

Gradient instabilities in Hall thruster plasmas

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Abstract: Hall thrusters plasma is prone to a number of instabilities related to gradients of plasma density and electron temperature, gradient of magnetic field as well as the equilibrium electron flow due to the equilibrium axial electric field. The theory of the gradient-drift instability due to gradients of density and magnetic field and equilibrium flow is reviewed. Quantitative corrections to earlier theories are provided and effects of a finite electron temperature and finite gradient of the equilibrium electron temperature are considered.

Nomenclature

m_e	= electron mass
m_i	= ion mass
T_e	= electron temperature
T_i	= ion temperature
n_e	= electron concentration
n_i	= ion concentration
n_0	= equilibrium concentration
k_x	= X component of the wavevector
k_y	= Y component of the wavevector
$k_\perp$	= perpendicular component of the wavevector
c	= vacuum speed of light
u_0	= electron equilibrium velocity
v_0	= ion equilibrium velocity
v_E	= electron $\mathbf{E} \times \mathbf{B}$ drift velocity
v_p	= electron diamagnetic drift velocity
v_{Te}	= electron thermal velocity
v_{Ti}	= ion thermal velocity
c_s	= ion sound velocity
B_0	= equilibrium magnetic field
E	= electric field

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- ϕ = electrostatic potential
- q = heat flux
- p = pressure

I. Introduction

Plasmas involving strong electron drift in crossed electric and magnetic fields are of great interest for a number of applications such as space propulsion and material processing plasma sources. In these devices, the strength of the external magnetic field is chosen such that electrons are magnetized $\rho_e \ll L$, but ions are not, $\rho_i \gg L$, where L is the characteristic length scale of the plasma region in the device. Electron and ion dynamics is mostly collisionless though inter-particle collisions (including those with neutrals) as well as with the walls can also be important. These conditions are met in a variety of plasmas, which for the purposes of this paper are loosely defined as Hall plasmas. These are typical conditions for plasma Hall Thrusters,^{1,2} which are high efficiency, low thrust engines used on many missions for satellite orbit corrections and planned for future interplanetary missions. Magnetron plasma discharges, which are widely used in materials processing for sputter deposition of metallic and insulating films, are also based on the electron drift in crossed electric and magnetic fields in the presence of non-magnetized ions. Similar conditions (of magnetized electrons and non-magnetized ions) are also typical for E-layer of plasma ionosphere, magnetic reconnection in laboratory and astrophysical plasmas, fast processes in pulsed power devices, as well as dynamics of the plasma boundary sheath in controlled fusion devices and plasma immersion ion implantation.

Despite many successful applications of Hall thrusters and other Hall plasma sources, some aspects of their operations are still poorly understood. A particularly important problem is anomalous electron mobility,³⁻⁶ which greatly exceeds classical collisional values. Hall plasma devices exhibit numerous turbulent fluctuations in a wide frequency range⁷⁻⁹ and it is generally believed that fluctuations resulting from plasma instabilities is likely one of the reasons leading to anomalous mobility. The first theoretical models to describe plasma instabilities in Hall plasmas were attempted at very earlier stage in the development of Hall plasma thrusters. It was shown that the combination of the magnetic field and density gradients may lead to instability in plasma with a finite electron current due to the crossed electric and magnetic field.^{10,11} This instability involved the equilibrium electric field as the main driving mechanism. It was shown later that the inhomogeneity of the electron flow further modifies the instability leading to the Rayleigh type instability and that plasma resistivity induces resistive instabilities of low-hybrid and Alfvén waves^{12,13} as well as resistive instabilities associated with axial ion flows.¹⁴ Kinetic studies of low-hybrid instability driven by wave resonances with cyclotron harmonics of the particle Larmor rotation have also been performed.¹⁵

