Numerical Analysis of Beam-Supported Detonation Propagating through Axisymmetric Tube

IEPC-2011-292

Presented at the 32nd International Electric Propulsion Conference,
Wiesbaden • Germany
September 11 – 15, 2011

H. Shiraishi[1]
Daido University, Nagoya, 457-8530, Japan

Abstract: Microwave-supported Plasma (MSP) is considered as the most important phenomena in a Microwave Propulsion, which is expected to be one of the most hopeful new space systems because of controllability and cleanliness come from non-combustibleness. In order to estimate the performance generated by MSP, validation and prediction by numerical simulation are very effective. Therefore, unsteady calculations of MSP are performed by modifying the programs for Laser-Supported Plasma (LSP). As a numerical method, TVD scheme taken into account of real gas effect, is applied. In this study, the geometry of detonation tube is applied as a more practical case.

Nomenclature

a_B	=	Bohr radius [5.292 × 10^{-11} m]
c	=	speed of light [2.998 × 10^{8} m/s]
$C_{p,s}$	=	specific heat at constant pressure for species *s* [J/(kg・K)]
D_s	=	effective diffusion coefficient for species *s* [m^2/s]
E	=	total energy (per unit volume) [J/m^3]
E_e	=	electronic energy (per unit volume) [J/m^3]
h	=	Planck's constant [6.626 × 10^{-34} J・s]
h_s	=	enthalpy per unit mass of species *s* [J/kg]
$h_{e,s}$	=	electronic enthalpy per unit mass of species *s* [J/kg]
I	=	laser intensity [W/m^3]
I_0	=	incident laser intensity [W/m^3]
$\hat{I}_{Ar^+}$	=	ionization potential of *Ar* [2.525 ×10^{-18}J]
k_B	=	Boltzmann constant [1.381 ×10^{-23} J/K]
k_f	=	forward reaction rate coefficient [m^3/s]
K	=	absorption coefficient [1/m]
M_s	=	molecular weight of species *s* [kg/mole]
m_s	=	mass species *s* per particle [kg/particle]
$\dot{n}_{e,s^+}$	=	number density rate of production of species *s* per unit volume by ionization [1/m^3/s]
p	=	pressure [Pa]
p_e	=	electron pressure [Pa]
p_0	=	initial pressure of cold gas [Pa]
Q_B	=	radiative energy rate by Bremsstrahlung [W/m^3]
Q_{IB}	=	radiative energy rate by Inverse Bremsstrahlung [W/m^3]
Q_T	=	relaxation term between heavy particles and electrons [W/m^3]

[1] Insert Job Title, Department Name, and email address for first author.

R	=	Rydberg constant [1.0973×10^7 m^{-1}]
R_0	=	universal gas constant [8.3145 J/mole/K]
T_0	=	initial temperature of cold gas [K]
T_h	=	heavy particle's temperature [K]
T_e	=	electron temperature [K]
u	=	velocity component in x-direction [m/s]
v	=	velocity component in r-direction [m/s]
w_s	=	mass rate of production of species s per unit volume [kg/m^3/s]
ε_i	=	i-th ionization energy per particles [J/particle]
$\nu_{e,s}$	=	effective collision frequency for electron and species s [1/s]
κ_e	=	frozen thermal conductivity for electronic energy[J/particle]
κ_{tr}	=	frozen thermal conductivity for translational energy of heavy particle [J/particle]
ρ	=	mixture of density [kg/m^3]
ρ_s	=	density of species s [kg/m^3]
ρ_0	=	initial density of cold gas [kg/m^3]
τ	=	viscous stress[N/(m・s)]

Subscripts

$(\ )_e$	=	electron
$(\ )_{ea}$	=	electron-atom collision
$(\ )_{ei}$	=	electron-ion collision
$(\ )_{h.p.}$	=	heavy particle
$(\ )_0$	=	initial cold gas

I. Introduction

Microwave Propulsion is expected to be one of the most hopeful new space systems because of controllability and cleanliness come from non-combustibleness. In order to estimate the performances generated by MSP, validation and prediction by numerical simulation are as effective as those by Laser-Supported Plasma (LSP). Therefore, applying numerical method for (LSP), which has been analyzed in a good many cases, is the most practical approach for MSP analysis.

