

Effect of an Additional, Parallel Capacitor on Pulsed Inductive Plasma Accelerator Performance

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A model of pulsed inductive plasma thrusters consisting of a set of coupled circuit equations and a one-dimensional momentum equation has been used to study the effects of adding a second, parallel capacitor into the system. The equations were nondimensionalized, permitting the recovery of several already-known scaling parameters and leading to the identification of a parameter that is unique to the particular topology studied. The current rise rate through the inductive acceleration coil was used as a proxy measurement of the effectiveness of inductive propellant ionization since higher rise rates produce stronger, potentially better ionizing electric fields at the coil face. Contour plots representing thruster performance (exhaust velocity and efficiency) and current rise rate in the coil were generated numerically as a function of the scaling parameters. The analysis reveals that when the value of the second capacitor is much less than the first capacitor, the performance of the two-capacitor system approaches that of the single-capacitor system. In addition, as the second capacitor is decreased in value the current rise rate can grow to be twice as great as the rise rate attained in the single capacitor case.

I. Introduction

PULSED inductive plasma accelerators are electrodeless space propulsion devices where a capacitor is charged to an initial voltage and then discharged as a high-current pulse through a coil, inductively coupling energy into the propellant. The field produced by this pulse ionizes the propellant, producing a plasma near the face of the coil. Once a plasma is formed it can be accelerated and expelled at a high exhaust velocity by the Lorentz force arising from the interaction of an induced plasma current and the magnetic field. While there are many coil geometries that can be employed to inductively accelerate a plasma, in this paper the discussion is limited to planar geometries where the coil takes the shape of a flat spiral. A recent review of the developmental history of planar-geometry pulsed inductive thrusters can be found in Ref. [1]. Two concepts that have employed this geometry are the Pulsed Inductive Thruster (PIT)^{2,3} and the Faraday Accelerator with Radio-frequency Assisted Discharge (FARAD).⁴

Pulsed inductive plasma accelerators possess many demonstrated and potential benefits,^{1,3} providing motivation for continued investigation. The electrodeless nature of these thrusters eliminates electrode erosion, mitigating the lifetime and contamination issues associated with conventional electric thrusters. Also, a wider variety of propellants is available for use when compatibility with metallic electrodes is no longer an issue. Pulsed inductive accelerators have demonstrated operation on propellants like ammonia, hydrazine, and CO₂, and there is no fundamental reason why they would not operate on other propellants like H₂O. It is well known that for a given propellant pulsed accelerators can maintain constant specific impulse I_{sp} and thrust efficiency η_t over a wide range of input power levels by adjusting the pulse rate to maintain a constant discharge energy per pulse. In addition, these thrusters have demonstrated operation

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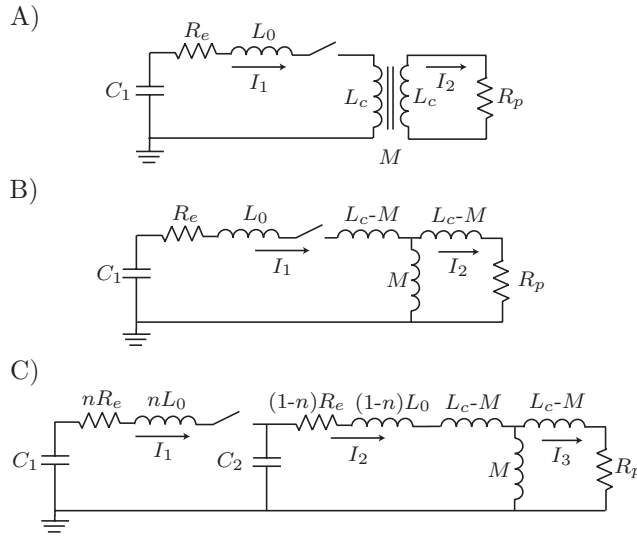


Figure 1. A) General lumped-element circuit model and B) equivalent circuit model of a pulsed inductive accelerator (after Ref. [2]). C) Equivalent electrical circuit model of an accelerator with a second, parallel capacitor.

in a regime where η_t is relatively constant over a wide range of I_{sp} . Finally, thrusters in this class have operated at high energy per pulse, and by increasing the pulse rate they offer the potential to process very high levels of power to provide relatively high thrust using a single thruster.

Pulse circuits for inductive thrusters have in the past typically been limited to a simple, ringing resistive-inductive-capacitive (RLC) configuration like that shown in Fig. 1A and redrawn in Fig. 1B as an equivalent circuit that is easier to model. However, as the field has developed proposed circuit topologies are becoming more complex.⁵ In this paper, we proceed with an investigation of the circuit shown in Fig. 1C where a second capacitor with value less than or equal to C_1 is inserted downstream of the switch. This study was motivated by a data set where the efficiency of a pulsed inductive thruster increased when the capacitor C_2 was added.⁶ Unfortunately, the value of C_1 was also increased when C_2 was added and previous work has shown that increasing C_1 can also increase the efficiency.⁷ The authors of Ref. [6] additionally noted that the voltage across C_2 could be approximately double the charge voltage across C_1 when $C_2 \ll C_1$. This result was interesting because it implied that the voltage and commensurate current rise rate in the coil could be increased by adding C_2 . A higher current rise rate can, in turn, produce stronger electric fields at the coil face and potentially lead to better inductive ionization of the propellant.⁵

