

Hall Thruster small scale plasma fluctuations : Qualifying 2D PIC Simulations against Collective Scattering Experimental Data

IEPC-2011-208

*Presented at the 32nd International Electric Propulsion Conference,
Wiesbaden, Germany
September 11–15, 2011*

Cyrille Honoré, * Sedina Tsikata [†] and Dominique Grésillon [‡]
Laboratoire de Physique des Plasmas CNRS-UPMC, École Polytechnique, Palaiseau France

Anne Héron [§]
Centre de Physique Théorique, École Polytechnique, Palaiseau, France

Jordan Cavalier [¶] and N. Lemoine ^{||}
Institut Jean Lamour, Université Henri Poincaré, Nancy, France

Hall thruster performance improvement requires a good understanding of electronic transport through the ion acceleration zone. In this zone, the electron mobility in the axial direction is higher than predicted by collisional models. Small-scale electronic instabilities could be a good candidate for explaining this anomalous transport. The collective Thomson scattering diagnostic allows direct plasma electron density fluctuation observation, at a fixed wave vector. Collective scattering measurements performed in front of a Hall thruster have shown the presence of small-scale millimetric fluctuation modes in the near-azimuthal (ExB drift direction) as foreseen by linear models. This small-scale azimuthal mode has a small axial inward component. The mode main orientation changes as a function of the distance between the thruster and the scattering volume. 2D PIC simulations (with axial and azimuthal directions) show a small a similar variation of the mode orientation, and also that this density fluctuation mode seems to be self-correlated for a long distance along the axial direction. The orientation variation of this mode in the axial direction could be linked to the ion flow acceleration in the thruster exit zone.

Nomenclature

$n(\vec{r}, t)$	= electron density
n_0	= mean density
$\vec{k}, \lambda$	= scattering wave vector and wavelength
V	= scattering volume
w	= laser Gaussian beam waist
θ	= scattering angle
α	= scattering wave vector orientation
$S(\vec{k})$	= static form factor
x, y	= thruster axial, azimuthal (ExB) coordinate
$S_{PIC}(\vec{k})$	= PIC simulation static form factor
α_{PIC}	= scattering wave vector orientation for PIC simulation geometry

*Research Engineer, LPP, cyrille.honore@polytechnique.edu

[†]CNES Postdoc, LPP, sedina.tsikata@polytechnique.edu

[‡]Senior Researcher, LPP, dominique.gresillon@polytechnique.fr

[§]Researcher, CPHT, Anne.Heron@cph.t.polytechnique.fr

[¶]Ph.D. student, IJL, jordan.cavalier@ijl.nancy-universite.fr

^{||}Assistant Professor, IJL, nicolas.lemoine@ijl.nancy-universite.fr

Introduction

The collective Thomson scattering diagnostic allows direct plasma electron density fluctuation observation, at a fixed wave vector. Collective scattering measurements performed in front of a Hall thruster¹ have shown the presence of millimetric fluctuation modes implicated in transport in the azimuthal, ExB drift direction, as foreseen by linear models.^{2,3} This instability is a drift instability, arising due to the velocity difference between the confined ExB drifted electrons and the axially accelerated ions.

The small-scale azimuthal mode seen by the collective scattering diagnostic is not purely azimuthal, but has a small axial inward component.⁴ This azimuthal mode main orientation changes with the distance between the thruster and the scattering volume. These observations are also made in PIC simulations. It is thought that the ion acceleration may contribute to the observed change in directivity.

In the first part of this paper, the collective scattering diagnostic is introduced, and experimental results concerning the azimuthal mode behavior along the thruster axis presented. In the second part 2D axial-azimuthal PIC simulation results are shown and compared to experimental observations.

I. Collective Scattering (CS) observations

The collective scattering is a non perturbative diagnostic used to observe small scale fluctuations in plasmas.⁵ It measures electron density fluctuation at a scattering wave vector. The scattered electric field is proportional to the electron density $n(\vec{r}, t)$ spatial Fourier transform at the scattering wave vector, $\vec{k}$, inside the scattering volume V :

$$n(\vec{k}, t) = \int_V n(\vec{r}, t) e^{-i\vec{k} \cdot \vec{r}} d^3\vec{r} \quad (1)$$

This scattering wave vector is determined through the Bragg relation by the laser's initial mono-mode wave vector $\vec{k}_i$ and the scattering angle θ . Fluctuation intensity is evaluated by the static form factor, defined from the scattering signal mean quadratic value (n_0 is the volume mean electron density) :

$$S(\vec{k}) = \frac{\langle |n(\vec{k}, t)|^2 \rangle_t}{n_0 V} \quad (2)$$

The normalization factor $n_0 V$ corresponds to the scattering signal mean quadratic value $\langle |n(\vec{k}, t)|^2 \rangle_t$ for incoherent electron spatial distribution.