Beam-supported Detonation (BSD), which is denominated LSD/MSD after the kind of an irradiated beam, is one type of Beam-supported Plasma (BSP) propagating with hypersonic flow. In BSD construction, exothermicity is supplied not by chemical reaction but by laser absorption. Therefore, the propellant is not required to be combustible. For example, an inert gas is available for propellant.

For MSD propagation, the dominant laser absorption mechanism is Inverse Bremsstrahlung: The laser energy is transformed into the kinetic energy of free electrons by photon absorption during collisions of electrons with ions or neutral particles, which is re-distributed among heavy particles through collisions. As a numerical method, semi-implicit Harten-Yee non-MUSCL modified-flux-type TVD scheme taken into account of real gas effects and thermal non-equilibrium. Though the ionization is an important process because Inverse Bremsstrahlung is mainly governed by electron density, ionization was considered as univalent for placing emphasis on the simplicity of the chemical reaction models.

I have suggested a numerical approach for MSD for planning Microwave Propulsion Systems by 1-dimensional model as a simple case in the previous study[1]. Though MSD propagation was able to calculate, the process has artificial points due to the simple geometry and the simple laser-irradiation. Moreover, MSD propagation is difficult to sustain because of easily exceeding the beam cut-off density in the simple geometry. Therefore the geometry of axisymmetric tube is applied as a more practical case.

II. Numerical Model and Assumption

A. Assumptions on Numerical Analysis

For the numerical analyses, the selection of laser beam and propellant is very important. Assumptions on this analysis are as follows:

(i) The plasma is electrically neutral having no charge separation.

An argon gas at room temperature is applied for propellant. Even if the gas is ionized and a plasma is generated, it has no charge separation

(ii) Chemical reactions considered are ionization, Bremsstrahlung and Inverse Bremsstrahlung.

When a laser beam is introduced into an ionized gas, the temperature rises up to several thousand degrees at least. At this temperature level, the most important laser absorption mechanism is Inverse Bremsstrahlung: The laser energy is transformed into the kinetic energy of free electrons by photon absorption during collisions of electrons with ions or neutral particles, which is re-distributed among heavy particles through collisions. In this model, ionization, Bremsstrahlung and Inverse Bremsstrahlung are the governing processes. Photo-ionization can be neglected because the photon energy is much smaller than the ionization potential of argon.

(ii) The thermal non-equilibrium is considered as 2-Temperature model.

In laser absorption, the electronic mode is firstly excited and a thermal non-equilibrium state easily arises. In this case, two-temperature model is available: The electron temperature related to electronic mode is separated from the heavy particle temperature related to the other modes.

Under these assumptions, chemical reactions are shown by

$$Ar + e^- \leftrightarrow Ar^+ + e^- + e^-, \tag{1}$$

$$Ar^+ + e^- + h\nu \rightarrow Ar^+ + e^-, \tag{2}$$

$$Ar^+ + e^- \rightarrow Ar^+ + e^- + h\nu. \tag{3}$$

The forward reaction rate is given by Nishida et al. [2]

$$k_f = 4\pi a_B^2 \bar{v}\left(\frac{R'}{k_B T_e}\right)^2 \frac{\exp(-q)}{q(q+1)}\left[\frac{1}{20+q} + \ln 1.35\left(\frac{q+1}{q}\right)\right]. \tag{4}$$

where $\bar{v} = \sqrt{\dfrac{8k_B T_e}{\pi m_e}}$, $R' = hcR$, $q = \varepsilon_1 / k_B T_e$.

B. Governing Equations

In this study, axisymmetric Navier-Stokes equations are changed into the finite-difference equations, using the TVD scheme where radiative energy transfer terms Q_{IB}, Q_B and Q_{EE} are treated implicitly. Concerning the effective diffusion coefficient, ambipolar diffusion [3] is considered.

