

Role of High-Frequency Waves in Process of Electron Conductivity in SPT with High Specific Impulse

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Abstract: Results of experimental and theoretical investigations of plasma high-frequency oscillation structure near discharge chamber exit plane of stationary plasma thruster with high specific impulse are presented. Spectral density maps of perturbations by means of two electromagnetic probes are obtained. Oscillation structure by using MHD equation system is considered. Contribution of high-frequency waves to anomalous electron transport in thruster plasma based on obtained data is discussed.

Nomenclature

B	=	magnetic field
E	=	electric field
k	=	wave vector
n	=	particles density
U	=	particles velocity
u	=	electron velocity
λ	=	wave length
ν	=	oscillation frequency
v	=	ion velocity
v_{ph}	=	phase velocity
ω	=	angular frequency
ω_c	=	electron cyclotron frequency
ω_p, Ω_p	=	electron and ion plasma frequencies respectively

I. Introduction

THERE are many theoretical and experimental papers dedicated to investigations of SPT (stationary plasma thruster) plasma instabilities¹. Interest to plasma instabilities is particularly caused by the problem of anomalous electron transport across magnetic field. Electron conductivity caused by near-wall interactions or due to electron-neutral or electron-ion coulomb collisions cannot explain observed electron current. Estimation shows that classical collision conductivity fails to describe electron current density in the area with low neutral particle density². Near-wall conductivity estimation is associated with difficulties of taking into account many factors such as secondary electron emission, magnetic field line curvature in near-wall area, complex structure of near-wall sheath, etc. Those difficulties considerably complicate estimations and interpretation of near-wall conductivity role in observed electron current. At the same time electron transport in the area of the thruster exit plane without walls is a free-answer question. At the other side plasma instabilities and perturbations are recently considered as a source of additional electron current. We suppose that high-frequency waves in plasma are of special interest because this type

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of waves possesses azimuthal component of electric field. The presence of the component leads to electron motion opportunity in axial direction when wave frequency is much less than gyro-frequency.

Present work is dedicated to experimental and theoretical investigations of high-frequency wave structure in the area near discharge channel exit plane of SPT with high specific impulse (~2000-2300 sec.). Additional electron transport mechanism caused by electron scattering by high-frequency waves is considered.

II. Experimental Facility

Investigation of high-frequency wave structure in discharge channel of SPT was carried out in the vacuum chamber with volume of 90 m³ with cryogenic pumping system. Pressure during experiments was less than 10⁻⁴ torr. The thruster used for experiments is a 1600 W range stationary plasma thruster with 85 mm mean diameter. Discharge voltage was varied in the range 400-550 V. The flow rate was kept equal 3.3 mg/s. The schematic diagram of the thruster which includes magnetic circuit with coils, insulating annular acceleration channel and gas-feeding anode is shown in Fig. 1. Two electromagnetic probes were used for experiment. Both probes represented current coils with few turns of wire and ferromagnetic core. Diameter of the coil was equal to 2.5 mm, length was equal to 5 mm. Probes were hidden in shielding ceramic tube, so they did not have a direct contact with plasma. To register phase shift, probes were installed along the radial direction and azimuthally shifted at $\pi/2$ (Fig. 1).

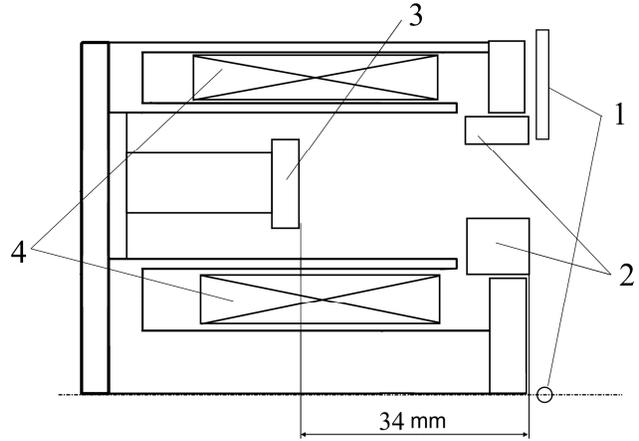


Figure 1. Scheme of probes installation: 1 – probes, 2 – ceramic insulators, 3 – gas-feed anode, 4 – magnet coils.

