Sheath Instability and Velocity-Space Diffusion in E×B Discharges with Strong Secondary Electron Emission

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Abstract: Sheath instability associated with intense secondary electron emission (SEE) is studied. We show that onset of the instability is given by relationship that the SEE coefficient of weakly confined electrons γ_{wc} becomes larger than unity. When plasma is hot and γ_{wc} >1 the plasma tends to exhibit the Relaxation Sheath Oscillations (RSO's), during which average electron energy changes periodically. Energy of weakly confined electrons rises until γ_{wc} =1 after which the sheath potential collapses and hot weakly confined electrons leave the plasma. Next, the large influx from cold SEE from the wall is trapped during the potential restoration. We show that velocity space diffusion, due to two-stream instability induced by the SEE beams, is responsible for effective heating of weakly confined electrons until γ_{wc} reaches unity and the process repeats itself. We identify the precise cause of RSO's and formulate existence criteria for RSO's in terms of global plasma parameters.

Nomenclature

$B = B_x$ = applied "radial" (normal to the walls) magnetic field in the discharge
e = electron charge
E_x = the plasma's self-generated electric field normal to the walls
E_z = applied background electric field (parallel to the walls)
H = width of the plasma (i.e. distance between the walls)
$dJ/d\Phi_c$ = differential conductivity of the sheath
m_e = electron mass
n_a = density of neutral atoms in the plasma
T_x = electron temperature in the direction normal to the walls
T_z = electron temperature in the direction of the applied electric field
V_D = drift velocty due to the crossed electric and magnetic fields
Φ = magnitude of the potential well that confines electrons in the plasma
Φ_c = confinement potential in equilibrium
ν_{turb} = frequency of turbulent collisions
ν_{en} = frequncy of electron collisions with neutral atoms
$\nu_{ratio} \equiv \nu_{turb}/\nu_{en}$

ε = impact energy of an electron striking the wall
$W_x = \frac{1}{2} m_e v_x^2 + e\Phi(x)$ = energy of electrons normal to the walls
$W_{//} = W_z + W_y = \frac{1}{2} m_e v_z^2 + \frac{1}{2} m_e v_y^2$ = energy of electrons parallel to the walls
Γ_{CE} = wall flux of "collision-ejected" electrons
Γ_{WC} = wall flux of "weakly confined" electrons
Γ_b = wall flux of "beam" electrons
$\Gamma_p = \Gamma_{CE} + \Gamma_{WC}$ = wall flux from "plasma" (non-beam) electrons
Γ_i = flux of ions to the walls
γ_{CE}, γ_{WC}, γ_b = secondary emission coefficient of each species of electron flux defined above

I. Introduction

E×B devices such as Hall thrusters and Penning type discharges are subject to various kinetic effects at low pressure where collisionality is low. The "sheaths" [1] at the negatively charged walls form a potential well of *magnitude* Φ_c that helps confine electrons. Electrons with energies exceeding $e\Phi_c$ can escape the plasma while the rest are trapped. Plasma electrons are often assumed to be Maxwellian, but if collisions that replace the escaped electrons are infrequent, the plasma is strongly depleted of electrons beyond the cutoff velocity. This "loss cone" effect, along with any applied beams [2], creates irregular electron velocity distribution functions (EVDF's) that can be subject to sheath instabilities which do not occur with a Maxwellian EVDF [3]. Also, when the loss cone is depleted, electrons bordering the loss cone, perturbed into it by fluctuations, form a substantial fraction of the total wall flux and have major influence on plasma behavior, as will be discussed.

Materials in contact with plasma may emit electrons due to bombardment from plasma electrons. Secondary electron emission (SEE) from device walls has been long known to increase energy losses by weakening the strength of the sheath [4]. This effect is critical for tokamak divertors [**5**], Hall thrusters [**6**], dusty plasmas [**7**], *etc*. Emitted electrons are often assumed to thermalize in the plasma [8], but in low collisionality, the SEE forms beams that propagate between the device's walls [9], causing additional effects, such as near wall conductivity [10]. SEE beams can cause two-stream instability. This is known to attenuate the beams by perturbing some electrons into the trapped plasma [11]. We will show the reverse effect, i.e. bulk electrons being driven to the loss region, also happens and has implications for wall energy losses under general conditions.

