charge is accumulated near the wall to produce a non-monotonic potential profile in the sheath and to reflect a portion of the emitted electrons back to the wall. The potential drop across the SCL sheath is much lower compared to a non-emitting wall, and the plasma electron flux to the wall is so intense that it can, for example, cause evaporation of the wall material. In the semi-bounded Maxwellian model, the SCL regime occurs if the emission coefficient reaches some threshold value $\gamma_{cr} \approx 1$, with corresponding critical electron temperature T_{cr} [1]. Due to enhanced wall losses in the SCL regime, this critical temperature becomes a virtual upper limit for the electron temperature with Maxwellian EVDF.

This traditional concept based on a Maxwellian EVDF fails for plasmas where the electron mean free path is comparable with or larger than the system size, as revealed in a number of experimental studies.²⁻⁵ In extensive numerical study of weakly collisional plasmas in Hall thrusters it was found that the EVDF in such plasmas is non-Maxwellian, strongly anisotropic, depleted at high energies, and even non-monotonic.^{6,7} The average kinetic energy of a majority of electrons, which are confined by the sheath potential barrier (referred to as the plasma bulk electrons), can be many times larger than that in the direction normal to the walls. The plasma bulk electrons reach the walls mostly after scattering off neutral atoms. These collisions are so rare that secondary electrons emitted from the walls propagate through the plasma almost freely, without energy exchange with the plasma electrons. Thus, a wall is bombarded by both the scattered plasma bulk electrons and the electrons emitted from the opposite wall.⁸

It appears that while the plasma is heated and the SEE intensity increases, the balance of electron and ion fluxes to the wall is maintained not through the formation of a double layer in the sheath, but through the modification of the EVDF of the plasma-beam system. This new balance mechanism creates an unusual situation in which the SCL sheath practically never develops. With sufficiently high heating of the plasma electrons, the system in question may enter into an oscillatory regime in which it switches quasi-periodically between SCL and non-SCL states. Such regime has been referred to as “relaxation sheath oscillations” (RSO). It was reported and analyzed in [9]. The present report is based on parametric studies of RSO, aimed to clarify how their onset and characteristics depend on the underlying plasma parameters.

In the studies quoted above, it was established that the following conditions are necessary for the RSO regime to occur. Firstly, the secondary electrons, emitted with relatively low energies on the order of few electron-volts, must gain sufficiently high energy in crossed \mathbf{E} and \mathbf{B} fields to produce enough secondary electrons at the opposite wall. This condition⁹ stems from the requirement that overall secondary emission coefficient of the plasma exceeds γ_{cr} and takes the form

$$\gamma_b > 1 - (\gamma_{max}/\gamma_{cr})(1 - \gamma_{cr}) , \quad (1)$$

where γ_b is the emission coefficient of the secondary beam and γ_{max} is the maximum of the emission yield, as a function of energy of primary electrons. The value γ_{cr} corresponds to SEE flux at which the electric field at the wall in the sheath becomes zero. It depends on the EVDF of both bulk and secondary populations; for a Maxwellian EVDF $\gamma_{cr} = 0.983$. This condition implies that, for SCL phase to exist, the average energy of the drift motion $m_e(E/B)^2$ has to exceed a certain threshold. Another, crucial condition for the existence of RSO is intense, anisotropic heating of the plasma electrons due to scattering on two-dimensional micro-turbulence in the plane perpendicular to the magnetic field. Such heating supplies energy to the plasma electrons residing near the top of the longitudinal potential well, causing their SEE yield to exceed unity when the plasma potential drops during the SCL phase. More specifically, the anisotropic EVDF should become such that the current-voltage curve of the sheath⁹ develops a branch with negative differential resistance. The heating rate is

$$\frac{d \langle w_{\perp} \rangle}{dt} = \nu_t V_d^2 H = \nu_t \left(\frac{E_z}{B_x} \right)^2 H , \quad (2)$$