Obtained signal was led through the shielded line (50 ohm coaxial cable) from the probe tip to the oscillograph Tektronix TDS3000B. Both transmission lines from probes were identical.

III. Experimental Results

It is necessary to note that main measured frequencies are in the range of few MHz. Perturbations correspond to wave train with unstable amplitude. During experiment it was obtained a number of oscillograms for each discharge voltage: 400 V, 450 V, 550V. For more precise investigation of obtained wave structure wavelet analysis was used.

Wavelet transform which was used to decompose a signal is given by:

$$\tilde{S}(a, \tau) = \int S(t) \frac{1}{\sqrt{a}} m\left(\frac{\tau-t}{a}\right) dt, \quad (1)$$

where a is scale of decomposition, $m(t)$ is the basis Morlet wavelet, given by:

$$m(t) = \exp(-t^2/2) \cos(5t). \quad (2)$$

Such method allows to explore single harmonics transformation in time. Phase shift of each harmonic was defined by means of cross-correlation function which is given by:

$$R(a, \tau) = \frac{\langle \tilde{S}_1(a, \tau) \tilde{S}_2(a, \tau) \rangle}{\sqrt{\tilde{S}_1^2(a, \tau)} \sqrt{\tilde{S}_2^2(a, \tau)}}. \quad (3)$$

During the analysis of obtained spectra three main frequency bands can be separated: ~3-4 MHz, ~4-10 MHz, ~15-17 MHz. The most intensive band at operation modes with discharge voltage 450 V and 550 V is the second band, for operation mode with discharge voltage 400 V is the first one. At the same time frequency of the second band has a decreasing behaviour when discharge voltage declines. Spectral densities of cross-correlation functions are shown in Fig. 2. Data are given for the second band only.