There exists a 1-D pulsed inductive acceleration model that employs a set of circuit equations coupled to a one-dimensional momentum equation.^{2,3} The model has since been nondimensionalized and used by Polzin *et al.*^{7,8} to define a set of scaling parameters and gain general insight into their effects on thruster performance. In this paper we modify the acceleration model to account for the presence of C_2 in the system and then nondimensionalize the equation set to identify any new nondimensional scaling parameters that arise for the new circuit topology. The current rise rate through the coil is computed for various cases, and it is used as a proxy measurement to gauge the ability of the coil to inductively ionize the propellant. Finally, we examine whether the addition of C_2 imposes potential benefits or detriments on thruster efficiency and I_{sp} .

II. Governing Equations

A. Physical Model

A circuit-based model of a pulsed inductive accelerator previously developed by Lovberg and Dailey² is modified to account for the addition of the second capacitor as shown in Fig. 1C. In the lumped-element circuit model shown in Fig. 1A,B, the external circuit (left side of the figure) possesses a capacitor bank with capacitance C_1 , external inductance L_0 , resistance R_e , and acceleration coil inductance L_C . The plasma also has an inductance equal to L_C and a resistance R_p . The two circuits are inductively coupled through

the coil, which acts as a transformer with mutual inductance M . The value of M is a function of the current sheet position z . In Fig. 1C, a second capacitor C_2 is added between the primary capacitor bank and the inductive coil. This has the effect of splitting the L_0 and R_e , with the value on the left and right sides of C_2 linearly scaled by the factor $n < 1$ and $(1 - n)$, respectively. Physically, we are assuming that the external inductance and resistance in each leg is scaled by the position of C_2 relative to C_1 and the coil, with n approaching unity as C_2 is moved nearer to the coil.

A set of circuit equations for Fig. 1C are written through the application of Kirchoff's law to each loop. Rearranging the equations and adding a statement for the time-rate of change of charge on each capacitor yields the following coupled set of first-order ordinary differential equations:

$$\begin{aligned}\frac{dI_1}{dt} &= \frac{V_1 - V_2 - nR_e I_1}{nL_0}, \\ \frac{dI_2}{dt} &= \left\{ L_C V_2 + (L_C I_3 + M I_2) \frac{dM}{dt} - (1 - n) R_e L_C I_2 - R_p M I_3 \right\} / \{ L_C [(1 - n) L_0 + L_C] - M^2 \}, \\ \frac{dI_3}{dt} &= \frac{I_2 \frac{dM}{dt} + M \frac{dI_2}{dt} I_3 R_p}{L_C}, \\ \frac{dV_1}{dt} &= -\frac{I_1}{C_1}, \\ \frac{dV_2}{dt} &= \frac{I_1 - I_2}{C_2},\end{aligned}\tag{1}$$

where V_1 and V_2 are the voltages on capacitors C_1 and C_2 , respectively.

The inductance of a planar, spiral inductive coil coupled to a plasma current sheet is difficult to compute analytically. However, based on experimental measurements it has been found² that the mutual inductance can be modeled using the simple function

$$M = L_C \exp\left(-\frac{z}{2z_0}\right),\tag{2}$$

which, when differentiated results in the following equation that governs the changing mutual inductance of the circuit:

$$\frac{dM}{dt} = -\frac{L_C}{2z_0} \exp\left(-\frac{z}{2z_0}\right) \frac{dz}{dt}.\tag{3}$$

As the current sheet moves forward, it entrains and accelerates any encountered gas. The propellant mass in the current sheet as a function of time can be written as

$$m(t) = m_0 + \int_{t=0}^t \rho_A v_z dt,\tag{4}$$

where $\rho_A = \rho_A(z(t))$ is the linear mass density distribution and v_z is the sheet velocity. The term m_0 represents the initial mass of propellant in the sheet while the integral term represents the mass accumulated by the sheet as it moves away from the acceleration coil.

The momentum equation for this system can be written as

$$\frac{L_C I_2^2}{2 z_0} \exp(-z/z_0) = \rho_A v_z^2 + m(t) \frac{dv_z}{dt}.\tag{5}$$

The left hand side represents the self-field electromagnetic force generated through the interaction of the current and the magnetic field. The first term on the right hand side in Eq. (5) is the momentum investment associated with 'snowplowing' propellant (i.e. accelerating the newly entrained propellant encountered by the sheet from rest to the sheet speed) while the second term involves further acceleration of the already entrained propellant.

