On electron inertia and current ambipolarity in magnetic nozzle models

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An axilsymmetric model of the supersonic plasma expansion in a divergent magnetic nozzle, which includes the relevant electron-inertia terms is presented. This generalized model presents the same mathematical structure and features than our extensively studied zero electron-inertia model. There is no room in the model for imposing current ambipolarity everywhere, and this is not fulfilled. The model confirms the trends obtained in a former linearized analysis about electron-inertia: this makes the plasma beam to diffuse outwards of the magnetic nozzle, thus hindering detachment. The generalized model has facilitated a quick revision of the electron-inertia detachment model of Hooper [J. Prop. Power 9, 757 (1993)] and the detection of a gross error in the manipulation of the equations, which leads to wrong solutions and conclusions. The error persists in recent followers of that model.

I. Introduction

A divergent magnetic nozzle, created by a longitudinal magnetic field, is being proposed as the acceleration stage for a magnetized plasma in advanced propulsion devices, such as the helicon thruster,[1–4] the applied-field magnetoplasmadynamic thruster,[5] and the VASIMR.[6] In order to understand the plasma physics in a propulsive magnetic nozzle we have being developing a two-dimensional(2D) plasma/nozzle model and its associated numerical code DIMAGNO. The model considers the stationary expansion of a fully-ionized, near-collisionless plasma (as the one we expect delivered from the production stage of a plasma thruster), with fully-magnetized electrons and partially-magnetized ions (which is the case to be expected for magnetic strengths of the order of 0.1 Tesla).

In a first group of works[7–11] we were devoted to analyze the main features of the plasma radial and axial expansion, the plasma currents, the thrust transmission, and the nozzle efficiency (i.e. the axial-versus-total energy flow). These studies have confirmed that a propulsive magnetic nozzle is able to both convert internal energy into axial directed energy and obtain additional thrust from the plasma beam, with the great benefit of no wall-contact and geometric versatility.

Then, in a second group of works, we have started to tackle the issue of plasma detachment from the nozzle,[12–14] which could be the penalty of using this, otherwise attractive, propulsive device. Since the nozzle magnetic lines close on themselves, once the plasma beam has been accelerated and before the turning point of the magnetic nozzle, the plasma jet needs to detach effectively from the magnetic lines. Experiments seem to confirm that (most of) the plasma detaches, but more measurements are needed to confirm this, detachment mechanisms are poorly known, and the efficiency of the detachment must be assessed.

Two detachment mechanisms of the plasma *from the guide magnetic field* have been modeled: magnetic-stretching detachment proposed by Arefiev and Breizman,[15,16] and electron-inertia detachment suggested by Hooper.[17] In the first case, the induced magnetic field would reinforce the applied field, thus stretching the effective magnetic nozzle. The plasma, by remaining attached to the nozzle resulting from the total magnetic

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field, would detach effectively *inwards* from the guide field nozzle. In the second case, Hooper claims that electron-inertia forces detach the plasma jet *inwards* from the guide-field nozzle. Resistive detachment[18] was also suggested as a third detachment mechanism, but no detailed model was developed.

In Ref. 13 we demonstrated that these three detachment mechanisms do not apply to a magnetic nozzle aimed for propulsion (i.e. with a near-sonic, hot plasma at its entrance). First, the induced magnetic field opposes the applied field and therefore reduces magnetization and increases the divergence of the effective nozzle. This has just been confirmed experimentally in Ref.[19] Second, a perturbation analysis showed that electron-inertia is a diffusive mechanism, comparable to resistivity, and both mechanisms detach the plasma plume *outwards* the magnetic streamtubes, therefore working against the axial expansion of the beam.

Current work is attempting mainly to unveil valid detachment mechanisms and, subsidiarily, to reinforce our disagreeing position with respect to magnetic-stretching and electron-inertia detachment theories. In Ref. 14 we show that plasma demagnetization, enhanced by the induced field, can be an effective detachment mechanism for a high density plasma. Here, it is the turn to deal directly with the model promoted by Hooper and to explain why we reach opposite conclusions to him. We will show that Hooper commits a mistake in the mathematical manipulation of the equations with fatal consequences, since it leads him to an ill-founded physical model and therefore to erroneous results and conclusions. Schmit and Fisch[20] and Little and Choueiri,[21,22] who tried to extend the model of Hooper to more general conditions, do not realize the mistake and are misled in their results too.