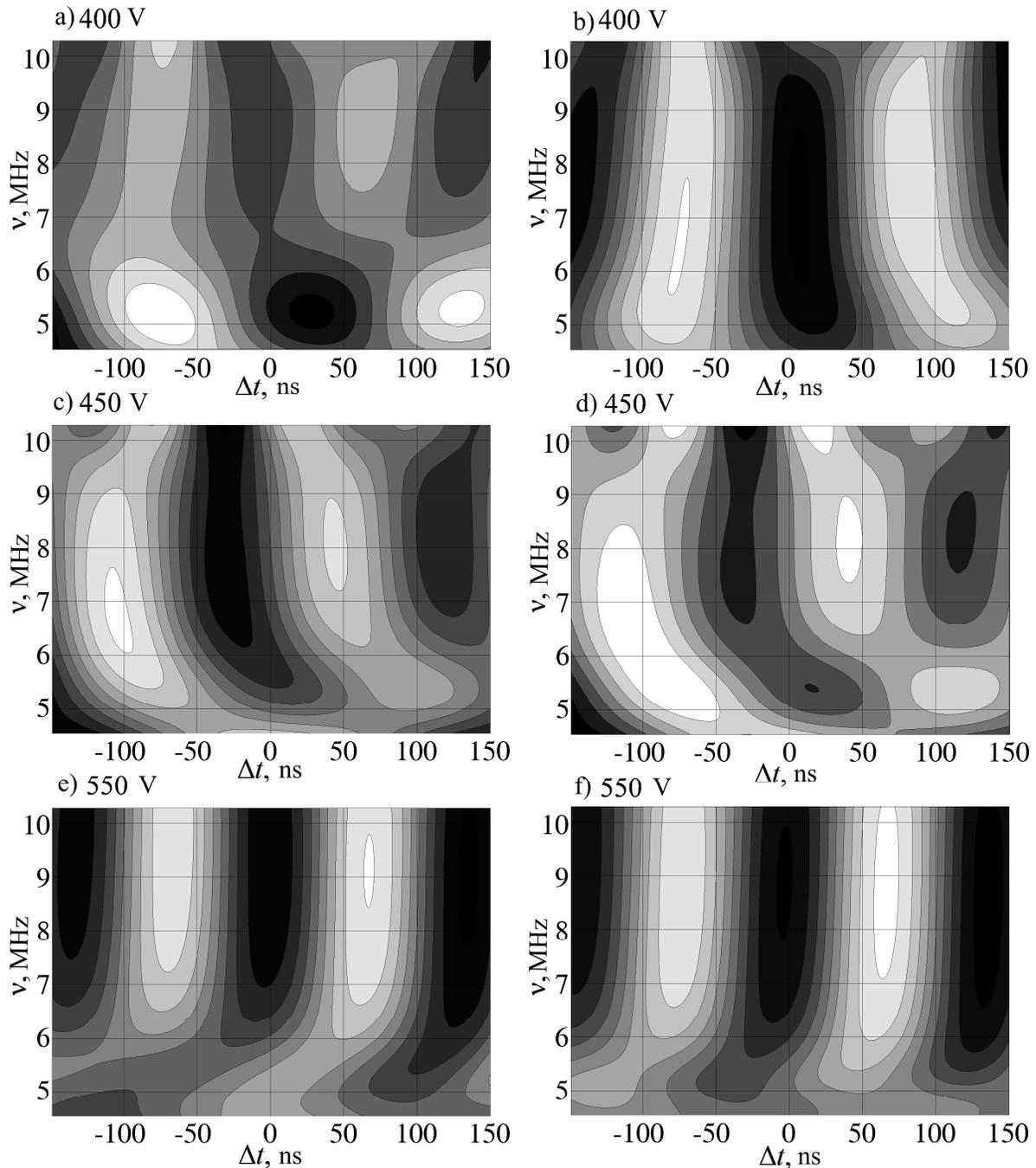


Figure 2. Spectral density of cross-correlation function for various operation modes. Two different points at time for each operation mode are presented.

Main frequency of the first band does not depend on discharge voltage and almost has not phase shift (~ 10 ns), thereby frequencies of this band have an external source. However this band disappeared when thruster was turned off. Therefore we can suppose that the source of the first frequency band is a unit of the supply systems, for example cathode. Perturbations of this frequency band are not purely azimuthal. But additional studying for more comprehensive investigation of the first frequency band is required. In present work we have chosen ~ 4 -10 MHz frequency band for the most precisely studying.

The second band can be characterized by variable peak frequency and presence of phase shift between probes. Phase shift occurrence shows that this type of perturbations is waves in plasma. Since thruster discharge chamber is closed

in azimuthal direction, wave length must possess a discrete values $\lambda = \pi d/m$, where d is thruster mean diameter, $m = 1, 2, 3, \dots$. Waves which lengths are satisfied to this condition are purely azimuthal. On the basis of data analysis shown in Fig. 2 we can conclude that for ~4-10 MHz frequency band two harmonics build up with wavelength corresponded to $m = 1$ and $m = 2$. In Table 1 phase velocities are resumed.

Table 1. Phase velocities for each operation mode. Values were calculated by two different ways: by means of phase shifts and in accordance with expression $v_{ph} = \lambda v$.

m ^a	400 V		450 V		550 V	
	on phase shift	on wavelength	on phase shift	on wavelength	on phase shift	on wavelength
1	-	-	17,2·10 ⁵ m/s	16,6·10 ⁵ m/s	19,7·10 ⁵ m/s	20,3·10 ⁵ m/s
2	6,2·10 ⁵ m/s	6,6·10 ⁵ m/s	6,9·10 ⁵ m/s	7,3·10 ⁵ m/s	9,8·10 ⁵ m/s	10,1·10 ⁵ m/s
^a Propagation number						

Velocities obtained by two different ways have a good agreement, at the same time phase velocity of each harmonic has a decline behavior when discharge voltage is decreased.

