

0-D Plasma Model for Orificed Hollow Cathodes

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Abstract: In this work it is developed a 0D-model of the plasma parameters within the orifice of orificed Hollow cathodes. The model is based on hydrodynamic considerations. In the poster presentation are exhibited the predictions of the model under different hollow cathode operation conditions and compared with the results of the original 0D-model of Mandell and Katz .

Nomenclature

D_a	=	ambipolar diffusion coefficient
F	=	gas flow rate
I_e	=	electron current in the orifice
I_{loss}	=	ion loss rate
I_{prod}	=	ion production rate
η	=	plasma resistivity
J_i^A	=	ion current density at the orifice inlet
J_i^B	=	ion current density at the orifice outlet
J_i^W	=	radial ion current density at the wall
L	=	orifice length
m	=	electron mass
M	=	atom mass
n	=	average plasma density
n^A	=	plasma density at the orifice inlet
n^B	=	plasma density at the orifice outlet
n_0	=	average neutral gas density
n_0^A	=	neutral gas density at the orifice inlet
n_0^B	=	neutral gas density at the orifice outlet
P_0	=	average neutral gas pressure
P_0^A	=	neutral gas pressure at the orifice inlet
P_0^B	=	neutral gas density at the orifice outlet
r	=	orifice radius
$\sigma(T_e)$	=	electron impact ionization cross section averaged over a Maxwellian distribution of velocities
$\sigma_{excit}(T_e)$	=	electron impact excitation cross section averaged over a Maxwellian distribution of velocities
σ_{CEX}	=	charge exchange cross section

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T_e	=	plasma electrons temperature
T_e^{in}	=	plasma electron temperature in the insert region
u_0	=	average neutral gas velocity
u_i^A	=	ion velocity at the orifice inlet
u_i^B	=	ion velocity at the orifice outlet
u_e	=	electron velocity
U_{ion}	=	potential of ionization by electron impact
U_{rad}	=	potential of excitation

I. Introduction

Hollow cathodes are known as one of the best high current plasma sources and are one of the critical components of most Electrostatic and Hall Ion Thrusters, where they provide electrons to neutralize the ion beam and/or to ionize the propellant. Electric and Hall ion thrusters have high efficiency and have been adopted as one of the most promising thrusters in satellites and spacecrafts^{1,2}.

Despite their high efficiency, electrostatic and Hall thrusters produce low thrust density (thrust per unit of area) and therefore need to operate for a long time to accomplish a mission in space applications. This implies in turn that Hollow cathodes have to operate continuously for long periods (thousands of hours³) and therefore the lifetime of those dispositives is a crucial issue. Orificed Hollow cathodes are used in electrostatic and Hall thrusters instead of open channel Hollow cathodes because the latter need a higher gas flow rate in order to operate properly, and this diminishes the overall efficiency of the thruster due to the required long time operation and consequently high propellant consumption. The orificed Hollow cathodes obstruct partially the gas flow rate and therefore use lower propellant flow for normal operation.

Plasma parameters measurements within Hollow cathodes and/or its orifice during operation are very difficult due to their small dimensions, high temperature and high current density within the orifice. It is then necessary elaborating models to simulate the plasma behavior within Hollow cathodes. The complexity inherent in the processes taking place within Hollow cathodes makes its study a very interesting and challenging field. Several models have been developed in the last decades¹⁻¹² in trying to predict the plasma behavior inside Hollow cathodes as a function of the mass flow rate, the required discharge current and Hollow cathode geometry.

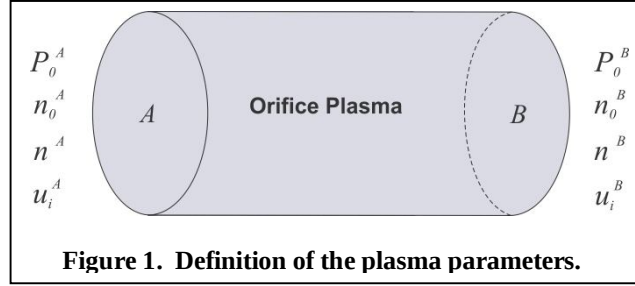
A 0-D model is essentially hydrostatic because all the body forces are in equilibrium producing no net movement of the bulk of the fluid. However across the orifice there exist dynamical forces which cause the plasma motion. In this work a new model is developed, and its predictions are exhibited during the poster presentation where they are analyzed and compared with those of the original Mandell and Katz (M&K) model. We develop a 0-D plasma model within the orifice based on hydrodynamic considerations. The strategy is to consider a non uniform distribution of some plasma parameters and express the resulting equations of the plasma movement in terms of averaged quantities properly defined. This approach allows one to obtain a set of algebraic equations that describe the behavior of the averaged values.