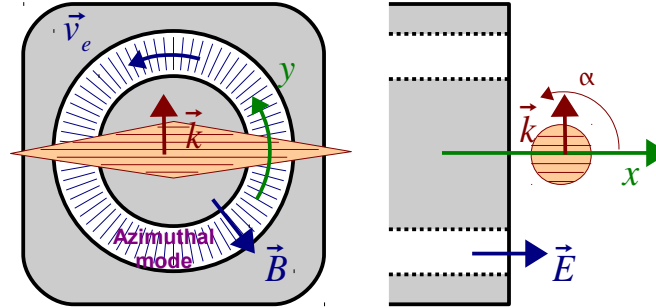


Figure 1. Collective scattering observation wave vector and volume in front of Hall thruster

In order to access millimeter scale instabilities in the low density thruster plasma, a high power CO₂ laser is used as the initial primary beam. Forward scattering is observed at small angle : θ varies from 5 to 15 mrad typically. The scattering volume is placed at least 7 mm in front of the thruster channel exit plane. Heterodyne detection is used to improve diagnostic sensitivity and to retain scattering signal phase information. The scattering volume V is defined by the crossing area of the laser primary and reference Gaussian beams. The beam waist w used here is 3.4 mm, and the common volume length depends on the θ scattering angle. Almost any scattering wave vector orientation α is accessible in the plane perpendicular to the laser beam. The collective scattering diagnostic is described more fully elsewhere.^{6,7}

Collective scattering is applied to the PPS®X000 thruster developed by Snecma inside PIVOINE facility. Figure 1 shows the typical scattering volume position, and the scattering wave vector orientation. Scattering volume covers both sides of the thruster channel, with opposite azimuthal velocities. But

former experiments have shown that the small scale azimuthal mode observed by collective scattering is moving along the ExB drift direction.⁴ Since we observe only the positive frequency modes, we select only modes present the right side of the channel.

Azimuthal mode direction variation along thruster axis

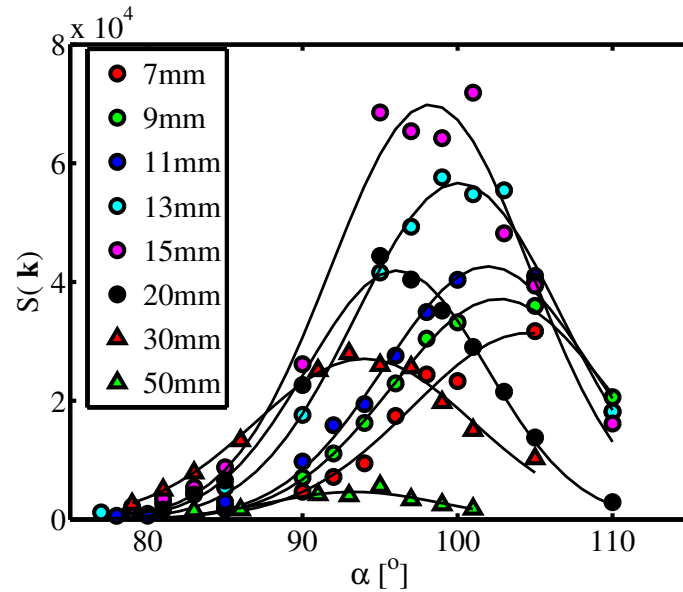


Figure 2. Collective scattering form factor variation with α scattering wave vector orientation, for different scattering volume positions x along thruster axis ($x = 0$ corresponds to the thruster exit plane position).

