

Development of Methods for Rapid Electric Propulsion System Design and Optimization

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Krzysztof Z. Pennar¹ and Pavlos G. Mikellides²
Arizona State University, Tempe, AZ, 85287

John K. Ziemer³
Jet Propulsion Laboratory, Pasadena, CA, 91108

Abstract: A solar electric propulsion low-thrust trajectory database is examined to formulate analytic models for use in rapid electric propulsion sizing tools. Analytic models are created based on characteristics integrated over the entire trajectory. Characteristic acceleration is defined for each trajectory, which in essence allows for a constant thrust assumption. Models are developed for near-Earth asteroid sample return mission, comet rendezvous, comet sample return, and Saturn fly by. Models follow a $1/x^2$ relationship between characteristic acceleration and optimum burn time. Given that these are solar electric missions the relationship provides physical insight. The models are then used in a rapid sizing tool which uses a genetic algorithm to find the maximum spacecraft payload. Optimized results are compared between missions and technology parameter sensitivity is explored. A comparison of variation of technology parameters within optimized results indicates which are more sensitive and therefore are system sizing drivers. The study showed that minimizing specific power has the largest affect on mass, followed by thruster efficiency, and thruster lifetime. Sensitivity trends were similar across each mission, regardless of destination. However, thruster lifetime was more sensitive in longer missions where more thrusters would be required.

Nomenclature

\bar{a}	=	mean acceleration [km/s/yr]
\bar{a}_{Char}	=	characteristic mean acceleration [km/s/yr]
α	=	alpha [kg/kW]
C_3	=	characteristic energy= v_∞^2
Comet-R	=	comet rendezvous
CSR	=	comet sample return
EPS	=	electric propulsion system
FB	=	fly by
GA	=	genetic algorithm
I_{sp}	=	specific impulse
LTTO	=	low thrust trajectory optimization

¹ Graduate Student, Mechanical and Aerospace Engineering, Pennar@asu.edu

² Associate Professor, Mechanical and Aerospace Engineering, Pavlos.Mikellides@asu.edu

³ Senior Engineer, Electric Propulsion Group, Jet Propulsion Laboratory, John.K.Ziemer@jpl.nasa.gov

M_o	=	initial spacecraft mass [kg]
NEA-SR	=	near Earth asteroid sample return
NEP	=	nuclear electric propulsion
SEP	=	solar electric propulsion
t_b^*	=	optimum burn time [yrs]
T_f	=	flight time [yrs]
ΔV	=	delta velocity [km/s ²]
η_p	=	propulsive efficiency
η_v	=	mission planning efficiency

I. Introduction

Electric propulsion (EP) spacecraft mission design and optimization is complex and computationally intensive because it requires concurrent design of the trajectory, power, and thruster systems. Specifically, trajectory design for EP devices requires multivariate optimization, because unlike chemical thrusters, EP systems (EPS) have thrust-to-weight ratios less than one. Therefore, the physics dominating trajectory design is different and more intricate. Programs that perform low-thrust trajectory optimization (LTTO) are employed to find optima via direct and indirect optimization search methods¹. Because of the expertise required to initialize and run LTTO programs it is not always effective to utilize existing LTTO programs for rapid multi-disciplinary EPS optimization. Instead less complex LTTO methods are favored in a rapid mission design environment. Since most deep space trajectories involve complicated maneuvers and variable thrust profiles such as sample returns, multi-destination missions, or gravity assisted missions^{2,3,4}, simple analytic solutions such as the multi-revolution “spiral” trajectory⁵ yields sub-optimal solutions. Recent research trends have created analytic models derived from a family or database of previously created optimized trajectories. Most notably for EP systems, analytic models have been created for Nuclear EP (NEP) missions to Mars, Jupiter, and Neptune^{6,7}. Building from that historic precedence, a Solar EP (SEP) mission database is examined in an attempt to develop simple analytic models for use in rapid EP sizing and optimization tools. Additionally, the intention of an analytic model is to explain trending within a database while being rooted in the governing physics of low thrust trajectories.

Previous deep space analytic models were limited to NEP systems, which assume constant thrust, constant power, circular, coplanar, optimum specific impulse trajectories. The SEP dataset is different because thrusting is not constant, and trajectories are not circular and coplanar. Nevertheless, by defining mission characteristic variables, analytic models can be created and applied to SEP family trajectories.

The paper outlines an approach to create analytic SEP trajectory models. Models are described for four different trajectories and are compared to existing analytic models. The models are then combined with a rapid EP sizing and optimization tool to study technology parameter sensitivity.

II. Low-Thrust Trajectory Analytic Modeling

A. Introduction to Datasets

Low-thrust trajectory optimization software, such as MALTO⁸, allow for the creation of trajectory databases. Trends can be found within these databases which form the basis of analytic models. Low-thrust trajectories are functions of launch date, flight time, launch vehicle, engine specific impulse, available power, throttle curve, number of engines and duty cycle. The database includes four different trajectories; each analyzed using four different thrusters. The database has more than 5,000 individual trajectories and was initially created to study the sensitivity affects on varying duty cycle⁹. However, for the purposes of creating analytic models a duty cycle of 80% is selected for all cases.

