Study of stochastic effects in a Hall effect thruster using a test particles Monte-Carlo model

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Abstract: For several years now, it has been shown that classical, collisional conductivity is not sufficient to explain the measured Hall thruster parameter. Adam *et al*. using a 2D (axial-azimuthal) Particle-in-cell (PIC) have demonstrated the possible role of fluctuations on electron transport. Previous studies of the interaction of a lower hybrid wave with an ion have revealed, under some conditions the enhanced heating of the particle. The present study analyzes the role of the force fields obtained from the PIC code, using a test particles Monte-Carlo model. For a quantitative description, we establish and use the mobility from the drift diffusion approximation. We show also that by replacing the complex fluctuating field by a wave with a single mode, the obtained mobility is too low to match the real current.

I. Introduction

Hall effect thrusters (HET) are ion sources used for space propulsion [1], [2]. A plasma is generated in a channel between two coaxial cylinders. Electrons are emitted by an external hollow cathode near the exhaust plane. The gas feeding anode is located at the other end of the cylinders. Their main characteristic is the use of a magnetic field which has two purposes. It traps the electrons and allow for a longer residence time in order to increase the ionization rate. The trapping greatly reduces the conductivity and consequently induces an electric field to ensure the current continuity. This field gives enough energy to accelerate the ions and to provide the thrust. The magnetic and the accelerating electric field together create a current in the azimuthal direction. This is the so-called Hall current.

The understanding of the de-trapping process is one of the major issue of this device. In the strong magnetic field region such as the exhaust region of a HET, the cross field electron mobility is proportional to the momentum-transfer frequency of the electron-neutral collisions. However, in a HET, the neutral atom density decreases strongly in the exhaust region. This results in a very low cross field mobility, insufficient to explain the experimental current. This phenomenon is usually called the "anomalous electron transport," because we need to consider others processes to explain the transport of electrons from the cathode to the anode [3-5]. This also point out the lack of theory on this subject. It was first attributed to electron-wall interactions [6-7] but it now seems that plasma turbulence may also

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play a major role in this anomalous transport. Experimental evidence of fluctuating signals in the azimuthal direction has been observed through antenna [8], probes [9-11] and Thomson scattering [12], [13] diagnostics.

In order to understand the electron transport in a HET, a Particle-In-Cell (PIC) model has been developed by Adam *et al.* [14]. This model will be referred as the PIC model in the rest of the paper. This model describes a flattened sector of cylinder in the azimuthal (θ is y) and axial directions (x). The axial boundaries in the axial direction are the anode and the cathode of a thruster. The azimuthal boundaries are periodic. The magnetic field is in the z direction and is imposed. The model is self-consistent and does not include any theory on wall interactions. Nevertheless, this model has successfully represented the physics of a HET, reproducing experimental measurements of some of the thruster characteristics such as the current, the time varying behaviour, the electric field profile or the plasma density [14]. We have reported on Figure 1 both the electron and collisional (electron-neutral) mobility calculated from the PIC model and averaged on several current oscillations [21]. The electron mobility is calculated from the momentum equation using the drift-diffusion approximation and reads :

$$\mu = -\mathbf{v}_e \Big/ \left(E_x + \frac{1}{n_e}\frac{\partial(n_e T_e)}{\partial x} \right) \tag{1}$$

In figure 1, we can see in the region of low neutral density (and consequently low collisional transport) the huge discrepancy between the collisional and the electron mobility. As expected, the profile of the collisional mobility follows the evolution of the neutral density. However, around the maximum of magnetic field (the middle of the canal, see Fig. 5), the electron mobility increases up to two orders of magnitude. Therefore, a physical mechanism must exist beyond the elastic collisions to explain such mobility.

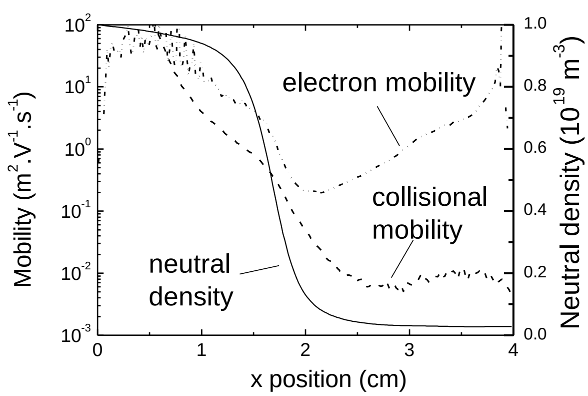

Figure 1: Electron and collisional mobility profiles and neutral density profile calculated with the PIC model.