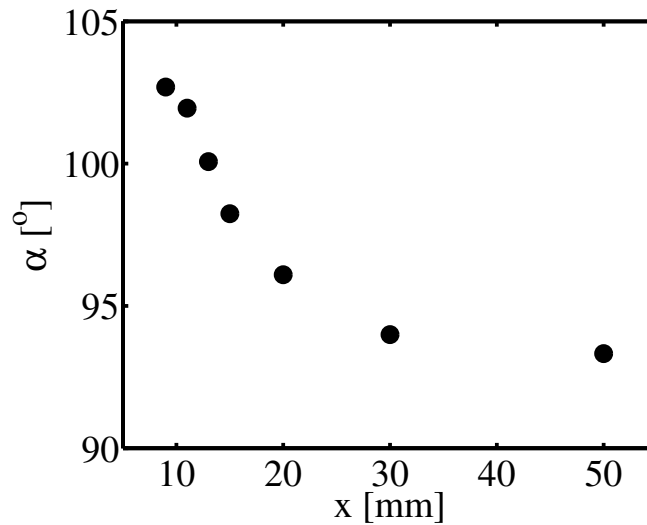


Figure 3. Main azimuthal mode α scattering wave vector orientation variation with scattering volume position x along thruster axis

First collective scattering observations in 2009 have shown the millimetric azimuthal ExB mode observed by the collective scattering has a slight inward direction. The optimum fluctuation α angle is larger than 90° .⁴ More recent observations show this orientation varies significantly when scattering volume moves along thruster axis. Figure 2 shows scattering signal form factor variations with the scattering wave vector α orientation angle and for different axial positions (scattering wave number is fixed $k = 5.8 \text{ rad mm}^{-1}$, $x = 0$ corresponds to the thruster exit plane position). Peak mean α angle variation with axial position is significant but smaller than each peak width. A Gaussian fit to the form factor distribution in α is used to determine accurately this mean angle. Figure 3 shows the fitted mean α variation with the axial x position. We observe that the fluctuation becomes less oblique for further axial positions.

Because the azimuthal mode appears to be convected by ions, the azimuthal mode main orientation could be close to the ratio of the azimuthal mode phase velocity to the ion velocity, for each axial position. The ion velocity can be deduced from α angle dependency of the axial fluctuation mode Doppler frequency (also observed by the collective light scattering).⁸ The azimuthal mode phase velocity can be deduced from the azimuthal fluctuation mode frequency.⁹

In the present case, The azimuthal mode velocity is close to 3.0 km/s for $x = 7$ mm and is slowly decreasing when the scattering volume is getting further from the thruster exit plane, down to 2.2 km/s for $x = 50$ mm. Unfortunately the α angle dependency of the axial fluctuation mode frequency is presently unknown. We can only estimate that the ion velocity is on the order of 15 km/s, and is almost constant along the thruster axis. Both azimuthal and axial observations are consistent with the observation that the azimuthal mode orientation angle α decreases with distance from the exit plane.

Mode form factor variation with thruster axis position

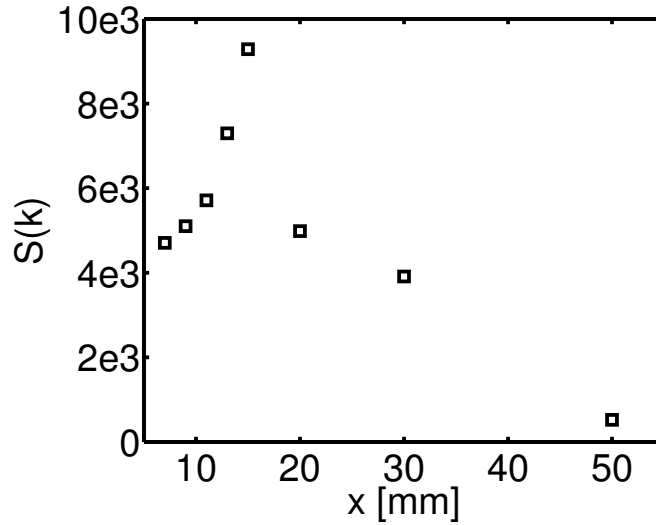


Figure 4. Collective scattering form factor, integrated over α direction angles for different scattering volume position x along thruster axis.

The azimuthal mode static form factor is not constant along the axial position. On figure 4 the azimuthal fluctuation mode form factor is integrated over α orientations for each axial position. The fluctuation form factor increases almost exponentially until $x = 15$ mm in front of the thruster exit plane. The form factor is doubled every 8 mm. The fluctuation form factor then decreases immediately at the same rate.

II. Axial and azimuthal 2D PIC simulations

In order to estimate how the drift instability can increase the small scale density fluctuations inside the thruster, axial (x) and azimuthal (y) 2D PIC simulations were performed.³ In these simulations, the magnetic field is imposed. It is perpendicular to the simulation plane and varies along the x axis, with a maximum value of 170 G at $x = 0$ mm. The electric acceleration is supplied by an imposed 300 V potential drop between the left ($x = -25$ mm) and right ($x = 15$ mm) borders. Primary electrons flow from the right border. Ions and secondary electrons are created by collisions between the primary electrons and the xenon neutral gas, treated as a fluid injected from the left border.