The database includes four different SEP mission scenarios: 1) Near-Earth asteroid sample return mission to asteroid 1989 ML. This is a moderate ΔV mission that has an approximate duration of 3 years with a minimum 90 day science period. The trajectory database was created using a neutral spacecraft mass of 1600kg. 2) Comet rendezvous mission to Tempel 1. This is a moderate ΔV mission with a small 800kg payload spacecraft. Rendezvous occurs within 1 year after perihelion. 3) Comet sample return to comet Tuttle-Giacobini-Kresak. The

spacecraft neutral mass is 1600kg and the ΔV requirement is large. The mission duration is approximately 8 years with a 180 day science window at the comet. An additional constraint, arrival V_∞ (9km/s maximum), is incorporated to allow for Earth entry. This dataset of trajectories approaches the limits of current SEP technology capabilities. 4) Saturn fly-by mission with a 4500kg neutral mass spacecraft requiring a large ΔV . The arrival V_∞ is again constrained to 9km/s at Saturn. The trajectory dataset for this mission uses a gravity assist (order Earth-Earth-Venus-Venus-Earth-Saturn). In addition, most of the thrusting occurs near Earth's orbit (.7-2AU). Table 1 summarizes the mission scenarios examined in the database.

Table 1. SEP Trajectory Database Highlights

Mission Scenario	Neutral Mass [kg]	TOF [yrs]	Typical ΔV [km/s]	Thrust Distance [AU]	Spacecraft Distance [AU]
Near-Earth Asteriod Sample Return- 1989 ML	1600	~ 3	3	1 - 1.6	1 - 1.6
Comet Rendezvous- Tempel 1	800	2.5 - 6	8	1 - 3	1 - 4
Comet Sample Return- TGK	1600	~ 8	12 - 16	1 - 4	1 - 5
Saturn Fly-By	4500	7.5 - 11	2 - 5	0.7 - 2	0.7 - 10

B. Analytic Modeling Method

Optimized SEP trajectories include a variable thrust profile, and variable specific impulse. However, net characteristics, such as the total kinetic energy output of the EP system, can be directly compared between a NEP and SEP system within the context of total trajectory requirements. Equation (1) shows that mean acceleration is the integral of the acceleration profile over the burn period.

$$\bar{a} \equiv \frac{1}{t_b^* - 0} \int_0^{t_b^*} a(t) dt = \frac{\Delta v}{t_b^*} \quad (1)$$

To best describe a trajectory a characteristic acceleration is defined, which is based on the mean acceleration of the spacecraft. Optimized results from the trajectory database include ΔV and optimum burn time. To find the characteristic acceleration it is necessary to decouple Earth departure C3. Increasing C3 decreases the amount of ΔV required by the EP system. Thus to normalize the results in the database equation (2) is used to find the optimum mean or characteristic acceleration at C3=0. The mission planning efficiency ($1/\eta_v$) is a measure of how efficiently Earth departure C3 reduces the spacecraft ΔV requirement. η_v is calculated numerically and differs based on the trajectory¹⁰. To simplify, a mean value of 0.8 is used for all the cases. Using equation (2), characteristic acceleration is calculated.

$$\bar{a}_{Char} = \bar{a} + \frac{v_\infty}{\eta_v t_b^*} \quad (2)$$

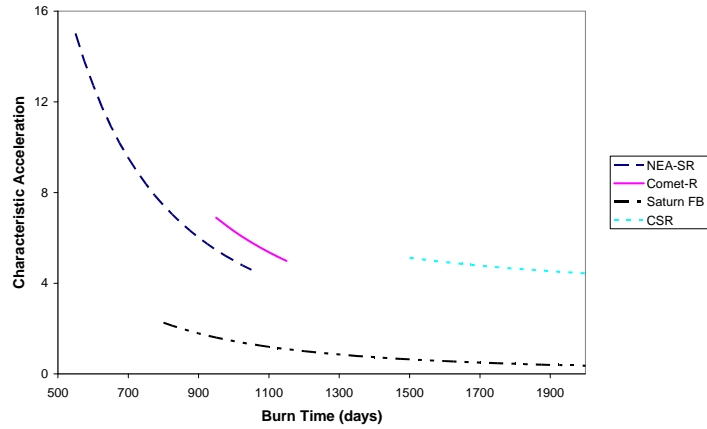


Figure 1. Characteristic Acceleration vs Optimum Burn Time for SEP Trajectories. Figure includes NEA-SR, Comet-R, Saturn FB, & CSR trajectories. Plotted lines are curve fits to optimized data series. Values are only evaluated where optimized trajectory data exists.

Next, the optimized characteristic accelerations are fit using an expression defined by equation (3). SEP solutions trend along burn time raised to the -2 power; which is similar to free-space radiation. This suggests a physical significance between burn time and characteristic acceleration. Figure 1 plots characteristic acceleration as a function of burn time for the available database value range.

$$\bar{a}_{Char} = \frac{A}{t_b^{*2}} + B \quad (3)$$

The various analytic models are similar to previously developed NEP models⁵ For comparison, Figure 2 adds the analytic model for Earth-Jupiter transfer using NEP. In addition, the figure uses data points to represent the database of optimized trajectories and the solid lines are extrapolations to show trending. Figure 2 shows that for very long and very short flight times optimized solutions begin to converge. Several observations are made when comparing the Jupiter trajectory family to the various SEP cases. As expected, it is observed that SEP missions stay within a feasible acceleration for shorter flight times, and the Jupiter NEP mission is more feasible at longer flight times. For both cases the most demanding spacecraft requirements are at short flight times.