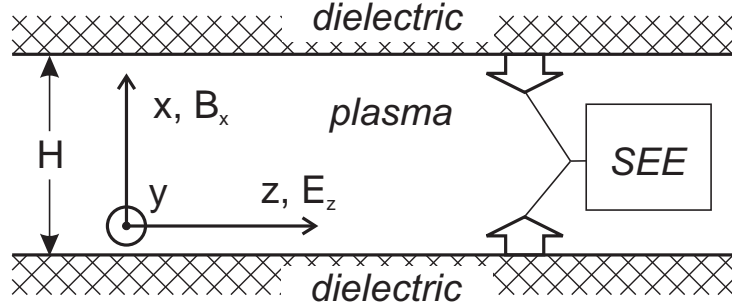


Figure 1. Schematic diagram of the simulation model. Positions of the walls are $x = 0$ and $x = H$.

where w_{\perp} is the average energy of the transverse motion, per “column” of unit area. In a spatially one-dimensional model of the HT acceleration region, the anisotropic heating is accounted for by introducing elastic scattering of the transverse ($\mathbf{y}-\mathbf{z}$) velocity with a set frequency ν_t . A number of simulations were performed with varying values of ν_t and E_z (which affect the heating rate directly), and of the channel width H . Three sample cases will be discussed presently.

II. Simulation Model

The relaxation oscillations are obtained in simulations carried out with an electrostatic particle-in-cell code EDIPIC.^{6,10} The code considers a plasma bounded by two dielectric walls with SEE (see Fig. 1). It resolves one spatial dimension normal to the walls, and three velocity components for electrons and ions. The code is based on a direct implicit algorithm.¹¹ The emission properties of the walls approximate those of boron-nitride ceramics.¹² In order to sustain high electron temperature, the plasma is immersed into crossed constant external electric and magnetic fields. Electrons perform elastic and inelastic (ionization and excitation) collisions with neutral atoms of constant density. The anomalous electron transport across the magnetic field^{13,14} is included via the additional “turbulent” collisions, which randomly scatter electrons in the plane parallel to the walls.¹⁵

In [9], the RSO regime was originally demonstrated in a simulation with the following parameters corresponding to typical Hall thruster conditions:⁵ the electric field parallel to the walls $E_z = 200$ V/cm, the distance between the walls $H = 2.5$ cm, the magnetic field directed normal to the walls $B_x = 100$ Gauss, the xenon neutral gas density $n_a = 10^{18}$ m⁻³, the initial plasma density $n_{e0} = 10^{17}$ m⁻³. In order to demonstrate the occurrence of the SCL regime, the increased frequency of turbulent collisions $\nu_t = 2.8 \cdot 10^6$ s⁻¹ was used, which enhanced the heating rate of electrons. In what follows, our additional examples are described.

Case 1

In this example, the distance between the walls (thruster channel width) is set to $H = 4.0$ cm, with other parameters being the same as cited above. The oscillation period T_{RSO} is approximately 170 ns, vs. 250 ns found for $H = 2.5$ cm. In this particular case, the decrease in T_{RSO} is roughly in inverse proportion to the increase in H , corresponding to the increased rate of turbulent heating (via increased lifetime of weakly confined electrons in the plasma potential well). However, it should be noted¹⁶ that the energy of primary electrons arriving at the walls also depends upon their phase of cyclotron rotation, and thus the dependence of T_{RSO} upon the model parameters is generally not monotonic. The results are shown in Fig. 2. The sawtooth profile of the average electron energy shows heating in the non-SCL phase and cooling in the SCL phase, which correspond to the variation in the sheath potential barrier between the two phases. It should be emphasized that RSO are a kinetic process, with EVDF evolving non-locally and showing a specific “breathing” structure in the velocity space.