The model lacks a plasma model to calculate temperature and resistivity and it does not contain any plasma ionization, or 'breakdown', model, instead simply assuming that there is plasma present at $t = 0$. These shortcomings were previously discussed in more detail in Ref. [7]. As in that previous work, the purpose here is to use the model to search for relevant nondimensional scaling parameters and gauge their general effect on performance, instead of looking for an exact matching of experimental data to the performance model. However, we do note that even with the shortcomings, the 1-D, circuit-based acceleration model has shown good qualitative and quantitative agreement with experimental data.^{1,3}

B. Nondimensionalization of the Model

The nondimensionalization strategy previously employed with success in Ref. [7] can also be used in this case. The following dimensionless terms are selected to aid in the conversion of the dimensional equations to a nondimensionalized set:

$$\begin{aligned} I_1^* &= \frac{1}{V_0} \sqrt{\frac{L_0}{C}} I_1, & I_2^* &= \frac{1}{V_0} \sqrt{\frac{L_0}{C}} I_2 \\ I_3^* &= \frac{1}{V_0} \sqrt{\frac{L_0}{C}} I_3, & t^* &= \frac{t}{\sqrt{L_0 C}}. \end{aligned} \quad (6)$$

Other dimensionless terms that arise in the course of nondimensionalizing the equation set are:

$$\begin{aligned} z^* &= \frac{z}{z_0}, & V_1^* &= \frac{V_1}{V_0}, & V_2^* &= \frac{V_2}{V_0}, \\ M^* &= \frac{M}{L_C}, & v_z^* &= \frac{\sqrt{L_0 C}}{z_0} v_z. \end{aligned} \quad (7)$$

The current sheet mass can be nondimensionalized as

$$m^* = m_0^* + \int_0^{t^*} \rho^* f(z^*) v_z^* dt^*, \quad (8)$$

where $m_0^* = m_0/m_{\text{bit}}$, $\rho^* = \rho_0 z_0/m_{\text{bit}}$, and m_{bit} is the total propellant mass per pulse.

Using these nondimensional terms, equations (1), (3), (4), and (5) are written in dimensionless form as,

$$\frac{dI_1^*}{dt^*} = \frac{1}{n} (V_1^* - V_2^*) - I_1^* \psi_1 \quad (9a)$$

$$\frac{dI_2^*}{dt^*} = \left[L^* V_2^* + (M^* I_2^* + I_3^*) \frac{dM^*}{dt^*} - I_3^* M^* L^* \psi_2 - (1-n) I_2^* L^* \psi_1 \right] / \left[((1-n) L^* + 1) - (M^*)^2 \right] \quad (9b)$$

$$\frac{dI_3^*}{dt^*} = M^* \frac{dI_2^*}{dt^*} + I_2^* \frac{dM^*}{dt^*} - I_3^* L^* \psi_2 \quad (9c)$$

$$\frac{dV_1^*}{dt^*} = -I_1^* \quad (9d)$$

$$\frac{dV_2^*}{dt^*} = C (I_1^* - I_2^*) \quad (9e)$$

$$\frac{dM^*}{dt^*} = -\frac{1}{2} \exp\left(-\frac{z^*}{2}\right) v_z^* \quad (9f)$$

$$\frac{dz^*}{dt^*} = v_z^* \quad (9g)$$

$$\frac{dv_z^*}{dt^*} = \left[\alpha (I_2^*)^2 \exp(-z^*) - \rho^* f(z^*) (v_z^*)^2 \right] / m^* \quad (9h)$$

$$\frac{dm^*}{dt^*} = \rho^* f(z^*) v_z^* \quad (9i)$$

The initial conditions for solving the nondimensional set of equations are as follows.

$$\begin{aligned} I_1^*(0) &= 0, & M^*(0) &= 1, \\ I_2^*(0) &= 0, & z^*(0) &= 0, \\ I_3^*(0) &= 0, & v_z^*(0) &= 0, \\ V_1^*(0) &= 1, & m^*(0) &= \frac{m_0}{m_{\text{bit}}}. \\ V_2^*(0) &= 0, \end{aligned} \quad (10)$$

C. Scaling Parameters

During the course of nondimensionalization, several new terms appear in Eqs. (9). These new terms are the relevant scaling parameters of the system of equations and are defined as

$$\begin{aligned} \mathcal{C} &= \frac{C_1}{C_2}, & L^* &= \frac{L_0}{L_C}, & \alpha &= \frac{C_1^2 V_0^2 L_C}{2 m_{\text{bit}} z_0^2}, \\ \psi_1 &= R_e \sqrt{\frac{C_1}{L_0}}, & \psi_2 &= R_p \sqrt{\frac{C_1}{L_0}}, \end{aligned} \quad (11)$$

which are similar to the parameters previously found for the single capacitor case.⁷ We proceed with a brief discussion of the meanings of the scaling parameters.

1. Inductance Ratio L^*

A pulsed inductive accelerator circuit possesses an external inductance L_0 and an acceleration coil inductance L_C . During a current pulse the moving plasma increases the circuit's inductance from L_0 to $L_0 + L_C$ (i.e., $L_C = \Delta L$). The fractional change of inductance, $(L^*)^{-1} = \Delta L / L_0$, in a pulsed electromagnetic accelerator provides a measure of efficiency,⁹ as this ratio is indicative of the fraction of energy that can be deposited into electromagnetic acceleration of the gas. In an efficient pulsed inductive accelerator the value of L^* must be much less than unity or $L_C \gg L_0$.