Typical electron density map

A typical electron density 2D map for a simulation snapshot time ($340\mu\text{s}$) is shown in figure 5. For $x \leq -8$ mm, the density increases from $10 \times 10^{17} \text{ m}^{-3}$ to $18 \times 10^{17} \text{ m}^{-3}$ mainly because of the ionization. For $x \geq -8$ mm, the electron density decreases because ions accelerate (while the ion flow rate $n_e v_i$ is maintained).

Density map shows millimetric fluctuation structures mainly in the azimuthal direction. These structures seem of small size and are quite oblique in the ionization zone. The structures have a larger

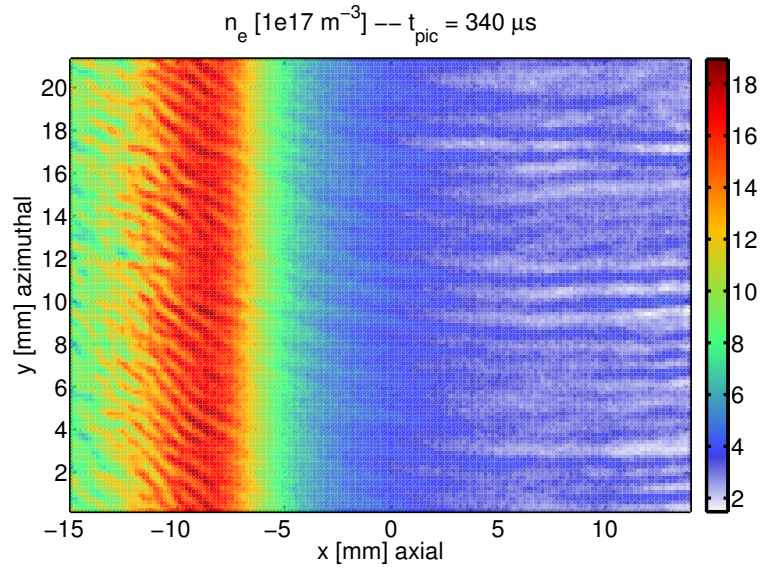


Figure 5. PIC simulation electron density map at snapshot time $340\mu\text{s}$

length scale in the acceleration zone. Their orientation changes from oblique to parallel to the azimuthal direction as ions accelerate.

We observe these structures have long correlation length along the axial direction in the acceleration zone. They are coherent all along this zone. Structures appear to be convected by the ion flow.

Density fluctuation convection by ions

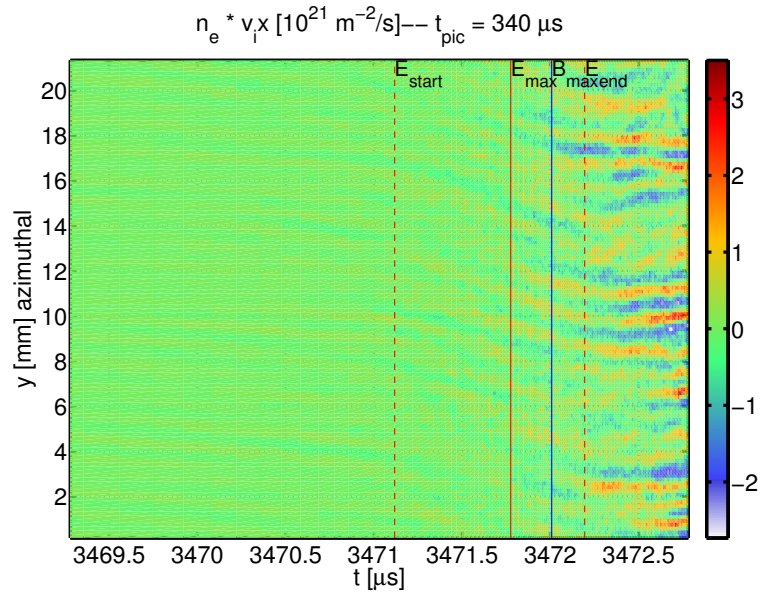


Figure 6. PIC simulation electron density map converted to a density flow representation. Axial coordinate is replaced with time by using ion convection property.