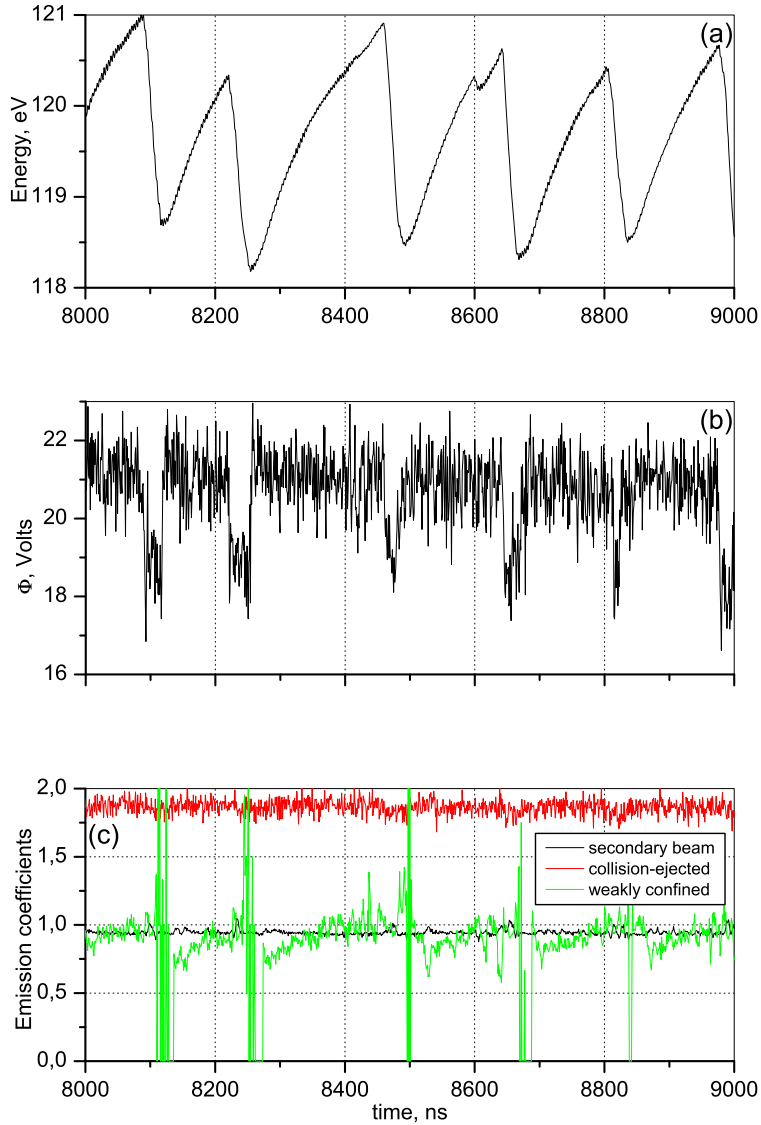


Figure 2. Relaxation sheath oscillations (RSO) observed for $H = 4.0$ cm, $\nu_t = 2.8 \cdot 10^6$ s $^{-1}$, $E_z = 200$ V/m, and $B_x = 100$ Gauss. Average kinetic energy of electrons in the plasma volume (a); electrostatic potential at the center $x = H/2$ (relative to the walls) (b); secondary emission coefficients for each of the three groups of electrons (c). The three groups of electrons are defined as follows: “collision-ejected” are the ones scattered into the loss cone from the bulk anisotropic distribution; the “weakly confined” portion is the one which escapes once the plasma potential collapses during the brief SCL phase; “secondary beam” refers to the electrons emitted from one wall and reaching another.