The central difference between Hooper's model and ours is that he *imposes* current ambipolarity everywhere whereas we show that its imposition makes the plasma model incompatible mathematically. Two other differences between the two models might be raised by someone to claim that both models are 'independent' and provide two 'independent truths'. The first one is that keeping electron-inertia is central in the derivation of Hooper's model, whereas for us electron-inertia is just a small effect, which does not modify the main features of the plasma/nozzle response. The second one is that Hooper considers only the case of a cold plasma at the nozzle entrance (a case of limited interest for a propulsive magnetic nozzle, since it misses its central role of incrementing thrust). Nonetheless, the cold-plasma case might still shed some right understanding on the detachment issue.

Here, we will proceed in the following way. First, we will derive the *nonzero* electron-inertia, hot-plasma model and we will demonstrate that that *zero* electron-inertia is just a regular limit of that model. Second, we will confirm that current ambipolarity is not fulfilled, *independently* of the presence or not electron-inertia effects. Third, we will show where lies the main mistake in Hooper's equations.

II. General model

A current-free, fully-ionized plasma jet is injected at the throat of a divergent magnetic nozzle created by both a external coil system *and* the magnetic field induced by plasma currents (which is solved as explained in Ref.14). In cylindrical coordinates (z, r, θ), the magnetic field is $\boldsymbol{B} = B_r\mathbf{1}_r + B_z\mathbf{1}_z$ and we adopt the convention $B_r, B_z > 0$. The cylindrical and magnetic frames of reference are $\{\mathbf{1}_z, \mathbf{1}_r, \mathbf{1}_\theta\}$ and $\{\mathbf{1}_\parallel, \mathbf{1}_\perp, \mathbf{1}_\theta\}$, with $\mathbf{1}_\parallel = \boldsymbol{B}/B = \cos\alpha\mathbf{1}_z + \sin\alpha\mathbf{1}_r$, $\mathbf{1}_\perp = -\sin\alpha\mathbf{1}_z + \cos\alpha\mathbf{1}_r$, and $\alpha(z, r)$ is the local magnetic angle. A magnetic streamfunction ψ exists, which satisfies

$$\nabla\psi = rB\mathbf{1}_\perp : \qquad \partial\psi/\partial z = -rB_r, \quad \partial\psi/\partial r = rB_z. \tag{1}$$

The plasma is assumed to satisfy the distinguished orderings

$$\lambda_{d0} \ll \ell_{e0} \ll R \ll \lambda_{ei0}, \qquad R\Omega_{i0}/u_{i0} = O(1) \tag{2}$$

where subscript 0 refers always to values at the center of the nozzle throat (located at $z = 0$); R is the plasma jet radius at the throat λ_d is the Debye length, ℓ_e is the electron gyroradius, λ_{ei} is the electron-ion collision mean-free-path, Ω_i is the ion gyrofrequency, and u_i is the longitudinal ion fluid velocity. These orderings imply that the plasma is quasineutral and near-collisionless, the magnetic field channels magnetized electrons (thus creating the nozzle effect) and ions are mildly or weakly magnetized (thus being tied to electrons mainly by the ambipolar electric field).[9]

A two-fluid model is adopted for the quasineutral plasma. Let $n \equiv n_i = n_e$ be the plasma density. For vectorial magnitudes, such as velocities $\boldsymbol{u}_k$ $(k = i, e)$ and current densities $\boldsymbol{j}_k$, it is convenient to separate the azimuthal component from the longitudinal ones, here denoted with a tilde: $\tilde{\boldsymbol{u}}_{\boldsymbol{i}} = \boldsymbol{u}_i - u_{\theta i}\mathbf{1}_\theta$, $\tilde{\boldsymbol{j}}_{\boldsymbol{e}} = \boldsymbol{j}_e - j_{\theta e}\mathbf{1}_\theta$, etcetera. The fluid equations for ions are

$$\nabla \cdot n\tilde{\boldsymbol{u}}_{\boldsymbol{i}} = 0, \tag{3}$$

$$m_i\tilde{\boldsymbol{u}}_{\boldsymbol{i}} \cdot \nabla(ru_{\theta i}) = reu_{\perp i}B, \tag{4}$$

$$m_i\tilde{\boldsymbol{u}}_{\boldsymbol{i}} \cdot \nabla\tilde{\boldsymbol{u}}_{\boldsymbol{i}} = -\nabla h_i - e\nabla\phi + \mathbf{1}_\perp eu_{\theta i}B + \mathbf{1}_r m_i u_{\theta i}^2/r, \tag{5}$$