2. Dynamic Impedance Parameter α

The dynamic impedance parameter α can be recast as the product of several important ratios:

$$\alpha = \frac{C_1^2 V_0^2 L_C}{2 m_{\text{bit}} z_0^2} = \frac{1}{8\pi^2} \frac{C_1 V_0^2 / 2}{m_{\text{bit}} v_z^2 / 2} L^* \left(\frac{2\pi \sqrt{L_0 C_1}}{L_0 / \dot{L}} \right)^2 \quad (12)$$

where $\dot{L}$ is the dynamic impedance, defined as $v_z L'$, and L' is the effective inductance per unit length, equal to L_C / z_0 . The ratio of the initial stored energy ($C_1 V_0^2 / 2$) to the plasma kinetic energy ($m_{\text{bit}} v_z^2 / 2$) is recognized as the inverse of thrust efficiency η_t^{-1} and will always be greater than one. The final term is the ratio of the natural period of the driving external circuit, $2\pi \sqrt{L_0 C_1}$, to the time interval, $L_0 / \dot{L}$, over which the plasma's motion increases the inductance of the circuit by one unit of L_0 . The former is the timescale on which the external circuit naturally operates, while the latter is the timescale on which the current sheet remains electromagnetically coupled to the acceleration coil. For a given configuration, there exists an optimum value of α where the electromagnetic coupling timescale is matched to the frequency of the external circuit, allowing for optimum transfer of stored electrical energy into directed kinetic energy.⁷

3. Circuit Parameters ψ_1 , ψ_2 and $\mathcal{C}$

The critical resistance ratios ψ_1 and ψ_2 were shown in previous work to control the nature of the current waveforms.⁷ The new parameter $\mathcal{C}$ also has an effect on the nature of the current waveforms. Proceeding with an analysis similar to that performed in Ref. [7], we search for limiting-case solutions to Eqns. (9a)-(9e). Decoupling the current sheet dynamics (i.e., the acceleration and sheet motion) from the problem allows us to apply the condition

$$M^* = 1, \quad \frac{dM^*}{dt^*} = 0,$$

which simplify the circuit equations. Under these assumptions, the circuit equations can be combined and rewritten as

$$\begin{aligned} \frac{d^2 I_1^*}{dt^{*2}} + \psi_1 \frac{dI_1^*}{dt^*} + \frac{(\mathcal{C} + 1)}{n} I_1^* &= \frac{\mathcal{C}}{n} I_2^*, \\ (1 - n) \frac{d^2 I_2^*}{dt^{*2}} + (1 - n) \psi_1 \frac{dI_2^*}{dt^*} + \mathcal{C} I_2^* &= \mathcal{C} I_1^* - \psi_2 \frac{dI_3^*}{dt^*}, \\ \frac{dI_2^*}{dt^*} - \frac{dI_3^*}{dt^*} &= L^* \psi_2 I_3^* \end{aligned} \quad (13)$$

If the right side of the third equation in (13) is small, then the induced current in the plasma mirrors the current in the coil

$$I_3^* \approx I_2^*.$$

This is the case where the coil and plasma currents are well-coupled, which is expected in efficient accelerators.

The third equation of (13) can be substituted into the second to yield

$$(1-n) \frac{d^2 I_2^*}{dt^{*2}} + [(1-n)\psi_1 + \psi_2] \frac{dI_2^*}{dt^*} + \mathcal{C}I_2^* = \mathcal{C}I_1^* - \psi_2^2 L^* I_3^* \quad (14)$$

This second-order non-linear ODE is similar to the one found in Ref. [7]. If the last term on the right-hand side is comparatively small, then the forcing term contains only I_1^* , which will never be small and cannot be neglected. We can gain insight by writing the homogeneous solution

$$I_2^* = A_0 \exp(-\Psi t^*) \sin \left(\left(\frac{\mathcal{C}}{(1-n)} - \Psi^2 \right)^{1/2} t^* \right), \quad (15)$$

where

$$\Psi \equiv \left(\psi_1 + \frac{\psi_2}{(1-n)} \right) / 2.$$

In this case, the damping is a function of the critical resistance ratios while the natural frequency is additionally a function of the parameter $\mathcal{C}$.

The term on the right hand side of the first equation in (13) will also never be small. However, we can write the homogeneous solution for I_1^* as

$$I_1^* = B_0 \exp \left(-\frac{\psi_1}{2} t^* \right) \sin \left(\left(\frac{(\mathcal{C}+1)}{n} - \frac{\psi_1^2}{4} \right)^{1/2} t^* \right). \quad (16)$$

Though these are only the homogeneous solutions to the simplified electrical response equations, there are several observations that can be made as a result of this exercise. The parameters ψ_1 , ψ_2 , and $\mathcal{C}$ control the electrical responses in the circuit. Increasing the parameter $\mathcal{C}$ increases the ringing frequency of both I_1^* and I_2^* , with the frequencies approaching each other as $\mathcal{C}$ becomes very large. To neglect the non-linear terms presented by the current I_3^* , the values of L^* and ψ_2 must be such that

$$\begin{aligned} \psi_2^2 L^* &\ll 1, \\ \psi_2 L^* &\ll 1. \end{aligned}$$

Finally, Eq. (9e) can be used to show that as $\mathcal{C}$ approaches infinity, the derivative dV_2^*/dt^* will only remain bounded if $I_1^* = I_2^*$. Physically, this is the case where C_2 is removed from the system, permitting the recovery of the response for the circuit shown in Fig. 1B.