Inhomogeneous plasma immersed in the external electric and magnetic fields (which are also inhomogeneous) is not in the thermodynamically equilibrium state and, generally, this deviation is a source of plasma instabilities. Plasma instability due to density and magnetic field gradients in Hall thruster was discovered in earlier papers. It is well known however, that magnetic field gradient in combination with temperature gradients is responsible for another powerful instability^{16,17} which is now considered to be a principal source of anomalous transport in toroidal magnetic confinement devices.¹⁸ In previous studies, electron temperature gradients were ignored in the theory of instabilities in Hall plasmas, however the temperature gradients are significant for typical Hall thruster parameters.¹⁹

The goal of this paper is to study the instabilities due to gradients of plasma density and plasma temperature for conditions typical for Hall thruster plasmas. We generically will refer to such instabilities as gradient-drift instabilities. It is worth noting here that inhomogeneous plasmas exhibit a wide class of eigenmodes induced by inhomogeneities of plasma density and temperature, generally called drift waves and instabilities.²⁰ However, conditions of Hall thrusters, particularly, the large ion Larmor radius, make its eigenmodes quite different from standard drift waves, e.g. particularly those widely studied in applications to fusion plasmas. We revisit previous works on the instability due to density gradients and show that quantitative corrections are required for accurate determination of the conditions for the instability and its characteristics (real part of the frequency and the growth rate). The effects of electron temperature fluctuations are also considered.

The paper is organized as follows. In Section II, the instability due to density gradient is studied and comparison with previous models is given. Section III discusses the effects of the electron temperature gradients and its role in the gradient drift instabilities. The summary is given in Section IV.

II. Magnetic field and density gradients instability

The gradients of magnetic field and plasma density were earlier identified as a source of robust instability in Hall plasma with an electron drift due to the equilibrium electric field. We consider this instability in this section and show that more accurate analysis leads to the quantitatively different result as compared to the previous work, though physical mechanisms behind the instability remains similar.

We consider the simplified geometry of a coaxial Hall thruster with the equilibrium electric field $\mathbf{E}_0 = E_0 \hat{\mathbf{x}}$ in the axial direction $\mathbf{x}$ and inhomogeneous density $n = n_0(x)$, $E_{0x} > 0$. Locally, the Cartesian coordinate (x, y, z) is introduced with z direction in the radial direction and y in symmetrical azimuthal direction. The magnetic field is assumed to be predominantly in the radial direction $\mathbf{B} = B_0(x) \hat{\mathbf{z}}$, where $\hat{\mathbf{z}}$ is a unit vector in z direction.

The ions are assumed un-magnetized so the magnetic field can be omitted in the ion momentum equation

$$m_i n_i \frac{d\mathbf{v}_i}{dt} = e n_i \mathbf{E} - \nabla p_i. \quad (1)$$

We assume $n_i = n_0 + \tilde{n}_i$ and $\mathbf{v}_i = \mathbf{v}_0 + \tilde{\mathbf{v}}_i$, and look for the solution in Fourier form $\sim e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)}$. The latter requires the Boussinesque quasi-classical approximation $k_x L_x \gg 1$, where $\mathbf{k} = (k_x, k_y, 0)$ is the wave-vector. Considering only electrostatic perturbations and isothermal ions one finds

$$\frac{\tilde{n}_i}{n_0} = \frac{e}{m_i} \frac{k_\perp^2 \phi}{(\omega - k_x v_0)^2 - k_\perp^2 v_{Ti}^2 / 2}. \quad (2)$$

Here $v_{Ti}^2 = 2T_i/m_i$, and $k_\perp^2 = k_x^2 + k_y^2$. The second term in the denominator of (2) is responsible for ion sound effect and Landau wave resonance. The fluid theory is only justified in non-resonant limit, $(\omega - k_x v_0)^2 \gg k_\perp^2 v_{Ti}^2$, so that the simplified limit will be used in the form

$$\frac{\tilde{n}_i}{n_0} = \frac{e}{m_i} \frac{k_\perp^2 \phi}{(\omega - k_x v_0)^2}. \quad (3)$$

This is a standard expression for ion density for un-magnetized ions as used in previous works.