Governing equations are as follows:

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial r} = \frac{\partial \mathbf{F}_v}{\partial x} + \frac{\partial \mathbf{G}_v}{\partial r} + \frac{\mathbf{H}}{r} + \mathbf{S}, \quad \mathbf{U} = \begin{pmatrix} \rho u \\ \rho v \\ E \\ E_e \\ \rho_i \end{pmatrix}, \quad \mathbf{F} = \begin{pmatrix} \rho u^2 + p \\ \rho uv \\ (E+p)u \\ E_e u \\ \rho_i u \end{pmatrix}, \quad \mathbf{G} = \begin{pmatrix} \rho uv \\ \rho v^2 + p \\ (E+p)v \\ E_e v \\ \rho_i v \end{pmatrix},$$

$$
\mathbf{F}_v = \begin{pmatrix} \tau_{xx} \\ \tau_{xr} \\ \tau_{xx} u + \tau_{xr} v + \kappa_{tr} \dfrac{\partial T_h}{\partial x} + \kappa_e \dfrac{\partial T_e}{\partial x} + \sum_j D_j h_j \dfrac{\partial \rho_j}{\partial x} \\ \kappa_e \dfrac{\partial T_e}{\partial x} + \sum_j D_j h_{e,j} \dfrac{\partial \rho_j}{\partial x} \\ D_i \dfrac{\partial \rho_i}{\partial x} \end{pmatrix},
$$

$$
\mathbf{G}_v = \begin{pmatrix} \tau_{rx} \\ \tau_{rr} \\ \tau_{rx} u + \tau_{rr} v + \kappa_{tr} \dfrac{\partial T_h}{\partial r} + \kappa_e \dfrac{\partial T_e}{\partial r} + \sum_j D_j h_j \dfrac{\partial \rho_j}{\partial r} \\ \kappa_e \dfrac{\partial T_e}{\partial r} + \sum_j D_j h_{e,j} \dfrac{\partial \rho_j}{\partial r} \\ D_i \dfrac{\partial \rho_i}{\partial r} \end{pmatrix},
$$

$$
\mathbf{H} = \begin{pmatrix} -\rho u v + \tau_{rx} \\ -\rho v^2 + \tau_{rr} \\ -(E+p)v + \left(\tau_{rx} u + \tau_{rr} v + \kappa_{tr} \dfrac{\partial T_h}{\partial r} + \kappa_e \dfrac{\partial T_e}{\partial r} + \sum_j D_j h_j \dfrac{\partial \rho_j}{\partial r} \right) \\ \kappa_e \dfrac{\partial T_e}{\partial r} + \sum_j D_j h_{e,j} \dfrac{\partial \rho_j}{\partial r} \\ -\rho_i v + D_i \dfrac{\partial \rho_i}{\partial r} \end{pmatrix}, \quad \mathbf{S} = \begin{pmatrix} 0 \\ 0 \\ Q_{IB} - Q_B \\ Q_{EE} + Q_{IB} - Q_B \\ w_i \end{pmatrix} \tag{5}
$$

where i indicates Ar, e^- or Ar^+ in the case argon gas is applied to propellant, so the total number of conservation is seven.

In the last term,

$$
Q_{EE} = Q_T - p_e\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial r} + \frac{v}{r}\right) + \frac{E_e + p_e}{\rho_e} w_e \tag{6}
$$

Q_T, the relaxation term between heavy particles and electrons, is given by

$$
Q_T = 3\rho_e R_0 (T_h - T_e) \sum_{i=h.p.} \frac{\nu_{e,i}}{M_i} - n_{e,Ar^+} \hat{I}_{Ar^+} . \tag{7}
$$

As real gases are considered, the specific heat C_p and enthalpy h are the functions of the temperature, which are calculated from the thermodynamic coefficients given by Gnoffo et al.[4] and Matsuzaki[5]. These chemically-reacting gasdynamic equations with strong source terms are solved by a semi-implicit Harten-Yee non-MUSCL modified-flux-type TVD scheme, where radiative energy transfer terms Q_{IB}, Q_B and relaxation term Q_T are treated implicitly.

