Azimuthal symmetry breaking, current filamentation and onset regime in MPD thrusters

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Recent experimental and theoretical investigations have pointed out the possible role of a *filamentation instability* in the inception and development of the critical operating regime of magnetoplasmadynamic thrusters known as *onset*. The onset regime affects these devices when operated at high ratios of total squared current over propellant mass flow rate, leading to strong performance degradation and intense thruster damage. In this work we review the previously cited investigations and analyze the theoretical effectiveness of a passive stabilization method, which consists in the insertion of longitudinal plates in the discharge channel in order to lock the azimuthal degree of freedom of the flow.

I.   Introduction

The onset regime of magnetoplasmadynamic (MPD) is among the most investigated aspects of electric propulsion, research activities devoted to its understanding spanning the last four decades. The impossibility to increase the imposed current level J above a critical threshold, the propellant mass flow rate $\dot{m}$ being held constant, became soon clear to the researchers in the field. This is a major drawback, as the thrust efficiency of these devices is known to increase with increasing $k = J^2/\dot{m}$ values, and this parameter is directly related to the specific impulse of the thruster and to the specific power put into the flow, i.e. electrical power per unit propellant mass flow rate.

The inception of the onset regime is accompanied by a series of events, the most important ones being:

- a transition from a homogeneous and diffuse current attachment to a so called *spotty* current pattern, characterized by the emergence of plasma channels connecting the cathode or the bulk of the plasma to the anode; this transition is clearly the footprint of azimuthal symmetry breaking

- strong fluctuations of thruster terminal voltage

- intense damage of thruster components (electrodes and insulators).

At the beginning of the vast amount of attempts made to achieve a complete understanding of MPD thruster onset, the inception of this critical regime was related to critical thresholds of integral parameters, i.e. expressions containing the driving parameters of the thruster (such as imposed current J, propellant mass flow rate $\dot{m}$ or applied magnetic field B_0), the physical properties of the employed propellant (such as ionization energy $\mathcal{E}_i$ or molecular/atomic mass $\mathcal{M}$), and geometrical quantities peculiar of the configuration being tested (such as ratio between mean anode and cathode radii r_a/r_c). It has been found that a stability criterion simply based on a threshold for the quantity $k = J^2/\dot{m}$, as previously stated, is insufficient to

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detect the onset inception for different thruster configurations, so that a first correction taking into account the propellant molecular mass was introduced for the critical k ratio

$$k_* \simeq \frac{(15 \div 33) \cdot 10^{10}}{\sqrt{\mathcal{M}}} \qquad \left[\frac{\text{A}^2\text{s}}{\text{kg}}\right]$$

and yet the geometrical configuration resulted to play an important role, as indicated for example by Boyle et al.[2] Other integral parameters have been introduced and related to onset inception, e.g. the ratio between the imposed current and the current which is necessary for full propellant ionization[5] or the ratio between the magnetic and gasdynamic pressure in the plasma at the sonic section,[19] but the presence of a critical threshold for the ratio $J^2/\dot{m}$ has mantained a more widely accepted validity.

A great wealth of research efforts[4, 17, 18, 23, 24, 27] has been dedicated to the attempt of relating the emergence of the onset regime to the inception of a number of plasma instabilities, and yet not being successful in explaining the complex phenomenology characterizing this critical regime. Among the most widely accepted causes of onset is the so called *anode starvation* hypothesis.[1, 5, 12, 21, 22] This is related to the presence of an inward directed component of the Lorentz force, which becomes higher as the driving parameter k is increased, leading to strongly reduced plasma density in the vicinity of the anode, and then preventing a stable current transport to the electrode. According to this theory, which is based on the widely accepted assumption that the electrical current to the anode is mainly carried by thermal electrons, at onset inception the so called *current saturation* occurs, which is supposed to trigger an unstable behavior of the device.