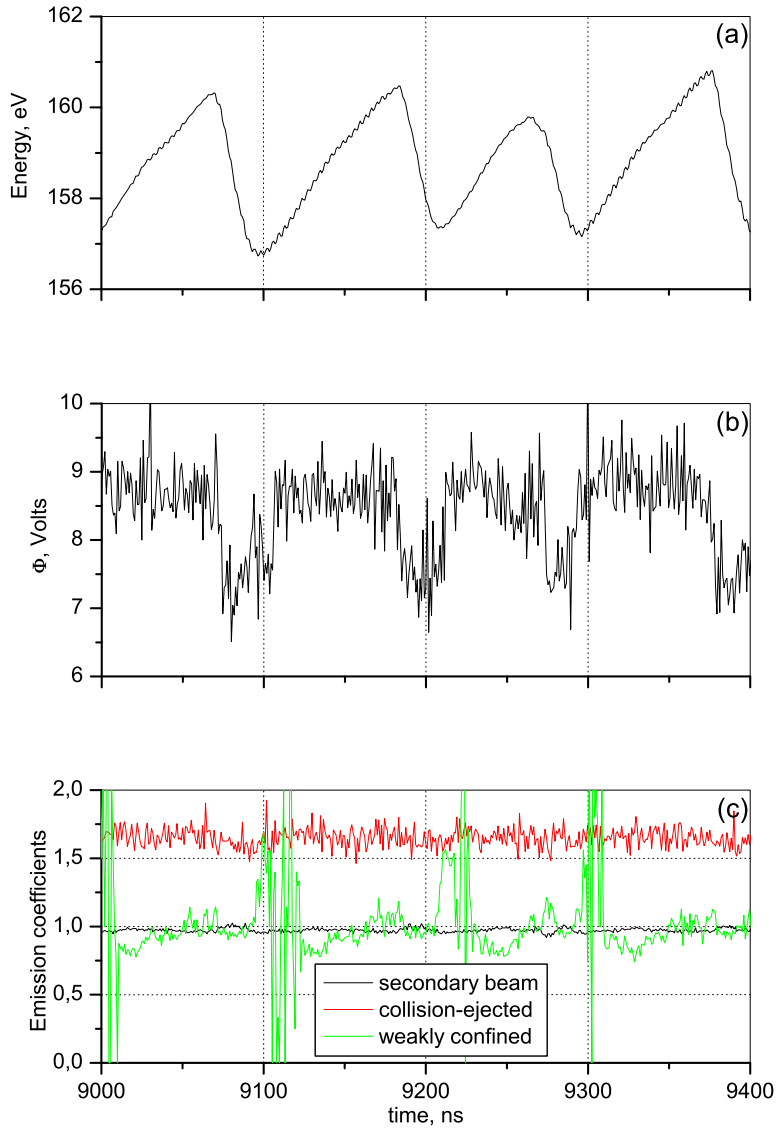


Figure 3. RSO for $H = 2.5$ cm, $\nu_t = 5.6 \cdot 10^6$ s $^{-1}$, $E_z = 200$ V/m, and $B_x = 100$ Gauss. Same panels as for Case 1.