III. Nondimensional Solutions

A. Solution Strategy

The set of coupled first-order ODEs given in Eqs. (9) can be solved numerically once the mass distribution and the set of scaling parameters given in Eqs. (11) are specified. The performance metrics chosen for this study are the exhaust velocity, v_z^* , and the thrust efficiency, which is written in terms of nondimensional parameters as

$$\eta_t = \frac{m^* (v_z^*)^2}{2L^* \alpha}. \quad (17)$$

Note that this corrects the equation given for efficiency in Ref. [7] where the square term was left off v_z^* (though it was included in the analysis contained in the reference).

In solving any set of first-order (in time) differential equations, it is important to know when the time histories of the computed variables (specifically v_z^* in our case) should be queried to calculate performance. This question is, in fact, critical to the evaluation of these accelerators. For our non-dimensional model, the integration period will end when one of the following two conditions is reached:

1. The end of the first half-cycle of the accelerator coil discharge is reached and the current I_1^* reverses in sign.
2. The sheet travels three characteristic lengths, $z^* = 3$.

The first condition is based on the fact that when the accelerator current goes through zero, it is going through a point of high dI/dt . While the acceleration model doesn't incorporate any ionization physics, it is well known that a new current sheet can form at the face of the coil during current reversals, causing what is known as a "crowbar discharge". If this occurs, the initial current sheet ceases to undergo acceleration. If at the time of current reversal the final current sheet mass m_f^* is less than the injected propellant mass bit m_{bit}^* , conservation of momentum can be used to correct the end-of-calculation final value of velocity $(v_z^*)_f$ to give the velocity as

$$v_z^* = \frac{m_f^*}{m_{\text{bit}}^*} (v_z^*)_f. \quad (18)$$

The second condition stems from the existence of a finite axial distance between the current sheet and the coil, beyond which the two are essentially decoupled electromagnetically. The separation distance $z^*=3$ is chosen as our cutoff for electromagnetic coupling as it represents an inductance change in the circuit of 95% of the coil inductance. Above this cutoff value, the integration yields no significant change to the calculated performance.

For the present study, a triangular (linearly decreasing) mass distribution is used. This is given by

$$\rho_A^* = \begin{cases} \rho_0^* (1 - z^*/\delta_m^*) & z^* \leq \delta_m^* \\ 0 & z^* > \delta_m^* \end{cases}$$

where $\rho_0^* = 2/\delta_m^*$, $\delta_m^* = \delta_m/z_0$, and δ_m is the (dimensional) depth of the propellant layer.

B. Solutions

1. Electrical Response as a Function of $\mathcal{C}$ and n

We first present representative waveforms showing the electrical behavior of the accelerator as a function of $\mathcal{C}$ in Fig. 2 and n in Fig. 3. The former gauges the effect of the capacitance ratio $\mathcal{C}$ while the latter gives insight into the effect of the position of capacitor C_2 in the line relative to C_1 and the coil. Waveforms for I_1^* , I_2^* and dI_2^*/dt^* are presented, where the latter provides a measure of the inductive voltage across the coil and are being used in this study as proxies for the ability of the coil to inductively ionize propellant.

In Fig. 2 we observe that currents I_1^* and I_2^* have the standard damped-sinusoidal response of an *LRC* circuit with a higher-frequency waveform superimposed on the response. The modulation in all the waveforms increases in frequency and decreases in amplitude as $\mathcal{C}$ is increased. For $\mathcal{C} = 1$ we observe a nearly resonant energy transfer between C_1 and C_2 with the rise rate of the current in the coil relatively low in comparison with the other cases. As $\mathcal{C}$ is increased to 100, the maximum value of dI_2^*/dt^* approaches 2.

The changing value of n appears to most affect the response of the current I_1^* in Fig. 3, with the amplitude of the oscillation superimposed on the wave decreasing with increasing n . We observe that for smaller n values dI_2^*/dt^* oscillates much longer and, like I_1^* , with greater amplitude, having the greatest peak value for the smallest n displayed. It is easiest to observe in Fig. 3G,H that the overall average behavior of I_1^* and I_2^* are roughly equivalent ($I_1^* \approx I_2^*$) for the cases where $\mathcal{C}$ equals 100, which was predicted in the limiting case solutions for $\mathcal{C}$ sufficiently large. The form and magnitude of the waveform for I_2^* does not appear to change much with n , implying that the acceleration process is generally unaffected by the position of C_2 in the line. For the remainder of this study we use $n = 0.2$ to obtain greater current rise rates.