Third band has an external source, because it was being observed even all power supply systems were turned off and frequencies of this band have no phase shifts for all operation modes and oscillograms.

It should be mentioned, that cross-correlation functions have some deviation from minimum for the second harmonic and from zero for the first harmonic.

We can conclude, that high frequency waves are observed in discharge channel SPT with high specific impulse. Wavelengths of such perturbations are close to the length of discharge chamber in azimuthal direction. Propagation velocities of the waves are close to the electron drift velocity.

IV. Azimuthal Wave Structure in SPT Plasma

We consider linearized equation system for cold two-component plasma without collisions, consisting of ions and electrons for description of obtained waves structure. We assume that electrons are magnetized, while ions are unmagnetized. Then we make appropriate assumptions in accordance with obtained experimental data.

Momentum and fluid equations in terms of the small perturbations are governed by:

$$\frac{\partial \mathbf{U}}{\partial t} + (\mathbf{U}_0 \cdot \nabla) \mathbf{U} + (\mathbf{U} \cdot \nabla) \mathbf{U}_0 = -\frac{e}{m} (\mathbf{E} + [\mathbf{U} \mathbf{B}_0] + [\mathbf{U}_0 \mathbf{B}]) \quad (4)$$

$$\frac{\partial n}{\partial t} + \nabla \cdot (\mathbf{U}_0 n) + \nabla \cdot (\mathbf{U} n_0) = 0 \quad (5)$$

Poisson equation for electric field is given by:

$$\nabla \cdot \mathbf{E} = \frac{e}{\epsilon_0} (n_i - n_e) \quad (6)$$

where \mathbf{U} is perturbations of ion and electron velocity, index «0» corresponds to unperturbed plasma parameters. Y axis was chosen along azimuthal direction, X – along axial direction and Z – along external magnetic field. Waves which are polarized along external magnetic field represent Langmuir waves with frequencies which strongly exceeds the band we are interested in. In this work we consider waves polarized in plane perpendicular to external magnetic field. Propagation direction of the waves is azimuthal direction. We try solution of the system in terms of small perturbations: $f(t, y) \propto f(x) \exp(-i(\omega t - k_y y))$. We also neglect wave's own magnetic field influence on wave propagation process. In this case equation system could be written as follows.

For electrons:

$$u_x = i \frac{e}{m} \frac{\omega - k_y u_0}{(\omega_c - \frac{\partial u_0}{\partial x}) \omega_c} E_x + \frac{e}{m} \frac{1}{\omega_c - \frac{\partial u_0}{\partial x}} E_y \quad (7)$$

$$u_y = -\frac{e}{m} \frac{1}{\omega_c} E_x + i \frac{e}{m} \frac{\omega - k_y u_0}{(\omega_c - \frac{\partial u_0}{\partial x}) \omega_c} E_y \quad (8)$$

Assume, that $\omega_c \gg \frac{\partial u_0}{\partial x}$, $\frac{\partial \omega_c}{\partial x} \ll \frac{\partial^2 u_0}{\partial x^2}$, then substitute velocities into flow equation:

$$n_e = \frac{e}{m} \frac{n_0}{\omega_c^2} \left(\frac{1}{\omega_c} \frac{\partial^2 u_0}{\partial x^2} - \frac{\partial \ln B_0 / n_0}{\partial x} - k_y \frac{\omega_c}{\omega - k_y u_0} - k_y \frac{1}{\omega - k_y u_0} \frac{\partial u_0}{\partial x} \right) E_x + n_0 \frac{e}{m} \frac{1}{\omega_c^2} \frac{\partial E_x}{\partial x} - \quad (9)$$

$$i n_0 \frac{e}{m} \frac{1}{\omega_c (\omega - k_y u_0)} \frac{\partial E_y}{\partial x} - i \frac{e}{m} \frac{n_0}{\omega_c^2 (\omega - k_y u_0)} \left(\frac{\partial^2 u_0}{\partial x^2} + \frac{\partial \ln n_0}{\partial x} \omega_c - k_y (\omega - k_y u_0) \right) E_y = 0$$