In Figure 2, we have plotted the spatial profile of the azimuthal electric field. It reveals the existence of organised micro-structures whose amplitudes are maximals in the region of low neutral density. It appears to be of great interest to investigate the interaction between these structures and the electrons. Karney *et al.* [17] carried out theoretical studies of single electrostatic wave-ion interaction. They show that in the presence of a lower hybrid wave, ions can experience strong acceleration if their initial velocities lie in a resonant band. In the context of HET, we will show that in the case of a single wave, some energization is indeed possible and enhance the collisional mobility but is not sufficient to match the electron mobility.

In section II, the MC model will be described. In section III we will present the results of the model. The theory developed by Karney will be analysed in the ExB field configuration of a HET in section IV. The MC results will be compared with the results of Karney theory in section V. We finally conclude in section VI.

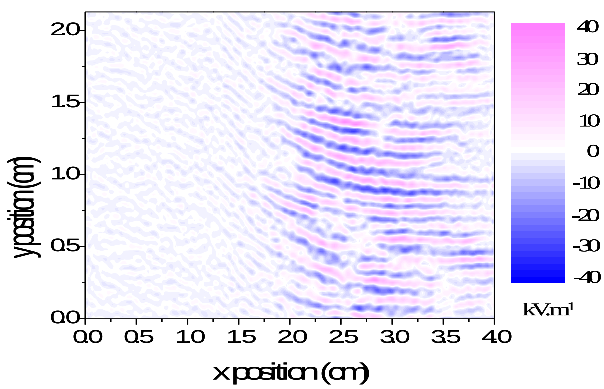

Figure 2: Two-dimensional spatial profile of the azimuthal electric field calculated with the PIC model.

II. Monte-Carlo model description

The PIC simulations reveal a time varying behaviour of the thruster as observed experimentally [14]. The MC particle test model we have developed is stationary, the time behaviour being not taken into account. We want to simplify the physical phenomena involved in the transport in order to analyze the micro-turbulence effects more easily. All the others ingredients of the PIC simulations are kept.

The total dimension of the cylinder in the azimuthal direction is not fully described in the PIC model to reduce the computation time. The size of the domain is x_{max} = 4 cm and y_{max} = 2.13 cm. The number of grid points is 240 and 128 in *x*- and *y*-directions. Primary electrons are injected one by one at *x* = 4 cm assuming a Maxwellian distribution with a temperature of 2 eV. Electron trajectories are integrated using classical Leap-frog scheme [18], the z direction is obviously ignored. As in the PIC model, only elastic and ionization collisions are taken into account using the same cross sections. After an ionizing event, the primary electron loses a fraction of energy corresponding to the ionization threshold and the secondary electron is created at rest (zero energy). One thousand primary electrons are injected at the cathode plane. The calculation stops when all the electrons (primary and secondary) cross the *x* = 0 plane. Boundaries in the y-direction are periodicals so that one electron that crosses the *y* = 2.13 cm (respectively, *y* = 0) plane is re-injected at *y* = 0 (respectively, *y* = 2.13 cm).

As we mentioned in section I, our model is not self-consistent: we used data computed from the PIC code. These data have been extracted at one arbitrary moment during the simulation. The input data of the MC model are profile of the density of neutrals, profile of the magnetic field which is only x-direction dependant, and the spatial profiles of the electric field in *x*- and *y*-directions (Figure 2 shows the y-direction of the electric field.)

III. MC model results

The profile of the electric potential has been calculated from the PIC simulation. We report in Figs. 2 and 3 the two dimensional profiles of the azimuthal and axial electric field. The profiles of neutral density and magnetic field are presented in Figs. 1 and 5, respectively.

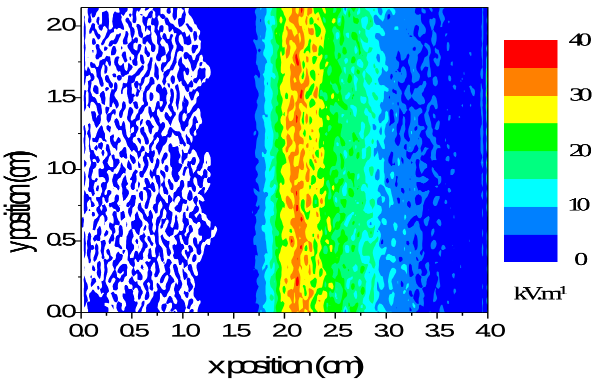

Figure 3: Two-dimensional spatial profile of the axial electric field calculated with the PIC model.