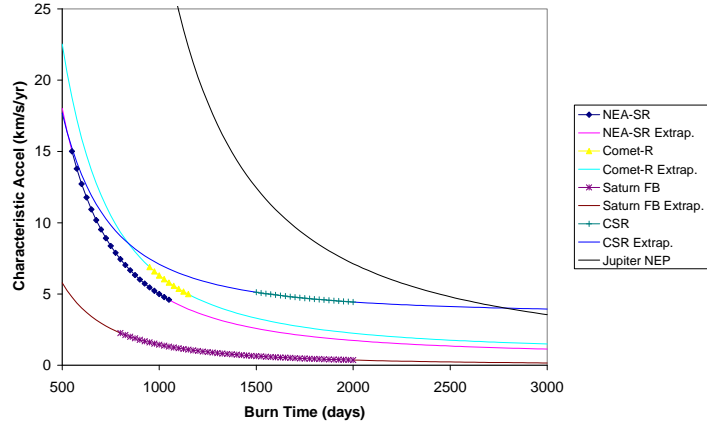


Figure 2. Extrapolated Characteristic Acceleration vs Optimum Burn Time. Dotted values show where optimized data exists in the database, solid lines are extrapolations. Jupiter NEP model added.

In addition to modeling characteristic acceleration, burn time is tied to flight time via a linear fit (equation 4).

$$t_b^* = A \cdot t_f + B \quad (4)$$

Fit parameters for each mission are presented in Table 2. In general, the fit parameters are similar and exhibit a scaling factor that is a function of the mission requirements. Examining the accuracy of the fit via the R-square term, the optimized trajectories trend the $1/x^2$ relationship and linear burn time relationship well. The NEA-SR mission is an outlier for the fit even though parameters A and B seem to trend the other missions well. This can be explained because there are two distinctly different optimum trajectory families within the dataset. The curve fit was assessed for the entire dataset. Although the data exhibits the same $1/x^2$ trend it has more scatter than the other cases, thus the reduced R-square value. Also, it is important to note that the Saturn mission does not have a burn time to flight time relationship because the flight time was fixed at 8 years for all cases. Therefore, only one relationship is derived and flight time is fixed for all further analysis.

Table 2. Fit Parameters for SEP Analytic Models

	Characteristic Acceleration Model Eq. (3)			Burn Time Model Eq. (4)		
	A ($\times 10^6$)	B	R-Square	A	B	R-Square
NEA-SR	4.345	-1.906	0.65	-22.6	69.24	0.77
Comet-R	5.411	0.8942	0.89	0.9037	0.2097	0.99
CSR	3.533	3.554	0.92	-10.35	87.77	0.97
Saturn FB	1.977	1.367	0.94	-	-	-

C. Analytic Model Derivation Example: Comet Sample Return

Figure 3 shows a CSR optimized trajectory example output plot from MALTO. The sample output provides a visualization of the trajectory including thrusting periods and magnitudes and arrival and departure times. The figure also displays details at key points and how spacecraft mass is changing over the trajectory.

The analytic model representing this trajectory is detailed in Figures 4 and 5. Figure 4 shows optimized trajectory data points for the CSR mission along with the generated curve fit and the 95% prediction bounds on the curve fit parameters. Figure 5 illustrates the linear relationship between optimum burn time and flight time. The appendix contains similar detailed plots for the other three missions studied.

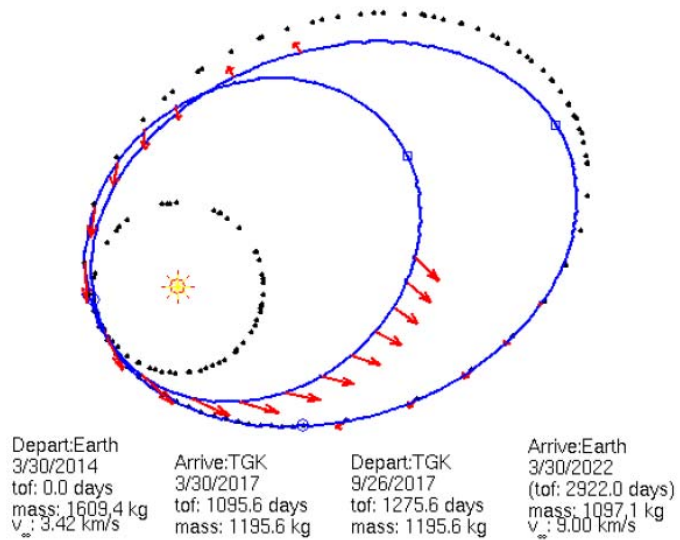


Figure 3. Example MALTO output for Comet Sample Return Mission.

