The Thrust of a Collisional Magnetized Plasma

IEPC-2011-045

Presented at the 32^{nd} International Electric Propulsion Conference,
Wiesbaden, Germany
September 11–15, 2011

A. Fruchtman* and G. Makrinich†
H.I.T. – Holon Institute of Technology, Holon 58102, Israel

Abstract: The acceleration of a partially ionized flow by a magnetic field pressure is explored, where the ions collide with neutral atoms during their acceleration. The configuration is similar to that of the Hall thruster. It is shown that, as momentum is delivered not only to ions but also to neutrals, the thrust per given electric power is larger than when ions are collisionless. The thrust over mass flow rate is lower when ions are collisional. Expressions in the collisional regime are derived for thrust over power, thrust over mass flow rate, specific impulse and efficiency.

Nomenclature

F	= thrust
P	= power
$\dot{m}$	= mass flow rate
$\overrightarrow{E}$	= electric field
$\overrightarrow{B}$	= magnetic field
$v_{e,i}$	= electron, ion drift velocity
$\mu_{e,i}$	= electron, ion mobility
$\nu_{e,i}$	= electron, ion collision frequency
m, m_i	= electron, ion (or neutral) mass
ω_c	= electron cyclotron frequency
$\Gamma_{e,i,N}$	= electron, ion, neutral particle flux density
S	= channel cross section area
d	= channel length
ϕ_A	= applied voltage
ϕ	= electric potential
n	= plasma density
I_{sp}	= specific impulse
η	= efficiency
g	= free-fall acceleration
λ	= mean free path of ion-neutral collision

*Professor, Faculty of Sciences, email: fnfrucht@hit.ac.il.
†Research Scientist, Faculty of Sciences, email: gennady@hit.ac.il.

I. Introduction

A major figure of merit in propulsion is the thrust per unit of deposited power, the ratio of thrust over power. We have recently demonstrated experimentally and theoretically[1,2,3] that for a fixed deposited power the momentum delivered by the electric force is larger if the accelerated ions collide with neutrals during the acceleration. The power deposited in the ion current depends only on the potential drop they cross. The momentum imparted, however, depends also on the residence time of the ions in the acceleration region. If ions collide with neutrals after they have been accelerated then the total momentum of ions and neutrals is the same as without collisions. However, if ions collide while they are being accelerated by the electric field, their residence time in the acceleration region is increased, due to their slowing-down collisions with neutrals, and the total momentum gained by the flow is enhanced. The electric force is being felt by the ions for a longer time, and the larger impulse gained by the ions is redistributed also in the neutral-gas mass through the ion-neutral collisions.

The main advantage of electric propulsion is the reduction of the propellant mass needed for a given space mission due to the higher exhaust velocities, relative to those achieved by chemical propulsion. When ions collide with neutrals during the acceleration, however, the average jet velocity is lowered. Therefore, the higher thrust for given power, F/P , is achieved for a collisional plasma at the expense of a lower thrust per unit mass flow rate, $F/\dot{m}$ ($\dot{m}$ is the mass flow rate), reflecting what is true in general, that the lower the flow velocity (and the specific impulse) is, the higher the thrust for a given power. This is the usual trade-off between having a large specific impulse and a large thrust. Broadening the range of jet velocities and thrust levels is desirable since there are different propulsion requirements for different space missions. Operation in the collisional regime can be advantageous for certain space missions, either for air-breathing propulsion[4], or for short periods of time when the thrust has to be increased and electric power is limited.

In this paper we summarize (in sections II and III) experimental results that demonstrate an increased thrust for a given power when a collisional plasma is accelerated across a magnetic field. We then analyze in sections IV and V the expected performance of a collisional magnetized plasma thruster relative to a collisionless magnetized plasma thruster.

II. The experimental system

The experiments described are carried out in a radial Plasma Source (RPS). We previously described the RPS and the measurements of the momentum carried by the flow[1,2]. The RPS, shown in Fig. 1, consists of a ceramic unit, a molybdenum anode, a magnetic-field generating solenoid, an iron core, a gas distributor and a cathode employed for neutralizing the ion flow. The heated cathode neutralizer is located 4cm from the RPS edge. All the measurements are taken for a mass flow rate through the cathode of 4sccm (Standard Cubic Centimeter per Minute). An argon gas is injected through the gas distributor in the anode. A voltage that is applied between the anode and the cathode ignites a discharge and accelerates the plasma ions radially-outward across the axial magnetic field. The momentum of the mixed ion-neutral jet is balanced by magnetic field pressure. The ions are accelerated by an applied electric field across a magnetic field, while electrons perform an $\overrightarrow{E} \times \overrightarrow{B}$ drift, as they do in the Hall thruster.