where ϕ is the ambipolar electric potential and $h_i \equiv n^{-1}\nabla(nT_i)$ is the barotropic function, with $h_i = T_i \ln n$ for isothermal ions and $h_i = T_{i0}\gamma_i(\gamma_i - 1)^{-1}(n/n_0)^{(\gamma_i-1)}$ for polytropic ions. The rest of symbols are conventional. Notice that $\tilde{\boldsymbol{u}}_{\boldsymbol{i}} \cdot \nabla$ is the derivative along the (meridian-projected) ion streamlines. From Eq. (3) an ion streamfunction ψ_i exists, which satisfies

$$\partial\psi_i/\partial z = -rnu_{ri}, \quad \partial\psi_i/\partial r = rnu_{zi}. \tag{6}$$

Also, Eq. (1) and Eq. (4) yield the conservation of total ion angular momentum,

$$rm_iu_{\theta i} + e\psi = D_i(\psi_i), \tag{7}$$

with $D_i(\psi_i)$ determined from entrance conditions at the throat. This equation relates, at $z = 0$, the magnetic and ion streamtubes (i.e. $\psi =$ const and $\psi_i =$ const) with the same cross section.

The fluid equations for electrons are

$$\nabla \cdot n\tilde{\boldsymbol{u}}_{\boldsymbol{e}} = 0, \tag{8}$$

$$m_e\tilde{\boldsymbol{u}}_{\boldsymbol{e}} \cdot \nabla(ru_{\theta e}) = -reu_{\perp e}B, \tag{9}$$

$$0 = -\nabla h_e + e\nabla\phi - \mathbf{1}_\perp eu_{\theta e}B + \mathbf{1}_r m_e u_{\theta e}^2/r. \tag{10}$$

The formulation is identical than for ions, except for the longitudinal inertia term $m_e\tilde{\boldsymbol{u}}_{\boldsymbol{e}} \cdot \nabla\tilde{\boldsymbol{u}}_{\boldsymbol{e}}$ in Eq. (10) is neglected, based on $m_e/m_i \ll 1$ and $\tilde{u}_e \sim \tilde{u}_i$. However, azimuthal inertia terms in Eqs. (9) and (10), involving $m_eu_{\theta e}$, are kept in order to recover the model of Hooper. Clearly these inertia terms are fully negligible if $u_{\theta e} \le O(\tilde{u}_e)$. As in the case of ions: h_e is the electron barotropic function; there is an electron streamfunction ψ_e satisfying

$$\partial\psi_e/\partial z = -rnu_{re}, \quad \partial\psi_e/\partial r = rnu_{ze}; \tag{11}$$

and the conservation of total electron angular momentum reads

$$rm_eu_{\theta e} - e\psi = D_e(\psi_e), \tag{12}$$

with $D_e(\psi_e)$ determined from conditions at the throat.

The equation for plasma longitudinal momentum is the sum of Eqs. (5) and (10),

$$m_i\tilde{\boldsymbol{u}}_{\boldsymbol{i}} \cdot \nabla\tilde{\boldsymbol{u}}_{\boldsymbol{i}} = -\nabla(h_i + h_e) + \mathbf{1}_\perp eB(u_{\theta i} - u_{\theta e}) + \mathbf{1}_r\frac{m_iu_{\theta i}^2 + m_eu_{\theta e}^2}{r}, \tag{13}$$

and is used instead of Eq. (5), because it does not involve the electric field, $-\nabla\phi$.

Equations (3)-(5) and (8)-(10) constitute a *complete* set of 8 scalar equations for the 8 plasma variables $\boldsymbol{u}_i$, $\boldsymbol{u}_e$, n, and ϕ. We will show this set to be a well-posed mathematical problem with a unique solution (for given throat conditions). Therefore, any equation added to the model makes it incompatible (unless it is an automatic consequence of the other 8 equations and their boundary conditions). Equations (3) and (8) yield the electron conservation law

$$\nabla \cdot (\tilde{\boldsymbol{j}}_i + \tilde{\boldsymbol{j}}_e) = 0. \tag{14}$$

Hooper substitutes this *scalar* equation by the more restrictive *vectorial* equation

$$\tilde{\boldsymbol{j}}_i + \tilde{\boldsymbol{j}}_e = \mathbf{0}, \quad \text{i.e.} \quad \tilde{\boldsymbol{u}}_{\boldsymbol{i}} = \tilde{\boldsymbol{u}}_{\boldsymbol{e}}, \tag{15}$$

known as *current ambipolarity* condition. Clearly, this procedure leads to a 9-equation model with 8 variables, which is presumably incompatible. This reasoning is independent on whether we keep the inertial terms on Eqs. (9) and (10) or we drop them.