The radiative energy transfer term Q_{IB}, which comes from Inverse Bremsstrahlung, is expressed as

$$
Q_{IB} = I(r,x,t) \times (K_{ea} + K_{ei}) \tag{8}
$$

where K_{ei} and K_{ea} are given by Johnston et al.[6] and Kemp et al.[7], respectively. They are functions of number density, electron temperature and wavelength of laser beam.

On the other hand, the energy loss Q_B, which is due to Bremsstrahlung, is given by integrating the expression from zero to infinity with respect to frequency.

$$Q_B = 1.43\times10^{-40} n_e^{\ 2} T^{\frac{1}{2}} \ . \tag{9}$$

C. Laser absorption model

In the presence of absorption, the local radiation intensity *I(r,x,t)* is governed by

$$\frac{d(IA)}{dn} = I(r,x,t)\times A\times(K_{ea}+K_{ei}) \ . \tag{10}$$

where *n* indicates the direction of laser beam.

D. Coordinative System of Analysis

Figure 1 and Table 1 show the analysis model for the axisymmetric 2-dimensional study, and physical conditions, respectively. The reason of lower pressure (density) in Case 3 is the cut-off, the critical density is proportional to the square of the inverse number of electron density. A laser beam passes from the right to the left, and the Microwave-supported detonation/ deflagration (MSD/ MSC) wave propagates through argon gas at room temperature (T_0 = 300K). In order to simulate the laser-supported plasma, a hot spot in which hot electrons are seeded, is set up to imitate the initial plasma. This hot spot model imitates the hot electrons, which are generated on a metal target by dielectric breakdown in an experiment[8]. The hot electron temperature is 15,000 K and only 1% seeding rate is applied because the number of seeded electrons has little effect on the Beam-supported detonation propagation results; however, a high seeding rate is not good for the simulations.

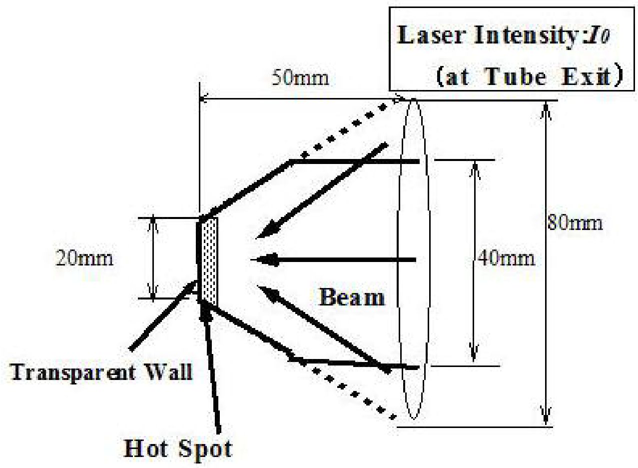

Figure 1. Schematic View of Axisymmetric Tube.

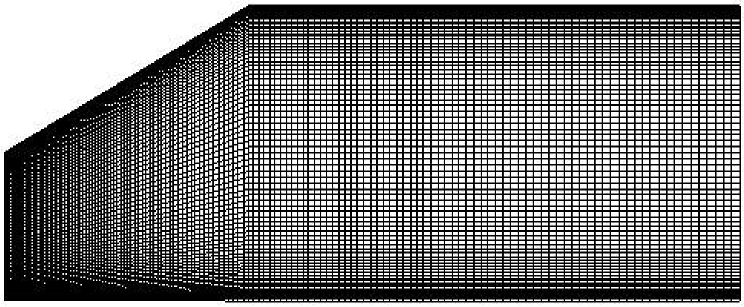

Figure 2. Computational Grid (81×151).

Table 1. Initial Condition for Analysis.