Recent investigations carried by Uribarri[20] on the Princeton Benchmark Thruster operated in the onset regime, have put in evidence the absence of characteristic frequencies in the fluctuating voltage traces (which was indeed a common claim in previous works[2, 9, 13, 22, 23]), their power density spectra sharing instead an inverse power law dependence on frequency, i.e. $\sim f^{-\delta}$, with an exponent in the range $1 < \delta < 2$. These results seemed to be independent from the particular anode material, thus excluding the hypothesis that local electrode melting and evaporation could play a major role in the onset phenomenology. These experimental results led Uribarri and Choueiri to build a heuristic model of the electrical interaction between anode spots and the anode sheath,[21] noting that these spectral trends are typical of brownian noise. In that model the anode sheath was schematized as an ideal capacitor randomly fed in time by the excess current which cannot be carried by the plasma in current saturation conditions, showing that such an elementary circuit could actually produce voltage traces similar to the observed ones. In the same work, it was also noted that the current pattern tends to be spotty at almost all $J^2/\dot{m}$ levels, although appearing mainly diffuse at the lowest ones (see Uribarri,[20] p. 100).

As previously stated, all the cited explanations and theories which have been put forward, however, fail to explain all the physical phenomena occurring during the onset regime. Nonetheless the authors of the present work have recently proposed a physical picture which seems promising in explaining and relating the cited phenomena.[7] This picture is based on the role played by the so called *filamentation instability*, a phenomenon which has already been investigated in other plasma physics contexts.[3, 8, 11, 14–16] In the next sections this recently proposed picture is described. In addition, some pereliminary calculations on a possible passive stabilization method will be described in Sec.(V).

II.   Azimuthal symmetry breaking

A.   Governing equations

The azimuthal stability of the plasma is studied by means of a simplified analytical model in which the equilibrium configuration (subscript 0) is assumed axi-symmetric with no plasma rotation and a purely azimuthal self-induced magnetic field. The plasma is assumed not to be viscous and to carry a radial current. A simplified two-fluid model (electrons, subscript e, and ions, subscript i) is introduced, making the following assumptions:

- a generic polytropic transormation is assumed for the plasma as a whole, i.e. $p\rho^{-\alpha} = $ const. $\Rightarrow T = T_0(n/n_0)^{\alpha-1}$, where the state equation $p = k_B n T$ was used ($T = T_i + T_e$); this allows to consider both adiabatic transformations ($\alpha = 5/3$, fast dynamics) and isothermal ones ($\alpha = 1$, slow dynamics)

- the radial current is carried only by electrons and their drifting velocity u_0 does not change substantially during filamentation; the axial component of current density is neglected

- since we expect a very peaked structure in the azimuthal direction in the presence of filaments, we neglect variations of plasma and field quantities along radial and axial directions with respect to variations along azimuthal direction, i.e. $\left|\dfrac{\partial a}{\partial r}\right|, \left|\dfrac{\partial a}{\partial z}\right| \ll \left|\dfrac{1}{r}\dfrac{\partial a}{\partial \theta}\right|$ for any physical quantity a

- we neglect the radial component of the magnetic field, as a consequence of previous assumptions; the assumption on electron radial velocity guarantees that the total current is constant (as it is during SF-MPD thruster onset); accordingly we assume that the azimuthal component of the magnetic field remains almost unchanged during filamentation, i.e. $B_\theta \simeq B_{\theta 0}$, even if in real operation this would not be the case since we do expect current concentration along the axial direction too. The inclusion of the axial coordinate is left for future work.

The derivation of the governing equations (i.e. continuity, azimuthal momentum and Ampère's equations) has been described in detail in previous publications[6,7] and will not be repeated here. The final point of that derivation, which is on the other hand the starting point of our analysis, is the following nonlinear partial differential equation describing the evolution in time and along the azimuthal coordinate θ of the axial component of the magnetic field B_z:

$$
\frac{\partial^2 B_z}{\partial t^2} + \frac{\dfrac{\partial}{\partial \theta}\left[\left(\dfrac{\partial B_z}{\partial t}\right)^2\right]}{\left(\mu_0 e r n_0 u_0 - \dfrac{\partial B_z}{\partial \theta}\right)} + \frac{\left(\dfrac{\partial B_z}{\partial t}\right)^2 \dfrac{\partial^2 B_z}{\partial \theta^2}}{\left(\mu_0 e r n_0 u_0 - \dfrac{\partial B_z}{\partial \theta}\right)^2} =
$$

$$
\frac{\alpha k_B T_0}{m_i r^2}\left(1 - \frac{\dfrac{\partial B_z}{\partial \theta}}{\mu_0 e r n_0 u_0}\right)^{\alpha-1} \frac{\partial^2 B_z}{\partial \theta^2} + \frac{\mu_0 e^2 n_0 u_0^2}{m_i} B_z - \frac{e u_0}{m_i r} B_z \frac{\partial B_z}{\partial \theta} \tag{1}
$$