III. The massless electron model

Since $m_e/m_i \sim 10^{-4}$ (and unless we presume $u_{\theta e}$ to be very large) the *natural* 8-equation model to discuss is the one with $m_e/m_i \to 0$. This requires only to drop from the above general model one term in Eq. (9) and another one in Eq. (10). The massless electron model is the one we have analyzed extensively in previous works. We summarize next the most relevant *mathematical and physical* results we have achieved.

On the mathematical side we have, first, that Eq. (9) yields

$$u_{\perp e} = 0, \tag{16}$$

and the alternative Eq. (12) becomes

$$-e\psi = D_e(\psi_e). \tag{17}$$

They state that electron streamtubes are magnetic streamtubes everywhere (relating univocally ψ and ψ_e). Second, Eq. (8) and $\nabla \cdot \boldsymbol{B} = 0$ yield

$$u_{\parallel e} = G_e(\psi_e)B/n, \tag{18}$$

with $G_e(\psi)$ determined from throat conditions. Third, the projection of Eq. (10) along $\mathbf{1}_\parallel$ yields a Boltzmann-type law for the electric potential

$$-e\phi + h_e = H_e(\psi_e), \tag{19}$$

with $H_e(\psi_e)$ determined from throat conditions. Fourth, the projection of Eq. (10) along $\mathbf{1}_\perp$ yields the electron azimuthal velocity

$$u_{\theta e} = -\frac{1}{eB}\frac{\partial H_e}{\partial \mathbf{1}_\perp} \equiv -\frac{r}{e}\frac{dH_e}{d\psi}. \tag{20}$$

This equation states that, first, $u_{\theta e}$ is the combination of the $E \times B$ and the diamagnetic drifts and, second, the angular velocity of the electron flow, $u_{\theta e}/r$, is frozen in the streamtubes. Fifth, the substitution of these electron magnitudes into the 4 ion equations, yields an hyperbolic set of equations for supersonic ions, wherefrom $\boldsymbol{u}_i$ and n are determined. It is worth to stand out that the four conservation equations for magnetized electrons are standard and have a well-founded physical meaning.

The most relevant physical features extracted from our model are the following:

(1) Partially-magnetized ions are not fully-channeled by the magnetic/electron streamtubes so that *current ambipolarity is not satisfied*, even when it is imposed as entrance condition at the throat.

(2) Due to the separation between electron and ion streamtubes, a strong electric field along $\mathbf{1}_\perp$ is formed to comply with quasineutrality. This leads to a strong perpendicular rarefaction and makes *quasi-1D models of the nozzle little adequate.*

(3) The perpendicular electron force balance consists of *an expanding pressure force, a confining magnetic force, and a confining electric force.*

(4) The confining magnetic force on electrons is intimately related to the *electron azimuthal current $j_{\theta e}$ being diamagnetic* (i.e. running opposite to the coils azimuthal electric current).

(5) In general, $j_{\theta e}$ *consists of a volumetric contribution within the plasma beam and a surface contribution at the plasma/vacuum edge.*

(6) For ions without azimuthal rotation at the nozzle throat, the *ion azimuthal current $j_{\theta i}$ is paramagnetic.*

(7) It is readily derived from basic physics that *positive plasma beam acceleration and thrust contribution in the diverging nozzle are achieved with a net diamagnetic plasma current.* [Points (4) to (7) seem to have been rediscovered lately by Little and Choueiri.[22]]

(8) Maximum thrust gain is achieved for a *sonic plasma flow at the throat* (so that there is maximum internal energy to be converted into axial directed energy) and *weakly magnetized ions* (so that perpendicular rarefaction is maximum). This case coincides with a minimum contribution of ion paramagnetic currents.

(9) Plasma detachment via magnetic stretching, as proposed by Arefiev and Breizman,[15] is related to a dominance of paramagnetic plasma currents and therefore has no interest for propulsive magnetic nozzles. Instead, plasma detachment in these nozzles is facilitated by demagnetization, enhanced by induced field effects.[14]

IV. Model with electron-inertia

We return here to the general model with the two electron inertia terms in Eqs. (9) and (10). Since these two terms have the product $m_e u_{\theta e}$, the relevance of electron inertia is going to depend on the value of $u_{\theta e}$ at the nozzle throat. Three cases can be distinguished.