A negative perturbation of the wall potential *magnitude* Φ_c allows some previously trapped electrons to overcome the sheath and reach the wall where they are absorbed. Usually, this recharges the wall, canceling the perturbation. But with SEE, in certain conditions, the extra influx may induce a secondary outflux larger than itself, leading to a net charge loss. That is, the conductivity of the sheath, $dJ/d\Phi_c$ is negative, the perturbation amplifies and the potential rapidly drops. Such behavior occurs in a recently discovered plasma regime with quasiperiodic sheath instabilities called Relaxation Sheath Oscillations [12]. During the potential drop each period, plasma parameters abruptly change and wall losses increase drastically. Minimizing losses is important in applications, so we investigate in detail why the instability occurs and under what plasma conditions it can exist.

II. Overview of Electron Kinetics in E×B discharges with Secondary Emission

The system under consideration is diagrammed in Fig. 1. We use the electrostatic direct implicit particle-in-cell code (EDIPIC) [13], to simulate a one-dimensional planar xenon plasma bounded by floating emitting walls. The main input variables are the uniform applied fields E_z and B_x, neutral gas density n_a, plasma width H and turbulent collision frequency ν_{turb}. Particle dynamics are governed by the plasma's self-generated field $E_x(x)$ and the E×B drift motion from the background fields. Electrons suffer Coulomb collisions, elastic collisions with

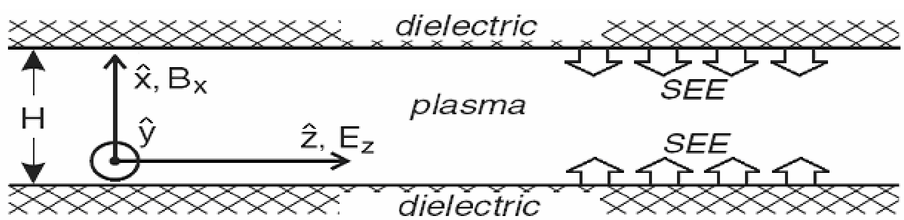

Figure 1. Simulation model. Plasma is uniform in y and z directions.

neutrals and "turbulent" collisions. We focus on the latter two here because Coulomb interactions only weakly affect the plasma [14]. Turbulent collisions, introduced to produce anomalous conductivity, scatter the y-z component of the velocity vector, i.e. rotate it at a random angle in the y-z plane [15]. This leads to displacement along E_z and an average parallel (y-z directed) energy gain of $<\Delta\varepsilon> = m_eV_D^2$ per scatter, where V_D=E/B is the drift velocity. Hence turbulent collisions can be used as a heat source, helping balance energy losses to the walls. Adjusting ν_{turb} is way of controlling the plasma temperature in EDIPIC, but the results presented are general, depending on the plasma properties alone regardless of how they are achieved. The coefficient γ represents the average number of secondaries produced by an electron striking the wall and depends on the impact energy, ε. EDIPIC models SEE with boron nitride ceramic walls. $\gamma(\varepsilon)$ is an increasing function of ε [16], well approximated by $0.17\varepsilon^{1/2}$.

Fig. 2 is a phase plot showing the arrangement of electrons in energy space in a typical simulation. The average energy parallel to the walls $<W_{//}> = <W_z+W_y>$ far exceeds $<W_x>$ and $e\Phi_c$ because the heating is anisotropic. Electrons with $W_x < e\Phi_c$ are trapped in the potential well. Beyond the cutoff at $W_x = e\Phi_c$ is the "loss region," as electrons with $W_x > e\Phi_c$ quickly escape to the walls. The high density area of the loss region is the beams, which are continuously replenished. Beam electrons are in drift motion with parallel energies spread between 0 and $2m_eV_D^2$.