Assume that frequency is high enough to neglect convective terms in ion momentum equation. Then ion density expression could be written as follows:

$$n_i = n_0 \frac{1}{\omega^2} \frac{e}{M} \frac{\partial E_x}{\partial x} + \frac{1}{\omega^2} \frac{e}{M} E_x \frac{\partial n_0}{\partial x} + i k_y n_0 \frac{1}{\omega^2} \frac{e}{M} E_y \quad (10)$$

Then obtained velocities and densities are substituted into equation (6) and small terms are neglected:

$$\left(\frac{\Omega_p^2}{\omega^2} - \frac{\omega_p^2}{\omega_c^2} - 1 \right) \frac{\partial^2 \varphi}{\partial x^2} - \left(\frac{1}{\omega_c} \frac{\omega_p^2}{\omega_c^2} \frac{\partial^2 u_0}{\partial x^2} - \frac{\omega_p^2}{\omega_c^2} \frac{\partial \ln B_0 / n_0}{\partial x} - \frac{\Omega_p^2}{\omega^2} \frac{\partial \ln n_0}{\partial x} - \frac{\omega_p^2}{\omega_c^2} \frac{k_y}{\omega - k_y u_0} \frac{\partial u_0}{\partial x} \right) \frac{\partial \varphi}{\partial x} \quad (11)$$

$$\left(\frac{k_y \omega_p^2}{\omega_c^2 (\omega - k_y u_0)} \frac{\partial^2 u_0}{\partial x^2} + k_y \frac{\omega_p^2}{\omega_c (\omega - k_y u_0)} \frac{\partial \ln n_0}{\partial x} + k_y^2 \left(\frac{\Omega_p^2}{\omega^2} - \frac{\omega_p^2}{\omega_c^2} - 1 \right) \right) \varphi = 0$$

Analysis of this equation is presented below.

A. High-frequency wave structure

In this section we are just interested in structure of azimuthal waves. Thereby all plasma parameter gradients are neglected.

For this case electron and ion density equations are governed by:

$$n_i = n_0 \frac{1}{\omega^2} \frac{e}{M} i k_x E_x + i k_y n_0 \frac{1}{\omega^2} \frac{e}{M} E_y \quad (12)$$

$$n_e = n_0 \frac{e}{m} \frac{1}{\omega_c (\omega - k_y u_0)} (k_x E_y - k_y E_x) + i \frac{e}{m} \frac{n_0}{\omega_c^2} (k_y E_y + k_x E_x) \quad (13)$$

Substitute these expressions into Poisson equation:

$$i(k_x E_x + k_y E_y) \left(1 - \frac{\Omega_p^2}{\omega^2} - \frac{\omega_p^2}{\omega_c^2} \right) = -\frac{\omega_p^2}{\omega_c (\omega - k_y u_0)} (k_x E_y - k_y E_x) \quad (14)$$

Electric field component ratio is given as follows:

$$\left| \frac{E_y}{E_x} \right| = \frac{\omega_c / (\omega - k_y u_0) + k_x / k_y}{1 - k_x \omega_c / k_y (\omega - k_y u_0)} \quad (14)$$

For purely azimuthal waves this ratio is determined by:

$$\left| \frac{E_y}{E_x} \right| = \frac{\omega_c}{(\omega - k_y u_0)}. \quad (15)$$

As a result, we can conclude that azimuthal waves are generally polarized along their propagation direction. Therefore perturbed currents are polarized across propagation direction of the wave, along thruster axis. Perturbations of magnetic field in such wave and external magnetic field must be co-directional.