In the new model the plasma is quasi-neutral and the plasma ions and neutral atoms are at the same temperature, while the plasma electrons have a higher temperature. The electron temperature and plasma density are determined as in the M&K model but the fluxes of particles across the orifice boundaries are not thermal. The neutral gas flow is determined assuming Poiseuille flow as in^{3, 4, 6, 7}, while the ion flux is determined from the momentum equation which under some simplifications, gives rises the ambipolar diffusion equation^{4, 5}. The ambipolar diffusion coefficient is calculated taking into account that charge exchange collisions of ions with neutrals is the dominant process. It is found that the M&K model with cold neutrals, and ions at the same temperature as the electrons, predicts a similar qualitative and quantitative behavior for the plasma density, plasma electron temperature and ion output as the new model, which assumes that the ions and neutrals have the same temperature which in turns is lower than the electron temperature. The explanation is that in the M&K model, the flux of ions across the orifice boundaries is thermal; then, in order to calculate such a flux, it is necessary to assume an energy distribution for the ions at some temperature, but the Bohm criterion relates the ion flux going out from the plasma through the plasma sheath, with the electron temperature. Therefore, the ion temperature has to be taken equal to the electron temperature in order to be consistent with the Bohm criterion. The new model does not need this assumption because the particle fluxes are calculated through the momentum equations. The two models predict also a similar response of the plasma parameters under rescaling of the electron discharge current, total flow rate and orifice dimensions. The quantitative predictions of the two models can differ by a factor of about two. In the new model the qualitative behavior of the plasma potential drop along the orifice depends on the radius of the orifice, while in the M&K model such dependence does not exist. In the new model, the plasma ions at the input of the orifice can be

dragged into the orifice by the gas flow, while in the M&K model the ions at that location are always driven by the electric field to the insert region.

II. Description of the Model

The plasma ion, neutral and electron temperatures are considered uniform along the orifice and the ion and neutral temperatures $T_i = T_0$ are supposed to be known values. At the orifice input the pressure is higher than at the exit and the neutral gas density will consequently be different at those locations (see Fig. 1). Radial variations in the neutral gas density are neglected in this model. The average neutral gas pressure and density are defined as: $\bar{P}_0 = (P_0^A + P_0^B)/2$ and $n_0 = \bar{P}_0/(eT_0) = (n_0^A + n_0^B)/2$. We also define the parameter $\alpha = P_0^B / P_0^A = n_0^B / n_0^A$.



Notice that this parameter is always in the range $0 < \alpha < 1$ and it measures the relative decrease in the pressure along the orifice. Using this parameter and the definition of the average neutral gas density one obtains $n_0^A = 2n_0/(1 + \alpha)$.

The model considers that the neutral gas flow is governed by the Poiseuille law which can be used to obtain a relation between the average neutral gas density n_0 and the axial velocity u_0 :

$$u_0 = \left[\frac{1 - \alpha}{1 + \alpha} \right] \frac{r^2 e T_0 n_0}{4L\xi} \quad (1)$$

The viscosity for the Xe gas as a function of the neutral gas temperature can be calculated as^{3, 4, 13}:

$$\xi = 2.3 \times 10^{-5} T_r^{0.71+0.29/T_r} ; \quad T_r = eT_0 / (k_B * 289.7) ; \quad T_r > 1 \quad (2)$$

One can see that the neutral gas velocity is determined by hydrodynamic forces and not by the thermal flux as in the original model of Mandel and Katz. It is also necessary to determine the ion fluxes through dynamical considerations as was done for the neutral gas. In a reference system linked to the flowing neutral gas bulk, the ions diffuse ambipolarly within the neutral and in this reference system the governing equation for the ion movement can be written as:

$$n(u_i - u_0) = -D_a \frac{dn}{dz} \quad (3)$$

or specifically at each one of the orifice exits:

$$\frac{1}{e} J_i^A = n^A u_i^A = n^A u_0 - D_a \left. \frac{dn}{dz} \right|_A ; \quad \frac{1}{e} J_i^B = n^B u_i^B = n^B u_0 + D_a \left. \frac{dn}{dz} \right|_B \quad (4)$$

The upstream and downstream directions are the directions towards the insert and towards the anode, respectively. Here the plus and minus signs are for the downstream and upstream ion current densities respectively, because inside the orifice one expects a larger ionization rate than in the insert region due to the higher electron temperature and neutral and plasma densities. Therefore the plasma density must decrease in the upstream direction from some place within the orifice. On the other side, the plasma density has to decrease near to the orifice exit because of the lower pressure and, respectively, the plasma density gradient is negative at this location. The ambipolar diffusion coefficient depends on the neutral gas density and one can evaluate the value of D_a at the average neutral gas density n_0 . At the orifice exit one has the following relations dictated by the total mass conservation:

$$\begin{aligned} n_0^B u_0 + n^B u_i^B &= 9.83 \times 10^{-8} F / (M \pi r^2) \\ \Rightarrow n_0^B u_0 + n^B u_0 + D_a \left| \frac{dn}{dz} \right|_B &= 9.83 \times 10^{-8} F / (M \pi r^2) \end{aligned} \quad (5)$$