Fluid theory is used also for electrons. The electrons are magnetized and conditions

$$\omega \ll \omega_{ce}, \quad \rho_e \ll L \quad (4)$$

are satisfied. The electron inertia is neglected. Under these conditions the perturbed electron density is found in the form²¹

$$\frac{n_e}{n_0} = \frac{\omega_* - \omega_D}{\omega - \omega_0 - \omega_D} \frac{e\phi}{T_e}. \quad (5)$$

Here, $\omega_D = k_y v_D$, $\omega_0 = k_y u_0$ and $\omega_* = k_y v_*$, where v_D is the magnetic drift velocity,

$$v_D = -2 \frac{cT_e}{eB_0} \frac{\partial}{\partial x} \ln B,$$

v_* is the electron diamagnetic drift velocity,

$$v_* = -\frac{cT_e}{eB_0} \frac{\partial}{\partial x} \ln n_0,$$

and u_0 is the electric drift velocity in the equilibrium electric field

$$\mathbf{u}_0 = -\hat{\mathbf{y}} \frac{cE_{0x}}{B_0}. \quad (6)$$

Invoking quasineutrality, equations (3) and (5), we obtain the following dispersion relation²¹

$$\omega - k_x v_0 = \frac{1}{2} \frac{k_\perp^2 c_s^2}{\omega_* - \omega_D} \pm \frac{1}{2} \frac{k_\perp^2 c_s^2}{\omega_* - \omega_D} \sqrt{1 + 4 \frac{k_x v_0}{k_\perp^2 c_s^2} (\omega_* - \omega_D) - 4 \frac{k_y^2}{k_\perp^2} \rho_s^2 \Delta}, \quad (7)$$

The instability will occur for

$$\frac{k_y^2}{k_\perp^2} \rho_s^2 \Delta > \frac{1}{4},$$

where

$$\Delta = \frac{\partial}{\partial x} \ln \left(\frac{n_0}{B_0^2} \right) \left[\frac{eE_0}{T_e} + \frac{\partial}{\partial x} \ln (B_0^2) \right], \quad (8)$$

and $\rho_s^2 = T_e m_i c^2 / e^2 B_0^2$ is the so called ion-sound Larmor radius.

The equation (5) is similar to the electrostatic limit in Refs. [10, 11]. However these authors did not included compressibility of electron diamagnetic drift. In the result, the ω_D term in the denominator of right hand side of (7) was absent in Refs. [10, 11].

The electron diamagnetic drift was included in Ref. [22], however only part of the electron compressibility was considered. As a result, our dispersion equation (7) is identical in structure to the equation (18) in Ref. [22], but numerical factors are different.

Equation (18) in Ref. [22] has the form

$$\omega - k_x v_0 = \frac{1}{2} \frac{k_\perp^2 \omega_{ci}}{l_B^{-1} - l_n^{-1}} \pm \sqrt{\frac{k_x v_0 \omega_{ci} k_\perp^2}{k_y l_B^{-1} - l_n^{-1}} + \frac{\omega_{ci}^2 k_\perp^2}{4k_y^2 (l_B^{-1} - l_n^{-1})^2} + \frac{ek_\perp^2 (E_{x0} + 3T_{e0}/el_B)}{m_i (l_B^{-1} - l_n^{-1})}}, \quad (9)$$

This equation reduces to (7) after replacement $(l_B^{-1} - l_n^{-1})$ with $(2l_B^{-1} - l_n^{-1})$, and $(E_{x0} + 3T_{e0}/el_B)$ with $(E_{x0} + 2T_{e0}/el_B)$. As it is explained in Ref. [21], these differences occur because of incomplete account of electron flow compressibility in Refs. [11, 22].