In order to check whether the changing structure orientation could be linked to ion acceleration, the axial position is converted to time, using the ion convection velocity. Density is replaced by density flow rate $n_e v_i$. Figure 6 shows that inside the acceleration zone, structure mode is oblique, with an almost constant slope. This slope corresponds to a velocity of the order of 3 km/s. This velocity is of the order as the typical azimuthal mode phase velocity observed by collective scattering at the end of the acceleration zone.

PIC electron density local Fourier transform

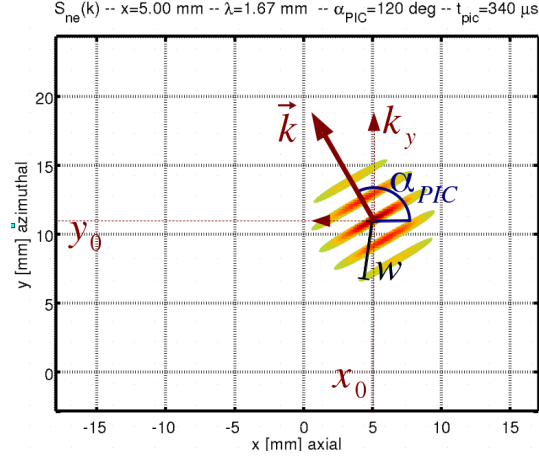


Figure 7. PIC simulation local Fourier transform geometry

In order to compare these simulations with the collective scattering observations, we have applied to these simulated electron density fields the same local spatial Fourier transform as performed by collective scattering, for each $\vec{r}_0 (x_0, y_0)$ volume position inside this plane and for a wide range of 2D wave vectors $\vec{k}$. The local volume is described with a Gaussian profile with a fixed w size, corresponding to the scattering laser Gaussian beam waist.

$$S_{PIC}(\vec{k}, \vec{r}_0, t) = \left| \int n(\vec{r}, t) e^{-i\vec{k} \cdot \vec{r}} e^{-2(\vec{r} - \vec{r}_0)^2 / w^2} d^2\vec{r} \right|^2 \quad (3)$$

Since particle number is lower in PIC simulations than in experiments, PIC form factor is not normalized as it is for experiments. Simulation and experimental Form factor levels can not be directly compared.

In order to reduce the numeric noise, and because the PIC simulation parameters are uniform along the azimuthal direction, the simulation form factor is averaged along the y direction.

The Fourier transform geometry is shown on figure 7. Because the magnetic field is not oriented the same way as for scattering experiments, the relation between both wave vector orientation is given by $\alpha_{PIC} = 180^\circ - \alpha$.

PIC electron density fluctuation structures

Figure 8 shows the form factor variation with the wave vector k_x and k_y components, for 4 different axial positions. For PIC simulations, the electron ExB drift velocity is in the direction of decreasing y . Drift millimetric instabilities are moving in the same decreasing y direction. Millimetric azimuthal modes for which k_x has the same sign as k_y ($\alpha_{PIC} < 90^\circ$) have a thruster inward direction (as observed for the collective scattering azimuthal mode).

The first position ($x = -11$ mm) (figure 8, first plot) corresponds to the ionization zone. We observe that the form factor is large for a large area of k values above 6.3 rad mm^{-1} ($\lambda \leq 1 \text{ mm}$) and for an oblique k direction around $\alpha_{PIC} = 60^\circ$. These modes are observed for x positions inside the ionization zone, and will disappear in the acceleration zone. On the figure 8, second plot ($x = -7$ mm) (beginning of the acceleration zone) appears another mode with a smaller k value ($k \sim 5 \text{ rad mm}^{-1}$, $\lambda \geq 1 \text{ mm}$). This new millimeter-scale mode is observed along the acceleration zone with a varying $\vec{k}$ wave vector (figure 8, third plot $x = -1$ mm). The azimuthal component k_y is constant, but the axial component k_x decreases as x grows. This effect on k_x is coherent with ion acceleration. A secondary mode appears with the same k_x value but larger k_y , around 5.2 rad mm^{-1} , moving the same direction. We observe both modes on figure 8, last plot after the end of the acceleration zone ($x = 10$ mm). The k_x component becomes slightly negative. For this position, the fluctuation mode is orientated slightly outward.

Ion density maps show almost the same behavior as electron density maps.