At the orifice input the relations are:

$$\begin{aligned} n_0^A u_0 + n^A u_i^A &= 9.83 \times 10^{-8} F / (M \pi r^2) \\ \Rightarrow n_0^A u_0 + n^A u_0 - D_a \left| \frac{dn}{dz} \right|_A &= 9.83 \times 10^{-8} F / (M \pi r^2) \end{aligned} \quad (6)$$

We define the average plasma density within the orifice as $n = (n^A + n^B) / 2$ and make the following approximation:

$$\left| \frac{dn}{dz} \right|_A \approx \left| \frac{dn}{dz} \right|_B \approx \frac{|\Delta n|}{L/2} \approx \frac{n}{L/2} = \frac{2n}{L} \quad (7)$$

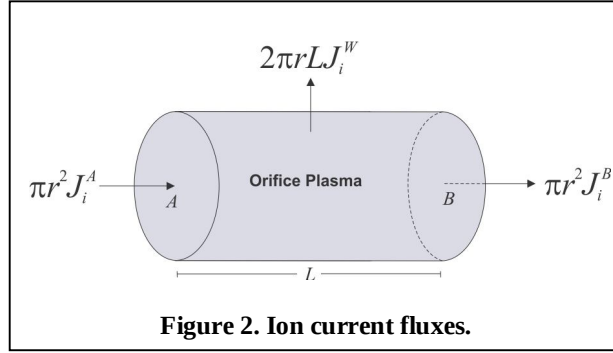
Also, we have assumed that the plasma density variation within the orifice is of the same order of magnitude as its average value. The sum of Eqs. (5) and (6) results in the continuity equation in terms of the average plasma and neutral gas density:

$$(n_0 + n) u_0 = 9.83 \times 10^{-8} F / (M \pi r^2) \quad (8)$$

The ion production and ion loss rate are defined as:

$$I_{prod} = e \pi r^2 L n n_0 \sigma(T_e) \sqrt{\frac{8eT_e}{\pi m}}; \quad I_{loss} = \pi r^2 (J_i^B - J_i^A) + 2 \pi r L J_i^w \quad (9)$$

The expression for the cross section for ionization by electron impact averaged over a Maxwellian distribution of velocities $\sigma(T_e)$ can be found in^{3, 4, 13}. Notice that in the ion loss the upstream ion current J_i^A is included with a negative sign. Let us note that this model does not suppose that the electric field is strong enough to drive ions in the upstream direction as it was supposed in the Mandell and Katz model. Thus, the results of the calculations will show the actual direction of the upstream current. Namely, if the upstream current is positive, then there are ions entering the orifice from the insert region, otherwise the ions are exiting from the orifice to the insert region.



The plasma ion density decreases from the axis towards the inner wall and this plasma density radial gradient leads to radial diffusion of the ions. Supposing plasma quasi-neutrality one can consider the plasma radial ambipolar diffusion. The radial plasma density gradient is estimated similarly to Eq. (7):

$$J_i^W = eD_a \left| \frac{dn}{dr} \right|_W \approx eD_a \frac{n}{r} \quad (10)$$

In steady state the ion generation rate is equal to the ion loss rate. Equations (9) and (10) result in an equation for the electron temperature as a function of the average neutral gas density and orifice dimensions:

$$I_{prod} = I_{loss} \Rightarrow n_0 \sigma(T_e) \sqrt{\frac{8eT_e}{\pi m}} = \frac{2D_a}{r^2} \left[1 + 2 \left(\frac{r}{L} \right)^2 \right] \quad (11)$$

Here the ambipolar coefficient is defined as:

$$D_a = \left(1 + \frac{T_e}{T_i} \right) \frac{\sqrt{eT_i / M}}{\sigma_{CEX} n_0} \quad (12)$$

For low ion temperatures the cross section for charge exchange is approximately $\sigma_{CEX} \approx 10^{-18} m^2$. The average ion-neutral collision frequency ν_{in} is enlarged by resonant charge exchange:

$$\nu_{in} = \sigma_{CEX} n_0 \sqrt{eT_i / M} \quad (13)$$

Finally, we consider, the energy balance for electrons similar to the model^{3, 4} except for the coefficient 5/2 which results from the sum of the total internal energy density ($3\pi r^2 n e T_e u_e / 2 = 3T_e I_e / 2$) flowing across the boundaries of the orifice and the flux of power due to the work done for the electron pressure ($\pi r^2 P_e u_e = \pi r^2 n e T_e u_e = T_e I_e$).

$$\frac{\eta L}{\pi r^2} I_e^2 = e \pi r^2 L n n_0 [\sigma(T_e) \mathcal{Y}_{ion} + \sigma_{excit}(T_e) \mathcal{Y}_{rad}] \bar{u}_e + \frac{5}{2} I_e (T_e - T_e^{in}) \quad (14)$$

The expression for the plasma resistivity η and the cross section for excitation with radiative de-excitation can be found in^{3, 4, 13}. The set of Eqs. (1), (8), (11) and (14) can be solved simultaneously to find the average neutral and plasma density, the electron temperature and the neutral gas velocity.

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