III. Measured plasma parameters and thrust

The plasma diagnostics system for the plasma potential included a Langmuir probe and an emissive probe. The emissive probe was built from a tungsten wire of 0.1mm diameter and of 12mm length. The plasma potential was measured at the emissive probe saturation potential during its heating. The Langmuir probe was used for the detection of any disturbance induced by the emissive probe in plasma. The Langmuir probe was built from a tungsten wire of 0.25mm diameter and of 5mm length. It was located at 5mm downstream near the emissive probe and measured a floating potential. The floating potential was measured by the Langmuir probe twice, first when the emissive probe is cool and a second time after its heating. Absence of a difference between those two measurements of floating potentials means that the disturbance induced by the emissive probe was not significant.

We employ a theory that we recently developed[5] for a cylindrical emissive probe to evaluate the plasma potential distribution in our Radial Plasma Source (RPS). Within the theory, the potential of a cylindrical emissive probe in plasma is calculated for an arbitrary ratio of Debye length to probe radius (Debye number).

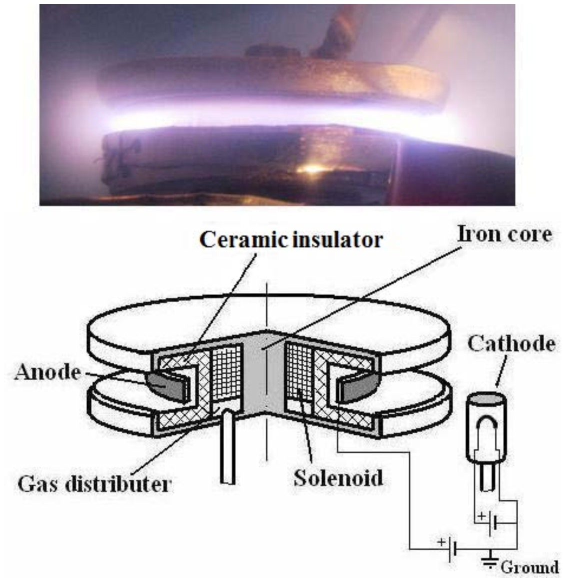

Figure 1. The RPS in operation and a schematic.

For a small Debye number, we derived analytical expressions for the potential drop and for the currents. The regime of a finite Debye number requires numerical calculation. For evaluating the plasma potential, we measured the potential of a floating probe once when the probe is cold and once when the probe is heated up to space-charge current saturation. We also measured the ion saturation current into a negatively biased Langmuir probe.

The analysis of the probe measurements using the theory and the inferred radial dependencies of the plasma parameters were recently described[6]. The radial dependencies of the electron temperature, the plasma density, and the plasma potential are shown in Figs. 2, 3, and 4. All these profiles are shown for two gas flow rates. The magnetic field intensity is maximal near the edge of the ceramic unit and its value there is 160G. The discharge current is 1.9A. It is shown in Fig. 4 that most of the potential drop is where the magnetic field is maximal.

The main measurement is that of the force exerted by the mixed ion-neutral jet on a thrust meter. These measurements are shown in Fig. 5 and were recently analyzed and explained[1,2]. As is shown in Fig. 5, if ion-neutral collisions are not taken into account, the measured thrust (force on our balance force meter) is larger than the maximal theoretically - expected thrust. The enhanced thrust was explained once ion-neutral collisions were included in the model[1,2]. Obviously, because of the cylindrical symmetry of the flow, the RPS will have to be modified in order to be used as a thruster. The increase of thrust through ion-neutral collisions has also been proposed for an ion thruster configuration[7].

IV. Collisionless ions - thruster performance

We examine the possibility of extending the regime of operation of thrusters that employ magnetic field pressure to the collisional regime. In this section and in the next section, we analyze the expected performance of thrusters that employ collisionless or collisional plasmas. For simplicity we assume a one-dimensional channel along which the plasma is accelerated by an applied voltage across a magnetic field that is perpendicular to the electric field. We analyze this acceleration region in which the ions and electrons flow in opposite directions. The ionization is assumed to occur outside the acceleration channel, so that no ionization occurs in the acceleration channel. The flows are assumed constant along the acceleration channel.