In Refs. [11, 22], the gradient of the ratio n_0/B_0 was identified as an important parameter controlling plasma stability. Full account of plasma compressibility results in modification of this parameter to n_0/B_0^2 .²¹ Typically the electric field in the acceleration zone is large so that

$$\frac{eE_{0x}}{T_e} > \frac{\partial}{\partial x} \ln (B_0^2). \quad (10)$$

Then the condition for the instability is

$$\frac{\partial}{\partial x} \ln \left(\frac{n_0}{B_0^2} \right) > l_c^{-1}, \quad (11)$$

where the parameter l_c is defined as

$$l_c \equiv \frac{k_y^2}{k_\perp^2} \rho_s^2 \left(\frac{eE_{0x}}{T_e} + \frac{\partial}{\partial x} \ln (B_0^2) \right). \quad (12)$$

Characteristic feature of the dispersion relation (9) is weak dependence of the real part of the frequency on the value of the equilibrium electric field, which enters only via the $k_x v_0$ term. For typical parameters $\gamma > \omega_r$. For generic case $l_n \simeq l_B \simeq l_T \simeq l_\phi$, the real and imaginary parts of the frequency scale as

$$\omega_r \simeq \omega_{ci} k_y L, \quad (13)$$

and

$$\gamma \simeq k_\perp c_s \sqrt{\frac{eE_{x0}}{(l_B^{-1} - l_n^{-1})}} \simeq k_\perp c_s \sqrt{\frac{e\phi_0}{T_e}}. \quad (14)$$

For weak electric field

$$\frac{eE_{0x}}{T_e} < \frac{\partial}{\partial x} \ln (B_0^2), \quad (15)$$

the weaker instability may set in for

$$4 \frac{k_y^2}{k_\perp^2} \rho_s^2 \frac{\partial}{\partial x} \ln \left(\frac{n_0}{B_0^2} \right) \frac{\partial}{\partial x} \ln (B_0^2) > 1. \quad (16)$$

III. Electron temperature effects

The instability discovered in Refs. [11,22] and revisited in Ref. [21] is caused by unfavorable combination of plasma density and magnetic field gradients. It can be generally referred as Rayleigh-Taylor type instability. It is well known however that such instabilities can be affected by temperature gradients which were neglected in Refs. [11,22]. Temperature gradient instabilities¹⁶ are the main source of anomalous plasma transport in fusion plasmas¹⁸ and may occur both in configurations with inhomogeneous magnetic field^{23,24} as well as in the uniform field. In the latter, case unstable modes are simply destabilized ion sound waves.

When fluctuations of electron temperature are included, the electron energy balance equation is used in the form

$$\frac{3}{2} \frac{dp}{dt} + \frac{5}{2} p \nabla \cdot \mathbf{v} + \nabla \cdot \mathbf{q} = 0, \quad (17)$$

which includes the electron diamagnetic heat flux

$$\mathbf{q} = -\frac{5}{2} \frac{cp}{eB_0} \mathbf{b} \times \nabla T. \quad (18)$$

The energy equations gives approximately

$$\frac{\tilde{T}_e}{n_0} = \frac{(2\omega_D/3 - \omega_{*T})}{\omega_0} \frac{e\phi}{T_e}, \quad (19)$$

where

$$\omega_{*T} = -\frac{k_y c}{eB_0} \frac{\partial T_{e0}}{\partial x} = -\frac{k_y c T_{e0}}{eB_0 l_T}. \quad (20)$$

These expressions show that for $\omega_0 \gg (\omega_{*T}, \omega_*, \omega_D)$, the perturbations of density and electron temperature are of the same order if $l_n \simeq l_B \simeq l_T$. When the temperature is included the growth rate becomes

$$\gamma \simeq k_\perp c_s \left(\frac{(\omega_D - \omega_*)}{\omega_0} + \frac{\omega_D (\omega_* + \omega_{*T}) - 5\omega_D^2/3}{\omega_0^2} \right)^{-1/2} \quad (21)$$

Generally, one can expect that for $\omega_{*T}/\omega_0 < 1$ the effect of temperature fluctuations on the growth rate will be small unless near marginal stability boundary $\omega_* = \omega_D$.