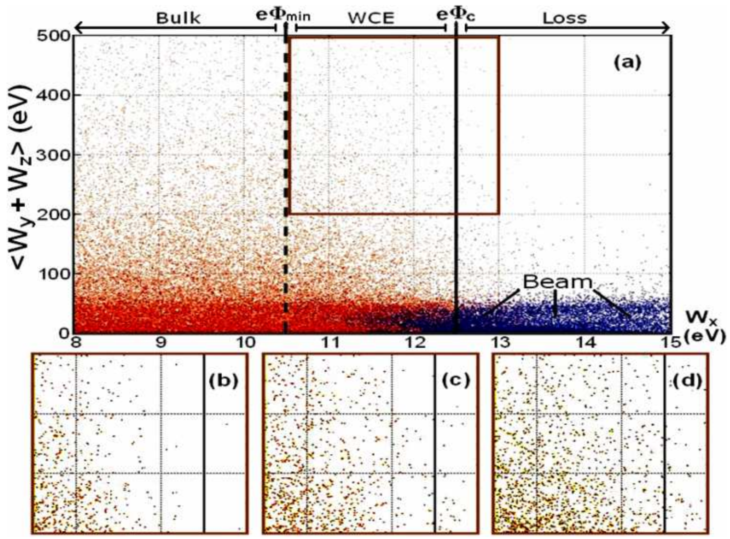

Figure 2. (a) Phase plot taken near the end of a stable state. (b), (c) and (d) show the boxed section of energy space in (a) at three times in a RSO period; near the beginning (b), middle (c) and end (d) of the stable state. Simulation parameters are E_z = 200V/cm, B_x = 0.01T, n_a = 10^{12} cm^{-3}, ν_{turb}=2.8×10^{6}s^{-1}, H=2.5cm. Red dots are plasma electrons and blue dots are secondaries.

The wall flux consists of four components. Trapped "bulk" electrons can escape when their velocity vectors are scattered into the loss region by collisions with neutrals. The flux from these collision-ejected electrons (CEE's) is denoted Γ_{CE}. Electrons with W_x slightly below $e\Phi_c$, called weakly confined electrons (WCE's), can be nudged by the potential fluctuations (typically ~1V) into the loss region. Simulations show WCE's produce a substantial flux, Γ_{WC}, similar in magnitude to Γ_{CE}. The potential well strength Φ limits the "plasma" electron flux $\Gamma_p = \Gamma_{CE} + \Gamma_{WC}$. Since the walls are floating, the equilibrium value Φ_c maintains $\Gamma_p = \Gamma_i$ where Γ_i is the ion flux. The beam fluxes Γ_b between the walls are equal and opposite in equilibrium, so they cancel. Γ_i is estimated by the Bohm criterion, so it is constant and unaffected by processes discussed here.

III. The Criterion of Instability

From the structure of this system, one can anticipate how it becomes unstable. Sheath instability occurs if a negative perturbation of the potential magnitude causes reduction of the (negative) surface charge σ_e at the wall, which further lowers the potential. ($d\sigma_e/d\Phi > 0$ is equivalent to $dJ/d\Phi < 0$ by charge conservation). Consider a perturbation $-\Delta\Phi$. Γ_{WC} increases as previously trapped electrons with $e(\Phi_c-\Delta\Phi) < W_x < e\Phi_c$ can suddenly reach the wall. Beams are emitted from the wall, at the "top" of the potential well, so $W_{x,b} > e\Phi_c$ and $\Delta\Gamma_b = 0$. Also $\Delta\Gamma_{CE}$, from the tiny increase in size of the loss cone, is negligible. So overall, σ_e should decrease if and only if $\gamma_{WC} > 1$. EDIPIC records temporal data of the average energies, wall fluxes and SEE yields for each component (CEE, WCE and beam). We have verified in all runs with RSO's that the parameter jumps marking instability each period occur when γ_{WC} reaches unity, see Fig. 3. γ_b, γ_{net} and γ_{CE} may change *in response* to the potential drop, but the *cause* of the drop is γ_{WC} reaching unity.