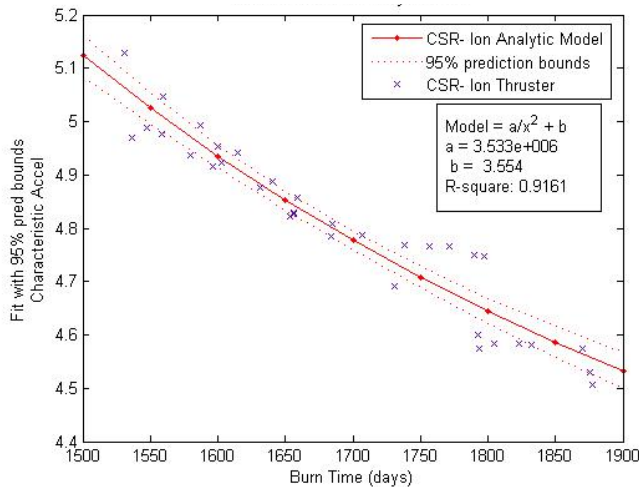


Figure 4. CSR Analytic Model Derivation. Figure includes data points calculated from the database and the curve fit.

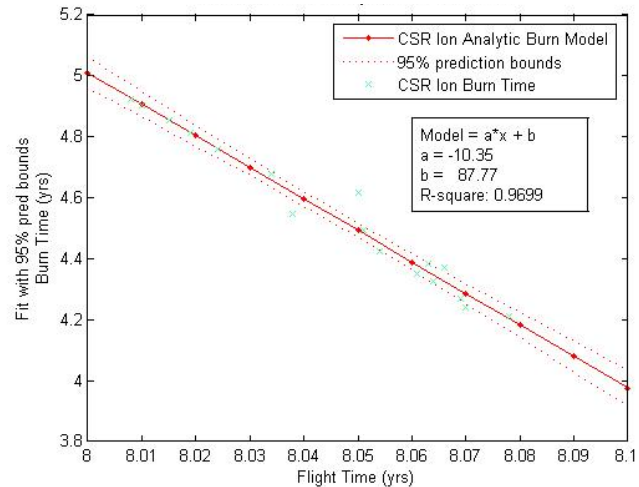


Figure 5. CSR Analytic Burn Time Model. Figure includes data points calculated from the database and the curve fit.

Using the CSR analytic model defined in Figures 4 and 5 ΔV can be solved for as a function of C3 and flight time. An example is shown below for $C3=10\text{km/s}^2$ and $T_f=8\text{yrs}$.

- 1) Calculate burn time

$$t_b^* = -10.35 \cdot t_f + 87.77 = 4.97 \text{ yrs}$$

- 2) Calculate the reference mean using

$$\bar{a}_{Char} = 3.533e6 / t_b^{*2} + 3.554 = 4.63 \text{ km/s/yr}$$

- 3) Using equation X, find the mean characteristic acceleration which includes positive Earth departure C3

$$\bar{a} = \bar{a}_{Char} - \frac{10}{0.8t_b^*} = 2.11 \text{ km/s/yr}$$

- 4) Next, calculate ΔV

$$\Delta v = \bar{a} \cdot t_b^* = 10.50 \text{ km/s}$$

The analytic model can be exported for use in a rapid EP system sizing tool. Within the sizing tool specific impulse can then be calculated one of two ways: 1) assuming optimum Isp trajectory and using the Stuhlinger equation to solve for Isp or 2) selecting a thruster and inputting Isp based on input power.

III. Sensitivity Study

A. Introduction

The four analytic SEP models in conjunction with the NEP Jupiter model span an expansive mission design space. The combination of analytic models provides an ideal condition for studying parameter sensitivities across very different missions. Employing a rapid EP system optimizer¹¹ there are six parameters that are varied: C3 [km/s^2], alpha [kg/kW], spacecraft power [kW], flight time [days], thruster efficiency [%], & thruster lifetime [days]. C3, flight time, and thruster input power are mission design functions which are chiefly dependent on the trajectory and EPS performance. Alpha, thruster efficiency, and thruster lifetime are technology parameters which are independent of trajectory. Isp is calculated at optimum. The goal is to understand which parameters, across the various missions, are most sensitive to maximizing payload mass. A more standard statistical sensitivity study or Taguchi robust design method is not used because solutions need to be system optimal.

To adequately explore this space an optimization program was written to study sensitivity. The optimization program employed is a genetic algorithm (GA). GAs are useful when objective functions are discontinuous, non-differentiable, stochastic, or highly nonlinear¹. In this case, the objective function is the EP system sizing tool. The GA begins by creating an initial random population. An individual consists of a fitness value and genes. Genes (or independent variables) are first assigned at random and evaluated by the objective function to find the fitness of the individual. Once an initial population is created the optimization program follows rules similar to natural selection. Within each generation parents are selected via various methods. The “tournament” method was used in this algorithm and it picks four individuals at random and chooses the most fit individual to be a parent for the next generation. Genes are swapped between parents to form children. In addition, mutation rules

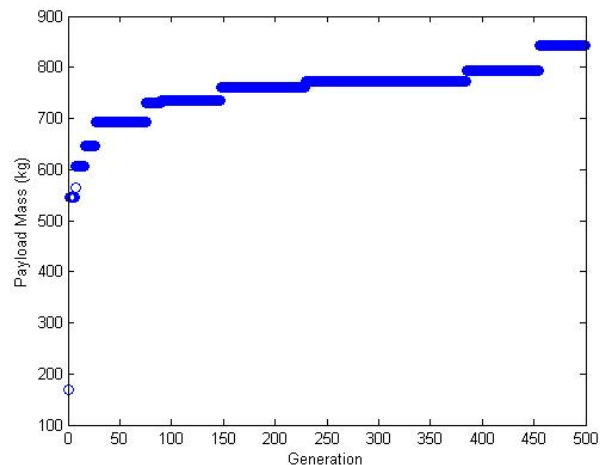


Figure 6. Maximum Payload Mass Observed in Each Generation. Sample run for the CSR mission.

are applied to introduce random variation into the population. Over multiple generations or iterations the population “evolves” and reaches an optimal fitness or solution. To illustrate how an optima is found, Figure 6 plots the most fit individual verses iterative generations.