We calculated different mobilities with the MC model. In Figure 4, we can see that the collisional mobility from the MC model matches the one from the PIC model with little difference. This is easy to understand because the collisional mobility $\mu \approx \frac{1}{B_r}\frac{v_m}{\Omega}$, depends on the collisional frequency which is related to the neutral density. The profile of the neutral density we chose at one arbitrary moment is close to the average value taken to calculate the collisional mobility (in dashed line in Figure 4). Moreover, during the transit time of the electrons in the PIC model, we can neglect the time evolution of the xenon density. Then, using a fixed neutral density profile does not affect much the mobility.

The most important result is that the other mobility, the electron mobility profile calculated with the PIC code is well reproduced by the simple MC model. The unresolved time behaviour of the instability seems to be of relatively small importance for the transport of electrons since the electric field is frozen. This indicates that most of the physic responsible for the electron transport is incorporated in the two dimensional micro-structures of the azimuthal fluctuating electric field. However, the difference between the PIC and MC mobilities is much more larger than for collisional mobility. This is again likely due to the time evolution of the azimuthal electric field during the transit of the electrons in the PIC model.

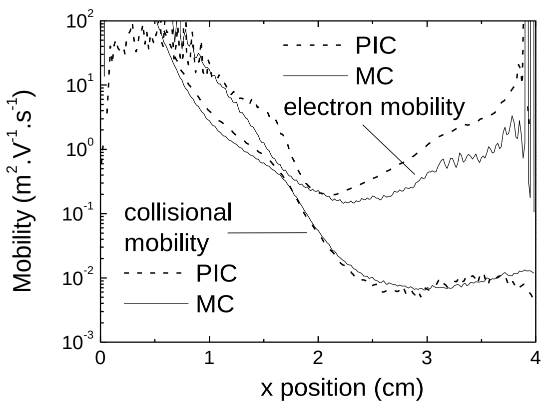

Figure 4: Electron and collisional mobility profiles calculated with the PIC and MC models.

IV. Description of the Karney model

Karney studied the motion of an ion interacting with a constant magnetic field and a perpendicular propagating single electrostatic wave [15-18]. Under certain conditions, the ion can gain net energy from the wave. This gain occurs during a resonant interaction with the wave and the ion motion becomes stochastic. Karney establishes two conditions which must be fulfilled by the ion and the wave. In the configuration of a HET, Ducrocq *et al.* applied Karney's work to derive the necessary two conditions for the electron to reach a stochastic regime [19].

An important requirement is needed: the wave frequency must be bigger than the gyro-frequency. During one cyclotron period, the electron undergoes many interactions. Thus, care must be taken to check the real wave frequency the electron "see" in its referential. In the thruster, the electrons drift in the azimuthal direction because of the combination of the E and B fields. This force introduces a Doppler shift $v_d = E_0/B$, E_0 being the accelerating electric field. Then the frequency of the wave in the electron referential is $\omega_d = \omega - k_y v_d$, where k_y is the wave number. The frequency of the wave is of the order of $10^6 s^{-1}$, the most visible mode of the wave is $k_y \approx 5600\ \mathrm{rad.m^{-1}}$ and the drift velocity is around $10^6 m.s^{-1}$. This gives us a frequency in the electron referential of $5.6.10^9 s^{-1}$ bigger than the maximum gyro-frequency $3.10^9 s^{-1}$.

From this change of referential, Drucocq *et al.* established that the electron should have a perpendicular velocity that verifies $v_\perp \geq -V_d - \Delta v$ to undergo a resonant interaction. Δv is the trapping velocity under which the electron is trapped by the wave. Moreover, once this condition is fulfilled, the amplitude of the fluctuating field E_f must equal $E_f \approx 1/4(\Omega/\mathrm{k_y v_d})^{1/3} E_0$ to reach the stochastic regime. Ω is the gyrofrequency.

When the electron has a resonant interaction with the wave with sufficient amplitude, a big amount of energy is transferred to the electron. The latter is liberated from its cyclotron trajectory and starts drifting stochastically. The axial electric field E_0 imposes the direction of the drift and pulls the electron toward the anode [20].