During the potential drop, the large number of WCE's reaching the wall increases the total incident electron flux and the corresponding beam outflux by an order of magnitude. When these beam electrons with γ_b <1 reach the opposite walls, there is a net absorption of electrons and the walls recharge. Some of these beam electrons become trapped when the potential restores. Overall, the net change in the plasma before vs. after instability is the hot WCE's with $\gamma_{WC} = 1$ in the "WCE region," $e\Phi_{min} < W_x < e\Phi_c$, are emptied and then replaced by colder trapped beam electrons (TBE's). Gradually, the WCE's regain energy until γ_{WC} reaches 1 again (see Fig. 2). The process repeats and we end up with periodic oscillations. We call the time interval during instability the unstable state and the "equilibrium" between instabilities the stable state. Notice in Fig. 3 how γ_{WC} increases over the stable state but γ_b, γ_{net} and Φ barely change. The WCE's drive the oscillations.

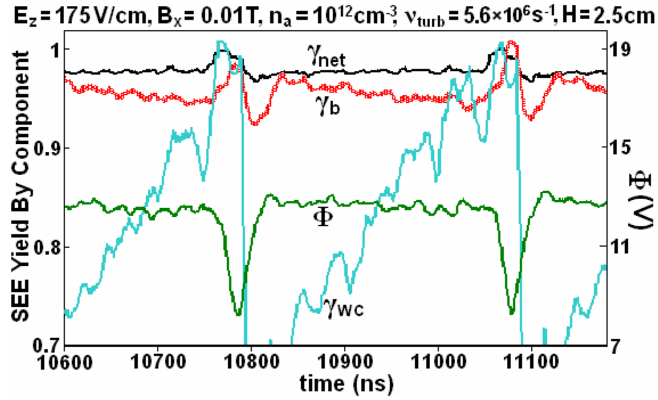

Figure 3. Time evolution of the SEE yield for each flux component. The potential is superimposed for comparison. Note $\gamma_{CE} \approx 2.3$ in this run (out of range above).

We now estimate what range of system parameters produce RSO's. There must be a curve of marginal stability, $\nu_{ratio}(V_D)$, where $\nu_{ratio} \equiv \nu_{turb}/\nu_{en}$ and $V_D = E/B$, dividing the $\{\nu_{ratio}, V_D\}$ plane is into two regions distinguishing plasmas hot enough to produce RSO's from those that are not. Marginal stability of the sheath is determined by $\Delta\sigma_e/\Delta\Phi_c = 0$, for a small perturbation $-\Delta\Phi$ at one wall that allows electrons with $e(\Phi_c\text{-}\Delta\Phi) < W_x < e\Phi_c$ to reach the wall. In terms of the *energy* distribution in the plasma center, $f_w(w_x, w_{//})$, we have

$$\int_0^{\infty}\int_{e(\Phi_c-\Delta\Phi)}^{e\Phi_c} f_w\left(w_x, w_{//}\right)\left(1-\gamma(\varepsilon)\right)dw_x dw_{//} = 0 \quad (1)$$

where $\varepsilon = [w_x - e(\Phi_c\text{-}\Delta\Phi)] + w_{//}$ is the impact energy of electrons at the wall. Note that this integral includes only WCE's. Incoming beam electrons were emitted at the top of the potential well, so $w_{x,beam} > e\Phi_c$ and $\Delta\Gamma_b = 0$. Also, $\Delta\Gamma_{CE} << \Delta\Gamma_{WC}$ because the density of CEE's in energy space is negligibly low compared to the bulk (compare loss and bulk regions in Fig. 2). For small $\Delta\Phi$, Eq. (1) reduces to

$$\Delta\Phi\int_0^{\infty} f_w\left(e\Phi_c, w_{//}\right)\left(1-\gamma(w_{//})\right)dw_{//} = 0. \qquad (2)$$