The first one corresponds to the plasma entering the divergent nozzle with $u_{\theta e}(z=0,r)=0$. In this case *the solution of the general model is the solution of the massless-electron model with* $u_{\theta e}=0$ *everywhere* (that is with $H_e = \text{const}$).

The second case corresponds to the plasma entering the divergent nozzle with a 'moderate' value of $u_{\theta e}(z=0,r)$, say $u_{\theta e}\sim u_i$. Then, the linear-perturbation analysis based on the solution for $m_e/m_i=0$, that we carried out in Ref.13, should be enough to assess electron-inertia effects. The main conclusion of that analysis is that electron inertia allows the development of an *outwards* velocity

$$u_{\perp e}\simeq m_e\frac{2u_{re}}{eB}\frac{u_{\theta e}}{r}. \tag{21}$$

This is the consequence of the diffusive character of electron inertia, which tends naturally to expand the plasma beam (instead of contracting it). Notice that since B is inversely proportional to the nozzle cross-section area, electron-inertia effects increase downstream proportionally to that area. [Our analysis of Ref.13 also showed that, in practical cases, electron-inertia is likely to be a more important effect than resistivity.]

Finally, the exact solution of the general model is presented next. This is likely to be needed for either large values of $u_{\theta e}$ or far downstream in the nozzle.

A. General analysis

When electron-inertia is included the solving of electron equations is less straightforward. The electron streamtubes differ from magnetic streamtubes and their shape must be determined. First, Eq. (12) yields $u_{\theta e}=[D_e(\psi_e)+e\psi]/(m_e r)$, that substituted into Eq. (10) leads to

$$0=\nabla(e\phi-h_e)-\mathbf{1}_\perp eB\frac{D_e(\psi_e)+e\psi}{m_e r}+\mathbf{1}_r\frac{[D_e(\psi_e)+e\psi]^2}{m_e r^3}. \tag{22}$$

After using Eq. (1) and some manipulation, this equation becomes

$$0=\nabla\left[e\phi-h_e-\frac{\left(D_e+e\psi\right)^2}{2m_e r^2}\right]+\frac{D_e+e\psi}{m_e r^2}\frac{dD_e}{d\psi_e}\nabla\psi_e. \tag{23}$$

The component of Eq. (23) parallel to electron streamtubes is

$$-e\phi+h_e+\frac{1}{2}m_e u_{\theta e}^2=H_e(\psi_e), \tag{24}$$

which is a recognizable generalization of Eq. (19). Then, the component of Eq. (23) perpendicular to electron streamtubes yields

$$\frac{D_e(\psi_e)+e\psi}{m_e r^2}\frac{dD_e}{d\psi_e}-\frac{dH_e}{d\psi_e}=0,$$

that is

$$e\psi=-D_e(\psi_e)+m_e r^2\frac{dH_e/d\psi_e}{dD_e/d\psi_e} \tag{25}$$

and

$$\frac{u_{\theta e}}{r}=\frac{dH_e/d\psi_e}{dD_e/d\psi_e}. \tag{26}$$

Equations (25) and (26) reduce, respectively, to Eqs. (17) and (20) in the massless-electron limit.

The main point here is that Eq. (25) is an implicit equation for the electron streamfunction $\psi_e\big(\psi(z,r),r\big)$ and therefore determines the shapes of the electron streamtubes without requiring the solution for the plasma density and the ion velocity. Once $\psi_e(z,r)$ is known, Eq. (26) yields $u_{\theta e}(z,r)$, which substituted in Eq. (13) allows the integration of the 4 ion hyperbolic equations and the determination of $\boldsymbol{u}_i$ and n. The last step is the determination of $\tilde{\boldsymbol{u}}_{\boldsymbol{e}}$ from Eq. (11).

Taking into account that $H_e(\psi_e)$ and $D_e(\psi_e)$ are expected to be negative and monotonic functions, the last term of Eq. (25) states that electron streamtubes separate *outwards* from magnetic streamtubes and the *separation increases with the nozzle cross-section area* (as predicted already by the perturbation analysis). Ion streamtubes are expected to continue separating *inwards* from magnetic streamtubes, so that *current ambipolarity is farther from being fulfilled when electron-inertia is added.*

A final observation is that $D_e(\psi_e) = \text{const}$ (and the constant can be set to 0 without loss of generality) is a singular case for expression (26). According to Eq. (12), it corresponds only to the particular conditions

$$u_{\theta e}(z=0,r) = \frac{e\psi(z=0,r)}{m_e r}. \tag{27}$$

and Eq. (26) imposes that $H_e(\psi_e) = \text{const} = 0$ too. The way to solve $\psi_e(\psi, r)$ for this very singular case has not been worked out completely since we believe it has no practical interest. First, it corresponds to a very particular spatial profile of $u_{\theta e}(z=0,r)$ at the throat. Second, the azimuthal velocity for this case seems extremely large: $u_{\theta e}/c_s \sim (R/\ell_{e0})(m_i/m_e)^{1/2}$ with c_s the plasma sound velocity.