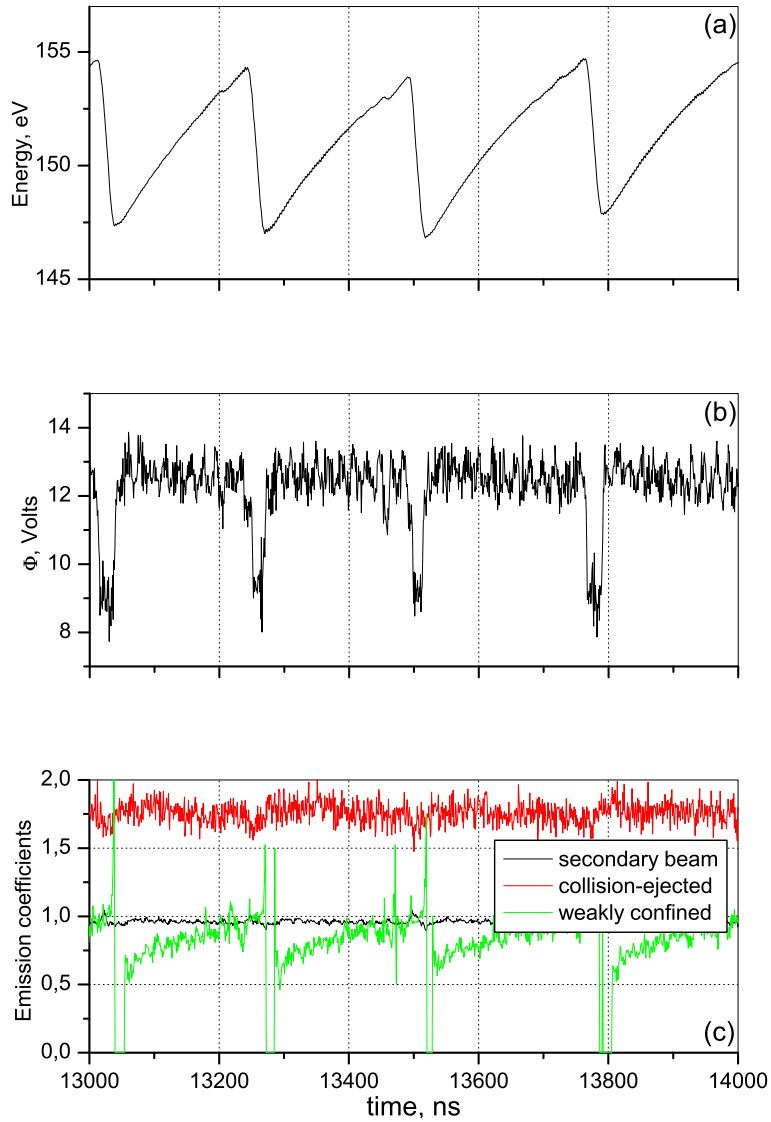


Figure 4. RSO for $H = 2.5$ cm, $\nu_t = 5.6 \cdot 10^6$ s $^{-1}$, $E_z = 175$ V/m, and $B_x = 100$ Gauss. Same panels as for Case 1.

Case 2

In the following case, the wall-to-wall distance H is 2.5 cm like in [9], while the frequency of turbulent scattering is doubled, $\nu_t = 5.6 \cdot 10^6 \text{ s}^{-1}$. The period T_{RSO} is approximately 100 ns, compared to the prediction of 125 ns based on the variation in the heating rate alone (the rate of energy gain per particle due to turbulent scattering is estimated as $m_e V_d^2 \nu_t$, but, as already noted, the actual energy gain depends on how long the electron remains trapped in the potential well during the non-SCL phase of the oscillations). The RSO waveforms for this case are shown in Fig. 3. The plasma potential during the quasi-stationary phase is lower, and the amplitude of its oscillations is smaller compared to Case 1.

Case 3

In the last example, the scattering frequency $\nu_t = 5.6 \cdot 10^6 \text{ s}^{-1}$ is kept the same as in the previous case, while the electric field is reduced: $E_z = 175 \text{ V/cm}$. Very regular oscillations spanning many periods were observed, with $T_{\text{RSO}} \approx 250 \text{ ns}$, as seen in Fig. 4. Compared to the preceding case, the electric field is reduced by a factor of $E_3/E_2 = 0.875$. At the same time, T_{RSO} is greater by a factor of 2.5, although an estimate based on heating rate predicts $1/(0.875^2) = 1.31$. This, as well as the results of our other simulations, suggests that gyrophase resonances should be playing a role in the dynamics of RSO. It should also be noted that, for this case, the SEE yield corresponding to the energy of the drift motion (17.6 eV) is about 0.8. This value would not be sufficient to create an SCL phase according to eq. (1) with $\gamma_{cr} = 0.983$ known for Maxwellian EVDF (and $\gamma_{max} = 3$, for boron-nitride). Given that RSO are present, one can use eq.(1) to obtain an upper bound on the actual γ_{cr} for the anisotropic EVDF, as a necessary condition. In this example, we find $\gamma_{cr} < 0.938$.

III. Conclusions

Our simulations, examples of which have been presented, give evidence that in the model of the acceleration region of a Hall thruster, RSO can exist within a broad range of parameters, as should be expected from the general analysis given in [9]. Intense anisotropic heating is the necessary driving factor. It has been verified that the RSO regime does not occur if the scattering due to anomalous transport is set to be isotropic. Our simulations qualitatively validate the criterion provided by eq.(1) even though γ_{cr} is not known exactly. It has been observed that, for sufficiently low values of E_z/B_x , oscillations do not exist for any realistic values of ν_t . For the cases at hand, no RSO were found for $E_z = 140 \text{ V/m}$ and $H = 2.5 \text{ cm}$. The threshold criterion related to the heating rate, based on eq.(2), is not known precisely at this time, but our studies confirm that it exists.

Acknowledgments

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