μ_0 being the vacuum magnetic permeability, e the elementary charge, r the mean radius of the channel, n_0 and T_0 respectively the equilibrium plasma number density and temperature, u_0 the electron radial drifting velocity, m_i the ion mass. Once the previous equation is solved for B_z, the plasma density and velocity can be calculated (as functions of θ) according to the following relations[6,7]

$$
n = n_0 - \frac{1}{\mu_0 e r u_0}\frac{\partial B_z}{\partial \theta}, \qquad v = \frac{1}{\mu_0 e n u_0}\frac{\partial B_z}{\partial t}.
$$

The axial component of the magnetic field is not present at equilibrium but can be generated by the axial symmetry breaking and by the inception of an azimuthal velocity component of the flow. This last point is very important as it is at the base of the passive stabilization method which will be described in Sec.(V).

B. Linear stability analysis and symmetry breaking

We perform here a linear stability analysis of the equation for the axial component of the magnetic field previously obtained. Neglecting all nonlinear terms in (1), it follows (recall $B_z = 0$ at equilibrium)

$$
\frac{\partial^2 B_z}{\partial t^2} = \frac{\alpha k_B T}{m_i r^2}\frac{\partial^2 B_z}{\partial \theta^2} + \frac{\mu_0 e^2 n_0 u_0^2}{m_i} B_z \tag{2}
$$

and then we assume a small sinusoidal perturbation of the form $B_z = \Re\left\{\widehat{B_z} e^{i(m\theta - \omega t)}\right\}$, where $\widehat{B_z}$ is the real-valued wave amplitude and m must be an integer number in order to respect the azimuthal 2π-periodicity, which is obviously to be accounted for even if the azimuthal symmetry is broken. Instability corresponds to the imaginary part of ω being positive. Proper substitution of the perturbation into (2) leads to the following quadratic equation for ω:

$$
\omega^2 = \frac{\alpha}{\tau_0^2}\left(m^2 - \frac{m_*^2}{\alpha}\right), \tag{3}
$$

where the characteristic time $\tau_0 = \sqrt{m_i r^2/(k_B T)}$ and the critical parameter $m_*^2 = (\mu_0 e^2 n_0 u_0^2 r^2)/(k_B T)$ have been introduced. Let $(\omega_r, \omega_i) \in \mathbf{R}$ be respectively the real and imaginary part of ω.

If $m^2 < (m_*^2/\alpha) \Rightarrow (m^2 - (m_*^2/\alpha)) < 0$, the system is unstable and the linear growth rate of the unstable $m-$th mode is $|\Im(\omega)| = \sqrt{|m^2 - (m_*^2/\alpha)|}/\tau_0$, while $\omega_r = 0$. Being $m_*^2 > 0$, it follows that this system is always unstable with respect to the $m = 0$ mode, implying a uniform rotation of the plasma in the thruster. Such a rotation has never been observed, to authors' knowledge, in usual operation of self-field thrusters, but this could also be a consequence of the fact that it has never been the scope of any dedicated experimental investigation. An exception is the work of Zuin et al.[25] on a thruster with a short hollow cathode and a flared anode, where $m = 0$ and $m = 1$ kink modes were observed both in applied and self-field configurations.