2. Nondimensional Performance

Contours of constant computed performance (η_t, v_z^*) and dI_2^*/dt^* found through solving the governing equations for a range of $\mathcal{C}$ and α values are presented in Fig. 4A-C. For a fixed $\mathcal{C}$ the efficiency possesses a local maximum with respect to α , which is consistent with the interpretation of α as a dynamic impedance matching parameter. The efficiency at a fixed α initially increases with increasing $\mathcal{C}$, eventually asymptoting at $\mathcal{C}$ between 10 and 100. The velocity increases with α and like the efficiency it also asymptotes as $\mathcal{C}$ is

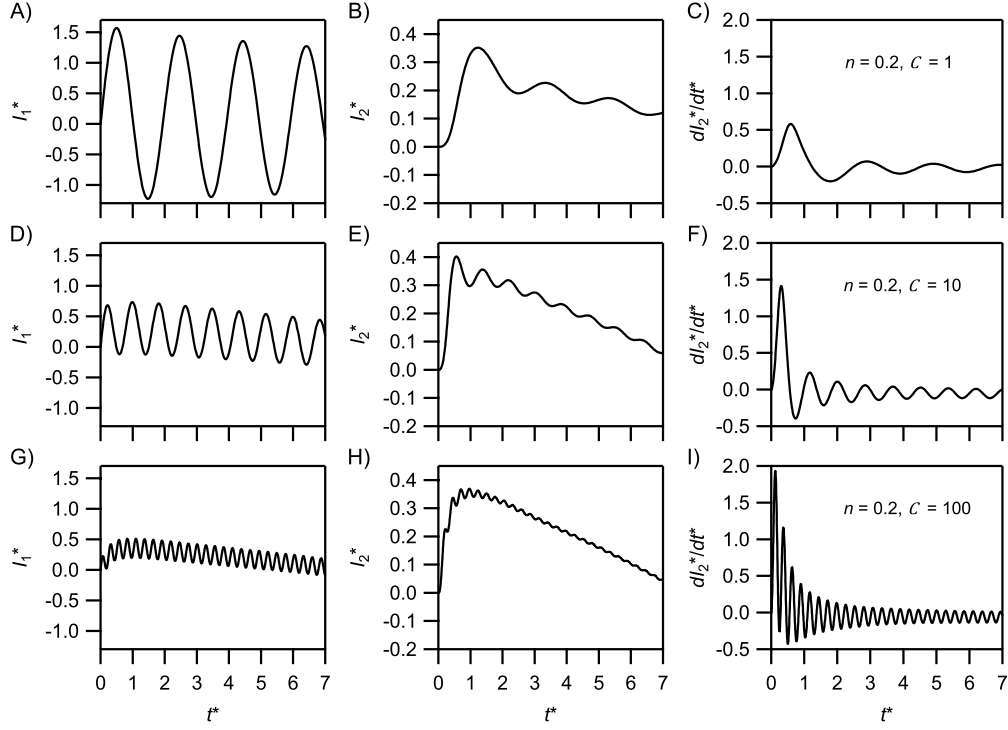


Figure 2. Waveforms for I_1^* , I_2^* , and dI_2^*/dt^* showing the variation in each for A)-C) $n = 0.2, C = 1$, D)-F) $n = 0.2, C = 10$, and G)-I) $n = 0.2, C = 100$.

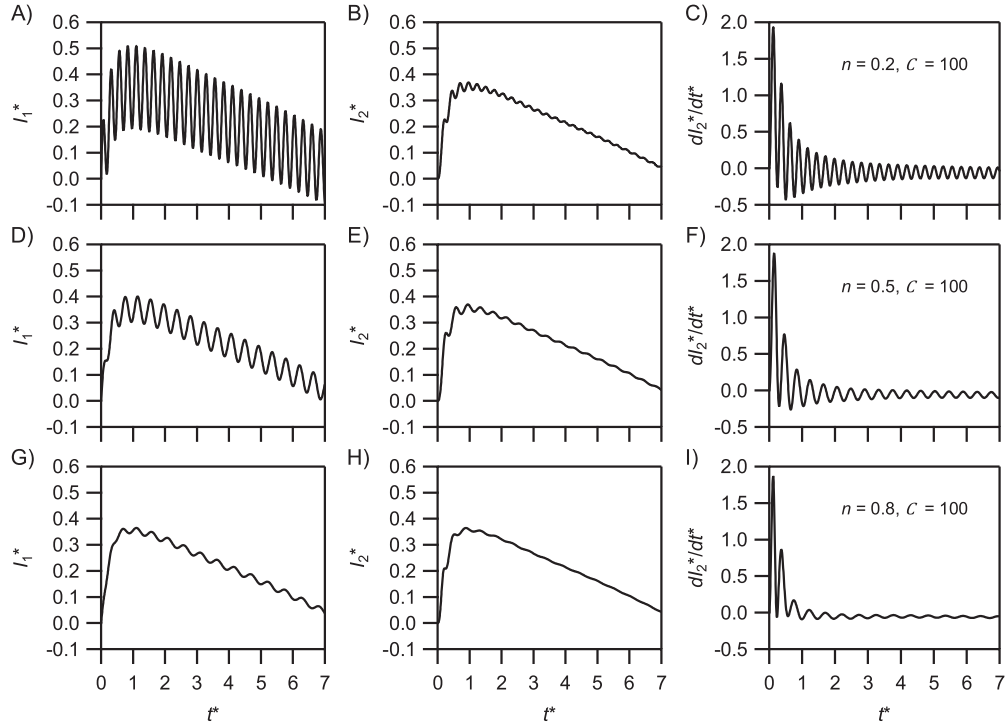


Figure 3. Waveforms for I_1^* , I_2^* , and dI_2^*/dt^* showing the variation in each for A)-C) $n = 0.2, C = 100$, D)-F) $n = 0.5, C = 100$, and G)-I) $n = 0.8, C = 100$.

increased. The current rise rate is a weak function of α , but increases very quickly as C is increased until asymptoting just below a value of dI_2^*/dt^* equal to 2.