B. Dispersion equation analysis

Equation (11) is rewritten in non-dimensional terms as follows:

$$A(x) \frac{1}{\Omega^2} + B(x) \frac{1}{\Omega - 1} = -C(x) \quad (16)$$

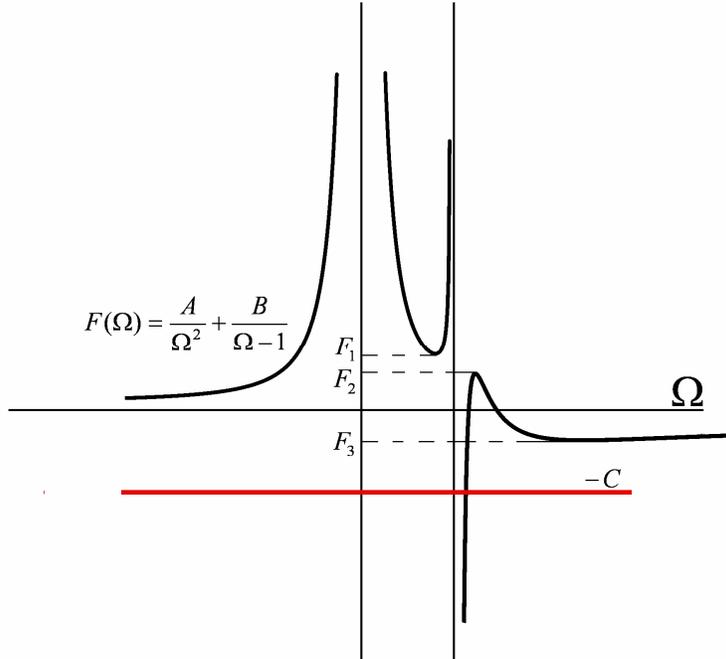
where $\Omega = \omega / k_y u_0$,

$$A(x) = \left(\frac{\Omega_p \omega_n}{u_0 k_y \omega_p} \right)^2 \left(\frac{1}{k_y^2 \varphi} \frac{\partial^2 \varphi}{\partial x^2} + \frac{1}{k_y^2 \varphi} \frac{\partial \varphi}{\partial x} \frac{\partial \ln n_0}{\partial x} - 1 \right), \quad (17)$$

$$B(x) = \frac{\omega_p^2}{\omega_c^2} \frac{1}{u_0 k_y^2} \left(\frac{1}{\varphi} \frac{\partial u_0}{\partial x} \frac{\partial \varphi}{\partial x} - \frac{\partial^2 u_0}{\partial x^2} - \frac{\partial \ln n_0}{\partial x} \omega_c \right), \quad (18)$$

$$C(x) = \left(\frac{\omega_p^2}{\omega_c^2} + 1 \right) \left(1 - \frac{1}{k_y^2 \varphi} \frac{\partial^2 \varphi}{\partial x^2} \right) - \frac{\omega_p^2}{\omega_c^2} \left(\frac{1}{\omega_c} \frac{\partial^2 u_0}{\partial x^2} - \frac{\partial \ln B_0 / n_0}{\partial x} \right) \frac{1}{k_y^2 \varphi} \frac{\partial \varphi}{\partial x}. \quad (19)$$

The easiest way to analyze this equation is a graphic approach. We consider near thruster exit plane area. In this area all unperturbed local parameters functions such as $n_0(x)$, $B_0(x)$, $E_0(x)$ are monotonic and decreasing. In the Fig. 3 characteristic graphs of the left and of the right side Eq. 16 are shown when $A(x) > 0$, $C(x) > 0$.



From the analysis of the figure it is clear that in general case Eq.16 has three roots. Complex roots appear when $F_1 < C < F_2$, $F_1, F_2 > 0$, and $-C < F_3 < 0$. Particularly, in case $A < -4B$ left side of Eq. 16 has only one root and criteria of instability become more simple. There are two possible cases: $A < 0$, $C < 0$ and $A > 0$, $C > 0$. There are sufficient instability conditions. Assume that $|k_y| \approx |\lambda| \gg |\partial \ln n_0 / \partial x|$, where we suppose that in the small neighborhood of a point x_0 $\varphi(x) \sim \lambda x$. Therefore only one of two previous cases is possible: $A < -4B$, $C < 0$. Criterion of instability for rather smooth function $\varphi(x)$ is given by following system:

Figure 3. Graphic view of Eq. 16 when $A > 0$, $C > 0$.

$$-\frac{1}{4}\left(\frac{\Omega_p}{u_0 k_y}\right)^2 < \frac{1}{u_0} \frac{\partial^2 u_0}{\partial x^2} + \frac{1}{u_0} \frac{\partial \ln n_0}{\partial x} \omega_c \quad (20)$$

$$k_y^2 < -\frac{\partial \ln B_0 / n_0}{\partial x} \lambda, \quad (21)$$

Determine the relative gradient of function as a natural logarithm derivative of function. On the base of the last expression analysis we can conclude that waves become unstable when their relative gradients (k_y , λ) are less then relative gradients of local plasma parameters. At the other side minimal value of the perturbation gradients is determined by maximal geometric size of the discharge consequently by size scale of device. Local plasma parameters gradients are generally determined by magnetic field configuration and input parameters such as mass flow rate and discharge voltage.

V. Electron Transport

Classical and near wall transport dependencies on magnetic field are given by following expression $1/B^2$. At the same time any deviation from this proportion is assumed to be anomalous transport appearance. In particular Bohm diffusion is usually related to anomalous transport, for which dependency on magnetic field is given by proportion $1/B$. Bohm diffusion comes from the results of enormous number of experiments and allows getting right tendencies and estimations. But this type of anomalous transport does not make clear physics of transport process in SPT plasma.

A. Classical case

For estimation of electron mobility across magnetic field we write momentum equation for electrons. Without pressure and ionization in homogeneous medium for stationary case the following system for X and Y is:

$$u_y = \frac{v}{\omega_c} u_y, \quad (22)$$

$$u_y = \frac{u_0}{1 + v^2 / \omega_c^2} \approx \frac{E_0}{B_0}, \quad (23)$$

where v is a frequency of electron-neutral collisions.

Data obtained in probe measurements in paper³ are used for the following estimations. Applied magnetic field is of the order of 250 G in our conditions, the electron gyro frequency $\omega_c = (3-7) \times 10^9$ Hz. Electron drift velocity is $u_0 = 10^6$ m/s. Electron-neutral collision frequency is governed by expression $\nu = n_n \langle \sigma_{ne} u_e \rangle \sim 10^6$ Hz, where n_n is neutral particles density, $\langle \sigma_{ne} u_e \rangle$ is a reaction rate constant. Density of neutral particles was chosen equal $n_n \sim 10^{13}$ 1/cm³ in accordance with calculations; this value is corresponded to mean value near discharge chamber shear. Ion-electron coulomb collisions are usually neglected in comparison with electron-neutral collisions in such estimations². As a result estimation for electron velocity magnitude across the magnetic field due to electron-neutral collisions is follow: $u_x = (0.1-1) \times 10^3$ m/s. Real electron motion velocity can be obtained using by following expression: $n_e u_x e S = (1 - \eta_I) I_d$ where S is a discharge channel square, I_d is a discharge current, η_I is an electron current efficiency, which is equal to ion plume current and discharge current ratio. These estimations are fair in the near thruster exit plane area. There are special experimental methods dedicated to studying such coefficients⁴. For described thruster this value is equal to 0.75-0.8. Thereby estimation for real electron velocity across magnetic field is follow: $u_x = (1.2-2.4) \times 10^3$ m/s.

It should be noted that neutral particle and electron densities were chosen equal to discharge channel volume averaged value in described estimations. Therefore they were overrated in comparison with thruster exit plane value. Thereby we conclude that transversal conductivity based on electron-neutral collisions is not in a position to explain real electron current.

B. Anomalous transport

It was shown, that classical mechanisms of electron transport across magnetic field cannot describe real electron current density. Therefore, other types of conductivity which could be associated with anomalous transport group should exist.