In general case, coupled equations for density and temperature can be solved giving the following general dispersion equation of the third order in ω

$$\frac{-(\omega - \omega_0)(\omega_D - \omega_*) + \omega_D(\omega_{*T} - 7\omega_*/3) + 5\omega_D^2/3}{(\omega - \omega_0)^2 - 10\omega_D(\omega - \omega_0)/3 + \frac{5}{3}\omega_D^2} = \frac{k_\perp^2 c_s^2}{(\omega - k_x v_{0i})^2}. \quad (22)$$

One can show²¹ that near the marginal stability boundary where $\omega_* = \omega_D$ and the electric field is eliminated as a driving term, the dispersion equation (22) has another class of instabilities which have real part of the frequency and the growth rate of the order of ω_0 . These instabilities require the condition $l_B^{-1} (l_T^{-1} - 4l_B^{-1}/3) < 0$.

IV. Summary

Understanding of the turbulent electron mobility requires the detailed knowledge of the spectra of unstable modes and their saturation levels. Quantitative information about the conditions for linear instabilities and mode eigenvalues (real part of the frequencies and growth rates) is thus of interest. Earlier works in instabilities in Hall thruster plasmas revealed the plasma density and magnetic field gradients as important sources for plasma instabilities. We have revisited this problem and derived somewhat modified criterion for this instability as discussed in Section II.

Note, that gradient density/gradient magnetic field driven modes (in neglect of temperature fluctuations) are mostly aperiodic modes with $\gamma \gg \omega_r$, with the real part of the frequency which scales linearly with the equilibrium magnetic field and not depend on the equilibrium electric field. Such features may be inconsistent with experimental observations^{4,9,12} that shows inverse dependence of the frequency on the magnetic field and show increase with the increase of the electric field. Therefore it appears that a different instability mechanism may be operative in Hall thruster plasmas.

We have extended the fluid model to include the dynamics of electron temperature. The inclusion of two moments, density and temperature, provides more accurate model of the electron response. The two moment model amounts to the two-pole approximation of the exact kinetic response²⁵ Such models were shown to be successful in describing a wide class temperature gradient modes in fusion plasmas.²⁵ Our analysis shows that for Hall plasmas conditions (with un-magnetized ions) the effect of temperature fluctuations is not significant in the regime where the strong $\mathbf{E} \times \mathbf{B}$ electron drift is the main driving mechanism for the instability. It is worth noting that even in this regime the amplitude of temperature fluctuations is of the same order as density.

The effects of temperature fluctuations were studied by employing a two-moment, two-pole approximation. An interesting feature of the two moment fluid model that it results in higher order (in frequency) dispersion equation (22). Such dispersion equation opens possibility of a high frequency unstable mode with the real part of the frequency that scales as ω_0 . We should note that two pole models, such as used in our work, provide a reasonably accurate description of the exact kinetic response away from the resonances.²⁵ The possible role of resonances has to be investigated with a kinetic model that will be reported somewhere else. Another important effect which was neglected in the current studies is the role of parallel electron dynamics in the direction of the equilibrium magnetic field. The models of the electron density and electron temperature used in our and previous papers completely neglect the parallel electron motion. Such an assumption requires $\omega \gg k_{\parallel} v_{Te}$, where $k_{\parallel}$ is the wave vector in the direction of the equilibrium magnetic field and v_{Te} is the electron thermal velocity. For $\omega \ll k_{\parallel} v_{Te}$, the parallel electron streaming will thermalize the electron density perturbation making them close to adiabatic $\tilde{n}/n_0 \simeq e\phi/T_e$. Electron temperature perturbations will also modified in this regime. Thermalization of the magnetic field lines² for perturbed quantities will strongly affect the gradient instabilities mechanisms.

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