It is worth comparing solutions to the two-capacitor case (Fig. 1C) with solutions to the single-capacitor case (Fig. 1A,B) to gauge the effect of the added capacitor C_2 on accelerator performance. Curves of computed performance (η_t , v_z^*) and the maximum value of dI_2^*/dt^* for C equal to 1.5, 5, 10, and 100 are presented in Fig. 4D-F. An additional curve on each graph appears for the single-capacitor case (with the maximum value of dI_1^*/dt^* given in Fig. 4F). We observe that there is not a large difference in the terminal exhaust velocity v_z^* of the single and two-capacitor cases for any value of C . The efficiency graph shows that the two-capacitor case asymptotes to the single-capacitor solution as C approaches 100, implying that a capacitor C_2 can be added without detrimentally affected thruster performance. Additionally, as C is increased the maximum current rise rate through the coil in the two-capacitor case asymptotes to a value that is twice the single-capacitor case.

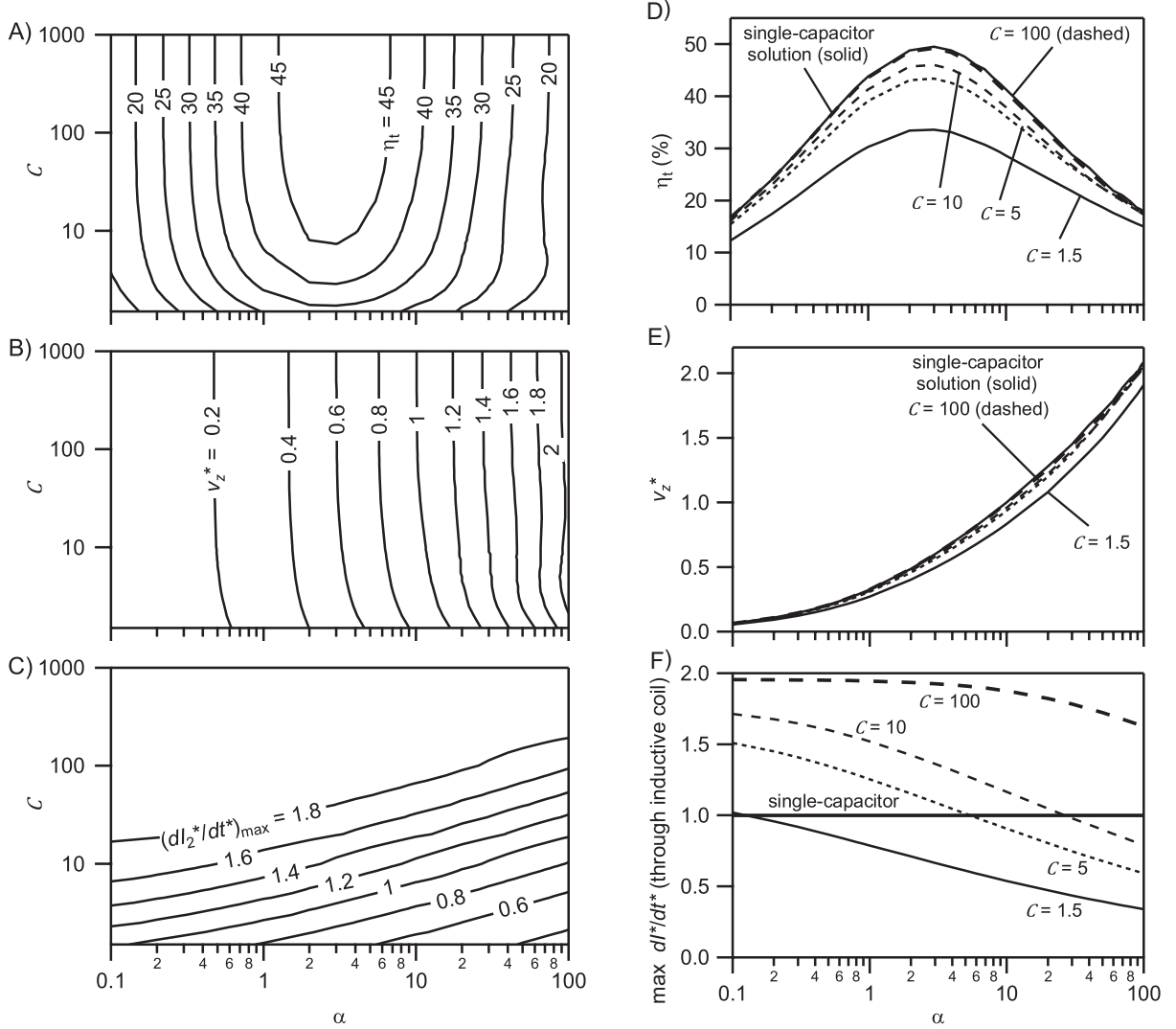


Figure 4. Contour plots of A) efficiency, B) nondimensional velocity, and C) maximum value of dI_2^*/dt^* as a function of varying C and α . Values of ψ_1 , ψ_2 , and L^* are 0.05, 0.13, and 0.121, respectively. Comparison of D) efficiency, E) nondimensional velocity, and F) maximum dI^*/dt^* through the inductive coil as a function of α for various values of C and for the single-capacitor case. The values of maximum dI_2^*/dt^* are given in F) except for the single capacitor case, where the maximum dI_1^*/dt^* is shown.