PIC electron density mode direction variation with axial position

As observed with collective scattering data, the PIC millimetric mode inside the acceleration zone is not purely azimuthal, but has a small axial component. The mode is oriented towards the thruster, except

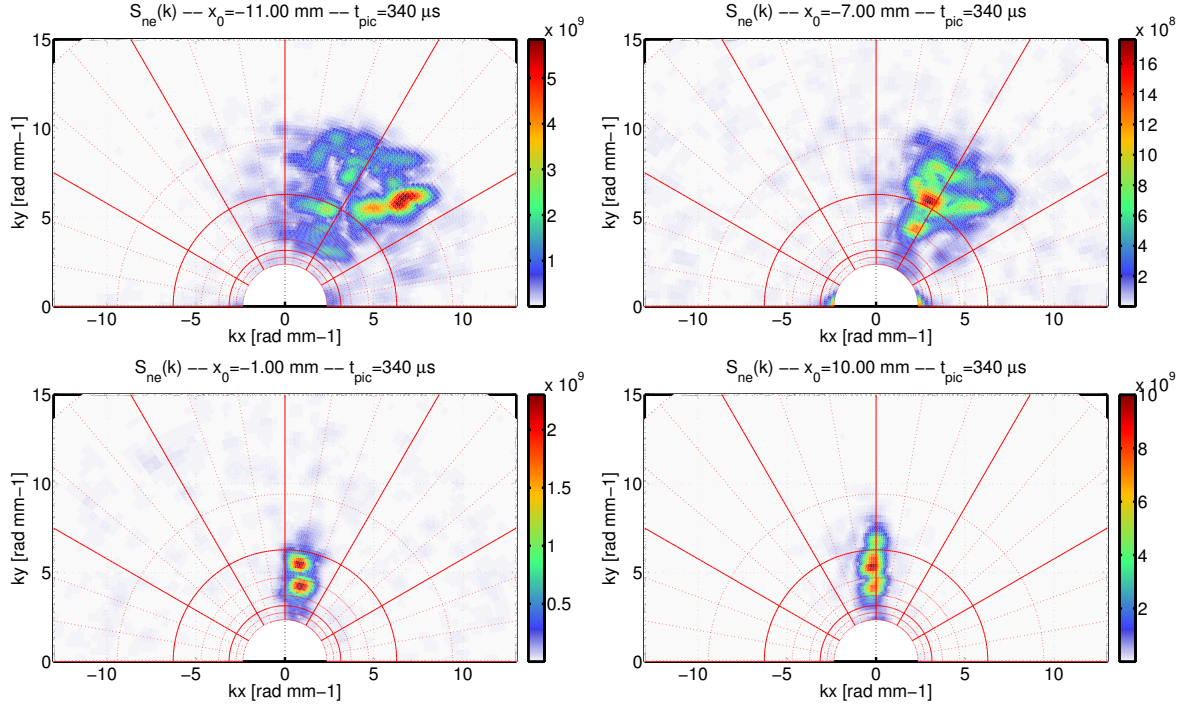


Figure 8. PIC electron density spatial spectra for different axial positions : $x = -11$ mm, $x = -7$ mm, $x = -1$ mm and $x = 10$ mm

for positions after the end of the acceleration zone (for $x \geq 4$ mm).

For PIC simulations, the mode orientation also becomes more azimuthal along the thruster axis. But the reason here is different : the azimuthal velocity seems to stay rather constant, but the ion velocity is increasing greatly in the acceleration zone.

PIC electron density mode direction variation with axial position

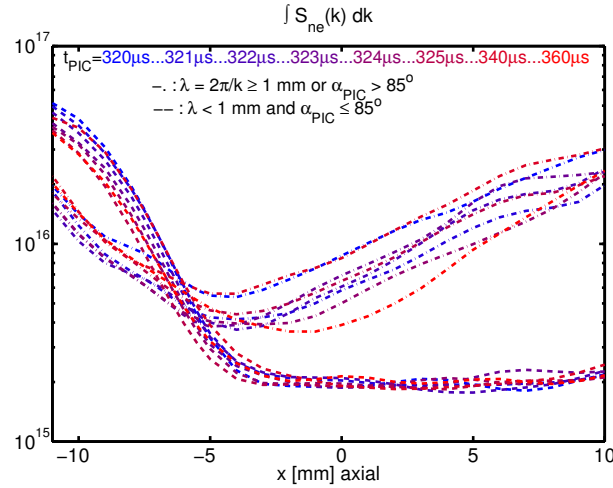


Figure 9. PIC form factor is integrated over $\vec{k}$, in 2 complementary k-space areas, for each axial position

This form factor is integrated over $veck$ in order to estimate the mode form factor variation with x . The k-space is divided in two parts. The first part corresponds to the area where the millimeter oblique mode is observed in the acceleration zone ($\lambda \geq 1$ mm or $\alpha_{PIC} \geq 85^\circ$). The second part corresponds mainly to ionization structures wave vector area ($\lambda \leq 1$ mm and $\alpha_{PIC} \leq 85^\circ$). Figure 9 shows results for different simulation snapshot times (320 to 360 μ s).