As the transition from a diffuse to a spotty conduction pattern is characterized by the presence of azimuthal variations of plasma quantities, we focus our attention on the unstable development of $m \geq 1$ modes. These modes can undergo an exponential growth only if at least $(m_*^2/\alpha) \geq 1$, and this allows us to obtain a criterion for the transition to the spotty conduction pattern:

$$\frac{m_*^2}{\alpha} = \frac{\mu_0 e^2 n_0 u_0^2 r^2}{k_B T} \geq 1 \ . \tag{4}$$

During the linear phase of the instability, any spurious axial component of the magnetic field (which could be created by any source of real-world noise) tends to bend the radial paths of the electrons, because of the interaction with the azimuthal magnetic field leading to an azimuthal component of the Lorent force (see Fig.(1)). Electrons (and ions as well in order to preserve quasineutrality) gather in specific locations, this concentration being opposed only by the induced azimuthal pressure gradient. The stabilizing effect of this pressure gradient, however, only depends on the equilibrium properties of the flow (i.e. plasma density and temperature), thus explaining why the instability we have found has a threshold in terms of plasma quantities for $m \geq 1$ modes. If conditions expressed in Eq.(4) are fulfilled, a positive feedback is created between the bending of electron trajectories and the increase of the axial magnetic field component which is responsible of such a deviation.

Note that low mode-number perturbations are more easily developed than high-m modes because the azimuthal pressure gradient, i.e. the pressure *force*, can be shown[6,7] to be proportional to the second derivative in θ of the axial magnetic field, which is proportional to m^2.

III.   Plasma filamentation

Plasma concentration in narrow conduction layers stops when the thermal pressure gradient between the inner and outer regions compensates the induced Lorentz force. We can analyze the new equilibrium of the plasma by removing all time derivatives in Eq.(1), which can then be written as

$$\alpha \left(1 - \frac{d\beta}{d\theta}\right)^{\alpha-1} \frac{d^2\beta}{d\theta^2} + m_*^2\beta \left(1 - \frac{d\beta}{d\theta}\right) = 0 \ ,$$

where the nondimensional axial magnetic field $\beta = B_z/(\mu_0 e n_0 u_0 r)$ was introduced.

This nonlinear differential equation is solved numerically with periodic boundary conditions and taking as a guess for β a sinusoidal waveform valid in the linear regime. We also introduce the nondimensional relative number density ξ, for which the following expression can be shown to hold:[6,7]

$$\xi = \frac{n - n_0}{n_0} = -\frac{d\beta}{d\theta} \ .$$

In Fig.(2-a) two solutions are shown for $m_* = 4.3$ and $\alpha = 1$, obtained taking $m = 3$ and $m = 4$, while in Fig.(2-b) two solutions are shown for $m_* = 5.3$ and $\alpha = 5/3$, with $m = 3$ and $m = 4$ respectively. The
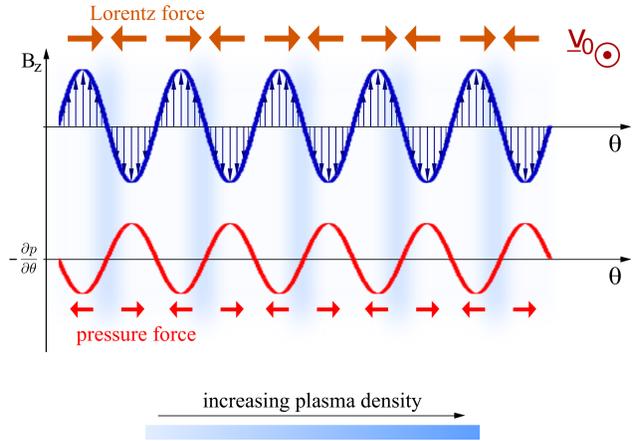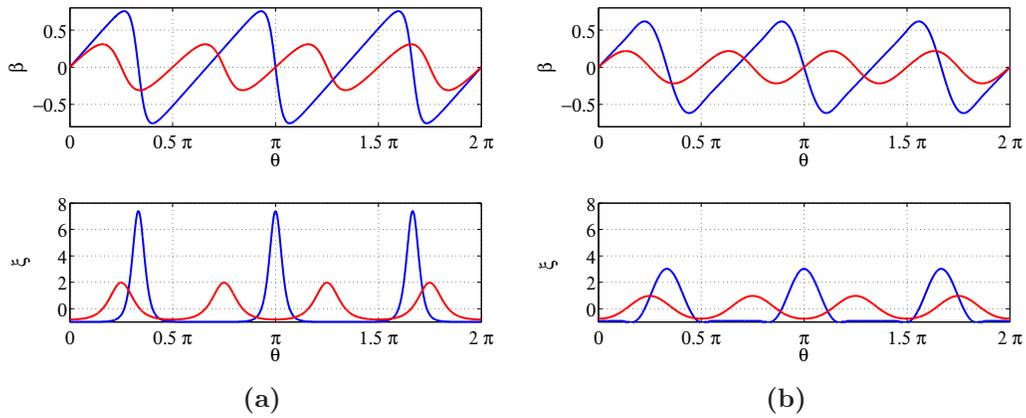
Figure 2: Steady nonlinear solutions of the nondimensional magnetic field β and the nondimensional relative number density ξ for (a) $m_* = 4.3$, $\alpha = 1$, $m = 3$ (blue curve) and $m = 4$ (red curve) and for (b) $m_* = 5.3$, $\alpha = 5/3$, $m = 3$ (blue curve) and $m = 4$ (red curve).