In addition to finding the optimal solution, the GA can identify parameter sensitivity. As a result of its random nature, each time the code is run, an optima is found that is slightly different from the preceding or subsequent run. By comparing the genes of optimal individuals a sensitivity analysis can be performed by examining variations within optimal genes. Genes that exhibit more fluctuations between optimal solutions are considered less sensitive, while genes that always reach the same solution across multiple runs are considered more sensitive or necessary to achieve an optimal state.

B. CSR Sensitivity Study

Prior to creating the initial population, each independent variable is constrained. Only individuals within these bounds can be created. Bounds are employed to prevent variables from exploding or reaching zero. For instance, in most optimization cases the optimum alpha was always at the lower bound. Additionally, boundaries prevent excessive extrapolation within the analytic trajectory model. Table 3 details the constraints for the CSR mission.

Table 3. CSR Mission Parameter Constraints

	Upper Bound	Lower Bound
<i>C3 (km/s²)</i>	50	0
<i>Alpha (kg/kW)</i>	200	50
<i>Thruster Power (kW)</i>	30	5
<i>Flight Time (days)</i>	3000	2500
<i>Efficiency (%)</i>	90	60
<i>Thruster Lifetime (days)</i>	3000	300

In consequence of the random nature of the algorithm five runs are performed for each case extending out 800 generations. The results show which genes become dominant; these are the more sensitive parameters. Table 4 shows results from the algorithm (Mo=2000kg). In addition, the mean optimal result is calculated along with the standard deviation. Examining the technology parameters, it is easily seen that alpha is pushed to the lower bound for all runs, efficiency is pushed to the upper bound for all runs, and thruster lifetime is selected based on burn time in order to minimize the required amount of thrusters. Thus, it varies from 2100 to 2500 days (5.75 – 6.85 yrs).

Table 4. GA Optimization Results- Comet Sample Return, Mo=2000kg

<i>Run</i>	1	2	3	4	5	Mean	Std Dev
<i>M/Mo</i>	0.40	0.40	0.42	0.40	0.41	0.41	0.01
<i>Payload Mass (kg)</i>	790.84	800.24	842.33	800.76	826.91	812.22	21.53
<i>C3 (km/s²)</i>	2.41	4.10	1.89	2.86	3.02	2.85	0.82
<i>Alpha (kg/kW)</i>	51.78	51.53	50.88	52.52	51.50	51.64	0.59
<i>Power (kW)</i>	9.07	8.58	8.07	8.58	8.99	8.66	0.40
<i>Flight Time (days)</i>	2920.04	2918.29	2912.74	2976.80	2908.19	2927.21	28.11
<i>Efficiency (%)</i>	0.89	0.83	0.90	0.89	0.90	0.88	0.03
<i>Thruster Lifetime (days)</i>	2419.17	2100.81	2199.64	2823.40	2926.28	2493.86	368.18

To better compare variables, each is non-dimensionalized and normalized such that the mean is 1. Then, the lowest and highest normalized values are plotted. The outcome is a plot that compares the variation of each variable about the common normalized mean. Examining Figure 7 the dominant technology parameter genes can be easily identified: 1) Alpha- lower values equate to more payload mass, thus the system will always optimize to the lowest possible value, 2) Efficiency- a 7% variation was noted in the optimum values, therefore, it ranks second to alpha, and 3) Thruster lifetime- varies by as much as 17% from the mean, making it the least dominant gene. Briefly examining the three mission design parameters along with the payload mass it is observed that the optimal solutions

did vary, albeit to a limited extent. C3 on the other hand varied more extensively about the mean. However, examining the values, the variations were only between 2 to 4 km/s². Thruster input power and flight time did reach optimal solutions. However, power varied more than flight time.

It should be noted that cost is not included in the model, only mass is evaluated at the objective function. For instance, adding thrusters to a spacecraft, because of inadequate demonstrated lifetime, would have a significant associated cost; the affect would be greater than a ~40kg mass decrease that is limited to the model herein. While cost is more straightforward to implement in the case of adding a thruster to a spacecraft, it is not so straightforward to evaluate development cost. Data does not exist that can quantify how much money it costs to increase an EP thruster efficiency by 1%, or the cost of increasing a thruster lifetime by 1 day. Thus, in an effort to avoid these different issues cost is omitted.