. The magnitude of the electron drift velocity across the magnetic field is

$$v_e = \mu_e E = \frac{e\nu_e}{m\omega_c^2} E, \tag{1}$$

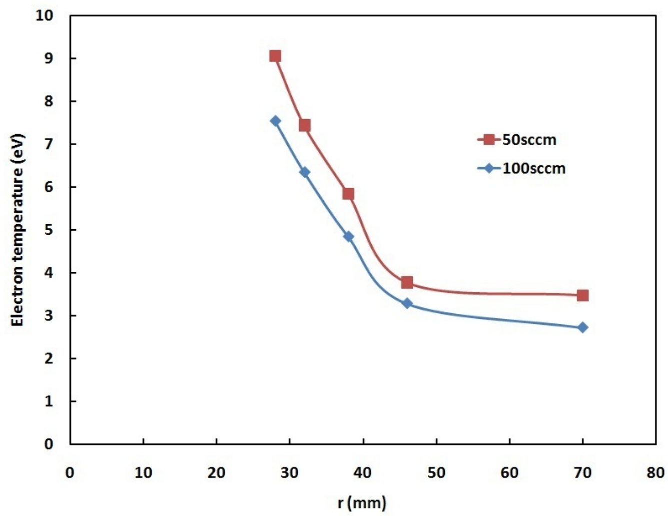

Figure 2. The calculated electron temperature versus the distance from the axis of symmetry of the RPS for the two gas flow rates.

and the electron (and ion) density is

$$n = \frac{\Gamma_e}{v_e} = \frac{\Gamma_i}{v_i} \tag{2}$$

The ion flow parameters depend on whether the ions are collisionless or collisional. Let us first discuss the case that the ions are collisionless, as is the usual case in the Hall thruster.

The electric force per unit area is balanced by the magnetic field pressure

$$nE = \frac{\Gamma_e m \omega_c^2}{e \nu_e}. \tag{3}$$

Until here nothing has been asuumed about the ion dynamics. Let us assume now that the ions are collisionless. The ion velocity is

$$v_i = \sqrt{\frac{2e\left(\phi_A - \phi\right)}{m_i}}, \tag{4}$$

so that the electric potential along the channel is

$$\sqrt{\frac{2e\left(\phi_A - \phi\right)}{m_i}} = \frac{\Gamma_e}{\Gamma_i} \int_0^x \frac{m\omega_c^2 dx'}{m_i \nu_e}. \tag{5}$$

The next relation determines the ratio between the electron and the ion fluxes and the inequality represents the condition for a high current utilization.

$$\frac{\Gamma_e}{\Gamma_i} = \frac{\sqrt{2e\phi_A/m_i}}{\int_0^d \left(m\omega_c^2/m_i\nu_e\right) dx} \ll 1 \tag{6}$$

Once the current utilization, the energy efficieny and the propellat utilization are high, we express the thrust as

$$F = m_i \Gamma_i S \sqrt{\frac{2e\phi_A}{m_i}} = \dot{m}\sqrt{\frac{2e\phi_A}{m_i}}, \tag{7}$$

whereas the power is approximately

$$P = e\Gamma_i S \phi_A = \dot{m}\frac{e\phi_A}{m_i} \tag{8}$$

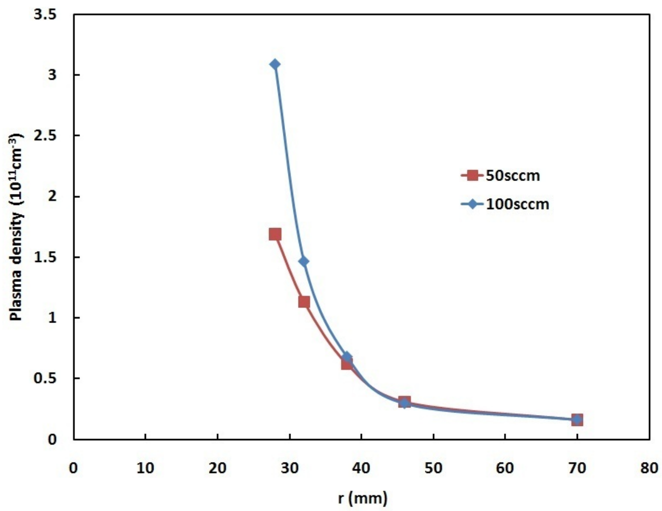

Figure 3. The calculated plasma density versus the distance from the axis of symmetry of the RPS for the two gas flow rates.