The millimetric azimuthal mode form factor decreases in the ionization zone. The millimeter mode grows exponentially along, and even beyond the end of the acceleration zone. This millimeter mode

intensity doubles each 6 mm axially in the acceleration. This value is of the same order as the collective scattering form factor growth rate in the same region.

PIC potential fluctuation structure

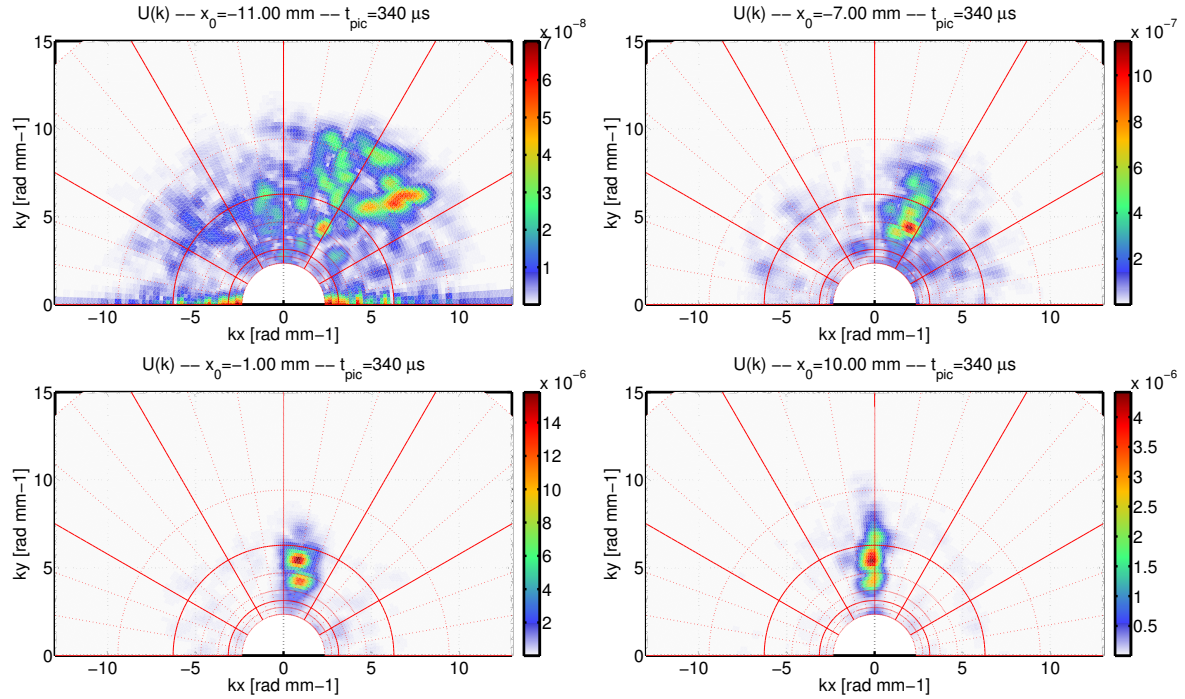


Figure 10. PIC potential spatial spectra for different axial positions : $x = -11$ mm, $x = -7$ mm, $x = -1$ mm and $x = 10$ mm

The same local Fourier transform treatment is applied to PIC simulation potential maps. Figure 10 shows the local spatial spectra for the same axial positions as for density spectra.

For $x = -11$ mm in the ionization zone (figure 10 first plot), no clear structure appears. The spectral intensity is small compared to the following spectra. On the 3 others spatial spectra on figure 10, 3 last plots), we observe the same spectral structures as seen on the electron density spatial spectra.