Laser Intensity	**Hot Spot (Initial)**	**Other Area (Initial)**	**Hot Spot Radius**	**Cross Section Radius of Laser Beam**
I_0 **= 8 kW/cm^2**	**1atm, 15000 K**	**0.01atm, 300 K**	**0.5 mm**	**40 mm (at Inlet) → 10 mm**

III. Calculation Results and Discussion

Figures 3 show the distributions of electron densities (areas shown in white color indicates $n_e > 5.0\times10^{21}$ m^{-3}) and pressures (areas shown in white color indicates p > 0.5atm), respectively. In this case, electron number density and pressure rises up on the hot spot near the left wall, where the beam is absorbed by inverse bremsstrahlung. As the absorption ratio increases with increasing ionization fraction, the shock front is generated by compression near the beam absorption zone, where the electron density becomes rich. The absorption zone propagates behind the shock front [Time (a)]. It becomes clear that the MSD wave propagates, where the beam absorption rate is almost 100%. When the absorption is weakened by decreasing the laser intensity caused by the geometric condition, the absorption zone departs from the shock front [Time (b)]. At this time, the MSD wave converts to an MSC wave the beam intensity is less than 50 kW/cm^2. Even after reaching the tube area, the propagations of the both fronts continue despite the diminished beam [Time (c)].

Figure 4 shows the x-t diagram along the line of the symmetric axis, where the dashed line and the solid line indicate the shock front and the ionization front, respectively. Though the shock front slows down by degrees, the ionization front keeps the velocity, 250m/s, in the tube area even by the diminished beam. Therefore, the axisymmetric divergent area, attached to the tube, is effective for sustaining Microwave-supported wave