The thrust per power is

$$\frac{F}{P} = \frac{2}{\sqrt{2e\phi_A/m_i}} = \frac{2}{v_0}. \tag{9}$$

Here

$$v_0 \equiv \sqrt{\frac{2e\phi_A}{m_i}}. \tag{10}$$

We also find that

$$\frac{F}{\dot{m}} = \sqrt{\frac{2e\phi_A}{m_i}} = v_0, \tag{11}$$

so that at this limit of negligible sources of inefficiency, the efficiency is

$$\eta \equiv \frac{F^2}{2\dot{m}P_T} = 1. \tag{12}$$

The specific impulse is

$$I_{sp} = \frac{v_0}{g}. \tag{13}$$

V. Collisional ions - thruster performance

We now assume that the ions are collisional. The electric force on the ions is balanced by the drag force due to ion-neutral collisions. We write the ion-neutral collison frequency as

$$\nu_i = \frac{v_i}{\lambda}, \tag{14}$$

so that the ion momentum balance is approximated as

$$0 = enE - nm_i\frac{v_i^2}{\lambda}, \tag{15}$$

whereas the neutral-gas momentum balance is

$$\frac{d}{dx}\left(m_i\Gamma_N v_N\right) = nm_i\frac{v_i^2}{\lambda}. \tag{16}$$

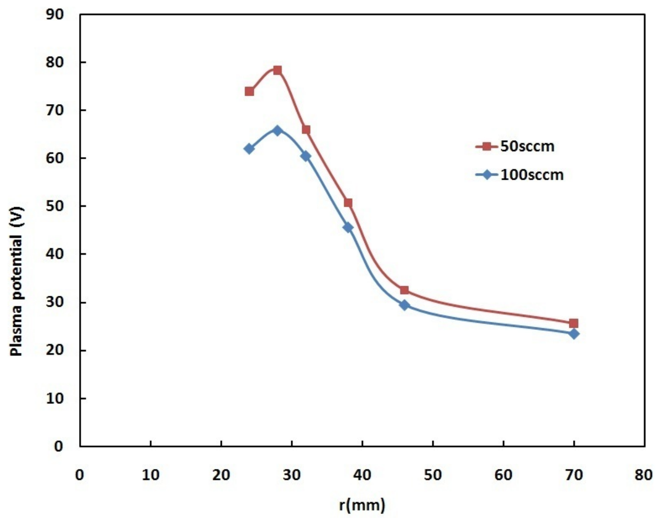

Figure 4. The calculated plasma potential versus the distance from the axis of symmetry of the RPS for the two gas flow rates.

The ion velocity is

$$v_i = \sqrt{\frac{eE\lambda}{m_i}}. \tag{17}$$

Equating the ion and the electron densities results in

$$E = \frac{\Gamma_e^2}{\Gamma_i^2}\frac{\lambda}{em_i}\left(\frac{m\omega_c^2}{\nu_e}\right)^2. \tag{18}$$

The potential distribution is

$$\phi_A - \phi = \int_0^x \frac{\Gamma_e^2}{\Gamma_i^2}\frac{\lambda}{em_i}\left(\frac{m\omega_c^2}{\nu_e}\right)^2 dx'. \tag{19}$$

This last relation determines the relative electron and ion fluxes in the collisional case:

$$\frac{\Gamma_e}{\Gamma_i} = \frac{\sqrt{2e\phi_A/m_i}}{\sqrt{\int_0^d 2\lambda\left(m\omega_c^2/m_i\nu_e\right)^2 dx}} \ll 1. \tag{20}$$

The inequality represents the condition for a high current utilization.

We estimate the thrust by writing the plasma density as

$$n = \frac{\Gamma_i}{\sqrt{eE\lambda/m_i}}, \tag{21}$$

so that

$$enE = \Gamma_i\sqrt{\frac{em_iE}{\lambda}} = \Gamma_e\frac{m\omega_c^2}{\nu_e}, \tag{22}$$

as we wrote above in Eq. (3). The thrust is therefore

$$F = S\int_0^d enEdx = m_i\Gamma_i S\sqrt{\frac{2e\phi_A}{m_i}}\frac{1}{\sqrt{\int_0^d 2\lambda\left(m\omega_c^2/m_i\nu_e\right)^2 dx}}\int_0^d \frac{m\omega_c^2}{m_i\nu_e}dx. \tag{23}$$