common pattern of the nonlinear steady solutions is the presence of more or less accentuated peaks in the solution of particle number density, in which the experimentally observed plasma filaments can be identified, thus confirming the filamentation nature of the instability analyzed up to this point.

A. Stability of the filamented pattern and mechanisms behind filament disruption

The fact that a stable *filamented* plasma configuration has never been observed, led us to investigate the stability of the previously found steady solutions. We have performed a standard linear stability analysis, whose details can be found in previous publications.[6,7] The result of this analysis showed that also these steady solutions, as the homogeneous equilibrium one, are unstable, and then giving confirmation that current filaments are actually *unstable* structures, as also Uribarri noted.[20] Moreover, other mechanisms can lead to the unstable dynamics of current filaments. First, they tend to be rapidly removed from the channel volume by the axial component of the Lorentz force (which is locally increased because of the strong current concentration), being then destroyed in the low-current density regions of the plume. In addition, current filaments can be somehow assimilated to micro z-pinches connecting the anode to the cathode or to the bulk of the plasma volume, and z-pinches are known to be subjected to several MHD instabilities (among which *sausage* and *tilting* instabilities). Finally, ohmic dissipation in the interior of the filaments tends to increase their internal pressure and to counteract the pinching action of the Lorentz force, and if the filament lifetime is sufficiently long, dissipation can actually play a role in filament disruption.

IV. Current filamentation and voltage fluctuations in MPD thrusters

In this section we describe how the filamentation instability described above, in connection with the unstable nature of plasma filaments and the saturation of current, concurs in the generation of fluctuations in the terminal voltage of the thruster.

The hypothesis recently developed by the authors[6,7] is that the thruster can operate in three different regimes, according to the behaviour of the plasma flow in the thruster and to its electrical response:

Below-onset regime: this regime should occur at the lowest $J^2/\dot{m}$ levels; SF-MPD thrusters can be thought, from the electrical standpoint, as nonlinear current-limited resistive elements, with a resistance growing almost quadratically with the imposed current, as the operating voltage of SF-MPD thrusters is known to increase with the third power of J in the full electromagnetic regime, as shown in Fig.(3-a).

Filamentation regime: in this regime, corresponding to intermediate values of $k = J^2/\dot{m}$, we suppose that plasma filamentation occurs (but the thruster current is not yet saturated) leading to the complex dynamics of plasma filaments previously described. Voltage oscillations can arise because of fluctuations of thruster inductance, as any current carrying circuit has a self-inductance L which can

be related to the energy stored in the magnetic field generated by the current itself[10] :

$$L = \frac{1}{J^2} \int \frac{\mathbf{B} \cdot \mathbf{B}}{\mu_0} \, dV \, , \tag{5}$$

the previous integration being performed over the entire current carrying volume. This shows that the voltage drop associated to the inductive part of the discharge, i.e. $\mathrm{d}(JL)/\mathrm{d}t$, fluctuates because the filamentation instability generates additional components of magnetic field (B_z in our model) which are not present in the homogeneous configuration, and at the same time these structures are destroyed because of the intrinsic unstable nature of plasma filaments. In the filamentation regime the thruster should have an almost inductive electrical response, and this kind of electrical behaviour has been observed by Di Vita et al.[4]