In current work we consider an electron transport opportunity based on unstable behavior of high frequency wave amplitude in SPT plasma. It was shown, that waves with frequency of the order of few MHz occur in HT plasma and propagate in azimuthal direction with nearly electron drift velocity. The waves are polarized in plane perpendicular external magnetic field. In case of unstable amplitude of perturbations, electron shift during one period of perturbation might occur, particularly when wave energy dissipation takes place. This scheme is illustrated in Fig. 4.

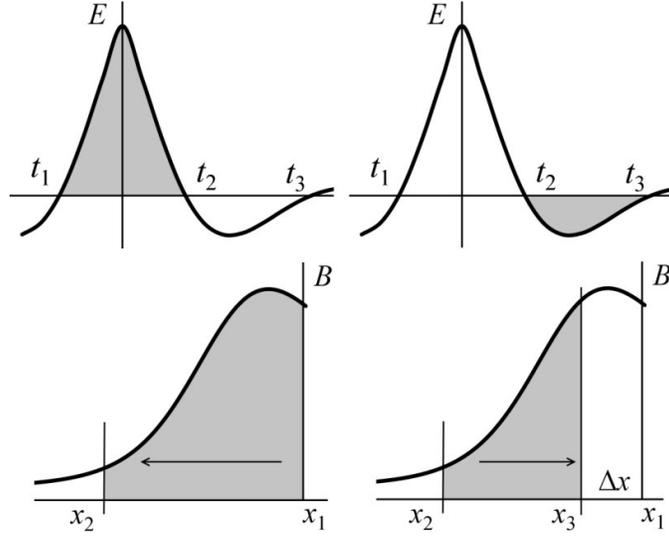


Figure 4. Scheme of electron shift occurrence during high frequency wave interaction

At the first half of period an electric field increases then decreases, at the same time electron moves along thruster axis from point x_1 to x_2 . This is a drift motion in crossed perturbed electric and external magnetic fields. Such motion occurs when following condition is satisfied $\omega \ll \omega_c$. At the next half of period electron moves in inverse direction, but does not reach initial point. In this case electron velocity perturbation is given by:

$$u(x) = \frac{E_y(t)}{B_0(x)} \Rightarrow \frac{dx}{dt} = \frac{E_y(t)}{B_0(x)}. \quad (18)$$

This expression can be rewritten:

$$\int_{x_1}^{x_2} B_0(x) dx = \int_{t_1}^{t_2} E_y(t) dt. \quad (19)$$

From the last equation we can conclude that area under electric field perturbation curve during first half of period must be equal to the area under magnetic field curve during electron motion. If area under electric field curve at second half of period decreases, than area under magnetic field should decline. It is possible only electron does not reach initial point.

If we assume, that electron shifts are sufficiently small and magnetic field during electron motion weakly changes, expression for estimation of electron shift during one period could be written as follows:

$$\Delta x = \frac{1}{B_0} \int_{t_1}^{t_2} E_y(t) dt. \quad (20)$$

where $t_1 - t_2 = 2\pi/\omega$.

It is clear, that shift could possess both positive and negative value. However in the presence of dissipative effects, such as electron-neutral or electron-wall collisions, moving at anode direction could gain an advantage.

VI. Conclusions

The experimental investigation of high frequency perturbations in SPT plasma has shown occurrence of azimuthal waves in discharge channel. For 1.6 kilowatt-range SPT the main band is 4-10 MHz. Wavelengths of such perturbations are close to length of discharge chamber in azimuthal direction $\lambda_1 = \pi d$ and $\lambda_1 = \pi d/2$, where d is a mean thruster diameter. Propagation velocities of the waves are of the order of electron drift velocity.

It has been shown that high frequency wave in SPT plasma has the same structure as a magnetic sound. Electric field is generally polarized in azimuthal direction; consequently electron current is generally polarized in axial direction.

It has been studied that waves would become unstable when their relative gradients (k , λ) are less than relative gradients of local plasma parameters. Instability development criterion has been obtained.

Probable conductivity mechanism based on unstable behavior of high frequency waves in HT plasma has been described.

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