If the parameters along the channel are constant, the thrust in the collisional regime is increased relative to the collisionless regime by

$$\sqrt{\frac{d}{2\lambda}}, \tag{24}$$

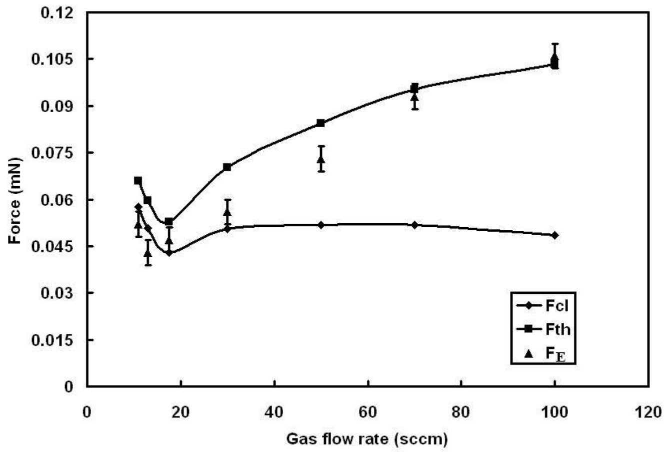

Figure 5. The calculated force without collisions F_{cl}, The calculated force with collisions F_{th}, and the measured net force F_E versus the gas flow rate. For details see Ref. [2]

as was shown in Refs. [1, 2] . For that increase not to cost too much power dissipated in the electron current, inequality (20) has to be satisfied. One could increase the neutral gas flow with an accompanying increase of the magnetic field pressure so that

$$\lambda \left(m\omega_c^2/m_i\nu_e \right)^2 \tag{25}$$

is kept constant. We therefore write again

$$P = e\Gamma_i S\phi_A, \tag{26}$$

and

$$\frac{F}{P} = \frac{2}{v_0} \frac{1}{\sqrt{\int_0^d 2\lambda \left(m\omega_c^2/m_i\nu_e \right)^2 dx}} \int_0^d \frac{m\omega_c^2}{m_i\nu_e} dx. \tag{27}$$

In the collisional regime we can increase F/P while we maintain the inequality (20).

The minimum flow rate includes the neutrals pushed by the ions through collisions. In order to estimate the mass flow rate we look at the neutral momentum equation, Eq. (16). Let us assume that all neutrals that are pushed by ions through collisions acquire a velocity v_N. We obtain a continuity equation for the neutrals of the form

$$v_N \frac{d}{dx}\Gamma_N = \Gamma_i \frac{v_i}{\lambda}. \tag{28}$$

A. Inelastic ion-neutral collisions

Let us first assume that the ion-neutral collisions are inelastic, so that

$$v_N = v_i. \tag{29}$$

In that case

$$\Gamma_N = \Gamma_i \int_0^d \frac{dx}{\lambda}. \tag{30}$$

The mass flow rate can be estimated as

$$\dot{m} = m_i \Gamma_i S \left(1 + \int_0^d \frac{dx}{\lambda} \right) \cong m_i \Gamma_i S \int_0^d \frac{dx}{\lambda}. \tag{31}$$

The thrust over mass flow rate is

$$\frac{F}{\dot{m}} = \frac{v_0}{\int_0^d dx/\lambda} \frac{1}{\sqrt{\int_0^d 2\lambda \left(m\omega_c^2/m_i\nu_e \right)^2 dx}} \int_0^d \frac{m\omega_c^2}{m_i\nu_e} dx. \tag{32}$$

The specific impulse is

$$I_{sp}=\frac{v_0}{g\int_0^d dx/\lambda}\frac{1}{\sqrt{\int_0^d 2\lambda\left(m\omega_c^2/m_i\nu_e\right)^2 dx}}\int_0^d\frac{m\omega_c^2}{m_i\nu_e}dx. \tag{33}$$

The efficiency is

$$\eta\equiv\frac{F^2}{2\dot{m}P_T}=\frac{1}{\int_0^d 2\lambda\left(m\omega_c^2/m_i\nu_e\right)^2 dx}\left(\int_0^d\frac{m\omega_c^2}{m_i\nu_e}dx\right)^2\frac{1}{\int_0^d dx/\lambda}. \tag{34}$$

We note that if all the flow parameters are constant, so is the electric field. In that case

$$\frac{F}{P}=\frac{2}{v_0}\sqrt{\frac{d}{2\lambda}};\quad\frac{F}{\dot{m}}=v_0\sqrt{\frac{\lambda}{2d}};\quad\eta=0.5. \tag{35}$$

The efficiency is only 0.5, since half of the kinetic energy acquired by the neutrals is thermal rather than directed kinetic energy that contributes to the thrust.