Filamentation-saturation regime: in this regime both current filamentation and current saturation occur, where by current saturation we mean a condition in which the current level imposed by the external power source, i.e. the Pulse Forming Network, is greater than the total electron thermal current which can be collected at the anode surface, given by the integration of electron thermal flux over the anode surface (more precisely the surface corresponding to the outer bound of the anode sheath), i.e.

$$J_{\mathrm{PFN}} \geq J_{\mathrm{sat}} = \int_{S_a} \frac{en}{4} \sqrt{\frac{8k_B T_e}{\pi m_e}} \, dS \, ,$$

In this regime no more than J_{sat} can actually be carried by the "resistive" part of the thruster, while the excess part $J_e = J_{\mathrm{PFN}} - J_{\mathrm{sat}}$ contributes to raise the anode sheath potential, which according to Uribarri and Choueiri can be schematized as an ideal capacitor, as shown in Fig.(3-b). Even if inductance oscillations should still be present, strong voltage fluctuations should arise as a consequence of the interaction between the complex dynamics of current filaments and the saturation of thruster current, according to the model proposed by Uribarri and Choueiri,[21] which has been previously discussed in Sec.(I). The lack of that model, i.e. what is the physical mechanism driving the random "switching" of the sheath-spot circuit, is solved by arguing that the *total saturation current*, i.e. $J_{\mathrm{sat}} = \int_{S_a} \frac{en}{4} \sqrt{\frac{8k_B T_e}{\pi m_e}} \, dS$, must be attributed this role. This quantity, which obviously depends indirectly on the driving parameters $J = J_{\mathrm{PFN}}$ and $\dot{m}$, fluctuates, increasing during plasma filamentation which builds up an increase of ohmic heating and then plama temperature because of plasma concentration, decreasing when plasma filaments are destroyed or carried out of the channel into the plume.
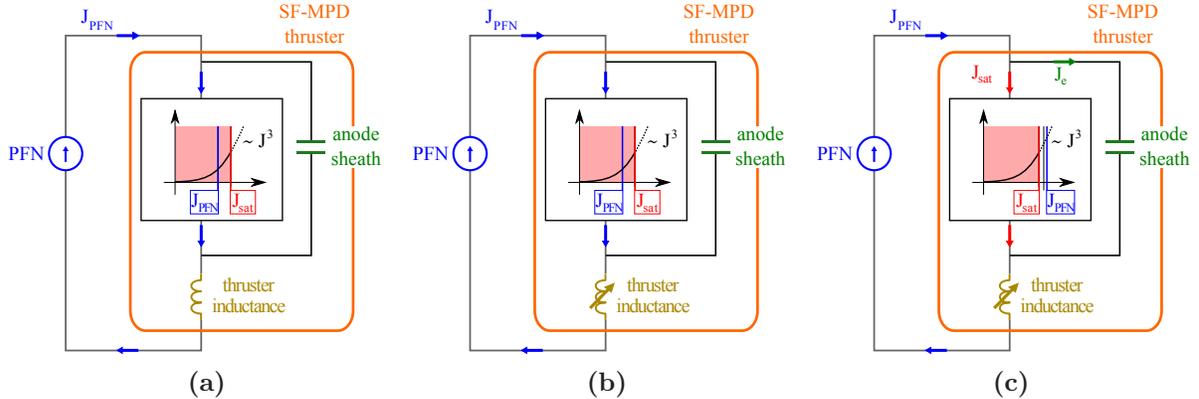
Figure 3: Schematic electrical circuit of SF-MPD thrusters operated (a) below onset conditions, (b) in the filamentation regime and (c) in the filamentation-saturation regime.

V.   Passive stabilization method

The method of passive stabilization which is here proposed, consists in the insertion of longitudinal insulating plates in the channel of the SF-MPD thruster. This method has already been employed to effectively

eliminate the inception of kink modes in applied-field MPD thrusters,[26] even if in that configuration the effects of preventing the rotation of the plasma and the build up of an azimuthal Hall current, both of which are known to give a substantial contribution to thrust generation in applied-field MPD thrusters, should be assessed.