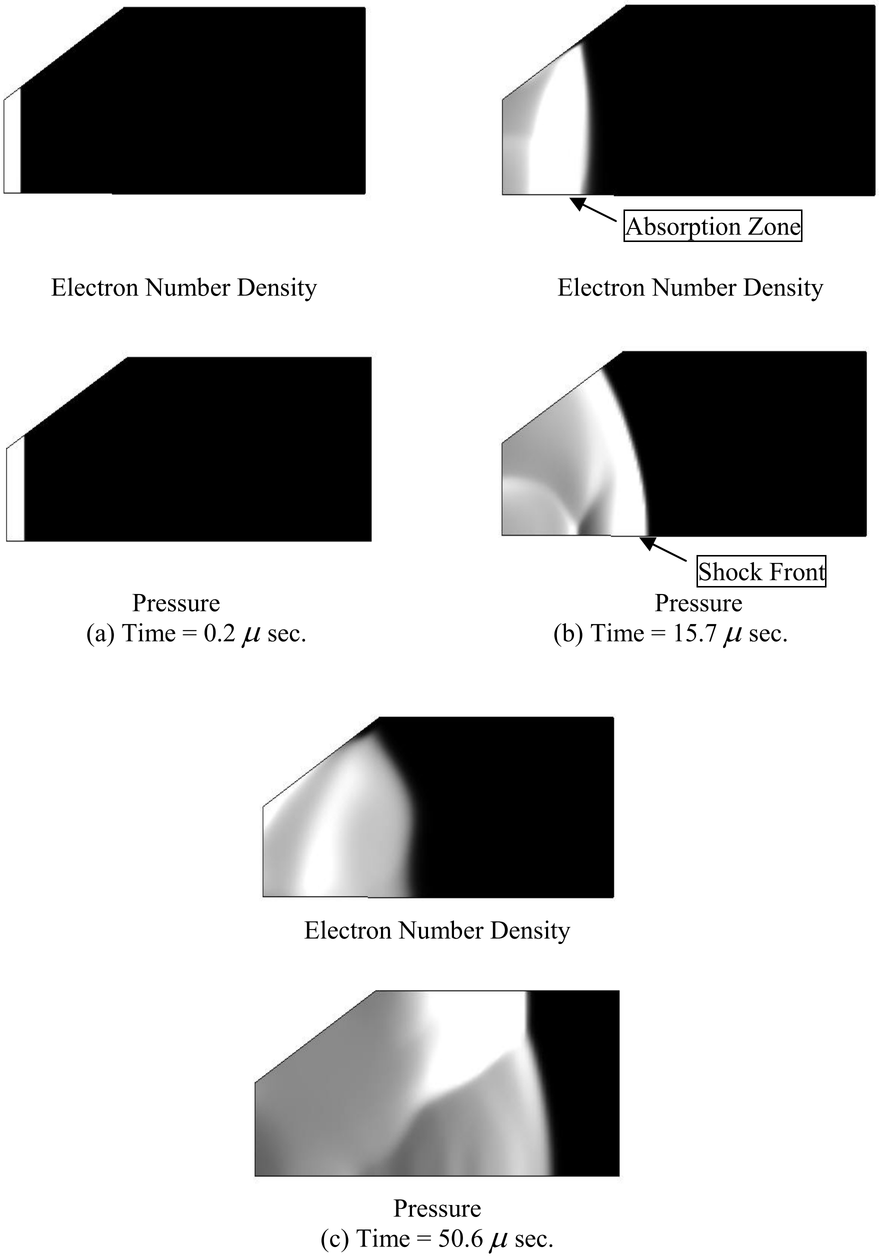

Figure 3. Pressure and Electron Number Density Distribution.

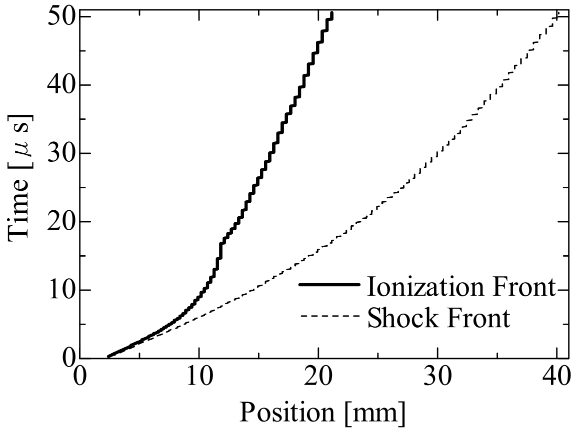

Figure 4. x-t diagram along the symmetric axis.

IV. Conclusion

In order to suggest a numerical approach for MSP for planning Microwave Propulsion Systems, Microwave-supported wave (MSD to MSC) driven by 100 μm microwave propagating through an axisymmetric tube is calculated by applying the approach of unsteady analyses for LSD waves.

The conclusion is as follows:

The axisymmetric divergent area, attached to the tube, is effective for sustaining Microwave-supported wave; Though MSD wave at once transitions to MSC because of the beam diminishing by geometric condition, the propagation diminishing is finished in the tube area.

References

[1] Shiraishi, H., "Unsteady Numerical Analysis of Microwave/Laser-Supported Plasma", Proceedings of 7th Int. Symp. on Beamed Energy Propulsion, 2011 (to be published).

[2] Nishida, M., Kaita, K. and Tanaka, K., "Numerical Studies of the Flow Field in a DC Arcjet Thruster", DGLR/AIAA/JSASS 20th International Electric Propulsions Conference, IPEC88-105, 1988, pp.591-597.

[3] Lee, J. H., "Basic Governing Equations for the Flight Regimes of Aeroassisted Orbital Transfer Vehicle", AIAA Paper 84-12729, 1984, pp.1-18.

[4] Gnoffo, P. A., Gupta, R. N. and Shinn, J. L., "Conservation Equations and Physical Models for Hypersonic Air Flows in Thermal and Chemical Nonequilibrium", NASA Technical Paper 2867, 1989, pp.1-57.

[5] Matsuzaki, R., "Specific Heat and Isentropic Exponent of High Temperature Gases, Exact Analytical Expressions", AIAA Paper 80-1532, 1980, pp.1-11.

[6] Johnston, T. W. and Dawson, J. M., "Correct Values for High-frequency Power Absorption by Inverse Bremsstrahlung in Plasmas", Phys. Fluids, **16**, 1973, p.722.

[7] Kemp, N. H. and Lewis, P. F., "Laser-Heated Thruster Interim Report", NASA Contractor Report, NASA CR-161665, 1980.

[8] Stuermer, E. and von Allmen, M., " Influence of laser-supported detonation wave on metal drilling with pulsed CO_2 lasers", **49**(11), 1978, pp.5648-5654.