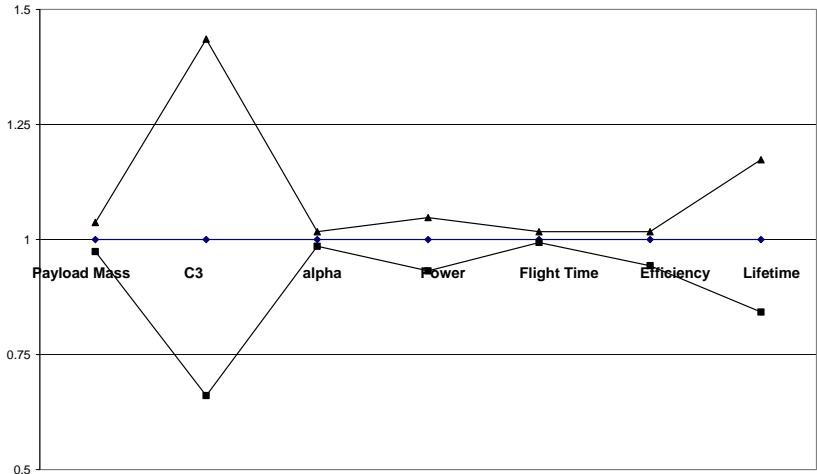


Figure 7. CSR Mission Design Parameter Sensitivity Study

C. General Sensitivity Study

The process outlined for the CSR mission was repeated for the remaining missions: NEA-SR, Comet Rendezvous, Saturn Fly-By, and Jupiter NEP transfer. Figure 8 plots the normalized maximum and minimum variables for each mission scenario. The lines are joined together to note trending across missions. C3 is not joined by lines because as

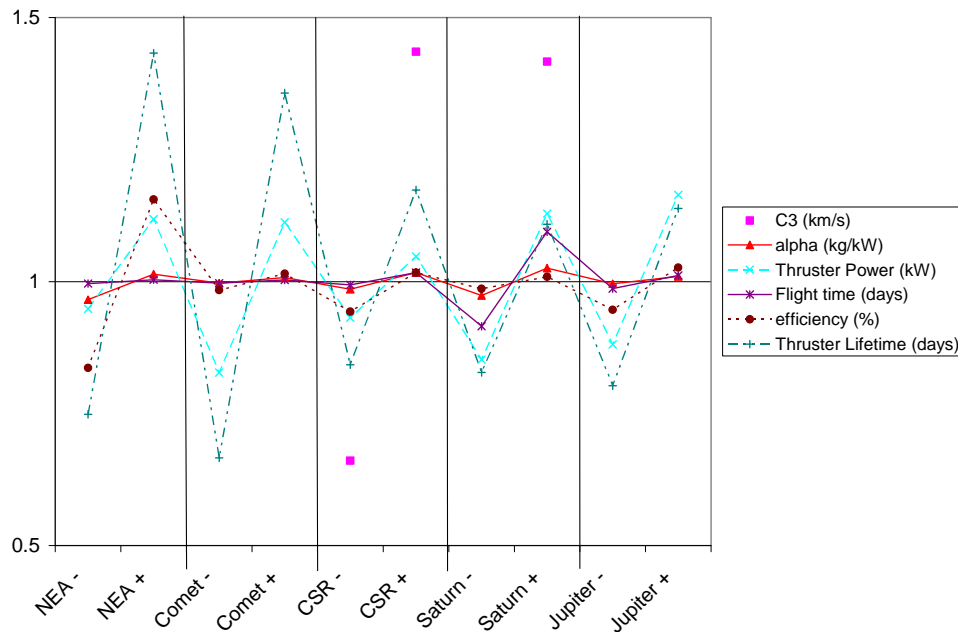


Figure 8. Mission Design Parameter Sensitivity Study.

seen in the CSR case, the actual variations are typically small even though it varies significantly about the mean. The results indicate while technology parameter sensitivity magnitude may vary across missions, in general, ranking and trending stays the same. For all system optimization cases alpha was always pushed to the lower constrained bound. Alpha is an important sizing parameter because changes are immediately reflected in spacecraft mass. Next, efficiency exhibits minimal variation, increasing efficiency proportionally decreases propellant mass. Third, thruster lifetime varies most significantly; however, the variation decreases with increasing ΔV . This shows that lifetime does not play a role until longer burn times are required.

Comparing flight time shows that there is an optimal flight time for each mission, with the only outlier being the Saturn Fly-By mission. However, this is expected because the fly-by mission has a fixed flight time. Last, comparing thruster input power shows uniform scatter over all missions. This shows that optimum input power can vary ~10-15% without a penalty in payload mass.

IV. Conclusion

Four SEP analytic optimum low-thrust trajectory models have been created from existing databases that vary flight time, power, and duty cycle. The models were developed such that they could be fed into rapid EP system sizing tools. Trends have been found within mission characteristics integrated over the entire trajectory family. Specifically, expressions were derived that present characteristic acceleration as a function of burn time, and burn time as a function of flight time. The first relationship trends a $1/x^2$ curve fit, while the second expression is linear. Curve fit parameters seem to follow a scaling law based on increasing ΔV .

Using the set of models, EP system sizing and optimization was performed using a sizing model with a genetic algorithm. The GA facilitated a study to determine technology parameter sensitivity at optimum across various missions. Sensitivity to alpha, thruster efficiency, and lifetime was examined and showed that for all the mission scenarios alpha is the most sensitive, followed by efficiency, and then by thruster lifetime.