Intuitively, WCE's strike the wall with negligible x-energy and Eq (2) is simply the integral over parallel energies of electrons at the edge of the bulk, which can be modeled as a 2-D Maxwellian of temperature $T_{//} = T_z$ [**9**]. Eq. (2) reflects our earlier heuristic explanation of $\gamma_{WC} = 1$ being the critical point of instability. Marginal stability reduces to the following condition, written as a velocity integral.

$$\gamma_{wc} \approx \gamma_{2D}(T_z) = \frac{m}{T_z}\int_0^{\infty} v\gamma\left(\frac{mv^2}{2}\right)\exp\left(\frac{mv^2}{2T_z}\right)dv = 1. \quad (3)$$

Ref. **9** gives estimates for T_z, T_x and Φ_c in terms of plasma parameters. The formulas are coupled, so T_z, given by,

$$T_z \approx k\left(1+\frac{\nu_{turb}}{\nu_{en}}\right)m_e\left(\frac{E}{B}\right)^2\left[1+\ln\left(\frac{H}{\lambda_{mfp,en}}\sqrt{\frac{T_z}{T_x}\sqrt{\frac{M_i}{2m_e}}}\right)\right] \quad (4)$$

must be calculated numerically. While the formulas are estimates within a factor of a few, the scaling is accurate, so better precision can be obtained by choosing a correction coefficient *k* so that Eq. (4) agrees with the value of T_z obtained in a simulation. Eq. (4) can then accurately predict T_z nearby in parameter space. We find the critical temperature T_{zcr} such that $\gamma_{2D} = 1$, is 44.4eV and with k = 3.34, the resulting curve $\nu_{ratio}(V_D)$ where $\gamma_{2D} = 1$, divides the parameter space into RSO and RSO-free regions, see Fig. 4. This gives credibility to the model because the curve predicts where RSO transition occurs in the ν_{ratio} direction over a range of V_D values using the *same k*. We conclude that RSO's should occur when,

$$T_z \geq 44.4eV \qquad (5)$$

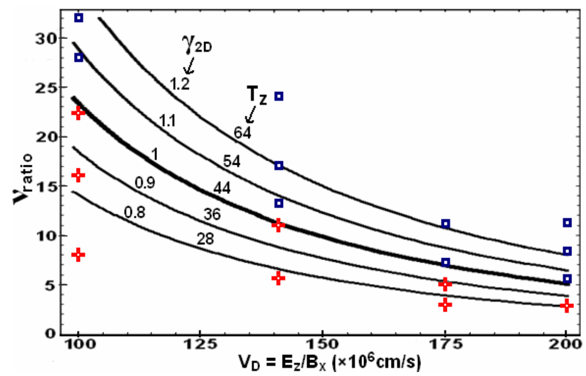

Figure 4. Simulations run over a range of parameters. Marks represent plasmas that featured RSO's (squares) or reached a stable steady state (pluses). Characteristic curves showing T_z and γ_{2D} vs. ν_{ratio} and V_D are superimposed. H is fixed at 2.5cm.

Previously, the regrowth of the WCE energy needed to drive oscillations was not well understood. We recently found that the WCE energy gain following instability that precipitates the next instability is driven by diffusion in velocity space that causes migration of hot electrons from the bulk through the WCE region to the loss region. Notice the "heating" of the WCE region occuring in Fig. 2 (b-d) shows electrons moving left to right, whereas turbulent collisions alone would heat electrons vertically. An additional energy source must exist.

The beams and high density of TBE's produce sharp peaks in the beam/WCE regions of the EV_xDF, which is a criterion for two-stream instability [**11**]. The resulting fluctuations drive relatively cold beam electrons leftward and hot bulk electrons rightward in energy space, in a diffusive manner that leads to average energy gain in the WCE region during the stable state. This is responsible for the trapping of beam electrons in Ref.**11**. But apparently the reverse effect leads to a flux of WCE's to the wall. Because the WCE's are hotter, in general, than the beam electrons, this can be thought of as a net energy loss.

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