IV. Discussion

The data presented in the previous section helped gauge the effect on performance that the addition of capacitor C_2 would have. Comparing the efficiency for the single and two-capacitor cases demonstrates that the performance above a certain value of $\mathcal{C}$ is not adversely affected by the addition of the capacitor C_2 . This result is somewhat expected because the overall behavior of current I_2^* (seen in Fig. 3) is relatively well-behaved, with only a small modulation superimposed on top of the main waveform, as opposed to the much larger excursions observed in Fig. 2 for smaller values of $\mathcal{C}$. In addition, at $\mathcal{C}$ of 100 the waveforms for I_1^* and I_2^* are approximately equivalent, implying that the presence of the capacitor C_2 is having little effect on the overall current through the inductive coil that is inducing fields in the plasma and acting on the gas to accelerate it.

As $\mathcal{C}$ is increased, the value of dI_2^*/dt^* asymptotes to twice the value of dI^*/dt^* through the coil in the single capacitor case. The reason for this can best be understood by focusing on the first current loop (I_1) in Fig. 1C. If there is no resistance in the loop, when $C_1=C_2$ ($\mathcal{C} = 1$), the charge from C_1 transfers completely to C_2 , with the current I_1^* peaking when the voltage on each capacitor is equal and going to zero as the voltage on C_2 equals the original voltage on C_1 . Consequently, the voltage across the coil is still roughly the same value as it would have been if C_2 was not present. This neglects resistive losses and doesn't account for the fact that much of the charge, instead of going through the coil from C_2 , instead transfers back to C_1 robbing potential current and dI_2^*/dt^* through the coil (a situation illustrated in Figs. 2A-C.).

We can best understand the case where $C_1 \gg C_2$ by further simplifying current loop I_1^* so it consists only of C_1 , C_2 , and nL_0 . (Neglecting the current loops I_2^* and I_3^* and their associated lumped electrical elements does not detract from the reasoning that follows and serves to simplify the explanation.) Initially, the voltage on C_1 changes very little as the discharge progresses. The maximum current I_1^* , corresponding to $dI_1^*/dt^* = 0$, occurs when the voltage on C_2 is approximately equal to the initial charge voltage on C_1 . As time progresses further the voltage on C_2 continues to rise, eventually reaching a theoretical maximum value that is approximately twice the initial voltage on C_1 when the current $I_1^* = 0$ and the value of dI_1^*/dt^* is a local minimum. Translating this to the full circuit, we observe that this would imply a voltage across the coil that is approximately twice the voltage possible in the single capacitor case, yielding a commensurate doubling of dI^*/dt^* through the coil.

V. Conclusions

We have used a nondimensional pulsed inductive acceleration model to study the effects of adding a second, parallel capacitor into the circuit topology. Inserting the second capacitor into the system was motivated by limited data showing that the voltage appearing across the accelerator coil could be increased significantly over the initial charge voltage. Increasing the voltage across the coil has been shown to be important in the process of inductively ionizing the propellant. The nondimensionalization of the modified model led to identification of new scaling parameters that are unique to the particular topology. The physical meanings of the new parameters and their effects on accelerator performance were explored through a series of theoretical arguments and numerical simulations. The value of current rise rate through the coil is used as a proxy measure for the voltage across the coil, with higher values of dI/dt potentially yielding better inductive propellant ionization. The following insights were gained in this study.

- As in the single capacitor case, there exists a value of the dynamic impedance parameter α for which thrust efficiency is maximized. This optimum corresponds to a matching of the driving circuits natural oscillation time scale to the residence time scale of the current sheet in the acceleration zone.
- The efficiency and maximum dI_2^*/dt^* through the coil suffer for lower values of capacitance ratio $\mathcal{C}$, with much of the charge transferring back and forth from C_1 to C_2 instead of going through the coil and performing work on the propellant.
- The efficiency of the two-capacitor system approaches the theoretical efficiency of the single capacitor topology for $\mathcal{C} \gg 1$, with the commensurate value of dI_2^*/dt^* asymptoting to a value twice as large as that possible in the single capacitor case.
- As $\mathcal{C}$ is increased, the gross behavior of the current waveforms for I_1^* and I_2^* match. The differences in these waveforms consists of a high-frequency modulation overlayed on the gross waveform. For a given

C the modulation is a function of n , which represents the position of C_2 relative to C_1 and the coil. The value of n greatly affects the level of modulation on I_1^* but only slightly affects I_2^* , implying very little change in the acceleration process as a function of n .

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