The scope of this technique is to lock the azimuthal degree of freedom of the flow, as we have seen in Sec.(II) that the azimuthal symmetry breaking first and the inception of the filamentation instability later, are a consequence of the tendency of the plasma to generate an azimuthal current or an azimuthal component of flow velocity. The channel is then divided into separated sub-channels by the plates, diminishing the effective length along the transverse (azimuthal) direction. The application of this passive stabilization technique to SF-MPD thruster, does not modify the structure of the equilibrium discharge, because at equilibrium none of the vector quantities (i.e. plasma current and velocity and electromagnetic fields) has a component along θ.

Here we present some preliminary calculations aiming at assessing the effectiveness of this technique in preventing the previously cited phenomena. These calculations are based on a linear stability analysis of the flow, taking the same assumptions as in Sec.(II).

The starting point is again the linearized version of Eq.(1)

$$\frac{\partial^2 B}{\partial t^2} = \frac{\alpha k_B T}{m_i r^2} \frac{\partial^2 B}{\partial \theta^2} + \frac{\mu_0 e^2 n_0 u_0^2}{m_i} B$$

(here we put $B_z = B$ for simplicity of notation) the difference from the previous analysis being that we are now going to solve it with different boundary conditions. Suppose indeed that we insert n plates in the channel of the the thruster. On the surface of the plates there can be no azimuthal component of flow velocity, and from the expression for velocity given in Sec.(A) we have in the linear regime

$$v = \frac{1}{\mu_0 e n u_0} \frac{\partial B}{\partial t} \quad \Rightarrow \quad v = -\frac{i\omega}{\mu_0 e n u_0} B \ .$$

Then the previous equation must not be solved with periodic boundary conditions but with homogeneous BCs in correspondance of two consecutive plates, i.e.

$$B(0) = B\left(\frac{2\pi}{n}\right) = 0 \ .$$

Note that in this case, because of the non-periodic BCs, it is not possible to expand B in a Fourier series along θ, but only in time, i.e. we will assume that $B = B(\theta) \exp(-i\omega t)$. The linear equation can then be written as

$$-\omega^2 B = \frac{\alpha}{\tau_0^2} \left[\frac{\partial^2 B}{\partial \theta^2} + \frac{m_*^2}{\alpha} B\right] \ ; \text{or}$$

$$\frac{\partial^2 B}{\partial \theta^2} + \frac{1}{\alpha}(m_*^2 + \omega^2 \tau_0^2) B = 0 \ , \qquad \text{with } B(0) = B\left(\frac{2\pi}{n}\right) = 0 \ ,$$

where use was made of the previously introduced τ_0 and m_*^2 parameters. With the change of variable $\phi = n\theta \Rightarrow \mathrm{d}\theta = \mathrm{d}\phi/n$ we find

$$B'' + \underbrace{\frac{1}{\alpha n^2}\left(m_*^2 + \omega^2 \tau_0^2\right)}_{=\lambda} B = 0 \ , \qquad \text{with } B(0) = B(2\pi) = 0 \tag{6}$$

which is the harmonic equation but with homogeneous boundary conditions. The following cases are possible (λ is assumed real):

- $\lambda > 0 \Rightarrow$ general solution $B = C \sin(\sqrt{\lambda}\phi) + D \cos(\sqrt{\lambda}\phi)$ and it is possible to satisfy the BCs

- $\lambda = 0 \Rightarrow$ general solution $B = C + D\phi$ but it is not possible to satisfy the BCs (only trivial solution $B = 0$)

- $\lambda < 0 \Rightarrow$ general solution $B = C \sinh(\sqrt{|\lambda|}\phi) + D \cosh(\sqrt{|\lambda|}\phi)$ but it is not possible to satisfy the BCs (only trivial solution $B = 0$).

The only case $\lambda > 0$ is then possible, this meaning that if the equilibrium plasma quantities and number of plates are such that $\lambda \leq 0$, then there cannot be an instability. The only kind of solutions satisfying the homogeneous BCs is

$$B_h = C_h \sin(h\phi) , \qquad \text{with} \quad h = \sqrt{\lambda} = 1, 2, 3, \ldots .$$

Here we are going to consider only the case of purely imaginary ω with positive imaginary part, i.e. $\omega = i\gamma$, with $\gamma \in \mathbb{R}^+$, which is the most interesting from the standpoint of the stability of the discharge.