Rapid comparison between missions helps gain insight into design and technology drivers. For the missions studied, thruster efficiency and lifetime was allowed to vary more significantly for lower ΔV missions. As a SEP system is pushed to physically limiting trajectories, EP performance parameters become increasingly important. With respect to power systems, the most direct decreases in mass are observed when alpha is decreased. While lower power density cells will always reduce overall mass, improvements are not trivial, especially with a deep space solar powered spacecraft.

Given the simplicity of the analytic models a more detailed continued examination of technology and mission parameters should be considered. In addition, analytic model fit parameters can be incrementally increased or decreased to fill a larger theoretical mission design space. This would help identify limits in current EP devices as well as mission enhancing and enabling technologies.

Appendix

Analytic models for NEA-SR, Comet Rendezvous, and Saturn Fly-By are detailed in the appendix.

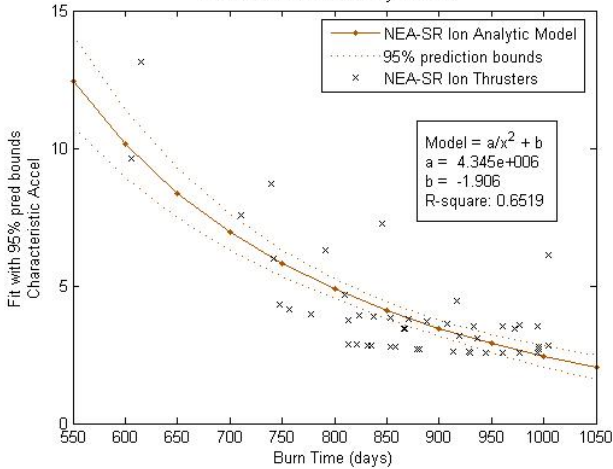


Figure A-1. NEA-SR Analytic Model Derivation.
 Figure includes data points calculated from the database and the curve fit.

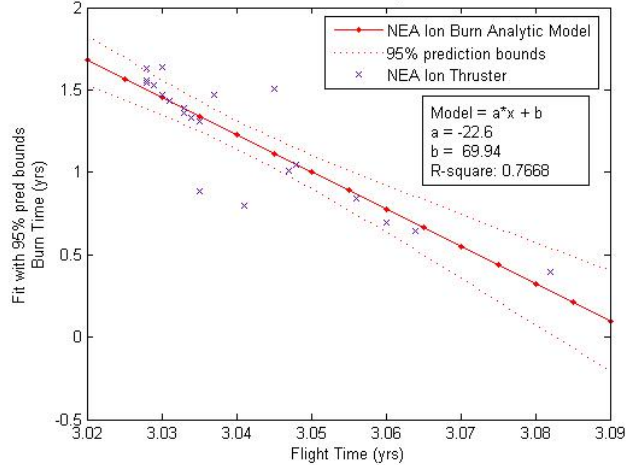


Figure A-2. NEA-SR Analytic Burn Time Model. Figure includes data points calculated from the database and the curve fit.

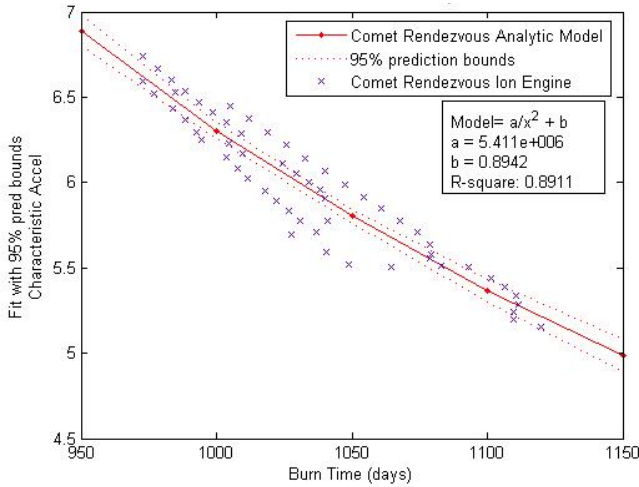


Figure A-3. Comet Rendezvous Analytic Model Derivation.
 Figure includes data points calculated from the database and the curve fit.

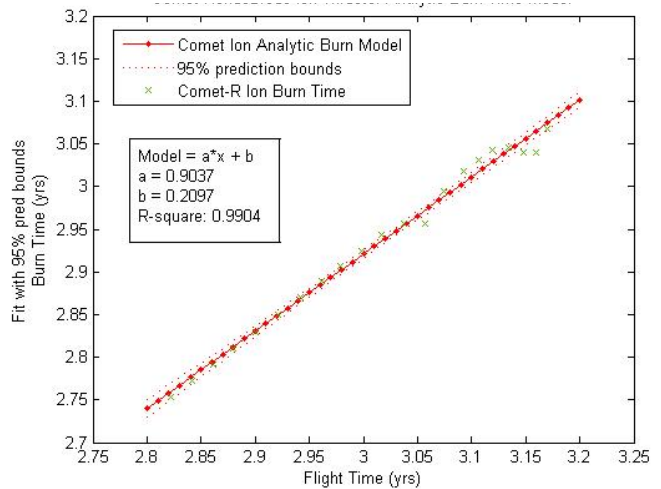


Figure A-4. Comet Rendezvous Analytic Burn Time Model. Figure includes data points calculated from the database and the curve fit.