V. Confrontation between the Karney theory and the MC results

Now we want to calculate the mobility in the case on a single electrostatic wave. We replace the fluctuating field by an imposed wave. Then the new electric field has the same amplitude than the previous one (PIC code) but has only one mode. As before, we do not take into account the temporal dimension of the field. It writes as:

$$E_y(x,y) = E_{y0}(x)\cos(k_y y) \qquad (2),$$

where the amplitude of the fluctuating field $E_{y0}(x) = \sqrt{2\left\langle E_y(x,y)^2 \right\rangle_y}$. $\langle \ldots \rangle_y$ indicates that the square of the amplitude is averaged along the y direction. The input axial accelerating field thus writes :

$$E_x(x,y) = E_{x0}(x) + \frac{1}{k_y}\frac{\partial E_{y0}(x)}{\partial x}\sin(k_y y) \qquad (3),$$

the amplitude of the accelerating field is calculated as $E_{y0}(x) = \langle E_x(x,y) \rangle_y$. The second term in eq. (3) is added to preserve $\nabla \times E = 0$. The profile of the amplitude of the electric field components are plotted in the Fig. 5.

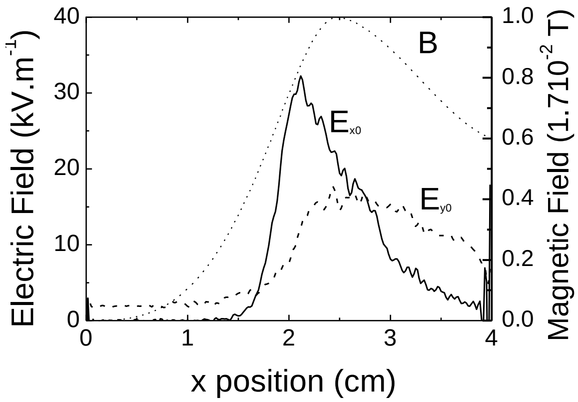

Figure 5: Profiles of the amplitude of the accelerating and fluctuating fields, and magnetic field magnitude used in the MC calculations.

We perform a parametric study: we change the wave number k_y and calculate the new mobility. Nevertheless, we are limited by the length of the domain and the number of grid points in the y-direction. k_y writes as $k_y = m(2\pi / y_{long})$, where y_{long} is the length in the y-direction and m is an integer. Previously, we said from Fig. 2 that the clearest mode was $k_y \approx 5600\ \text{rad.m}^{-1}$. Actually, this is the smallest k-mode that can fulfill the condition $\omega_d = \omega - k_y v_d > \Omega$. It corresponds to a m-number equals to 19. The upper bound is m=64 as we have 128 points in the y-direction.

In Fig. 6, we plot the mobility from the PIC code and several different mobilities calculated from the single electrostatic wave with different wave numbers. As we can see, it is not possible to recover the same mobility with only one mode. Even if some energy transfer occurs between the wave and the particle in the second half of the channel, its amount remains limited. We conclude that the theory developed by Karney can not explain such an energy transfer we obtain in the Hall thruster.

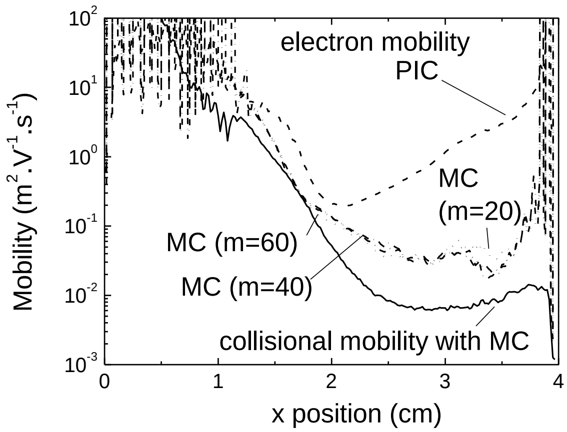

Figure 6: Electron mobility profiles calculated with the PIC and MC models for different single mode waves. The collisional mobility profile is also plotted.

VI. Conclusion

We have developed a Monte-Carlo model using 2D profiles of fields and neutral density of an existing PIC code as input. Our aim was to study the interaction of electrons with the field. This allowed us to get rid of the time-consuming calculation of fields quantities to focus on the motion of electrons. Even if the electric field was frozen, we have been able to recover mobilities relatively close to the PIC code. The discrepancy has been attributed to the unresolved time-component of the field. We have also study the interaction of an electron with a single electrostatic wave in the frame on a theory developed by Karney. Despite a proven transfer of energy to the electron, a single mode wave is actually insufficient to energize the electron. The mobilities are too low compared to the real ones. We may invoke other processes such as multi-mode electrostatic wave [22] and especially, two-mode electrostatic wave, which have theoretically proved to be a more efficient way to energize particles [23].

Acknowledgments

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