In this case

$$\lambda = \frac{1}{\alpha n^2} \left(m_*^2 - \gamma^2 \tau_0^2 \right) .$$

As it was told before, in order to obtain a valid solution, i.e. in order to have *instability*, it must be $\lambda > 0 \Rightarrow \gamma^2 < m_*^2 \tau_0^2$, and also $\lambda = h^2 , \quad h = 1, 2, 3, \ldots$, or

$$\frac{1}{\alpha n^2} \left(m_*^2 - \gamma^2 \tau_0^2 \right) \geq 1 .$$

In order to prevent the inception of the instability we can impose

$$\frac{1}{\alpha n^2} \left(m_*^2 - \gamma^2 \tau_0^2 \right) < 1 \Rightarrow n^2 > \frac{m_*^2}{\alpha} \left(1 - \frac{\gamma^2 \tau_0^2}{m_*^2} \right) .$$

The previous result shows that a sufficiently high number of plates can effectively stabilize the discharge. The worst case, requiring a greater number of plates to achieve stabilization, is for low-m, and then low-γ, modes, just as in the case without insulating plates. In the limit $\gamma \to 0$ it must be

$$n^2 > \frac{m_*^2}{\alpha} .$$

Note that being $\alpha \simeq 1$, it should be $n > m_*$. So, as it was in the case of stability without plates, the important point is to make the parameter m_* the lowest possible in the channel, which essentially means to reduce the local plasma current density and to increase the local plasma number density. In particular, obtaining an almost-constant axial distribution of the current density in the thruster is of primary importance (and this would also be preferable to reduce ohmic dissipation in the plasma). To illustrate this point, we can calculate the minimum number of plates required to stabilize, in accordance with the previous analysis, a SF-MPD thruster in the following configuration:

- active (current carrying) length: 10 cm
- anode radius: 5 cm
- imposed current 20 kA
- plasma average number density: 10^{21} m^{-3}
- plasma average density: 2 eV.

Recall that the critical parameter m_*^2 can be rewritten as

$$m_*^2 = \frac{\mu_0 j_0^2 r^2}{n_0 k_B T}$$

j_0 being the equilibrium (homogeneous) plasma current density. It can be verified immediately that in this case the required number of plates is $n = 2$. This example shows that the proposed method is in principle a feasible method to stabilize the thruster discharge and avoid plasma filamentation, but it requires a proper design of the entire thruster in order to obtain suitable values of the plasma quantities in the channel.

VI. Conclusions

In this work we have reviewed a recently theory proposed by the authors to explain the complex phenomenology of the unstable operating regime affecting MPD thrusters known as onset.

The linear stability analysis of a simplified plasma configuration we have carried, clearly shows that SF-MPD thrusters are actually prone to azimuthal symmetry breaking and to the development of a filamentation instability which can be accounted for the observed transition from a homogeneous and diffuse current pattern to a spotty one, characterized by the presence of tiny plasma channel, filaments, whose presence has been clearly identified by the nonlinear steady solutions of our problem. Moreover, our analysis gives both quantitative and qualitative indication of the intrinsic unstable nature of the plasma filaments.

These results, as well as previous experimental investigations, led us to assume that two types of voltage fluctuations, and two regimes of voltage hash, can actually exist, which are both characterized by current filamentation, but differing in that the thruster current can be saturated (filamentation-saturation regime) or not (filamentation regime). In the first regime, voltage fluctuations should come from variations of thruster inductance, while in the second regime fluctuations of anode sheath potential should add, the mechanisms of continuos charging/discharging of the sheath begin a consequence of fluctuations of the total current density, which are ultimately driven by the formation and distruction of plasma filaments.

Finally, we have proposed a method to stabilize the thruster behavior, consisting in the insertion of longitudinal plates in the channel in order to lock the azimuthal degree of freedom of the flow. We have proved that this method is theoretically effective, and still it requires a proper design of the thruster in order to be a feasible technical solution.

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