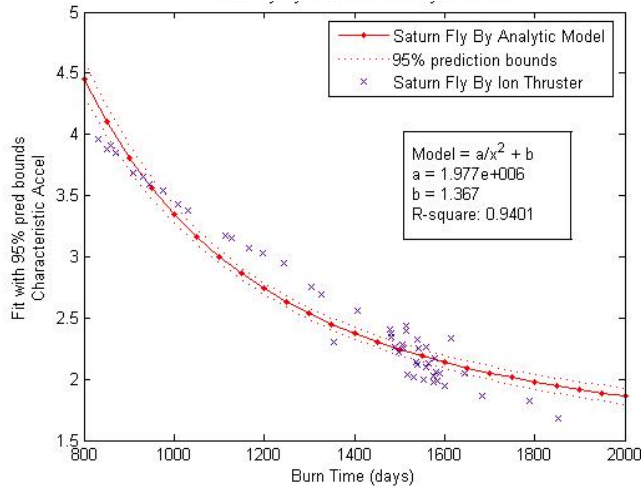


Figure A-5. Saturn Fly-By Analytic Model Derivation.
 Figure includes data points calculated from the database and the curve fit.

Details from GA optimization of the other four analytic trajectory models:

Table A-1. GA Optimization Results- NEA-SR, Mo=2000kg

Run	1	2	3	4	5	Mean	Std Dev
<i>M/Mo</i>	0.86	0.86	0.87	0.86	0.87	0.86	0.01
<i>Payload Mass (kg)</i>	1722.62	1717.39	1731.62	1715.44	1741.12	1725.64	10.68
<i>C3 (km/s)</i>	0.34	0.06	0.21	0.63	0.13	0.27	0.22
<i>Alpha (kg/kW)</i>	54.11	51.82	54.37	54.41	53.61	53.66	1.08
<i>Thruster Power (kW)</i>	3.20	3.54	3.00	3.06	3.02	3.16	0.22
<i>Flight Time (days)</i>	1066.10	1067.72	1061.78	1069.40	1063.76	1065.75	3.04
<i>Efficiency (%)</i>	0.77	0.88	0.67	0.85	0.64	0.76	0.11
<i>Thruster Lifetime (days)</i>	1543.20	2954.10	1551.04	2052.75	2208.28	2061.87	580.42

Table A-2. GA Optimization Results- Comet Rendezvous, Mo=2000kg

Run	1	2	3	4	5	Mean	Std Dev
<i>M/Mo</i>	0.50	0.50	0.50	0.49	0.50	0.50	0.00
<i>Payload Mass (kg)</i>	996.23	999.51	994.96	987.09	996.88	994.93	4.69
<i>C3 (km/s)</i>	3.61	1.08	5.28	4.58	1.91	3.29	1.77
<i>Alpha (kg/kW)</i>	50.07	50.01	50.13	50.56	50.07	50.17	0.22
<i>Thruster Power (kW)</i>	7.54	7.53	5.68	5.61	7.54	6.78	1.03
<i>Flight Time (days)</i>	1194.60	1191.88	1199.28	1194.76	1197.24	1195.55	2.82
<i>Efficiency (%)</i>	0.88	0.90	0.89	0.88	0.87	0.89	0.01
<i>Thruster Lifetime (days)</i>	1952.30	1438.97	1310.44	2670.03	2466.70	1967.69	602.92

Table A-3. GA Optimization Results- Saturn Fly-By, Mo=5000kg

Run	1	2	3	4	5	Mean	Std Dev
M/Mo	0.73	0.74	0.73	0.73	0.74	0.73	0.00
Payload Mass (kg)	3640.57	3682.07	3658.85	3660.03	3676.54	3663.61	16.39
C3 (km/s)	2.29	1.45	1.47	0.78	2.09	1.61	0.60
Alpha (kg/kW)	50.12	52.80	52.40	50.72	51.42	51.49	1.12
Thruster Power (kW)	10.94	9.69	8.26	10.16	9.40	9.69	0.99
Flight Time (days)	1733.81	1882.99	1662.29	1986.92	1812.66	1815.73	126.58
Efficiency (%)	0.88	0.89	0.88	0.89	0.87	0.88	0.01
Thruster Lifetime (days)	2210.44	2001.17	2624.93	2569.31	2679.21	2417.01	296.13

Table A-4. GA Optimization Results- Jupiter NEP, Mo=5000kg

Run	1	2	3	4	5	Mean	Std Dev
M/Mo	0.73	0.73	0.73	0.73	0.74	0.73	0.00
Payload Mass (kg)	3655.96	3669.26	3659.24	3643.36	3678.00	3661.17	13.20
C3 (km/s)	0.370037	1.05	0.22	1.46	0.62	0.75	0.51
Alpha (kg/kW)	50.47505	50.57	50.48	50.74	51.18	50.69	0.29
Thruster Power (kW)	8.559856	6.97	9.17	6.94	7.74	7.87	0.98
Flight Time (yrs)	19.75858	19.79	19.30	19.55	19.42	19.57	0.21
Efficiency (%)	0.866727	0.89	0.82	0.88	0.89	0.87	0.03
Thruster Lifetime (days)	2102.7	2981.64	2718.36	2852.57	2441.04	2619.26	351.48

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