V. On models with current ambipolarity

Once we have established and discussed the model with electron-inertia, we are ready to show where and why Hooper equations get wrong. Hooper assumes that electrons have no rotation when injected at the nozzle throat, i.e. $u_{\theta e}(z=0,r) = 0$, leading to $D_e(\psi_e) = -e\psi$ at $z=0$. The *correct solution* for these conditions was derived before and has $u_{\theta e} = 0$ everywhere and electron streamtubes coincide with magnetic streamtubes while ion streamtubes do not, i.e. *there is neither current ambipolarity nor detachment.*

Instead, Hooper commits the gross mistake of treating the function $D_e(\psi_e)$ as a constant (his $-e\psi_0$) and thus *misses* the term with $dD_e/d\psi_e$ in Eq. (23). The consequence is fatal because he then misses Eq. (25), which is crucial for determining the electron streamtubes. The one-equation vacancy left by Eq. (25) misleads him to supplant it with the ambipolarity condition. The consequence is a nonphysical model, yielding conclusions widely opposed from those of the correct solution pointed out in the former paragraph.

The very only case where the model of Hooper could claim a possibility of being correct is for the singular conditions of Eq. (27), when Eq. (25) provides no information. Still we believe that current ambipolarity is not satisfied. Indeed, the solution that arises from adding current ambipolarity is rather weird. We illustrate it with the simple case of a cold plasma ($h_i = h_e = 0$) and a zero-divergence nozzle, having $B_r = 0$, $B_z = \text{const} = B_0$, $\psi = B_0 r^2/2$. Then, one has

$$u_{\theta e}(z,r) = u_{\theta e}(0,r) = \frac{eB_0}{2m_e} r, \qquad e\phi(z,r) = \frac{1}{2} m_e u_{\theta e}^2 = \frac{e^2 B_0^2}{8m_e} r^2. \tag{28}$$

This electric potential profile is not natural since it expands radially electrons instead of confining them as in the rest of cases. The effect of the unnatural radial electric field on the ion beam is to contract it until collapsing in a point. *This convergence of the ion beam would be the detachment mechanism claimed by Hooper.*

Schmit and Fisch[20] adopt the same cold plasma model of Hooper and discuss plasmas with non-zero injection angular velocity profiles. Little and Choueiri[21,22] extend the model of Hooper to include (in a simplified way) the electron pressure. Both works continue ignoring dD_e/ψ_e [and dD_i/ψ_i too] and adding current ambipolarity to the model.

VI. Conclusions

A model that generalizes our previous one by including the relevant electron-inertia terms has been derived and analyzed. It has been shown that the zero electron-inertia limit is a regular limit and a parametric continuation of solutions is expected to be continuous. The general model has the same mathematical structure and features than the zero inertia one. First, electron equations reduce to conservation equations, which are easily matched to the ion differential equations. Second, ion equations continue to be a set of hyperbolic differential equations to be solved with the method of characteristic surfaces. Third, the model does not leave room for imposing the current ambipolarity condition, and this is not fulfilled. This non-fulfilment is by no means anecdotic, as results show. Two interesting properties of the zero-inertia model are preserved in the general model: (a) if the electron azimuthal current is zero at the entrance, it remains zero everywhere (except in the plasma/vacuum current sheet); and (b) the electron streamtubes and the electron longitudinal flux are determined exclusively from the electron equations. In addition, the general model confirms the two main trends of electron-inertia, obtained in our former linearized analysis:[13] (a) electron-inertia expands outwards the plasma beam and therefore is negative for detachment; and (b) although electron-inertia effects are expected quite small near the nozzle entrance, they grow downstream proportionally to the nozzle cross-section area.

Finally, it has been demonstrated that Hooper commits a mistake in the mathematical manipulation of the equations. The error is fatal since it eliminates one crucial electron equation from the model and leads him to fill the false vacancy with current ambipolarity. Consequently, the resulting solutions and conclusions are nonphysical.

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