B. Elastic ion-neutral collisions

Let us now assume that the ion-neutral collisions are elastic, so that the ion loses its all kinetic energy while colliding and transferring that kinetic energy to a neutral particle as forward directed kinetic energy. Since ions are accelerated from rest to that velocity, the relation between the velocities is

$$v_N=2v_i. \tag{36}$$

In that case

$$\Gamma_N=\Gamma_i\int_0^d\frac{dx}{2\lambda}. \tag{37}$$

The mass flow rate can be estimated as

$$\dot{m}=m_i\Gamma_i S\left(1+\int_0^d\frac{dx}{2\lambda}\right)\cong m_i\Gamma_i S\int_0^d\frac{dx}{2\lambda}. \tag{38}$$

The thrust over mass flow rate is

$$\frac{F}{\dot{m}}=\frac{v_0}{\int_0^d dx/\left(2\lambda\right)}\frac{1}{\sqrt{\int_0^d 2\lambda\left(m\omega_c^2/m_i\nu_e\right)^2 dx}}\int_0^d\frac{m\omega_c^2}{m_i\nu_e}dx. \tag{39}$$

The specific impulse is

$$I_{sp}=\frac{v_0}{g\int_0^d dx/\left(2\lambda\right)}\frac{1}{\sqrt{\int_0^d 2\lambda\left(m\omega_c^2/m_i\nu_e\right)^2 dx}}\int_0^d\frac{m\omega_c^2}{m_i\nu_e}dx. \tag{40}$$

The efficiency is

$$\eta\equiv\frac{F^2}{2\dot{m}P_T}=\frac{1}{\int_0^d 2\lambda\left(m\omega_c^2/m_i\nu_e\right)^2 dx}\left(\int_0^d\frac{m\omega_c^2}{m_i\nu_e}dx\right)^2\frac{1}{\int_0^d dx/\left(2\lambda\right)}. \tag{41}$$

We note that in this elastic case, if all the flow parameters are constant, as is the electric field, we obtain that

$$\frac{F}{P}=\frac{2}{v_0}\sqrt{\frac{d}{2\lambda}};\quad\frac{F}{\dot{m}}=v_0\sqrt{\frac{2\lambda}{d}};\quad\eta=1. \tag{42}$$

The efficiency depends on the nature of the ion-neutral collisions, to what extent they are elastic.

VI. Conclusion

We have presented experimental results that show an increase of thrust per power when the plasma flow is collisional. The analysis presented here could serve as a basis for studies how to optimize the performance of a thruster that employs a collisional plasma by modifying the profiles of the magnetic field and of the neutral-gas density.

Acknowledgments

This research was supported by the Israel Science Foundation, Grant no. 864/07.

References

[1]Makrinich, G., and Fruchtman, A., "Experimental Study of a Radial Plasma Source," *Phys. Plasmas*, Vol. 16, 2009, pp. 043507-1 - 043507-8.

[2]Makrinich, G., and Fruchtman, A., "Enhancement of Electric Force by Ion - Neutral Collisions," *Appl. Phys. Lett.*, Vol. 95, 2009, pp. 181504-1 - 181504-3.

[3]Fruchtman, A., "The Thrust of a Collisional-Plasma Source," *IEEE Transactions on Plasma Science,*, Vol. 39, No. 1, 2011, pp. 530 - 539.

[4]Diamant, K. D., "Microwave Cathode for Air Breathing Electric Propulsion," *31st International Electric Propulsion Conference*, Ann Arbor, Michigan, USA, Sep. 20 - 24, 2009, IEPC Paper No. 2009 - 015.

[5]Fruchtman, A., Zoler, D., and Makrinich, G., "Potential of an emissive cylindrical probe in a plasma," *Physical Review E - Rapid Communication*, Vol. 95, 2011, pp. 025402-1 - 025402-4.

[6]Fruchtman, A., Zoler, D., and Makrinich, G., "Theory of a cylindrical emissive probe," *20th International Symposium on Plasma Chemistry*, Philadelphia, Pennsylvania, July 24-29, 2011.

[7]Burton, K., E., Jr., Paris, Texas, " Atmospheric Fueled Ion Engine", U.S. Patent No. 6,145,298, November 14, 2000.