PIC potential mode direction variation with axial position

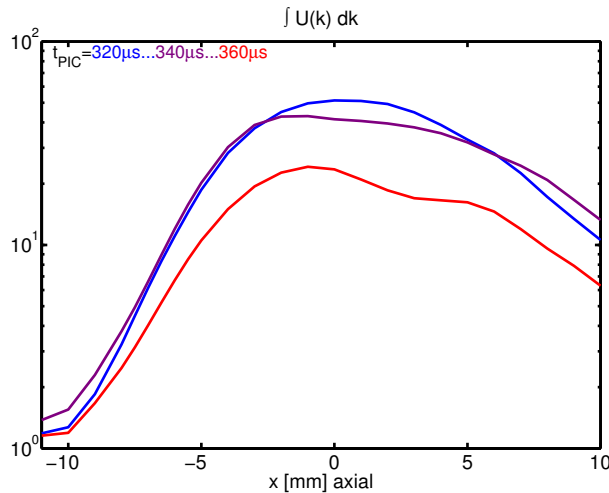


Figure 11. PIC local Fourier transform intensity integrated over k-space, for each axial position

Potential local Fourier transform intensity is also integrated over k-space (figure 11) : if the potential spatial spectral structures in the acceleration zone are very close to the electron density spectral

structures, their intensities behave very differently. The potential structures increase before the electron density structures, and faster. Potential structures are present in the acceleration zone where the axial electric field is high. The evolution of density structures is slower because of ion inertia.

Conclusion

As observed with collective scattering, an azimuthal millimetric mode is present in the axial-azimuthal 2D PIC simulation. 2D PIC simulations show this mode is coherent along the thruster axis. Collective scattering experiments and PIC simulations show that the mode intensity grows in the acceleration zone (and even beyond).

Directivity analysis shows that the small scale mode seen by collective scattering is not purely azimuthal, but has a small axial inward component. 2D PIC simulations (with axial and azimuthal directions), also show an axial component in the same direction. For collective scattering and simulations, this angle gets closer to the azimuthal direction for positions further from the thruster, where ion convection velocity is larger and azimuthal mode phase velocity decreases.

Acknowledgment : This work was performed in the framework of the collaborative-research program GdR 3161 CNRS-CNES-Snecma-Universities “Propulsion par Plasma dans l’Espace”

References

- ¹S. Tsikata, N. Lemoine, V. Pisarev, and D. Grésillon, ‘Dispersion relations of electron density fluctuations in a Hall thruster plasma, observed by collective light scattering’, *Phys. Plasmas* **16** (2009), 035506.
- ²J.-C. Adam, A. Heron, and G. Laval, ‘Study of stationary plasma thrusters using two-dimensional fully kinetic simulations’, *Phys. Plasmas* **11** (2004), p 285.
- ³A. Ducrocq, J. C. Adam, A. Héron, and G. Laval, ‘High-frequency electron drift instability in the cross-field configuration of Hall thrusters’, *Phys. Plasmas* **13** (2006), 102111.
- ⁴S. Tsikata, C. Honoré, N. Lemoine and D. Grésillon, ‘Three-dimensional structure of electron density fluctuations in the Hall thruster plasma: ExB mode’, *Phys. Plasmas* **17** (2010), 112110.
- ⁵E. Holzhauser And J. H. Massig, ‘An Analysis Of Optical Mixing In Plasma Scattering Experiments’, *Plasma Physics*, **20** (1978), p. 867.
- ⁶S. Tsikata, C. Honoré 1 , D. Grésillon , and N. Lemoine, ‘*Collective Light Scattering for the Study of Fluctuations in Magnetized Plasmas: The Hall Thruster Case Study*’, *Contrib. Plasma Phys.* **51**, No. 2-3 (2011), p119.
- ⁷S. Tsikata, ‘Small-Scale Electron Density Fluctuations In The Hall Thruster, Investigated By Collective Light Scattering’, Ph D Thesis, Ecole Polytechnique (2009).
- ⁸S. Tsikata, C. Honoré and D. Grésillon, N. Lemoine and J. Cavalier, ‘Observation of ion beam properties in the Hall thruster plasma using an axial electron density fluctuation mode’, *Proceedings of the 32nd International Electric Propulsion Conference*, Wiesbaden, Germany (2011), IEPC-2011-225
- ⁹J. Cavalier, N. Lemoine, G. Bonhomme, S. Tsikata, C. Honoré and D. Grésillon, ‘Estimation of the electron temperature and density in the plasma of a Hall thruster’, *Proceedings of the 32nd International Electric Propulsion Conference*, Wiesbaden, Germany (2011), IEPC-2011-337